Bandwidth- and Power-Efficient Multi-Carrier Multiple Access*

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***Abstract*— Orthogonal Frequency Division Multiple Access (OFDMA) gains increasing attention for broadband, high data rate wireless/wireline communications. In this paper, we propose a novel unitary precoded (UP) OFDMA scheme for uplink applications, that increases the system bandwidth efficiency, while preserving constant modulus transmissions. Theoretical analysis for the proposed UP-OFDMA with channel coding quantifies the performance improvement introduced by unitary precoding. It provides guidelines for practical system designs, and reveals performance gap between the proposed system and the single user bound. Simulation results confirm that the proposed system improves performance considerably relative to conventional OFDMA.**

I. INTRODUCTION

Broadband wireless/wireline applications require effective handling of inter symbol interference (ISI) that arises when high-rate transmissions propagate through time dispersive channels. Converting the ISI channel into a set of parallel flat fading channels and therefore considerably reducing the equalization complexity, Orthogonal Frequency Division Multiplexing (OFDM) has found widespread applications [9]. Orthogonal Frequency Division Multiple Access (OFDMA) is OFDM's counterpart for multiuser communications, and inherits the attractive features of OFDM. Originally proposed for cable TV networks [6], it is now being considered for IEEE802.16a [5], and ETSI BRAN [2] standards.

In its simplest form, each OFDMA user transmits information symbols using one complex exponential (subcarrier) that retains orthogonal to other users' subcarriers even when passing through multipath fading channels. As a result, Multi-User Interference (MUI) is suppressed deterministically, regardless of the underlying ISI channels. The performance of OFDMA degrades if the user-specific channel exhibits deep fades on the information-carrying subcarrier. Error control codes and frequency hopping are usually deployed [6] to robustify performance. On the other hand, more than one subcarriers can be assigned to one user to support high rate applications. However, simultaneous transmission of multiple subcarriers results in non-constant modulus signaling, that reduces the power amplifier efficiency at the transmitter.

Recently, redundant linear precoding across subcarriers has been proposed in the so-termed Generalized Multi-Carrier (GMC)-CDMA [9], which improves performance considerably over conventional OFDMA. However, the redundant precoding in GMC-CDMA leads to bandwidth efficiency loss, especially for long channels. The transmissions are generally non-constant modulus, as well.

In this paper, we propose a novel unitary precoded (UP) OFDMA scheme, that achieves higher bandwidth efficiency than both conventional OFDMA and GMC-CDMA. Even with multiple subcarriers per user, UP-OFDMA maintains perfectly constant modulus transmissions and is thus highly power efficient. To evaluate the performance improvement introduced by unitary precoding, we carry out a theoretical performance analysis for the proposed system with channel coding. This analysis discloses substantial advantage of unitary precoding, and quantifies power savings of UP-OFDMA over conventional OFDMA. It also provides useful guidelines for selecting the precoder size for practical systems, and reveals a small performance gap of UP-OFDMA from the single user bound, that is achieved by single user spread spectrum transmissions.

Notation: Bold upper letters denote matrices, bold lower letters denote column vectors; $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively. $\mathbf{I}_K$ denotes the $K \times K$ identity matrix, and $\mathbf{F}_N$ stands for the $N \times N$ FFT matrix with the $(p+1, q+1)$st entry $(1/\sqrt{N})\exp(-j2\pi pq/N)$, $\forall p, q \in [0, N-1]$.

II. PRELIMINARIES

A. Channel Model

We focus on quasi-synchronous uplink transmissions over wireless channels, where mobile users attempt to synchronize with base-station's pilot waveform and thus the relative asynchronism among users is small. The discrete time baseband equivalent channel can be modeled as a finite impulse response (FIR) filter, which incorporates the asynchronism, the transmit- and receive filters, and the underlying physical channel [9]. Let $\tau_{max,a}$ denote the maximum asynchronism, and $\tau_{max,s}$ stand for the maximum delay spread, the channel order can be determined by $L \geq \lceil (\tau_{max,a} + \tau_{max,s})/T_c \rceil$, where T_c is the chip duration.

B. OFDMA

The uth user out of a maximum N users in OFDMA places its information symbols on the subcarrier: $\mathbf{f}_u := (1/\sqrt{N})[e^{j0}, e^{j2\pi u/N}, \ldots, e^{j2\pi u(N-1)/N}]^T$, where the symbol period $T_s = NT_c$ determines the subcarrier bandwidth:

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$1/T_s = 1/(NT_c)$. The ith transmitted chip block is thus $\mathbf{x}_u(i) = \mathbf{f}_u s_u(i)$, where $s_u(i)$ is ith data symbol of user u. To avoid inter block interference, a cyclic prefix (CP) of length L is inserted at the transmitter, and is removed at the receiver.

Since each subcarrier is an eigen-function of the FIR channel, the received block after CP removal is

$$\mathbf{y}(i) = \sum_{u=0}^{N-1} H_u(\rho_u)\mathbf{f}_u s_u(i) + \mathbf{e}(i), \quad (1)$$

where $\rho_u := e^{j2\pi u/N}$, and $H_u(\rho_u)$ is the channel response $H_u(z) := \sum_{l=0}^{L} h_u(l)z^{-l}$ evaluated at the subcarrier frequency ρ_u, $\mathbf{e}(i)$ is the additive white Gaussian noise. The uth user's signals can then be separated by exploiting the orthogonality among subcarriers:

$$y_u(i) = \mathbf{f}_u^H \mathbf{y}(i) = H_u(\rho_u)s_u(i) + e_u(i), \quad (2)$$

where $e_u(i)$ is the remaining noise component.

Let us now examine the properties of uplink OFDMA transmissions. First of all, each user in OFDMA maintains constant modulus transmissions, and is thus power efficient. Taking CP into account, the bandwidth efficiency, which is defined as the maximum number of transmitted symbols per subcarrier, is

$$\eta_1 = \frac{N}{N+L}, \quad (3)$$

which approaches 100% when $N \gg L$. But when L is large and N is moderate, the bandwidth efficiency may suffer. As evidenced by (2), the performance of OFDMA may degrade severely when the underlying channel undergoes deep fading around ρ_u. To cope with deep channel fades, incorporation of error control codes together with frequency hopping is imperative for OFDMA.

C. GMC-CDMA

Linear precoding across OFDMA subcarriers has been proposed as an alternative approach to mitigate channel fades in Generalized Multi-Carrier CDMA systems [9]. Instead of one subcarrier, $J > 1$ subcarriers $\{\mathbf{f}_{u,j}\}_{j=0}^{J-1}$ are assigned to user u, to transmit $K > 1$ information symbols simultaneously. Specifically, the ith information block $\mathbf{s}_u(i) := [s_u(iK+0, \ldots, s_u(iK+K-1)]^T$ is precoded using a $J \times K$ tall precoder $\mathbf{\Phi}$ to obtain $\mathbf{x}_u(i) := \mathbf{\Phi}\mathbf{s}_u(i)$. The J entries of $\mathbf{x}_u(i)$ are transmitted over J subcarriers, respectively. In the absence of noise, symbol recovery is guaranteed regardless of the channel zero locations provided that $J \geq K + L$ [9].

By using J subcarriers simultaneously, GMC-CDMA does not possess constant modulus transmissions, in general. For a maximum number of N users, NK symbols are transmitted over $N(K + L)$ orthogonal subcarriers. Taking CP into account, the bandwidth efficiency is thus:

$$\eta_2 = \frac{NK}{N(K+L)+L} \approx \frac{K}{K+L}. \quad (4)$$

To achieve high bandwidth efficiently, it is preferable to choose K as large as possible [9]. However, in practice, the choice of K is also constrained by other factors (cf. section III). Thus, for channels with large L, GMC-CDMA incurs considerable bandwidth efficiency loss.

D. Spread Spectrum (SS) OFDM: single user bound

GMC-CDMA introduces redundancy by using J subcarriers to transmit K symbols. In the extreme case, all the N available subcarriers can be used to transmit one symbol, which is essentially SS OFDM proposed in [4]. To make up for the considerable rate loss inherent in SS-OFDM, multicarrier (MC) CDMA has been proposed in [13], where different users share all the subcarriers and are distinguished by their signature codes. MUI thus appears, that needs to be suppressed by joint multiuser detection (MUD). The performance of MC-CDMA is upper-bounded by SS OFDM, because the latter corresponds to the best scenario where multi-user interference has been correctly detected and subtracted. Similarly, the performance of SS-OFDM also serves as the upper bound for GMC-CDMA. We will compare the performance of UP-OFDMA with that of SS-OFDM, to demonstrate the effectiveness of the unitary precoding.

III. Unitary Precoded OFDMA

A. Bandwidth and power efficient transmissions

Distinct from *redundant precoding* utilized by GMC-CDMA [9], we propose here non-redundant *unitary precoding* across OFDMA subcarriers. For comparison, we allow for the same number of maximum users N. Therefore, our multiuser system will have $P = NK$ subcarriers. Specifically, each user will be allocated K subcarriers $\{\rho_{u,k}\}_{k=0}^{K-1}$ to transmit K information symbols. The ith $K \times 1$ information block $\mathbf{s}_u(i)$ is precoded by a $K \times K$ matrix $\mathbf{\Theta}$ to obtain: $\mathbf{x}_u(i) = \mathbf{\Theta}\mathbf{s}_u(i)$, with its entries transmitted on K distinct subcarriers. Collecting the outputs on those K subcarriers for user u, we arrive at the equivalent block input-output relationship as

$$\mathbf{y}_u(i) = \mathbf{\Lambda}_u(i)\mathbf{\Theta}\mathbf{s}_u(i) + \mathbf{e}_u(i), \quad (5)$$

where $\mathbf{\Lambda}_u(i) := \text{diag}(H_u(\rho_{u,0}), \ldots, H_u(\rho_{u,K-1}))$ collects the channel frequency response during the ith block, and $\mathbf{e}_u(i)$ is additive white Gaussian noise. The block index i on $\mathbf{\Lambda}_u(i)$ indicates that the channel's frequency response could change from block to block due to either frequency hopping, or, slow channel variations.

With each user transmitting K symbols over $P + L$ chips, the maximum bandwidth efficiency of UP-OFDMA is

$$\eta_3 = \frac{NK}{P+L} = \frac{N}{N+L/K} \approx 1. \quad (6)$$

Comparing (6) with (3) and (4), we have established

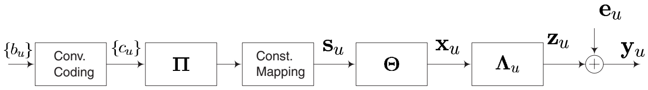

Fig. 1. The equivalent system model for coded UP-OFDMA

Proposition 1: *The proposed system enjoys higher bandwidth efficiency than both OFDMA and GMC-CDMA: $\eta_3 > \eta_1$, and $\eta_3 > \eta_2$.*

With respect to OFDMA, our UP-OFDMA offers a K-fold decrease in the effective channel order, and is thus particularly useful for channels with large order L.

Now we need to assign subcarriers to different users and choose the precoder $\mathbf{\Theta}$ judiciously. The optimal precoder $\mathbf{\Theta}$ shall yield the best performance. And it is desirable to maintain constant modulus transmission, as in OFDMA. It turns out that the two goals can be achieved simultaneously as follows.

We assign equi-spaced subcarriers to each user, i.e., subcarriers $\{\rho_{u,k} = e^{j2\pi(u+kN)/NK}\}_{k=0}^{K-1}$ are assigned to user u. The idea is to separate the subcarriers as much as possible, so that they are (almost) uncorrelated. We will choose the unitary precoding matrix as in [3, 11]:

$$\mathbf{\Theta} = \mathbf{F}_K\mathbf{A}, \tag{7}$$

where $\mathbf{A} := \text{diag}\{1, e^{j\pi/K}, \ldots, e^{j(K-1)\pi/K}\}$ is a diagonal matrix with unit-amplitude diagonal entries. The precoder (7) achieves the best performance among all square linear precoders for most even values of K's, e. g., $K = 2, 4, 6$ [11].

Proposition 2: *The equi-spaced subcarrier assignment together with the precoder* (7) *leads to perfectly constant modulus UP-OFDMA transmissions.*

Proof: Suppose that the symbols in $\mathbf{s}_u(i)$ are drawn from a PSK constellation, and thus have constant modulus. The transmitted signal is $\mathbf{z}_u(i) = \mathbf{F}_P^H\mathbf{\Psi}_u\mathbf{\Theta}\mathbf{s}_u(i)$, where $\mathbf{\Psi}_u$ is the $P\times K$ subcarrier selection matrix, having columns the K unit vectors with the only nonzero entry positioned according to the assigned subcarriers. With an equally spaced subcarrier assignment, it can be verified that $\mathbf{F}_P^H\mathbf{\Psi}_u$ simplifies to $\mathbf{F}_P^H\mathbf{\Psi}_u = \mathbf{\Gamma}\mathbf{F}_K^H$ where $\mathbf{\Gamma} := [\mathbf{\Gamma}_0, \ldots, \mathbf{\Gamma}_{N-1}]^T$, and the $K \times K$ matrix $\mathbf{\Gamma}_n := (1/\sqrt{N})\text{diag}(w^{K(n-1)u}, w^{(K(n-1)+1)u}, \ldots, w^{(KN-1)u})$ with $w := e^{j2\pi/P}$. Therefore, $\mathbf{z}_u(i) = \mathbf{\Gamma}\mathbf{A}\mathbf{s}_u(i)$, with each entry having constant amplitude. ∎

So far, we have developed an uncoded system, that is both power- and bandwidth- efficient. Since error control coding is always employed in practical systems, we will analyze a convolutionally encoded UP-OFDMA system. The equivalent system model with convolutional coding is depicted in Fig. 1. Specifically, the information bits are encoded by the convolutional encoder and interleaved. After constellation mapping and unitary precoding, the precoded symbols go through parallel flat-fading subchannels. We will analyze the performance of coded UP-OFDMA, to illustrate the benefit induced by unitary precoding, and to quantify the power savings over conventional OFDMA in a simplified fading channel.

B. Performance Analysis

We consider optimal Maximum Likelihood (ML) decoding at the receiver to carry out the theoretical analysis. In practice, effective turbo decoders can be deployed. Related analysis has been provided in [10] for single user OFDM systems, in Rayleigh fading channels. Our analysis here extends to multiuser scenario, and the more general Rician fading channels.

Let $\underline{\mathbf{c}} := (c[0], c[1], \ldots)$ denote one realization of the coded bit sequence, $\underline{\mathbf{s}} := (s[0], s[1], \ldots)$ the corresponding symbol sequence after interleaving and constellation mapping, and $\underline{\mathbf{y}} := (y[0], y[1], \ldots)$ the received sequence. Similarly let $\underline{\hat{\mathbf{c}}}$, $\underline{\hat{\mathbf{s}}}$, and $\underline{\hat{\mathbf{y}}}$ be the corresponding quantities for another realization. Assume that $\underline{\mathbf{c}}$ and $\underline{\hat{\mathbf{c}}}$ differ in d bits. With the interleaver designed properly, these d bits are scrambled such that no two bits fall into the same symbol block. This assumption is based on the fact that the block size K is small in practice; the validity of this assumption will also be corroborated using simulation results. Suppose that after interleaving, these d different symbols, labeled as $s[n_1], \ldots, s[n_d]$, fall into blocks $\mathbf{s}[b_1], \ldots, \mathbf{s}[b_d]$, with $m_1, \ldots, m_d$ describing the positions of these erroneous symbols in their respective blocks; i.e., $n_w = b_wK + m_w$. Recalling (5) and dropping the user index, the serial equation is

$$y[b_wK + k] = \lambda[b_wK + k]\boldsymbol{\theta}_k^T\mathbf{s}[b_w] + e[b_wK + k], \tag{8}$$

where $\lambda[b_wK + k]$ is $(k + 1, k + 1)$st entry of the diagonal matrix $\mathbf{\Lambda}[b_w]$, $\boldsymbol{\theta}_k^T$ is the $k + 1$st row of $\mathbf{\Theta}$, $\mathbf{s}[b_w]$ is the b_wth symbol block, if $\underline{\mathbf{s}}$ is transmitted. Similarly, we have $\hat{y}[b_wK + k]$ corresponding to $\underline{\hat{\mathbf{s}}}$.

Define $\bar{\mathbf{s}}[b_w] := \mathbf{s}[b_w] - \hat{\mathbf{s}}[b_w]$, and $\bar{y}[b_wK+k] := y[b_wK+k] - \hat{y}[b_wK + k]$. Notice that only one symbol discrepancy occurs in each of the d inconsistent blocks. Therefore, $\boldsymbol{\theta}_k^T\bar{\mathbf{s}}[b_w] = \theta_{k,m_w}\bar{s}[n_w]$, where θ_{k,m_w} is the (k, m_w)th entry of $\mathbf{\Theta}$. We then obtain

$$\bar{y}[b_wK + k] = \lambda[b_wK + k]\theta_{k,m_w}\bar{s}[n_w]. \tag{9}$$

Each single error $\bar{s}[n_w]$ leads to K errors in received symbols, which intuitively illustrates that precoding introduces diversity. Since outside these erroneous blocks, $\underline{\mathbf{y}}$ and $\underline{\hat{\mathbf{y}}}$ are the same, the Euclidean distance between them is:

$$\begin{aligned} D^2(\underline{\mathbf{y}}, \underline{\hat{\mathbf{y}}}) &= \sum_{w=1}^{d}\sum_{k=0}^{K-1} |\lambda[b_wK + k]\theta_{k,m_w}\bar{s}[n_w]|^2 \\ &\geq (\delta^2/K)\sum_{w=1}^{d}\sum_{k=0}^{K-1} |\lambda[b_wK + k]|^2 \end{aligned} \tag{10}$$

where δ is the minimum distance of the symbol constellation. The derivation of (10) takes into account that each entry of $\mathbf{\Theta}$ has amplitude $1/\sqrt{K}$.

Thus, for each channel realization, the pairwise error probability that $\underline{\hat{\mathbf{c}}}$ is decided when $\underline{\mathbf{c}}$ is actually transmitted, can be upper bounded as:

$$\begin{aligned}\mathrm{P}_E\{\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}|\mathbf{h}\} &= \mathcal{Q}\left(\sqrt{D^2(\underline{\mathbf{y}}, \hat{\underline{\mathbf{y}}})/(2N_0)}\right)\\ &\leq \mathcal{Q}\left(\sqrt{(\delta^2/2KN_0)(\sum_{w=1}^{d}\sum_{k=0}^{K-1}|\lambda[b_wK+k]|^2)}\right).\end{aligned}$$

This conditional pairwise error probability needs to be averaged over all channel realizations. Without loss of generality, we assume that the physical channel contains one line-of-sight (LOS) Rician path, and the rest as non-LOS Rayleigh paths. The channel's frequency response $\lambda[b_wK+k]$ will be Rician faded with the same Rician factor $\mathcal{K}$ across different subcarriers. In practice, due to the transmit-receive filters and chip-rate sampling, $\lambda(b_wK+k)$ may be Rician faded with carrier-specific Rician factors. For illustration purposes, we will adopt the simple Rician fading channel model, assuming that the frequency response on the K subcarriers are independently and identically Rician faded, with Rician factor $\mathcal{K}$. We further suppose that the interleaving is sufficiently long, so that the error blocks experience independent fading. (The other extremes case would be that, no interleaving is available and the channel varies so slowly such that the error blocks experience block fading. For a treatment of such channels, please cf. [12].) These two assumptions are rather idealistic, and can only be approximately valid in practice. Nevertheless, the results for such a simplified channel provide theoretical insights, illustrate the benefit of unitary precoding, and offer practical guidelines for the block size.

Using the Chernoff bound $\mathcal{Q}(x) \leq \frac{1}{2}\exp(-x^2/2)$, and by averaging over random channels [8], we arrive at the average pairwise error probability:

$$\mathrm{P}_E(d) := \mathrm{P}_E\{\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}\} \leq \frac{1}{2}\left(\frac{1+\mathcal{K}}{\gamma_1}\cdot e^{-\mathcal{K}(1-\frac{\mathcal{K}+1}{\gamma_1})}\right)^{Kd}. \quad (11)$$

where $\gamma_1 := 1+\mathcal{K}+\delta^2/(4KN_0)$. The union bound on the bit error rate can then be obtained as,

$$\mathrm{P}_b \leq \sum_{d=d_f}^{\infty} B_d \mathrm{P}_E(d) = \sum_{d=d_f}^{\infty}\frac{B_d}{2}\left(\frac{1+\mathcal{K}}{\gamma_1}e^{-\mathcal{K}(1-\frac{\mathcal{K}+1}{\gamma_1})}\right)^{Kd}, \quad (12)$$

where the (B_d, d) pair is the bit distance spectrum of the convolutional code [7]. At sufficiently high SNR, $\mathrm{P}_b \sim (\gamma_1)^{-Kd_f}$ indicates that the diversity order of our UP-OFDMA is Kd_f, which amounts to a multiplicative diversity order enhancement due to precoding at the transmitter. The special case with $K=1$ reduces to conventional OFDMA.

For $K \geq 2$, we define the SNR gain G_K as the decrease in SNR for UP-OFDMA to achieve the same prescribed error performance as the conventional OFDMA. Targeting this prescribed performance, let δ_1 and δ_K be the minimum constellation distance needed for OFDMA and UP-OFDMA, respectively. The SNR gain G_K can be obtained by equating the average performance in(12) for both systems. As nonlinear equations must be solved to obtain G_K, no closed form is possible. However, this could be circumvented by approximating the Rician-$\mathcal{K}$ distribution using the Nakagami-m distribution, with the two factors related as $m = (1+\mathcal{K})^2/(1+2\mathcal{K})$ [8, p.23], where $\mathcal{K} \geq 0$ and $m \geq 1$. Notice that Rayleigh fading corresponds to $m=1$, or $\mathcal{K}=0$. Hence, similarly, we get the pairwise error probability using the Nakagami distribution,

$$\mathrm{P}_E(d) \leq \frac{1}{2}\left(1+\frac{\delta^2}{4mKN_0}\right)^{-mKd}. \quad (13)$$

And similarly, the bit error rate can be bounded by

$$\mathrm{P}_b \leq \frac{1}{2}\sum_{d=d_f}^{\infty} B_d\left(1+\frac{\delta^2}{4mKN_0}\right)^{-mKd}. \quad (14)$$

We verify that these two bounds in (12) and (14) are almost identical in the performance ranges considered ($> 10^{-6}$). Therefore, the distribution approximation is well justified in our case, and allows us to find G_K in closed-form.

Plugging δ_1 and δ_K into (14) for OFDMA and UP-OFDMA, the SNR gain G_K to achieve the same performance can be readily obtained as:

$$G_K = \frac{\delta_1^2}{\delta_K^2} = \frac{\delta_1^2}{4mKN_0\left[(1+\frac{\delta_1^2}{4mN_0})^{\frac{1}{K}}-1\right]}. \quad (15)$$

We now quantify the performance gap of UP-OFDMA with respect to the single user bound. We assume a rich scattering environment, where the $L+1$ channel taps are uncorrelated. As asserted in [14], the performance of SS-OFDM can be achieved with only $L+1$ equi-spaced subcarriers. Therefore, the single user bound as described in Section II-D can be quantified by setting $K=L+1$ in (12) and (14). Following that, we define the SNR gap between our UP-OFDMA, and the single user bound as

$$\varepsilon_K := \frac{\delta_K^2}{\delta_{L+1}^2} = \frac{K\left[(1+\frac{\delta_1^2}{4mN_0})^{\frac{1}{K}}-1\right]}{(L+1)\left[(1+\frac{\delta_1^2}{4mN_0})^{\frac{1}{L+1}}-1\right]}. \quad (16)$$

Fig. 2 depicts the SNR gain G_K for different block sizes K, and typical values of the Nakagami factor m (or the Rician factor $\mathcal{K}$), where we set $\delta_1^2/N_0 = 11$ dB, which amounts to $E_b/N_0 = 6.7$dB for a rate $2/3$ code and BPSK modulation. It is evident that G_K saturates quickly as K increases. Most performance improvement is observed for $K \leq 10$. The SNR gain of the single user SS-OFDM over conventional OFDMA can be found in Fig. 2, by setting $K=L+1$. For all the different m's, the additional performance improvement by increasing $K=4$ to $K=L+1$ (the single user bound) is less than 1 dB even for very large L. For sparse channels, the performance gap decreases further, since those $L+1$ taps

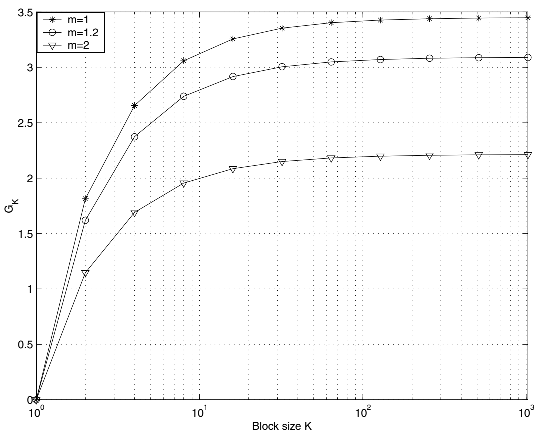

Fig. 2. The SNR gain of UP-OFDMA over OFDMA

become highly correlated. We therefore conclude that our UP-OFDMA is very effective, with each user having performance close to the single user bound. Also, notice that the single user bound as well as the SNR gap ε_K decreases as m increases; i.e., when the channel condition improves.

From a performance perspective, K should be chosen as large as possible. However, in practice, the choice of K is limited by many factors. First, the channel must be ensured time invariant during each block of duration $(NK + L)T_c$. Increasing K arbitrarily, would make this assumption no longer valid. Second, the decoding complexity increases when K increases. All these motivate and well justify choosing a small K for UP-OFDMA. In practice, K can be chosen to be smaller than 12, e.g., $K = 4, 8$. The performance gap from the single user bound is then less than 1 dB with ideal uncorrelated fading channels.

IV. Simulation Results

We present numerical results in this section. We assume $P = 64$ subcarriers in the system and allocate $K = 4$ subcarriers per user. We use the rate 2/3 convolutional code with generator polynomial $[3\ 1\ 0; 2\ 3\ 3]$, and bit distance spectrum polynomial $B(z) = 0.5z^3 + 3z^4 + 8z^5 + 23z^6 + \ldots$. BPSK modulation is used, and each frame contains 192 information bits. A block interleaver of dimension 18×16 is employed. At the receiver, we adopt the turbo decoding algorithm of [10]. For all simulations, the bit error rate (BER) after three iterations will be plotted.

We simulate a rich scattering environment, with the channels independently Rayleigh faded from block to block (an idealized fast fading scenario that can be approximated through sufficiently long interleaving together with frequency hopping). Fig. 3 depicts the simulated bit error rate (BER) of UP-OFDMA with $K = 4$, compared with conventional OFDMA with $K = 1$. Performance of single user SS-OFDM is plotted with an underlying FIR channel of length 8. For reference, the union bounds in (12) are also plotted. First, we observe that the simulated BERs lie within 1 dB of the

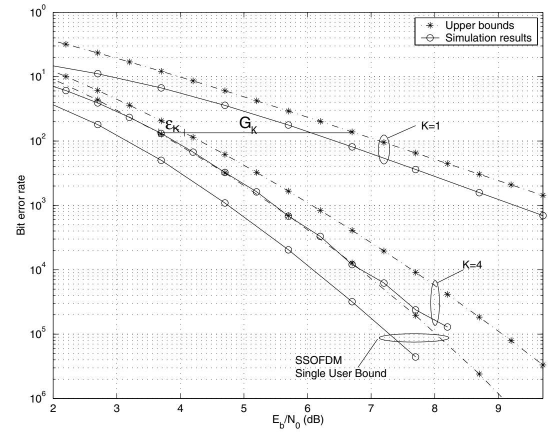

Fig. 3. Simulation Performance for UP-OFDMA

corresponding union bounds, which validates our analytical result. When K increases from 1 to 4, UP-OFDMA outperforms the conventional OFDMA considerably. At the same time, UP-OFDMA is less than 1 dB away from the single user SS-OFDM, which is the lower bound achievable in the presence of multipath fading. Recall that Fig. 2 is plotted with $\delta_1^2/N_0 = 11$dB, or $E_b/N_0 = 6.7$dB for OFDMA. The same $G_K = 2.7$dB and $\varepsilon_K = 0.4$dB can be observed from Fig. 3, in agreement with Fig. 2.

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