Maximum-Diversity Transmissions over Time-Selective Wireless Channels*

Xiaoli Ma and *Georgios B. Giannakis*

Dept. of ECE, University of Minnesota, Minneapolis, MN 55455, USA

Abstract— **Carrier frequency-offsets and mobility-induced Doppler shifts introduce time-selectivity in wireless links. Doppler-RAKE receivers have been developed for collecting the resulting diversity gains only for spread-spectrum systems. Relying on a basis expansion model of time-selective channels, we find that the maximum achievable Doppler diversity is determined by the rank of the correlation matrix of the channel's expansion coefficients. We also prove that RAKE reception can not collect maximum diversity gains, unless the transmission is appropriately designed. Finally, we design such block precoded transmissions to ensure maximum diversity gains, and provide thorough simulations to corroborate our theoretical findings.**

I. INTRODUCTION

Modeling temporal channel variations and coping with time-selective fading are important and challenging tasks in mobile communications. Time-selectivity arises due to oscillator drifts, phase noise, multipath propagation, and relative motion between the transmitter and the receiver.

Several approaches for modeling mobile time-varying channels are available [1,4,8,9]. Jakes in [4] proposed a random model by assuming an isotropic receive antenna, and a large number of incident scattered reflections arriving with uniformly distributed angles. Building on Bello's time-frequency sampling approach [1], a so-termed multipath-Doppler canonical model was advocated in [8]. Prior to [8], a more general Basis Expansion Model (BEM) for time-frequency selective channels was derived in [9]. Temporal variations over a finite time interval were expressed as a linear combination of polynomial bases in [2]. Relying on parsimonious BEM parameterizations, various equalizers were constructed in e.g., [3]. However, selecting a number of Fourier bases to capture dominant channel variations, and linking the BEM with the canonical and Jakes' models were left open, and constitute this paper's starting point.

Our main theme however, concerns diversity techniques that are known to offer valuable counter measures against fading [7]. Frequency-selective channels offer multipath diversity, while time-selective channels can provide Doppler diversity. A time-frequency generalization of the RAKE receiver was proposed in [8] to exploit the joint multipath-Doppler diversity. However, [8] focused on multi-user spread spectrum (SS) systems. In addition to non-SS point-to-point links, our distinct contribution relative to [8] is an analytical formula of the maximum achievable diversity order for transmissions over correlated channels, along with the transmitter design that is necessary to achieve it. Specifically, we first prove that the maximum Doppler diversity for time-selective frequency-flat channels is $Q+1$, where Q is the number of bases in the BEM. Subsequently, we design linearly precoded transmissions to collect this maximum diversity gain. We show that the rank of the channel correlation matrix decides the maximum diversity. Time- and frequency-selective channels are considered in [5].

Notation: Upper (lower) bold face letters will be used for matrices (column vectors). Superscript $^{\mathcal{H}}$ will denote Hermitian, * conjugate, ^{T} transpose, and † pseudo-inverse. We will reserve $\star$ for convolution, and $E[\cdot]$ for expectation with respect to all the random variables within the brackets. We will use $[\mathbf{A}]_{k,m}$ to denote the $(k,m)th$ entry of a matrix $\mathbf{A}$, and $[\mathbf{x}]_m$ to denote the m^{th} entry of the column vector $\mathbf{x}$; diag$[\mathbf{x}]$ will stand for a diagonal matrix with $\mathbf{x}$ on its main diagonal.

II. TIME-SELECTIVE CHANNEL MODEL

Most wireless links experience multipath propagation where a number of reflected or scattered rays arrive at the receiving end [7]. Time-selectivity arises when all the rays arrive at the receiver almost simultaneously with a common propagation delay that can be set to zero without loss of generality. Each ray is characterized by an attenuation and a phase shift [4]. The time-varying (TV) impulse response of the resulting physical channel at baseband is thus expressible as:

$$c^{(ch)}(t) = \sum_m a_m e^{j\theta_m} e^{j2\pi f_m t}\delta(t), \qquad (1)$$

where $\theta_m \in [-\pi, \pi)$ denotes the phase, f_m is a possible frequency offset (capturing Doppler and/or oscillator drifts), a_m is the amplitude of the mth path, and $\delta(\cdot)$ denotes Dirac's delta function. The overall channel is defined as the convolution of the transmit-filter, $c^{(tr)}(t)$, the receive-filter, $c^{(rec)}(t)$, and the physical channel, $c^{(ch)}(t)$. Therefore, by defining $\alpha_m := (c^{(tr)} \star c^{(rec)})(0)a_m$, we arrive at the following continuous-time baseband equivalent channel [c.f. (1)]

$$c(t) := (c^{(ch)} \star c^{(tr)} \star c^{(rec)})(t) = \sum_m \alpha_m e^{j\theta_m} e^{j2\pi f_m t}. \qquad (2)$$

Note that if $\omega_m := 2\pi f_m$, and m takes on a finite number of values, then (2) coincides with the BEM in [3,9].

Eq. (2) captures deterministically a time-selective channel over a finite time horizon; but it is also possible to link it with existing random channel models. Towards this objective, define the maximum Doppler shift (spread) as $f_{\max} := v f_0/c$, where v is the mobile's velocity relative to the base station, c is the speed of light, and f_0 stands for the carrier frequency. With β_m uniformly distributed over $[-\pi, \pi)$, we can use $f_m := f_{\max}\cos(\beta_m)$ to capture omni directional arrivals of the rays at the receiver end. Then (2) reduces to the so called Jakes' model [4, p. 65]:

$$h_J(t) = \sum_m \alpha_m e^{j\theta_m} e^{j2\pi f_{\max}\cos(\beta_m)t}. \qquad (3)$$

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If the scattering is rich, then the number of paths can be as large as infinity. Albeit useful for performance analysis studies, Jakes' model presents formidable challenges to channel estimation because it entails prohibitively large (theoretically infinite) number of parameters. In the following, we will by-pass this problem through a simplifying BEM approximation.

We will consider block transmissions with block length N, over channels with maximum Doppler spread $f_{\max}$ defined as for the Jakes' model. For a given $f_{\max}$ and a sampling period T_s, by properly choosing N, we can always ensure that $f_{\max}NT_s \geq 1$.

Considering that $c(t)$ in (2) is time-limited over NT_s with approximate bandwidth $1/(NT_s)$, we can sample its Fourier transform

$$C(f) = \sum_m \alpha_m e^{j\theta_m} \delta(f - f_m) \ , \tag{4}$$

with period $1/(NT_s)$, to obtain

$$C(q/(NT_s)) = \alpha_{m_q} e^{j\theta_{m_q}} \delta(f - q/(NT_s)), \tag{5}$$

where $q \in [-Q/2, Q/2]$, and m_q is the corresponding path index after frequency-sampling. The parameter Q dictates the number of bases in our BEM, and is defined as ($\lceil\cdot\rceil$ denotes integer ceiling):

$$Q := 2\lceil f_{\max} T_s N \rceil. \tag{6}$$

Restricting the frequency-domain samples to the interval $[-Q/2, Q/2]$ gives rise to the TV impulse response:

$$c_B(t) = \sum_{q=-Q/2}^{Q/2} \alpha_{m_q} e^{j\theta_{m_q}} e^{j2\pi q/(NT_s)t} \ , \quad t \in [0, NT_s) \ . \tag{7}$$

Sampling $c_B(t)$ in the time-domain with sampling period T_s, yields the discrete-time BEM:

$$h_B(i) := c_B(iT_s) = \sum_{q=-Q/2}^{Q/2} \alpha_{m_q} e^{j\theta_{m_q}} e^{j2\pi q i/N}. \tag{8}$$

The number of discrete-time complex exponential bases $\{e^{j2\pi q/N}\}_{q=-Q/2}^{Q/2}$ determines the number of time-domain block replicas we obtain in the frequency domain, and justifies intuitively the diversity order that emerges due to Doppler.

Sampled representations similar to $h_B(i)$ were reported in [8]. However, our derivation and interpretation is not limited to multi-user SS systems. Model (8) can be viewed as a special case of the BEM in (2), since it entails only a small finite number of bases. We will illustrate by simulation in Section V that these bases capture the main channel variations. But before that, we will study diversity issues pertaining to time-selective channels modeled as in (8).

III. Block Transmissions and Diversity

In this section, we adopt the BEM given in (8) as our time-selective channel model. For brevity, we drop the subscript $_B$ of channel realizations. We also assume that:

A1) *The maximum Doppler spread $f_{\max}$ is bounded, known to both transmitter and receiver, and invariant per transmission burst of NT_s seconds;*

A2) *A new realization of our randomly fading time-selective channel $h(i)$ is considered for each received block of size N; hence, the BEM channel coefficients depend on the block index; i.e., $\alpha_{m_q,\lfloor i/N \rfloor} \exp(j\theta_{m_q,\lfloor i/N \rfloor})$;*

A3) *The BEM coefficients $\alpha_{m_q,\lfloor i/N \rfloor} \exp(j\theta_{m_q,\lfloor i/N \rfloor})$ are zero-mean complex Gaussian random variables.*

Since the maximum velocity of mobiles can be experimentally measured, A1) is practical. Although $h(i)$ in (8) was viewed as deterministic for equalization purposes in [3,9], we adopt A2) here in order to evaluate the diversity order, and design maximum diversity transmissions irrespective of the specific channel realization encountered. When transmissions experience rich scattering and no line-of-sight is present, the validity of A3) is assured by the Central Limit Theorem.

Under A1)-A3) we can rewrite the BEM of (8) in a more compact form as:

$$h(i) = \sum_{q=0}^{Q} h_q(\lfloor i/N \rfloor) e^{j\omega_q (i \bmod N)}, \tag{9}$$

where $h_q(\lfloor i/N \rfloor) := \alpha_{m_q,\lfloor i/N \rfloor} \exp(j\theta_{m_q,\lfloor i/N \rfloor})$, and $\omega_q := 2\pi(q - Q/2)/N$.

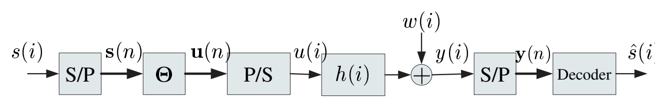


Fig. 1. Discrete-time baseband equivalent model

Figure 1 depicts the discrete-time equivalent baseband model when communicating through the time-selective channel (9). The information bearing symbols $s(i)$ are drawn from a finite alphabet, and parsed into blocks of size K. We use two arguments (n and k) to describe $i = nK + k$ for $k \in [0, K-1]$, and denote the $(k+1)$st entry of the n^{th} block as $[\mathbf{s}(n)]_k := s(nK + k)$. Each block $\mathbf{s}(n)$ is linearly precoded by an $N \times K$ matrix $\boldsymbol{\Theta}$ to yield $N \times 1$ symbol blocks $\mathbf{u}(n) = \boldsymbol{\Theta}\mathbf{s}(n)$. After parallel to serial (P/S) multiplexing, the blocks $\mathbf{u}(n)$ are transmitted through a time-selective channel $h(i)$ modeled as in (9). The i^{th} received sample $y(i)$ can be written as:

$$y(i) = h(i)u(i) + w(i), \tag{10}$$

where $w(i)$ denotes complex additive white Gaussian noise (AWGN) with mean zero, and variance $N_0/2$. The received samples $y(i)$ are serial to parallel (S/P) converted to form $N \times 1$ blocks: $\mathbf{y}(n) = [y(nN), y(nN+1), \ldots, y((n+1)N-1)]^T$. The matrix-vector counterpart of (10) can be expressed as

$$\mathbf{y}(n) = \mathbf{H}(n)\boldsymbol{\Theta}\mathbf{s}(n) + \mathbf{w}(n), \tag{11}$$

where $\mathbf{H}(n) := \sum_{q=0}^{Q} h_q(n)\mathbf{D}(\omega_q)$ is a diagonal matrix, with $\mathbf{D}(\omega_q) := \mathrm{diag}[1, \exp(j\omega_q), \ldots, \exp(j\omega_q(N-1))]$. By plugging $\mathbf{H}(n)$ into (11), we obtain

$$\mathbf{y}(n) = \sum_{q=0}^{Q} h_q(n)\mathbf{D}(\omega_q)\boldsymbol{\Theta}\mathbf{s}(n) + \mathbf{w}(n) = \boldsymbol{\Phi}_s(n)\mathbf{h}(n) + \mathbf{w}(n), \tag{12}$$

where $\mathbf{h}(n)$ and $\mathbf{\Phi}_s(n)$ are defined, respectively, as

$$\mathbf{h}(n) := [h_0(n), \ldots, h_Q(n)]^T, \quad \text{and}$$
$$\mathbf{\Phi}_s(n) := [\mathbf{D}(\omega_0)\mathbf{\Theta s}(n), \ldots, \mathbf{D}(\omega_Q)\mathbf{\Theta s}(n)]. \quad (13)$$

Similar to [10], that dealt with frequency-selective channels, we will resort to a pair-wise error probability (PEP) analysis technique to examine the best achievable diversity of time-selective channels. Since the ensuing analysis is based on a single block, we drop the block index n for convenience.

Assuming that perfect channel status information (CSI) is available, and that maximum likelihood (ML) decoding is employed at the receiver, we consider the PEP, $P(\mathbf{s} \to \mathbf{s}'|\mathbf{h})$, that a block $\mathbf{s}$ is transmitted but is incorrectly decoded as $\mathbf{s}' \neq \mathbf{s}$. The PEP can be approximated using the Chernoff bound as:

$$P(\mathbf{s} \to \mathbf{s}'|\mathbf{h}) \leq \exp(-d^2(\mathbf{x}, \mathbf{x}')/4N_0), \quad (14)$$

where $\mathbf{x} := \mathbf{H\Theta s}$, $\mathbf{x}' := \mathbf{H\Theta s}'$, $d^2(\mathbf{x}, \mathbf{x}') := \|\mathbf{x} - \mathbf{x}'\|^2 = (\mathbf{x} - \mathbf{x}')^{\mathcal{H}}(\mathbf{x} - \mathbf{x}')$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}'$, and $N_0/2$ is the noise variance.

With $\mathbf{e} := \mathbf{s} - \mathbf{s}'$ denoting the symbol error vector, we can express the distance between $\mathbf{x}$ and $\mathbf{x}'$ as:

$$d^2(\mathbf{x}, \mathbf{x}') = \|\mathbf{H\Theta e}\|^2 = \|\mathbf{\Phi}_e \mathbf{h}\|^2, \quad (15)$$

where $\mathbf{\Phi}_e$ is defined as $\mathbf{\Phi}_s$ with $\mathbf{e}$ replacing $\mathbf{s}$. The channel correlation matrix and its rank will be denoted, respectively, by:

$$\mathbf{R}_h := E[\mathbf{h}\mathbf{h}^{\mathcal{H}}], \text{ and } r_h := \text{rank}(\mathbf{R}_h). \quad (16)$$

Eigenvalue decomposition of $\mathbf{R}_h$ yields:

$$\mathbf{R}_h = \mathbf{V}_h \mathbf{D}_h \mathbf{V}_h^{\mathcal{H}}, \quad (17)$$

where $\mathbf{D}_h := \text{diag}[\sigma_0^2, \sigma_1^2, \ldots, \sigma_{r_h-1}^2]$, and $\mathbf{V}_h$ is a $(Q+1) \times r_h$, para-unitary matrix satisfying $\mathbf{V}_h^{\mathcal{H}}\mathbf{V}_h = \mathbf{I}_{r_h}$. If we define the normalized channel $\bar{\mathbf{h}}$ as $\bar{\mathbf{h}} := \mathbf{D}_h^{-\frac{1}{2}}\mathbf{V}_h^{\mathcal{H}}\mathbf{h}$, its correlation matrix will be given by $E[\bar{\mathbf{h}}\bar{\mathbf{h}}^{\mathcal{H}}] = \mathbf{I}_{r_h}$; i.e., the coefficients $\bar{h}_q$ are i.i.d. Gaussian distributed with zero mean, and unit variance. Let us also define the matrix $\mathbf{A}_e := (\mathbf{V}_h\mathbf{D}_h^{\frac{1}{2}})^{\mathcal{H}}\mathbf{\Phi}_e^{\mathcal{H}}\mathbf{\Phi}_e\mathbf{V}_h\mathbf{D}_h^{\frac{1}{2}}$. Since $\mathbf{A}_e$ is Hermitian, there exists a unitary matrix $\mathbf{V}_e$, and a real non-negative definite diagonal matrix $\mathbf{D}_e$ such that $\mathbf{V}_e\mathbf{A}_e\mathbf{V}_e^{\mathcal{H}} = \mathbf{D}_e$. The $r_h \times r_h$ diagonal matrix $\mathbf{D}_e := \text{diag}[\lambda_0, \ldots, \lambda_{r_h-1}]$, holds on its diagonal the eigenvalues of $\mathbf{A}_e$, that satisfy $\lambda_q \geq 0$, $\forall q \in [0, r_h - 1]$. The vector $\tilde{\mathbf{h}} = \mathbf{V}_e\bar{\mathbf{h}}$ has correlation identical to $\bar{\mathbf{h}}$ because $\mathbf{V}_e$ is unitary. Thus, $d^2(\mathbf{x}, \mathbf{x}')$ can be written in terms of the eigenvalues of matrix $\mathbf{A}_e$ as

$$d^2(\mathbf{x}, \mathbf{x}') = \sum_{q=0}^{r_h-1} \lambda_q |\tilde{h}_q|^2. \quad (18)$$

To compute the upper bound on the average PEP in (14), we simply average the channel-conditioned PEP

$$P(\mathbf{s} \to \mathbf{s}'|\mathbf{h}) \leq \exp\left(-\sum_{q=0}^{r_h-1} \lambda_q |\tilde{h}_q|^2/4N_0\right),$$

over the independent Rayleigh distributed $|\tilde{h}_q|^2$'s. The resulting average PEP is bounded as follows

$$P(\mathbf{s} \to \mathbf{s}') \leq \prod_{q=0}^{r_h-1} \left(1 + \frac{\lambda_q}{4N_0}\right)^{-1}. \quad (19)$$

If $r_a(e) := \text{rank}(\mathbf{A}_e)$, then $r_a(e)$ eigenvalues of $\mathbf{A}_e$ are nonzero; we denote these eigenvalues as $\lambda_0, \ldots, \lambda_{r_a(e)-1}$. We have from (19) that at high SNR,

$$P(\mathbf{s} \to \mathbf{s}') \leq \left(\prod_{q=0}^{r_a(e)-1} \lambda_q\right)^{-1} \left(\frac{1}{4N_0}\right)^{-r_a(e)}, \quad (20)$$

where $r_a(e)$ is the diversity gain, and $\left(\prod_{q=0}^{r_a(e)-1} \lambda_q\right)^{1/r_a(e)}$ is the coding gain for the error pattern $\mathbf{e} = \mathbf{s} - \mathbf{s}'$ (see also [10]). In this paper, we will focus on diversity analysis. Since $r_a(e)$ depends on $\mathbf{e}$, taking a conservative approach we will define the diversity gain of our system as:

$$G_d := \min_{\forall \mathbf{e} \neq 0} r_a(e), \quad (21)$$

which means that $\forall \mathbf{e} \neq \mathbf{0}$, we have $r_a(e) \geq G_d$. Because $\mathbf{A}_e$ is an $r_h \times r_h$ matrix, we also have that $r_a(e) \leq r_h, \forall \mathbf{e} \neq \mathbf{0}$. Thus, we have established the following proposition:

Proposition 1 *If the correlation matrix of the BEM channel coefficients in* (16) *has rank* r_h*, then the maximum diversity gain of the time-selective channel in* (9) *is* $G_d = r_h$*. When* $\mathbf{R}_h$ *has full rank* $r_h = Q + 1$*, the maximum diversity gain is* $G_d = Q + 1$*; i.e., the number of bases in the BEM determines the maximum diversity-based system performance.*

For transmissions over frequency-selective channels, [10] proved that carefully designed linear precoding implemented via a matrix $\mathbf{\Theta}$ having entries over the complex field guarantees maximum multipath diversity. We will show that maximum Doppler diversity can be achieved with linearly precoded transmissions over time-selective channels as well.

IV. Maximum diversity Transmissions

To assure maximum diversity gain, the matrix $\mathbf{A}_e := (\mathbf{V}_h\mathbf{D}_h^{\frac{1}{2}})^{\mathcal{H}}\mathbf{\Phi}_e^{\mathcal{H}}\mathbf{\Phi}_e\mathbf{V}_h\mathbf{D}_h^{\frac{1}{2}}$ needs to have full rank $\forall \mathbf{e} \neq \mathbf{0}$. If $N \geq Q + 1$, the matrix $\mathbf{\Phi}_e$ (defined as in (13)) is tall. To guarantee that $\mathbf{A}_e$ has full rank $\forall r_h \in [1, Q + 1]$, we need $\mathbf{\Phi}_e^{\mathcal{H}}\mathbf{\Phi}_e$ to be full rank. Because rank$(\mathbf{\Phi}_e^{\mathcal{H}}\mathbf{\Phi}_e)$ =rank$(\mathbf{\Phi}_e)$, the maximum diversity is achieved if and only if $\mathbf{\Phi}_e$ has full rank, $\forall \mathbf{e} \neq 0$. Now let us go back to the definition of $\mathbf{\Phi}_e$ to devise design criteria for $\mathbf{\Theta}$ that guarantee the full rank of $\mathbf{\Phi}_e, \forall \mathbf{e} \neq \mathbf{0}$.

Recall that $\mathbf{u} := \mathbf{\Theta s}$ (see also Fig. 1). By defining $\mathbf{u}_e := \mathbf{\Theta e}$, the matrix $\mathbf{\Phi}_e$ can be expressed as:

$$\mathbf{\Phi}_e = [\mathbf{D}(\omega_0)\mathbf{u}_e, \ldots, \mathbf{D}(\omega_Q)\mathbf{u}_e] = \mathbf{D}(\mathbf{u}_e)[\mathbf{d}_0, \ldots, \mathbf{d}_Q], \quad (22)$$

where $\mathbf{D}(\mathbf{u}_e) := \text{diag}[\mathbf{u}_e]$, and $\mathbf{d}_q := [1, \ldots, \exp(j\omega_q(N-1))]^T$, for $q \in [0, Q]$. Because $[\mathbf{d}_0, \ldots, \mathbf{d}_Q]$ is a tall column-wise Vandermonde matrix, and the ω_q's are all equi-spaced, by defining $\overline{\omega} := \omega_q - \omega_{q-1}$, we can verify that

$$[\mathbf{d}_0, \ldots, \mathbf{d}_Q] = \mathbf{D}(\omega_0)[\mathbf{d}^0, \ldots, \mathbf{d}^Q], \qquad (23)$$

where $\mathbf{d}^q := [1, \exp(j\overline{\omega}q), \ldots, \exp(j\overline{\omega}q(N-1))]^T$ is the q-tuple Hadamard product of $\mathbf{d}$. Notice that $[\mathbf{d}^0, \ldots, \mathbf{d}^Q]$ is a Vandermonde matrix with distinct generators both column-wise and row-wise; hence, *any* $Q+1$ *rows of the matrix* $[\mathbf{d}^0, \ldots, \mathbf{d}^Q]$ *are linearly independent*, which establishes the following proposition:

Proposition 2 *For linearly precoded transmissions over a time-selective BEM obeying the input-output relationship* (12)*, the maximum diversity gain* G_d *is achieved with a precoder* $\mathbf{\Theta}$ *if and only if there exist at least* $Q+1$ *non-zero entries in* $\mathbf{u}_e = \mathbf{\Theta}\mathbf{e}, \forall \mathbf{e} \neq \mathbf{0}$.

Proof: Suppose $\mathbf{u}_e = \mathbf{\Theta}\mathbf{e}, \forall \mathbf{e} \neq \mathbf{0}$ has at least $Q+1$ non-zero entries. Then without loss of generality, let the first $Q+1$ non-zero entries of $\mathbf{u}_e$ be $\overline{\mathbf{u}} = [u_{n_0}, \ldots, u_{n_Q}]^T$, where $n_q \in \{0, \ldots, N-1\}$, and u_{n_q} denotes the n_q^{th} entry of $\mathbf{u}_e$. Selecting the corresponding rows $\{n_q\}_{q=0}^{N-1}$ of the matrix $[\mathbf{d}_0, \ldots, \mathbf{d}_Q]$, we obtain from (22) and (23) a $(Q+1)\times(Q+1)$ matrix $\mathbf{D}(\overline{\mathbf{u}})\overline{\mathbf{D}}(\omega_0)\mathbf{V}_1$, where the $(Q+1)\times(Q+1)$ Vandermonde matrix $\mathbf{V}_1$ is formed by the corresponding $Q+1$ rows of $[\mathbf{d}^0, \ldots, \mathbf{d}^Q]$, and $\overline{\mathbf{D}}(\omega_0)$ is a diagonal matrix with those selected $Q+1$ entries from $\mathbf{D}(\omega_0)$ on its main diagonal. Since rank$(\mathbf{D}(\overline{\mathbf{u}})\overline{\mathbf{D}}(\omega_0)) = Q+1$ and rank$(\mathbf{V}_1) = Q+1$, we deduce that rank$(\mathbf{\Phi}_e) = (Q+1), \forall\ \mathbf{e} \neq \mathbf{0}$.

Next, we prove the "only if" part by contradiction. Suppose that for some $\mathbf{e}$, $\mathbf{u} = \mathbf{\Theta}\mathbf{e}$ has only $\bar{Q}+1 < Q+1$ non-zero corresponding entries, that we collect in $\overline{\mathbf{u}} = [u_{n_0}, \ldots, u_{n_{\bar{Q}}}]^T$. Then similar to the "if" part, we can group the non-zero rows in a matrix $\mathbf{D}(\overline{\mathbf{u}})\overline{\mathbf{D}}(\omega_0)\mathbf{V}_1$. Now this $\mathbf{V}_1$ is a $(\bar{Q}+1)\times(Q+1)$ matrix, while $\overline{\mathbf{D}}(\omega_0)$ is a $(\bar{Q}+1)\times(\bar{Q}+1)$ matrix. It follows immediately that rank$(\mathbf{V}_1) = \bar{Q}+1 < Q+1$, and hence rank$(\mathbf{\Phi}_e) < Q+1$, which implies that the maximum diversity can not be achieved. ■

In the following, we will introduce two classes of linear precoders which satisfy the necessary and sufficient condition of Proposition 2.

Tall Vandermonde Precoders: Choose $N \geq K+Q$ points $\{\rho_n\}_{n=1}^N \in \mathbb{C}$, where $\mathbb{C}$ denotes the complex field, such that $\rho_m \neq \rho_n, \forall m \neq n$. Then the Vandermonde precoder $\mathbf{\Theta} \in \mathbb{C}^{N\times K}$ is defined by $[\mathbf{\Theta}]_{n,k} = \rho_n^k$. Any non-zero error vector $\mathbf{e}$ with length K can be equivalently viewed as the coefficients of a polynomial $\psi_e(x)$ with highest order $K-1$. It can be verified that $\mathbf{\Theta}\mathbf{e} = [\psi_e(\rho_1), \ldots, \psi_e(\rho_N)]^T$. Since $\psi_e(x)$ has at most $(K-1)$ roots, $\{\psi_e(\rho_n)\}_{n=1}^N$ have at most $(K-1)$ zero elements; i.e., at least $N-(K-1) \geq Q+1$ non-zero elements. Therefore, these tall Vandermonde precoders satisfy the condition of Proposition 2, and can thus achieve maximum Doppler diversity order.

As a special case, we can choose $\mathbf{\Theta} = \mathbf{F}^{\mathcal{H}}\mathbf{T}_{zp}$, where $\mathbf{F}^{\mathcal{H}}$ is the N-point inverse fast Fourier transformation (IFFT) matrix with $[\mathbf{F}]_{m,n} = (1/\sqrt{N})\exp(-j2\pi mn/N)$, and $\mathbf{T}_{zp} := [\mathbf{I}_K\ \mathbf{0}_{K\times Q}]^T$. Applying this precoder to the symbol block $\mathbf{s}$ consists of two steps: first, the information block $\mathbf{s}$ is padded with Q zeros (via $\mathbf{T}_{zp}$), and then the zero-padded block is processed by an IFFT (via $\mathbf{F}^{\mathcal{H}}$). Tall Vandermonde precoders are redundant, but guarantee maximum diversity regardless of the constellation. In contrast, our next class of precoders will be non-redundant but constellation-specific.

Square Vandermonde Precoders: It has been proved that if the information symbols in $\mathbf{s}$ are drawn from a QAM constellation, there always exists a square ($K = N$) Vandermonde matrix $\mathbf{\Theta}$ such that $\mathbf{u}_e = \mathbf{\Theta}\mathbf{e}$ has at least one nonzero entry $\forall \mathbf{e} \neq \mathbf{0}$ [11]. In this case, the maximum diversity is achieved whenever $K \geq Q+1$ with non-redundant linear precoders.

As an example, for a 4-QAM constellation, and block length $N = K = 5$, the square precoder $\mathbf{\Theta}$ ensuring maximum Doppler diversity is [11]: $[\mathbf{\Theta}]_{n,k} = \rho_n^k$, where $\rho_n = 2^{1/(2N)}e^{j(\pi/4+2\pi n)/N}$, $n = 0, \ldots, N-1$, and C is a normalizing constant given by: $C = (2^{1/N}-1)^{1/2}$.

Spread-Spectrum Precoders: For SS transmissions, each block contains one symbol ($K = 1$), and the precoder $\mathbf{\Theta}$ reduces to an $N \times 1$ vector $\boldsymbol{\theta}$. From Result 2, we know that in order to achieve full diversity, $\boldsymbol{\theta}$ must contain at least $Q+1$ non-zero entries, which implies that the length of the SS precoder $\boldsymbol{\theta}$ should be at least $Q+1$. Therefore, any SS transmission with spreading factor less than $Q+1$ cannot guarantee the maximum diversity order, even if ML decoding is employed. This result shows that for a single user, the time-frequency approach of [8] can collect the full diversity at the receiver provided that: i) the chip period $T_c = T_s$ is chosen to ensure that $2f_{\max}T_s < 1$; and, ii) the spreading gain N is sufficiently large to satisfy $2f_{\max}NT_s \geq 1$, and $N \geq Q+1$. Unfortunately, for point-to-point links, the class of SS precoders consumes more bandwidth relative to the first two classes. Furthermore, for multiple access, the SS system in [8] may not guarantee full diversity due to the presence of multiuser interference.

All three classes of precoders can guarantee maximum diversity. It can be seen that square-Vandermonde precoders have the highest bandwidth efficiency; however, they are constellation-dependent. Tall-Vandermonde precoders on the other hand, are constellation-independent. The SS precoders consume the most bandwidth efficiency among the three. A natural question at this point is, which precoder offers the best BER performance. It is difficult to fully address this question, because diversity decides only the slope of the BER curve (in dBs) at high SNR. Hence, in selecting the best-performed $\mathbf{\Theta}$, one should also consider the coding gain as well as the kissing number. But these considerations go beyond the scope of this paper, and will be left for future research.

V. Simulated Performance

We now present simulations to test the fitting of BEM to time-varying channels, and confirm the performance of our maximum diversity transmissions.

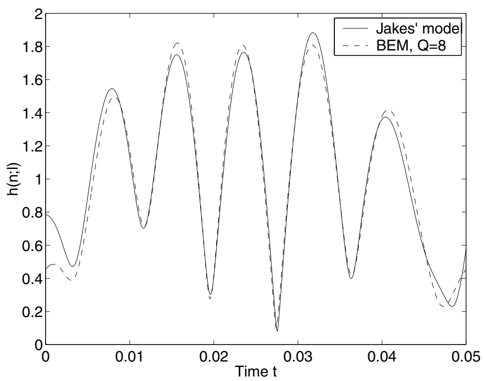
Test case 1 (BEM justification): We have introduced both

Jakes' model (3), and the BEM (9). Here we will test how accurately the latter can approximate the former.

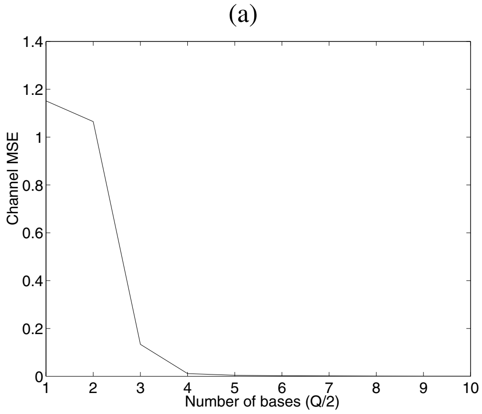
First, we consider fast fading channels, with carrier frequency $f_0 = 1.8$ GHz, and symbol duration $T_s = 50\mu s$. With maximum speed $v_{\max} = 48$ km/hr, the maximum Doppler shift is $f_{\max} = 80$ Hz. The block length N is selected as $1,000$. The Jakes' model is generated by (3), with parameters: $Q_J = 200$, $\beta_m = 2\pi m/(Q_J + 1)$; and the ϕ_m's are drawn independently from the uniform $[-\pi, \pi)$ distribution $\forall m \in [-Q_J/2, Q_J/2]$. We find from (6) that $Q = 8$, and estimate the BEM coefficients $\{h_q(l)\}$ using the least-squares algorithm of [6] with the known exponential bases. From Figure 2 (a), we observe that when $Q = 8$, the BEM in (9) approximates closely Jakes' model. In Figure 2 (b), we plot the mean-square error between $h_J(i)$ and $h_B(i)$:

$$e_{mse} = \frac{1}{N}\sum_{i=1}^{N} |h_J(i) - h_B(i)|^2. \quad (24)$$

for $Q = 8$. Adopting $Q > 8$ shows no significant difference in the accuracy of the fit.



(a)



(b)

Fig. 2. BEM validation

Test case 2 (Time-varying frequency-flat channels): We use our BEM in (9) to generate channels, and check our diversity claims of Section IV with QPSK modulated transmissions; Q assumes different values. We test both classes of precoders. For the tall Vandermonde class, we choose $K = 16$, $N = K + Q$, and $\mathbf{\Theta} = \mathbf{F}_N^{\mathcal{H}} \mathbf{T}_{zp}$. For the square Vandermonde class, we select $\mathbf{\Theta}$ as in Section IV with $Q = 4$, and $N = K = Q + 1$. The channel coefficients are generated as i.i.d. zero-mean Gaussian random variables with variance $\sigma_q^2 = 1/(Q + 1)$. Every point of the BER curve is averaged over $1,000$ channel realizations. Figure 3 shows the performance of ZF and MMSE equalizers, along with the near-ML equalizer implemented with the SD algorithm (see [10] for a detailed description of these equalizers). We notice that MMSE outperforms ZF by more than 1 dB when $Q = 2$, while SD achieves almost the full diversity at high SNR. As Q increases, the system diversity increases, and the BER performance improves accordingly.

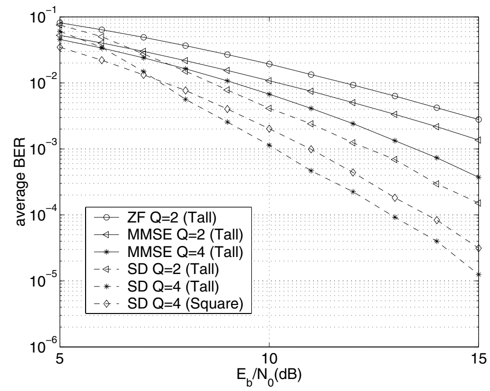


Fig. 3. Time-selective only channels

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