WEIS 2004

An Insurance Style Model for Determining the Appropriate Investment Level against Maximum Loss arising from an Information Security Breach

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Traditional Capital Budgeting

$$NPV = \sum_{\substack{\text{all possible} \\ \text{values of t}}}^{\substack{\text{planning} \\ \text{horoizon}}} \frac{\lambda E[L]}{(1+k)^t} - I$$

I	Investment in information security
E[L]	Change in expected loss from introducing investment at level I
λ	Probability of a breach
k	Appropriate discount rate
t	Time

Traditional Capital Budgeting

$$\text{NPV} = \sum_{\substack{\text{all possible}\\ \text{values of t}}}^{\substack{\text{planning}\\ \text{horoizon}}} \frac{\lambda\, E[L]}{(1+k)^t} - I$$

Select the level of I to optimise NPV

Level of I has to influence one or both of:

- Distribution of L
- Value of λ

Appropriate value of k

- Reflect riskiness of future cash flows

Aim

Investment model:

- Inclusion of time
- Distribution of losses
- Possibility of multiple breaches within planning horizon
- Appropriate discount rate

Level of investment influences the distribution of losses

- Later, allow investment level to influence λ

Conceptual Model

Investment in information security limits the maximum loss from a security breach to Q

Savings from the investment = $\max\{ L - Q, 0\}$

Conceptualise model as binomial option pricing framework

- Apply standard binomial option pricing theory
- Extend binomial model to continuous equivalent

Model Notation

L Loss incurred when breach occurs

L^+ Upper loss with probability p

L^- Lower loss with probability 1- p

λ Probability of a breach per unit period

F Investment in information security

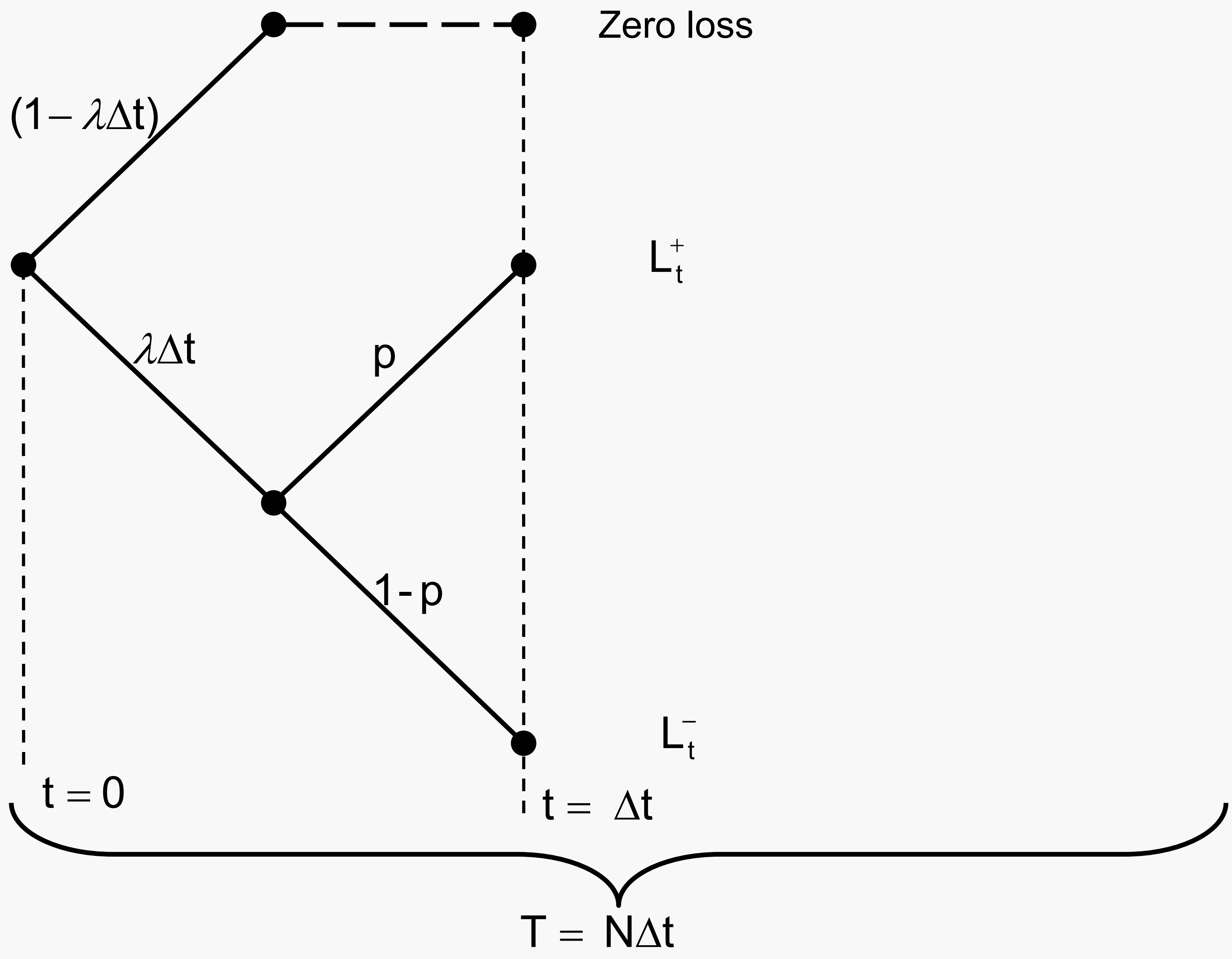
Consider a small interval Δt

Define :

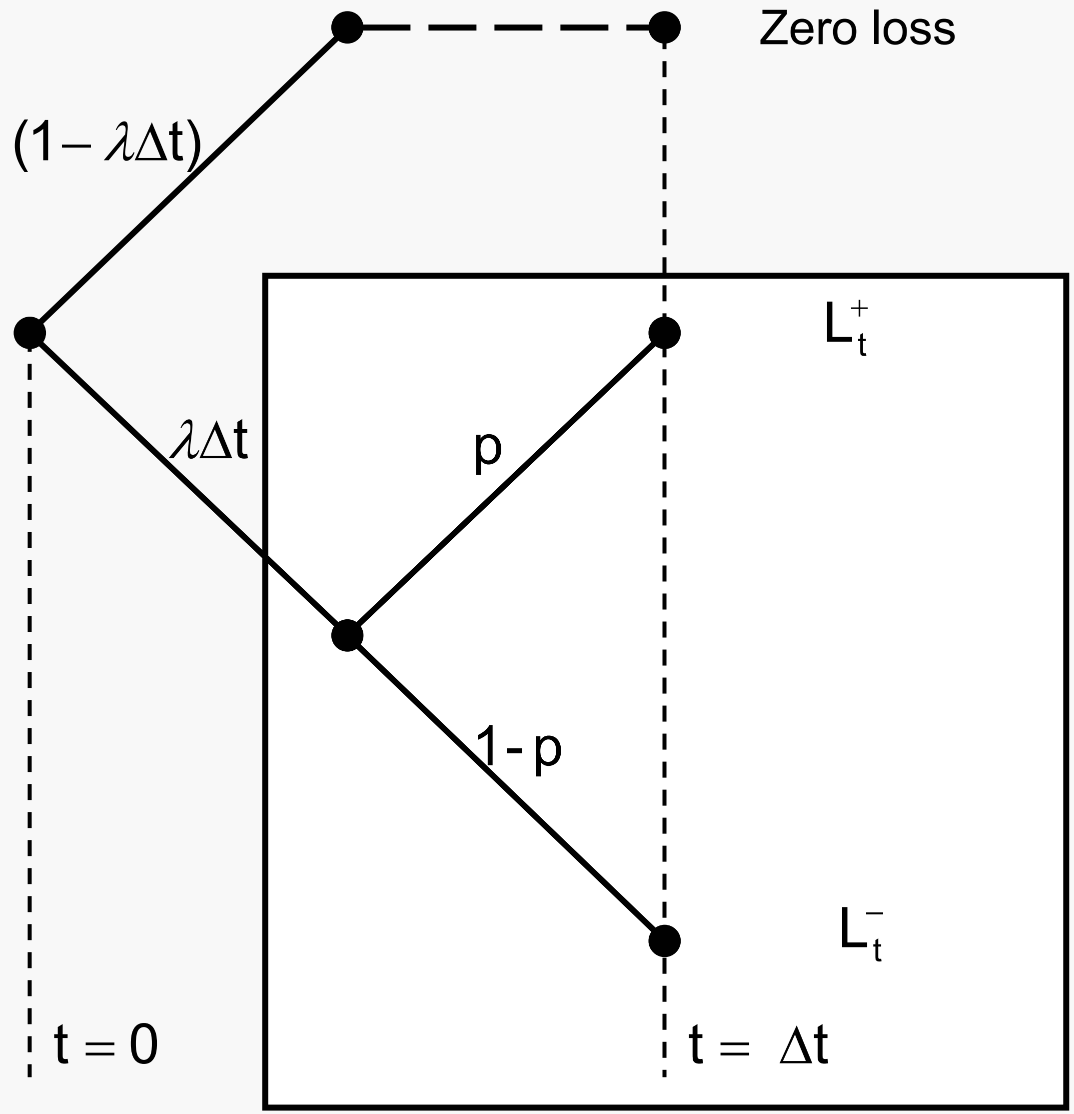
$\tau = n\Delta t$

$T = N\Delta t$ where $N > n$

Conceptual Model

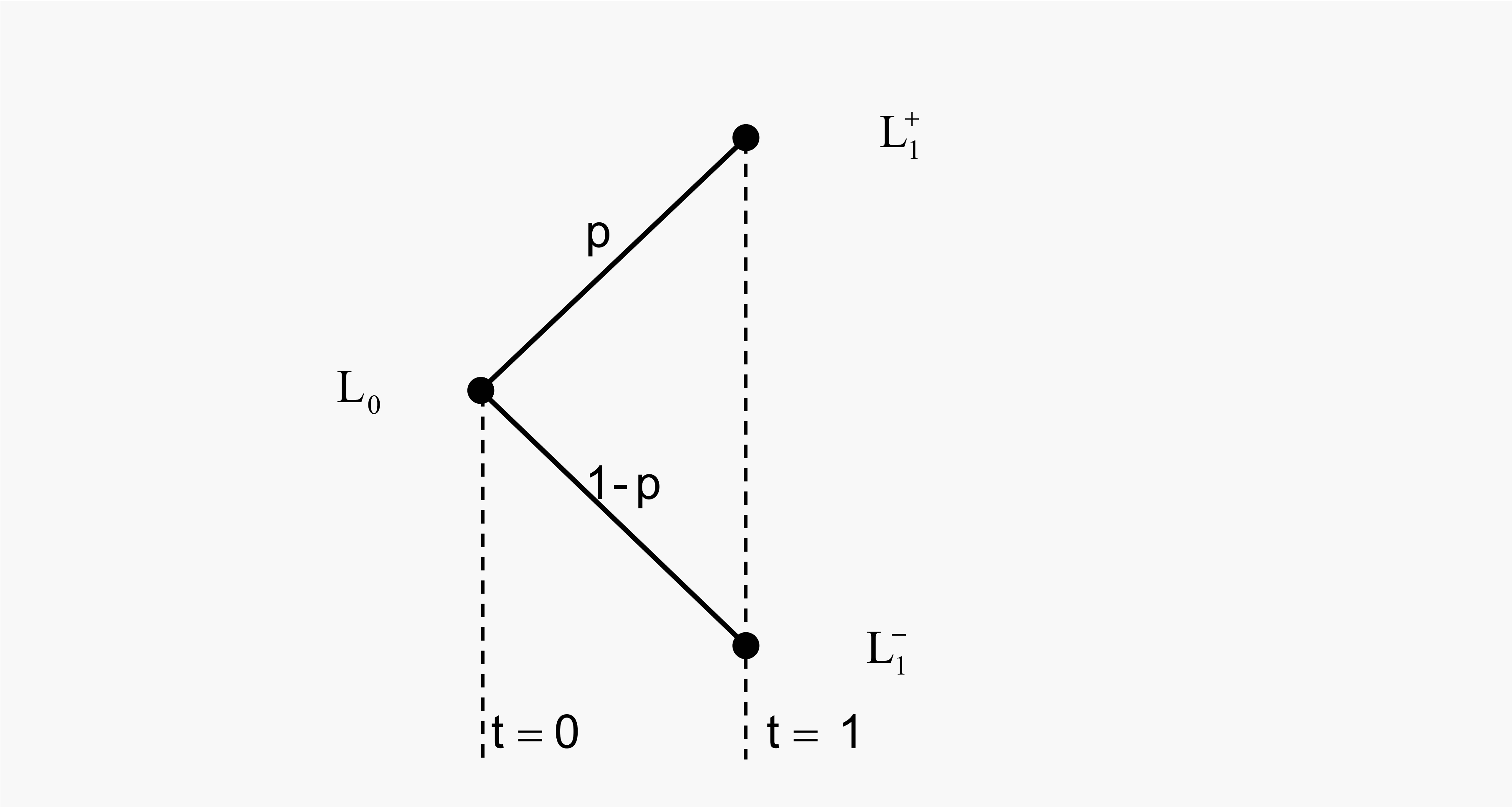


Conceptual Model



Assume λ and p are independent

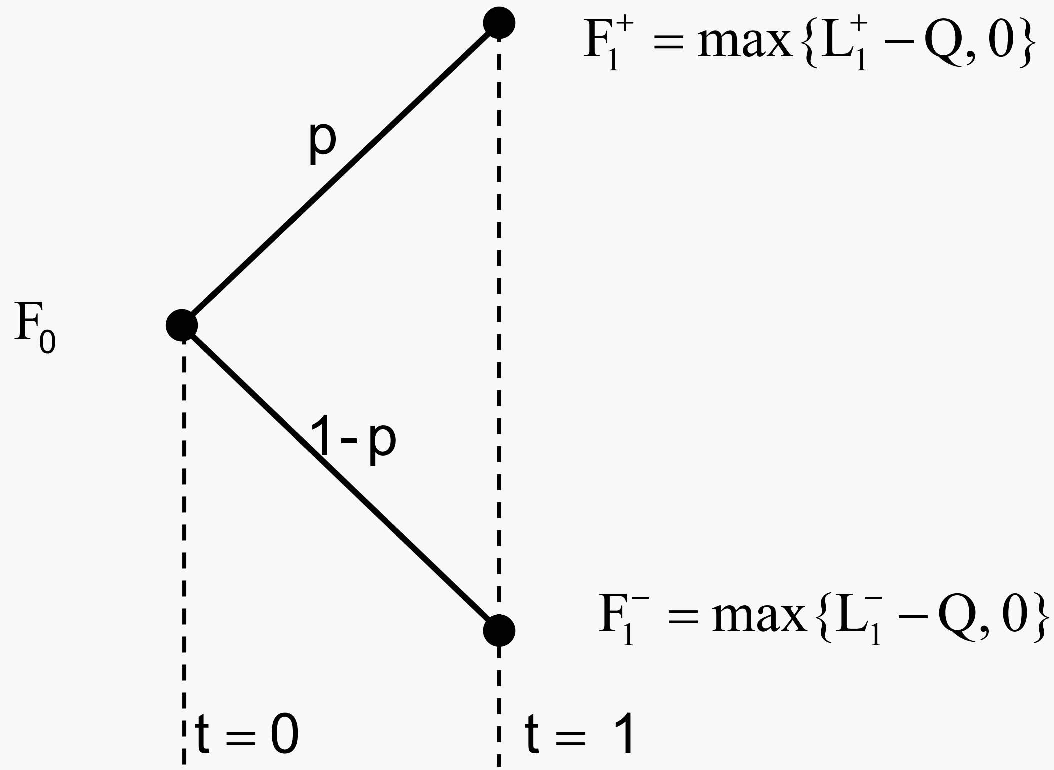
Conceptual Model



Set Δt=1 with no loss in model complexity

Conceptual Model

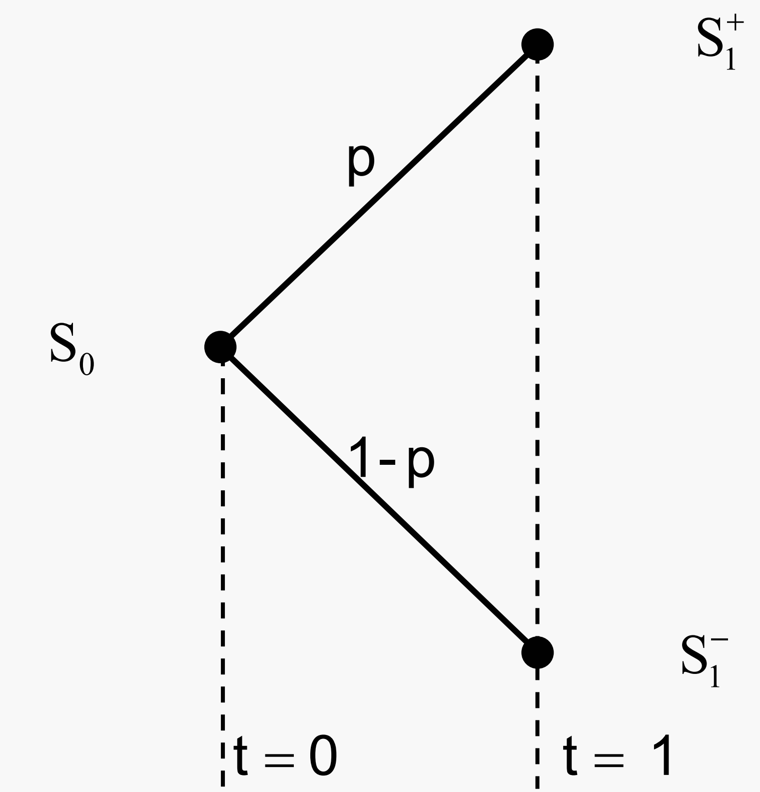
Value of investment F to confer a degree of protection against breach such that firm suffers no greater loss than Q



Not possible to discount expected value at risky rate

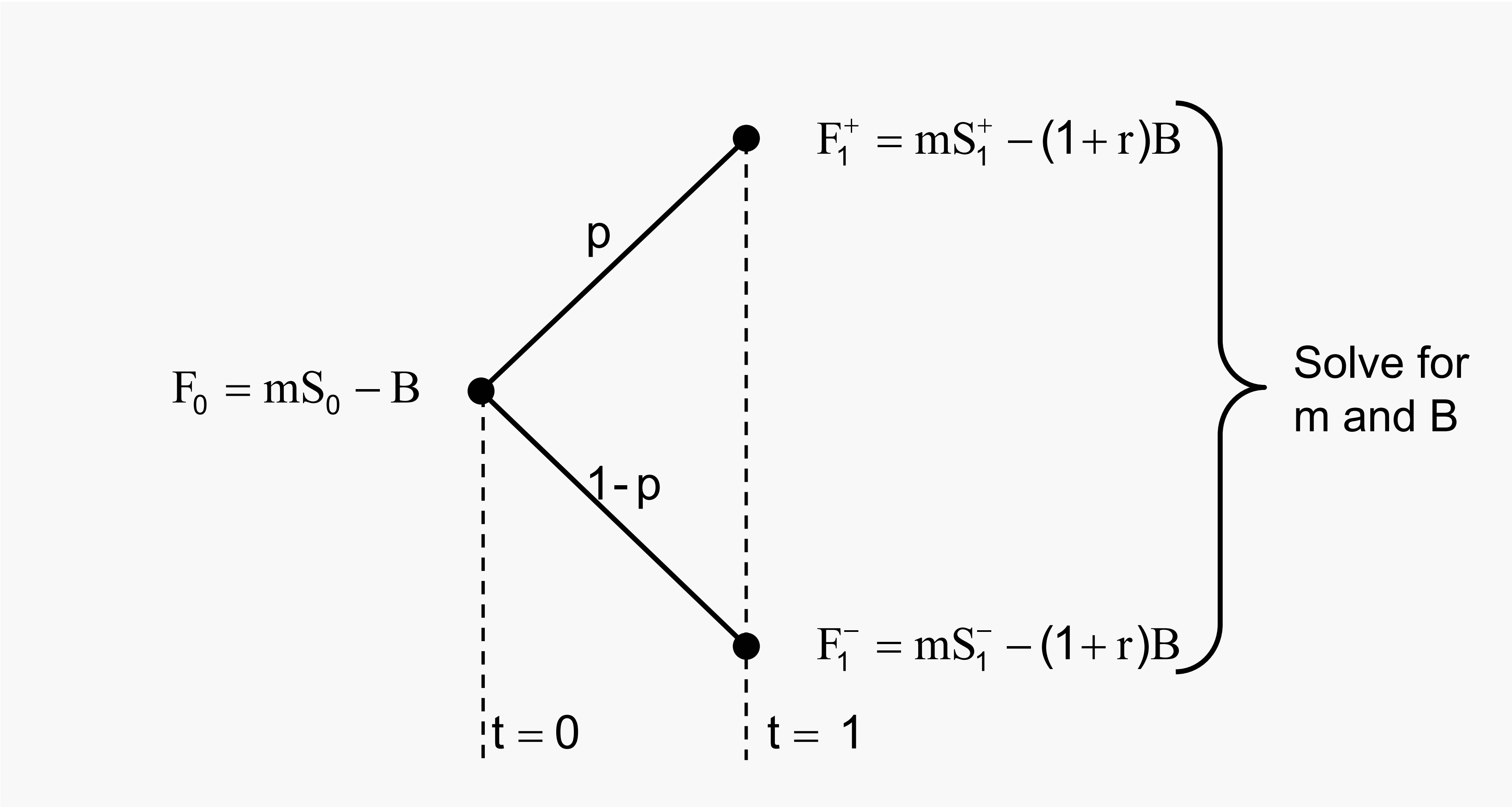
Conceptual Model

Twin security (spanning portfolio) S exists perfectly correlated with L



Conceptual Model

Construct a portfolio composed of buying m units of S partially funded through borrowing B at riskless rate r



Conceptual Model

$$m = \frac{F_1^+ - F_1^-}{S_1^+ - S_1^-}$$

$$B = \frac{mS_1^- - F_1^-}{1+r}$$

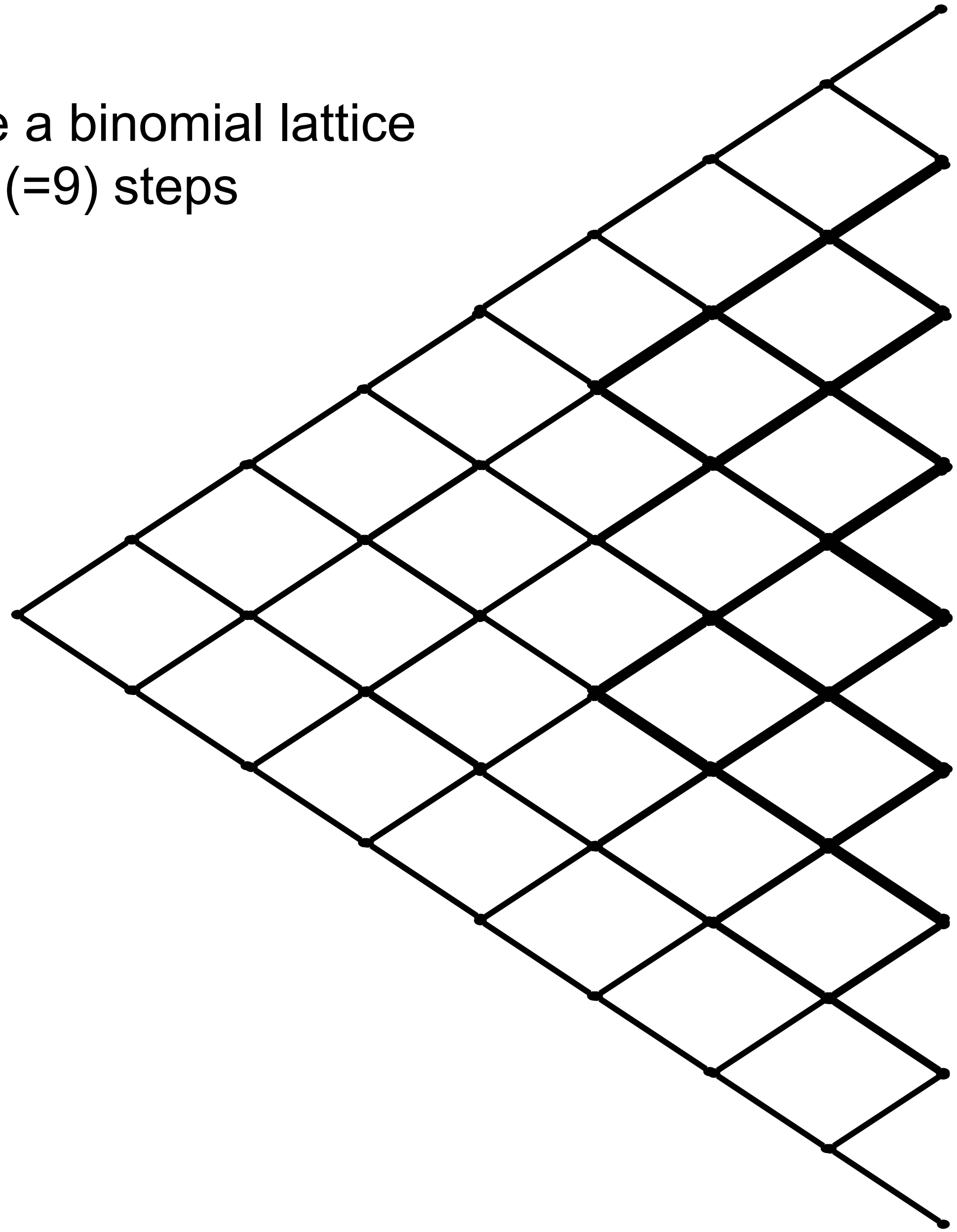
$$F_o = mS_0 - B$$

$$q = \frac{(1+r)S_0 - S_1^-}{S_1^+ - S_1^-}$$

$$F_o = \frac{qF_1^+ + (1-q)F_1^-}{1+r}$$

Conceptual Model

Create a binomial lattice with n (=9) steps



At each final node, derive F^s_n and apply certainty equivalent probabilities and riskless rate to determine present value of F

Allow n to become very large

Conceptual Model

Derive F_0 from the Binomial probability model

Equivalently apply the log-normal approximation

$$F_0 = L_0 N(x) - Q(1+r)^{-\tau} N(x - \sigma\sqrt{\tau})$$

$$x = \frac{\ln\left(L_0 / Q(1+r)^{-\tau}\right)}{\sigma\sqrt{\tau}} + 0.5\sigma\sqrt{\tau}$$

Define $u = S_1^+ / S_0$ and $d = S_1^- / S_0$ where $d = 1/u$

$$\sigma = ln(u)/\sqrt{h}$$

h is length of period from 0 to 1, $h = \tau / n$

Conceptual Model

$$F_0 = L_0 N(x) - Q(1+r)^{-\tau} N(x - \sigma\sqrt{\tau})$$

Black – Scholes option pricing formula

F_0 is value of European call option

Exercise (strike) price Q

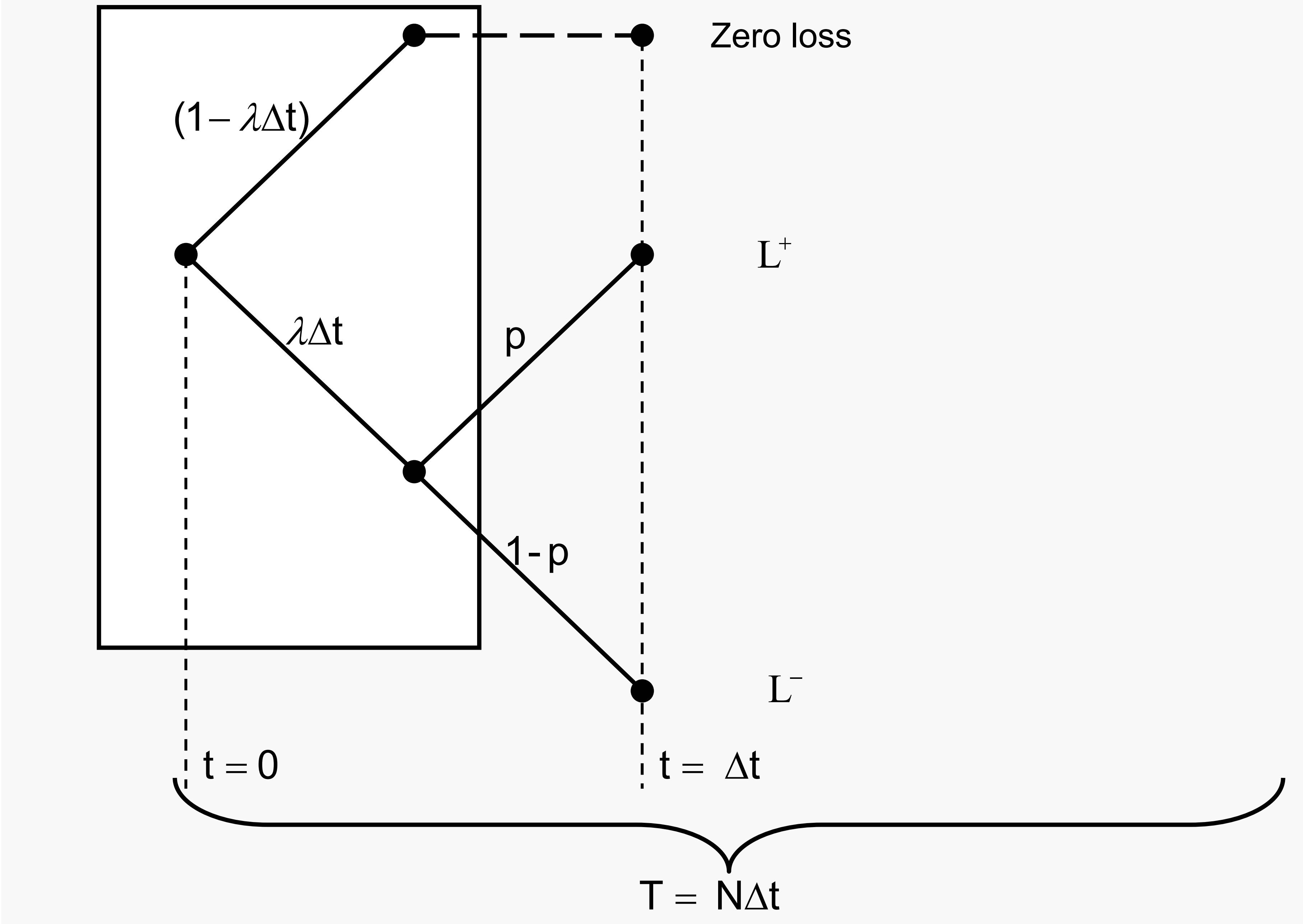
Time to expiration is

F_0 denotes the cost of protection against a loss of at least Q

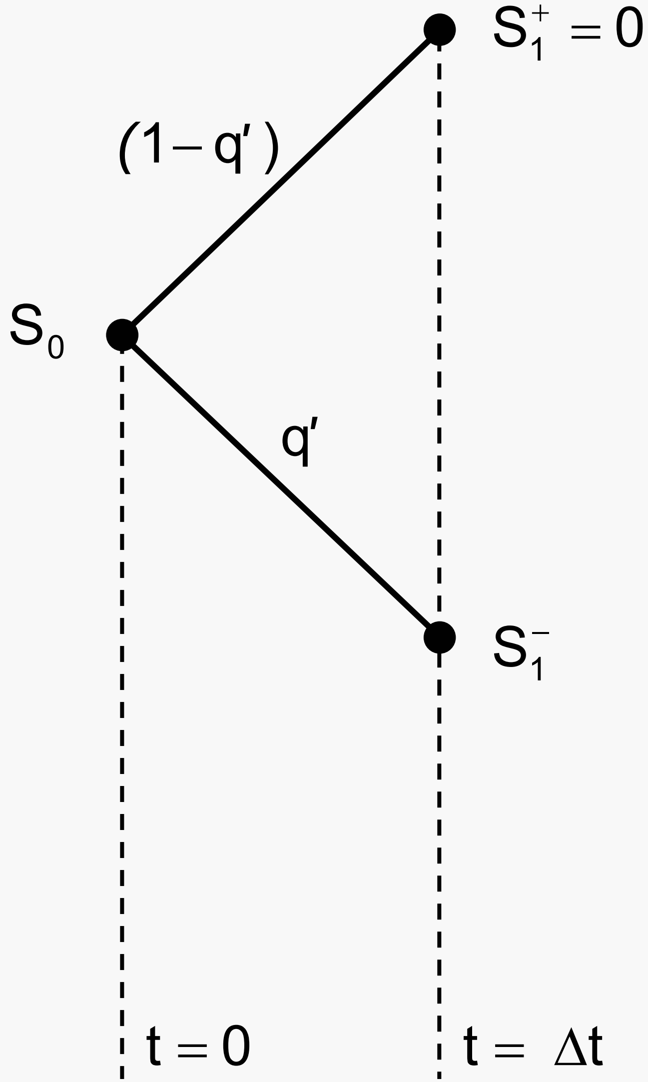
Loss only occurs at time τ

Assumes a breach will take place

Conceptual Model



Conceptual Model



$$S_o = \frac{\hat{E}[S_1^s]}{1+r\Delta t} = \frac{(1-q')S_1^+ + q'S_1^-}{1+r\Delta t}$$

$$= \frac{q'S_1^-}{1+r\Delta t}$$

$$\lambda\Delta t \approx S_0 / S_1^-$$

$$q' = \lambda\Delta t(1+r\Delta t) \approx \lambda\Delta t$$

Conceptual Model

Probability of a breach at t = nΔt

Probability of a breach at stage n, preceded by j breaches and (n-j-1) no breaches in a specified order:

$$= (\lambda \Delta t)^{j} (1 - \lambda \Delta t)^{n-1-j} \lambda \Delta t$$

Probability of a breach at stage n, preceded by j breaches and (n-j-1) no breaches in any order:

$$= \binom{n-1}{j} (\lambda \Delta t)^{j} (1 - \lambda \Delta t)^{n-1-j} \lambda \Delta t$$

Probability of a breach at stage n:

$$= \sum_{j=0}^{n-1} \binom{n-1}{j} (\lambda \Delta t)^{j} (1 - \lambda \Delta t)^{n-1-j} \lambda \Delta t = \lambda \Delta t$$

Conceptual Model

$F_0(Q, t)$ denotes the cost of protection against a maximum loss of Q

Loss only occurs at time $t = n\Delta t$

Assumes a breach will take place

Cost of protection against a maximum loss of Q at time $t = n\Delta t$:

$$= \lambda \Delta t \times F_0(Q, t)$$

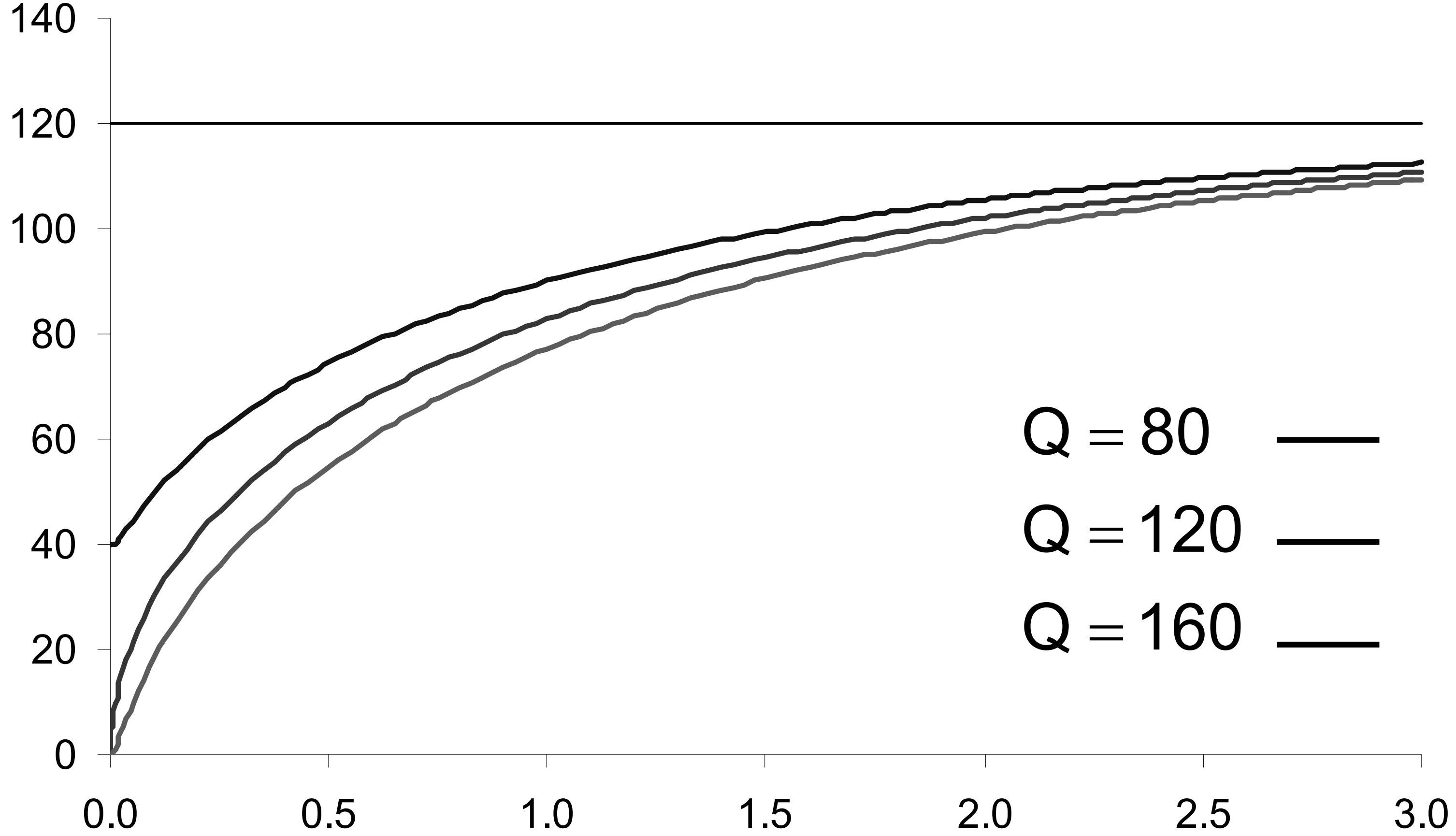
Cost of protection against a maximum loss of Q over horizon t=0 to t=T:

$$= \int_0^T \lambda F_0(Q, t) dt$$

Profiles

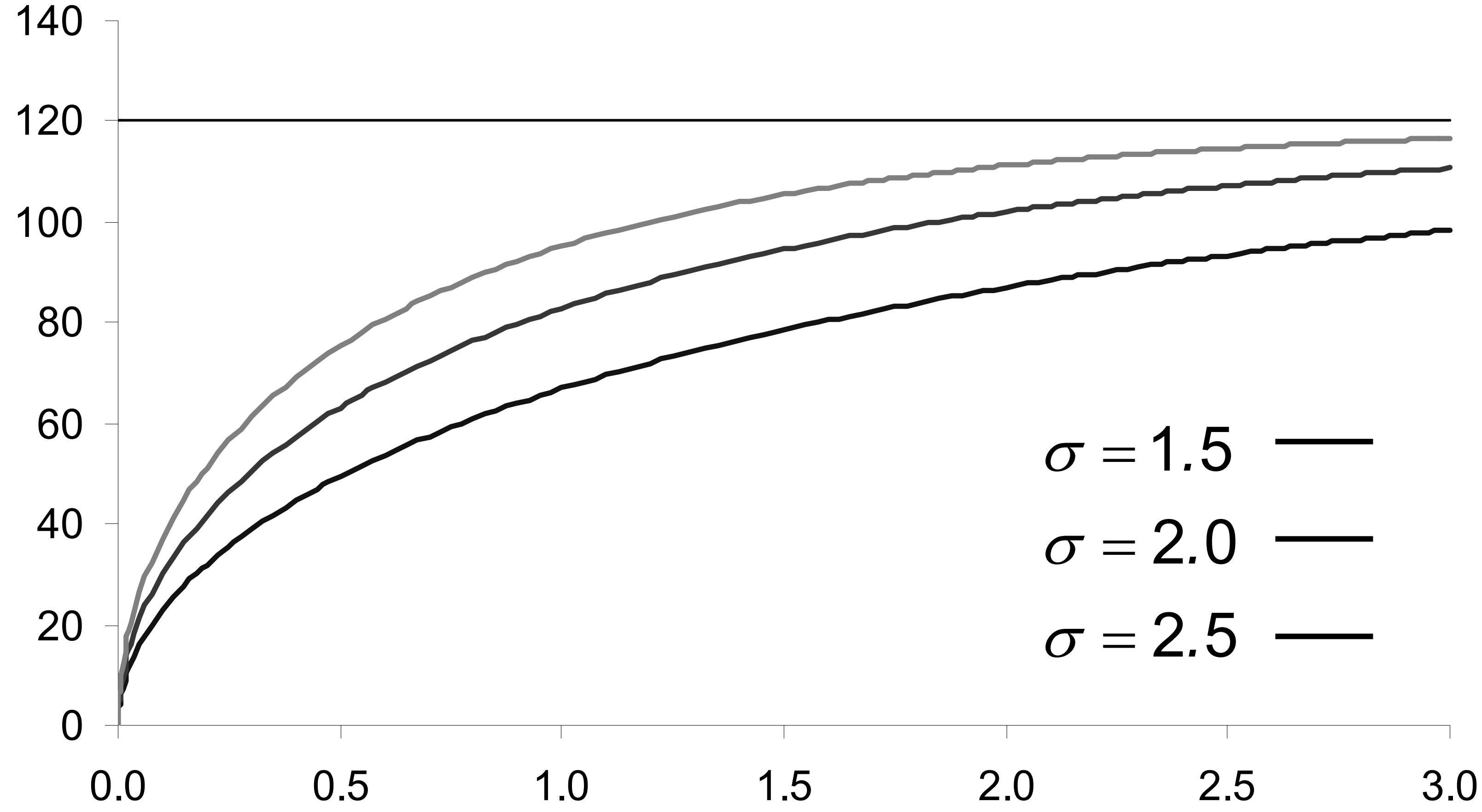
Profile of F_0 against time (years)

$L_0 = 120$ $r = 5\%$ $\sigma = 2$



Profile of F_0 against time (years)

$L_0 = 120$ $r = 5\%$ $Q = 120$



Profiles

Cost of protection against a maximum loss Q over horizon t=0 to t=T:

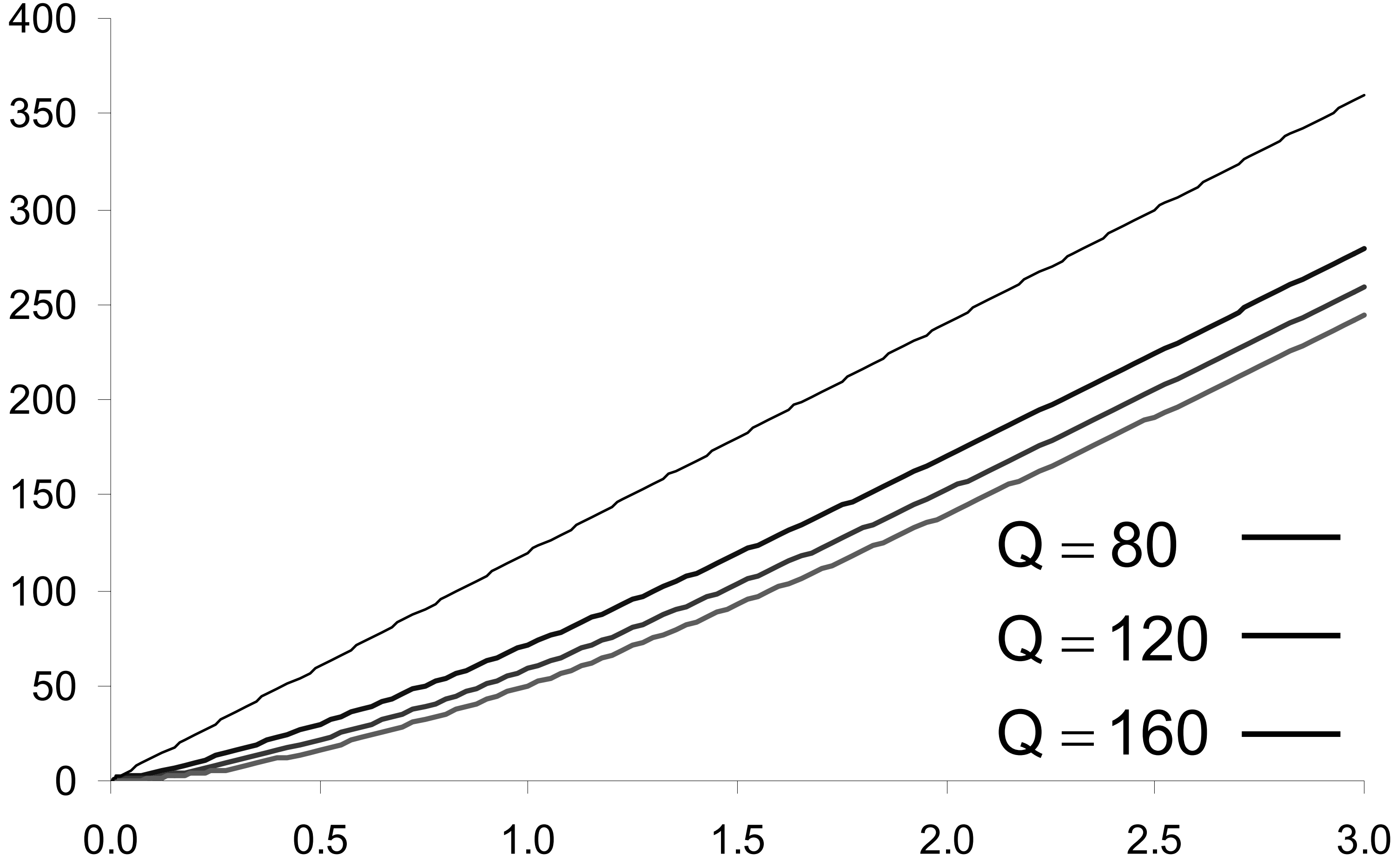
$$= \int_0^T \lambda F_0(Q, t) dt$$

Define H_0 as integral of F_0 over interval t=0 to t=T:

$$H_0 = \int_0^T F_0(Q, t) dt$$

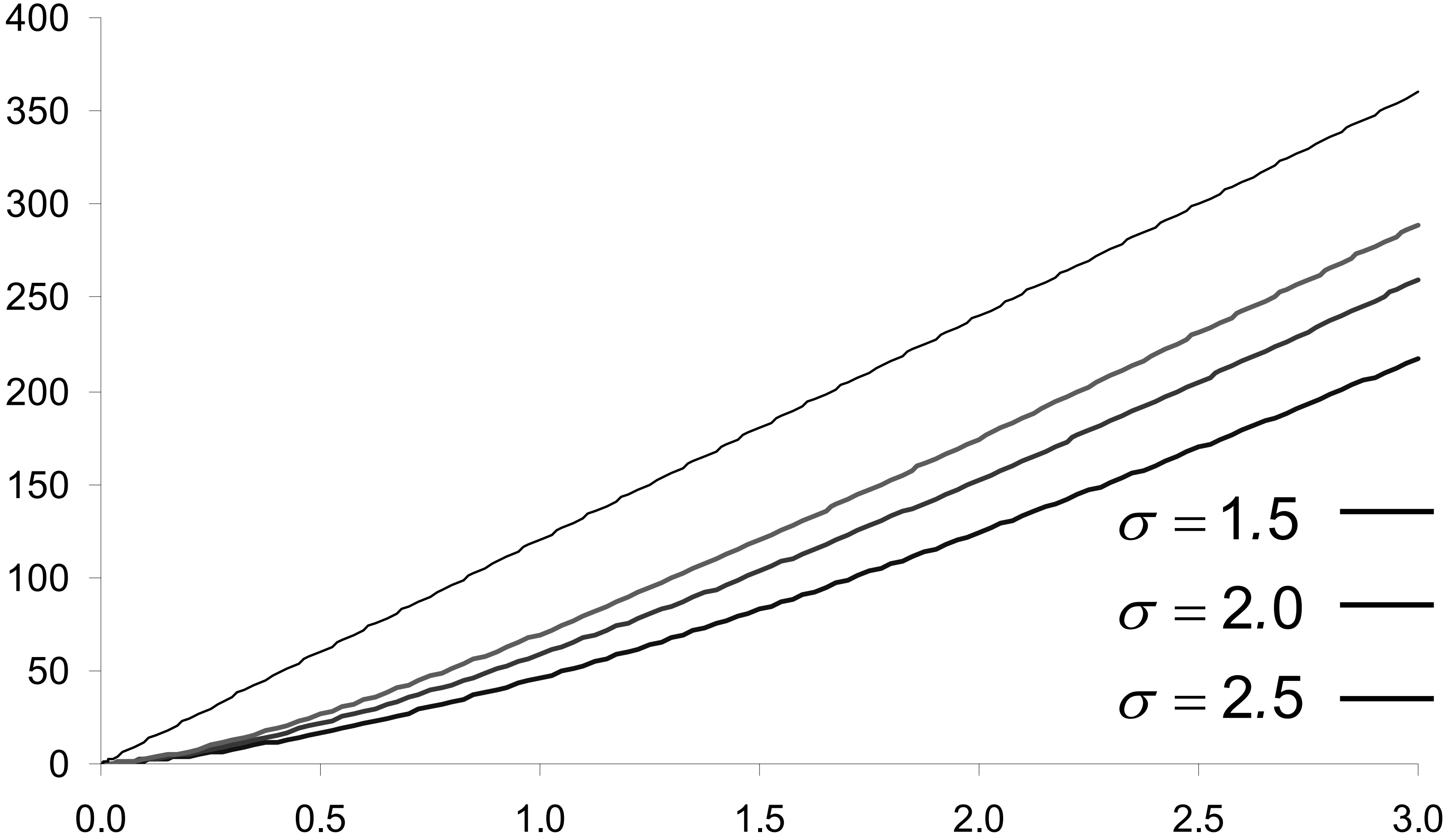
Profile of H_0 against time (years)

$L_0 = 120$ $r = 5\%$ $\sigma = 2$



Profile of H_0 against time (years)

$L_0 = 120$ $r = 5\%$ $Q = 120$



Investment Level Influences λ

Cost of protection against a loss of at least Q over horizon t=0 to t=T:

$$I_0 = \int_0^T \lambda F_0(Q, t)dt = \lambda H_0$$

Assume:

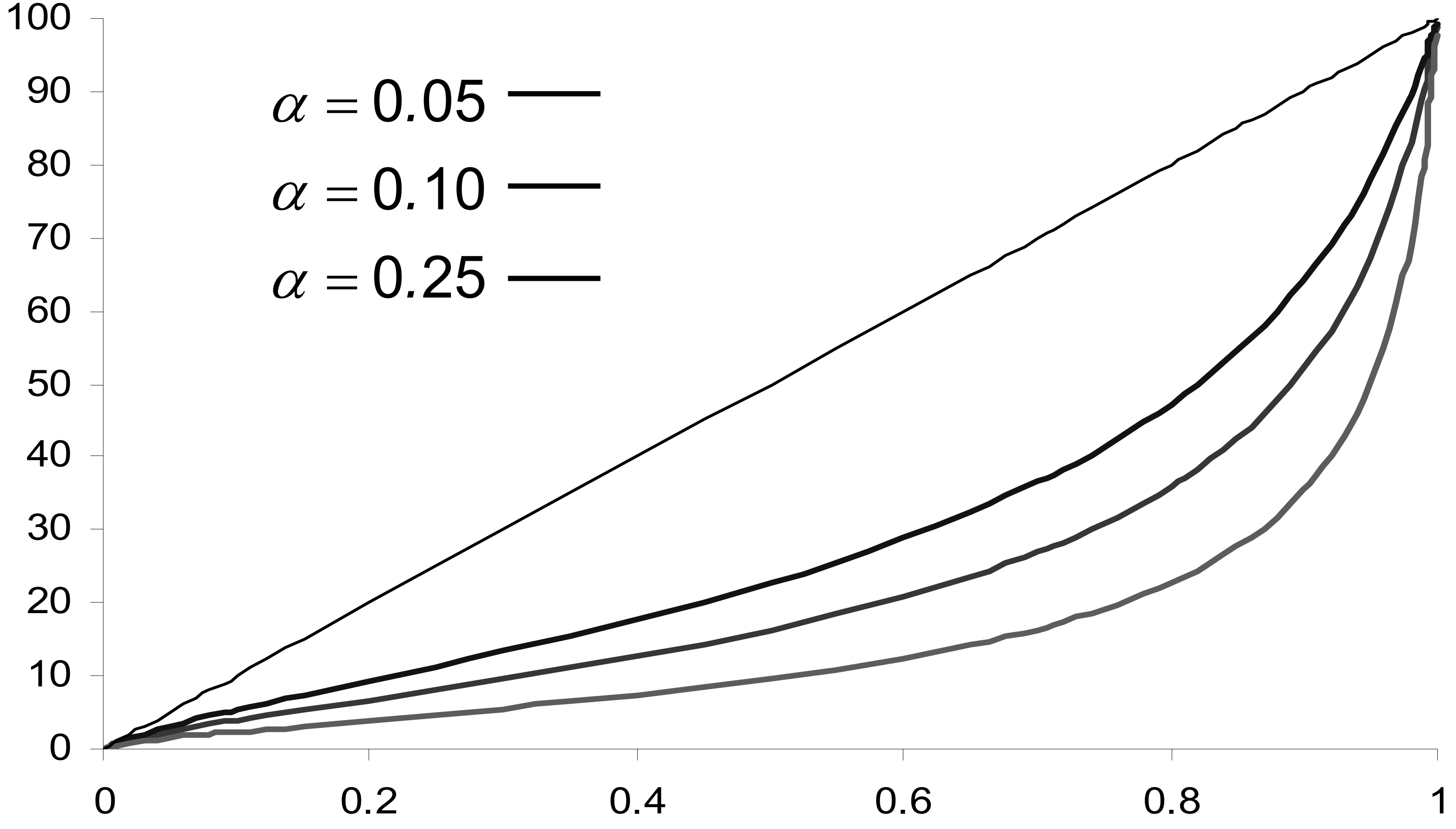
$$\lambda = \lambda_0^{(\alpha I_0 + 1)}$$

Cost of protection against a loss of at least Q over horizon t=0 to t=T:

$$I_0 = \lambda_0^{(\alpha I_0 + 1)} \times H_0$$

Profile of I_0 against Lambda-zero

$H_0 = 100$



Conclusions

Maximum loss

- Lowering the maximum loss incurred increases the investment level

Planning horizon

- Investment level is non-linear in short term

NPV or time dependent framework

- Apply the justified appropriate discount rate

Expectation of loss is important

- BUT, so is the variability

Discussion

- Twinning security
- Independence between λ and p
- Firm investment behaviour
- Investment allocation
- Benefit function

Twinning Security

What is at risk?

- Theft and fraudulent acquisition of assets
- Reputation
- Information integrity
- Operational breakdown and failures
- Compensation
- Resources to revive normality

} Extent that loss correlated with firm's equity

Adopt real options framework

- Use subjective estimates

Independence between λ and p

Treatment of λ as a constant

Independence condition can be relaxed

P Boyle (1988)

A Lattice framework for option pricing with two state variables

Journal of financial and Quantitative Analysis, 23 1-26

Mathematically more complicated

Firm Investment Behaviour

Assumption of firm rationality

Prospect Theory: D Kahneman and A Tversky (1979), D Kahneman, J Knetschen and R Thaler (1991)

Firms adopt risk-loving solution in the face of losses

- Preference for uncertain higher loss than a certain lower loss
- Risk aversion falls after a period of zero security breaches
- Risk aversion increases after a security breach
- Risk loving to recoup a substantial past loss

Investment Allocation

Apportionment of investment across information security devices and mechanisms

- Upfront versus continuous expenditures
- Division between mitigating losses and probability reduction

Benefit Function

Future benefits from investment akin to insurance

- Inappropriate
- Investigations with random benefits

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