Optimal Adaptive Precoding for Frequency-Selective Nagakami-m Fading Channels *

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Abstract

DMT transmissions with optimal power and bit loading are suitable for wired-line applications but have high complexity when it comes to wireless time-varying environments. Adaptive modulation on the other hand, assumes that training sequences are available to provide an accurate estimate of the channel parameters, while the channel statistics allow to evaluate average performance. Random channel modeling is a powerful tool for assessing wireless systems performance, but can be also be instrumental in optimizing the modulation. We develop optimal loading strategies for frequency selective fading, assuming OFDM modulation and by modeling the channel impulse response as an FIR filter whose taps are Nagakami-m correlated fading processes. The design minimizes the BER for a given average transmit power. Channel statistics need to be updated at a very slow rate when compared to the exact channel status information (CSI), which reduces complexity of our adaptive OFDM scheme compared to a standard DMT approach. This also alleviates the need of training and allows us to incorporate partial channel knowledge in the design. Interestingly, our derivations identify the optimal solution for the limiting case where the channel transfer function is exactly known at both transmitter and receiver.

Keywords: *Transceiver Design, Optimal Modulation, OFDM Techniques.*

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1 Introduction and motivation

The optimal block-modulation for wide-band frequency-selective channels is OFDM with appropriate bit and power loading across the frequency bins, according to the water-filling principle [7, 12, 6]. This transmission scheme is adopted in DSL standards to increase transmission rate over the traditional copper twisted pair links. By duality, for narrowband time-selective, flat fading channels, maximum information rate is achieved by optimally distributing power and bits across the time bins (see e.g., [4],[15],[10]). Both schemes assume that the channel is perfectly known at the both receiver and the transmitter ends. This assumption is nearly satisfied when accurate channel estimation is available with a feedback channel, or, with time division duplex (TDD) transmissions, where transmitter and receiver use the same carrier frequency. Recently, the adaptive modulation idea was extended to time *and* frequency selective channels using OFDM [16]. Extension of adaptive modulation techniques to OFDM is challenging because the average power and the bits should be loaded in a jointly optimal fashion across different frequency bins and time blocks. It is also important to remark that unlike adaptive modulation on flat fading channels [15], DMT cannot achieve asymptotically capacity because the optimal modulation conveys the information symbols through the channel eigenfunctions instead of the Fourier basis [7]. In [2], it was shown that for deterministic underspread *frequency selective* time-varying channels, optimal precoding corresponds to transmitting information through a class of *channel-dependent* FM modulated signals, that obey the water-filling principle in the time-frequency domain. However, if for the duration of an OFDM symbol the channel can be considered stationary, OFDM modulation will still convert the approximately time-invariant channel to a set of parallel flat fading subchannels. This is the reason why OFDM transmission appears to be particularly suitable for Wireless Local Area Networks (WLAN) (see e.g., HIPERLAN/2 in [9]). Optimal bit/power loading can be implemented, but estimating the channel and updating the transmit power/bit distribution and the corresponding decision rules at the receiver entails extra complexity that grows rapidly in broadband mobile scenarios. Apart from the cyclic prefix, channel estimation requires pilot tones, an overhead which reduces the system efficiency. This renders the DMT schemes adopted in DSL standards less appealing for wireless applications. Moreover, WLAN operate with high carriers and wide bandwidth which already provide a potential for high rates. Therefore, quality of service (QoS) requirements, measured by the link BER, and reduced complexity and power consumption of the user terminals, can be more relevant optimization criteria than maximizing information rate.

We are thus motivated to design a low complexity transmission scheme that minimizes the BER for a given average transmit power and is based on a statistical characterization of the channel rather than on the instantaneous channel parameters. Although in a different setup, our idea is similar in spirit to the optimal design in [5] for transmit diversity in correlated Rayleigh fading: specifically, in our scheme i) the transmitter and the receiver are only estimating the statistical parameters of the channel (which can be accomplished without training) and the maximum delay spread is assumed to be known; ii) differential encoding (DPSK) is used to cope with the unknown phase of the channel frequency response at the receiver; iii) the transmitter tracks the channel statistical parameters and optimally loads power in order to minimize *average probability of error* at the receiver.

We model the channel as an FIR filter of order L time-varying impulse response $h(n, l)$. For an input $u(n)$, the channel output samples are

$$y(n) = \sum_{l=0}^{L} h(n,l)u(n-l) + v(n), \qquad (1)$$

where $v(n)$ is AWGN. We assume that $|h(n,l)|$ are Nagakami-m correlated fading processes.

Interestingly, the optimal power loading result comes in a simple closed form expression for the transmit power that has to be assigned to each frequency bin: the resulting power distribution is similar to an inverse water-filling or to a zero-forcing type of loading performed over the *average* channel power spectrum.

2 System model

Sampling the OFDM signal at the Nyquist rate, the channel (which includes the effects of time asynchronism, multipath and transmit-receive filters) can be modeled as an FIR filter of finite order L approximately equal to the product between the maximum delay spread and the transmission bandwidth. The information symbols are parsed into consecutive I-long blocks $s(n) := (s(nI), \ldots, s(nI + I - 1))^T$, and are mapped through a precoding matrix $\boldsymbol{F}$ to the vector $\boldsymbol{u}(n) = \boldsymbol{F}\boldsymbol{s}(n)$ whose entries are modulated and transmitted through the channel. The columns of $\boldsymbol{F}$ are complex exponentials, $\{\boldsymbol{F}\}_{n,i} := (\Phi_i/\sqrt{I})\, e^{j\frac{2\pi}{I}i(n-L)}$, $i \in [0, I-1], I > L, n \in [0, P-1]$, with the coefficients Φ_i in $\{\boldsymbol{F}\}_{n,i}$ used to load different powers across subchannels and blocks. Matrix $\boldsymbol{F}$ is tall ($P \times I$ where $P := I + L$), because its rows are augmented by the addition of the cyclic prefix (or suffix) that has length L, equal or greater than the channel order; thus, each transmitted block $\boldsymbol{u}(n) := (u(nP), \ldots, u(nP+P-1))^T$ has length P. After discarding the prefix at the receiver, inter-block interference (IBI) free data blocks $\boldsymbol{y}(n) := (y(nP+L), \ldots, y(nP+P-1))^T$ are obtained. Assuming that $h(nP + k, l) \approx h(nP, l)$, for $k = 0, \ldots, P-1$, the kth entry of $\boldsymbol{y}(n)$ is

$$\begin{aligned} y(nP+k) &\approx \sum_{l=0}^{L} h(nP,l) \sum_{i=0}^{I-1} \{\boldsymbol{F}\}_{k-l,i} \{\boldsymbol{s}(n)\}_i \\ &+ \ v(nP+k) \qquad (2) \\ &= \sum_{i=0}^{I-1} H(nP, f_i) e^{j\frac{2\pi i}{I}(k-L)} \Phi_i s_i(nI) \\ &+ \ v(nP+k), \end{aligned}$$

where $k = L, \ldots, P-1$, $\{f_i = i/I\}_{i=0}^{I-1}$ indicates normalized frequency and $s_i(nI) := s(nI + i)$. Subsequently, an I point FFT is performed on $\boldsymbol{y}(n)$ and the μth FFT output sample is

$$\begin{aligned} Y(nP, f_\mu) &:= \frac{1}{\sqrt{I}} \sum_{k=L}^{P-1} e^{-j\frac{2\pi}{I}\mu k} y(nP+k) \qquad (3) \\ &\approx H(nP, f_\mu)\Phi_\mu s_\mu(nI) + V(nP, f_\mu), \end{aligned}$$

where $V(nP, f_\mu) := (1/\sqrt{I}) \sum_{k=L}^{P-1} \exp(-j\frac{2\pi}{I}\mu k) v(nP+k)$ is also AWGN with variance N_0. To obtain (3) we assumed that $h(nP + k, l) \approx h(nP, l)$ for $k = 0, \ldots, P-1$ and neglected the effect of inter-symbol interference (ISI), which would arise if the channel was time-selective during the OFDM symbol interval.

We model the channel impulse response as multivariate Nagakami-m fading processes. The channel response at each frequency bin is: $H(nP, f_i) = \mathrm{e}_i^{\mathcal{H}} \boldsymbol{h}(n)$, with $\mathrm{e}_i := (1, \ldots, \exp(j\pi/IL))^T$. Since $H(nP, f_i)$ is a linear combination of the channel taps, we have that $H(nP, f_m)$ is also Nagakami-m fading with the same fading parameter m. Defining the SNR at the decision level as

$$SNR_i := \gamma_i(n)|\Phi_i|^2, \qquad (4)$$

where

$$\gamma_i(n) := \frac{2E_s}{N_0}|H(nP, f_i)|^2, \qquad (5)$$

it follows that $\gamma_i(n)$ is gamma distributed [13]:

$$f_{\gamma_i(n)}(x) = \frac{m^m x^{m-1}}{\bar{\gamma}_i^m} \exp\left(-\frac{mx}{\bar{\gamma}_i}\right). \quad (6)$$

Notice that the model of $H(nP, f_i)$ implicitly assumes stationarity of $h(n,l)$ with respect to n. Extensive studies have shown that the Nagakami-m model in (6) provides the best fit for land [3] and indoor fading [14], from which we deduce that (6) is flexible enough to capture the nature of the random variations of the channel taps. Depending on the noise level and on the speed of the variation of $|H(nP, f_i)|^2$ from block to block, the transmitter will be able to know more or less accurately the true $\gamma_i(n)$. As $m \to \infty$, the gamma distribution tends to a Dirac delta around $\bar{\gamma}_i$, which corresponds to the transmitter knowing deterministically the $\gamma_i(n)$'s at all subcarriers.

If $\forall i \in [0, I-1]$ the p.d.f. in (6) has identical $\bar{\gamma}_i = \bar{\gamma}$, any optimization algorithm based on this model would lead to the trivial solution of uniform power distribution across frequency. However, it is reasonable to assume that this situation is unlikely to happen. For example, it is well known that in broadband communications the multipath channel has time varying impulse response $h(n,l)$ that is usually correlated and has a decaying power profile in l. Thus, defining

$$\{\boldsymbol{R}_{hh}(q,n)\}_{k,l} := E\{h^*(qP,k)h(qP+nP,l)\}, \quad (7)$$

for $k, l = 0, \ldots, L$, and assuming that $h(n,l)$ is stationary with respect to n, we have that $\boldsymbol{R}_{hh}(q,n) \equiv \boldsymbol{R}_{hh}(n)$ and

$$E\{|H(qP, f_i)|^2\} = \mathbf{e}_i^{\mathcal{H}} \boldsymbol{R}_{hh}(0)\mathbf{e}_i, \quad (8)$$

which is different for every i unless $\boldsymbol{R}_{hh}(0) \propto \boldsymbol{I}$; but this is impossible, unless the channel paths are uncorrelated.

Even not considering the the intrinsic model of the time varying impulse response $h(n,l)$, it is widely recognized that the coefficients $|H(qP, f_i)|^2$ can be modeled as Nagakami-m fading coefficients, in some cases also by approximating with appropriate parameters other fading distributions. Since this model could potentially fit other random channel behaviors it is a good candite to represent, for appropriate values of m, our uncertainty on the true values of $|H(qP, f_i)|^2$. The higher is m the least is our uncertainty on the channel frequency response the more we rely on the values of $E\{|H(qP, f_i)|^2\}$ as true values for $|H(qP, f_i)|^2$.

3 Optimal Loading

The average probability of error on the ith subcarrier for DPSK transmissions over Nagakami-m fading is given by (see e.g. [13])

$$\bar{P}_e(f_i) := E\{P_e(n, f_i)\} = \frac{1}{2}\left(1 + \frac{\bar{\gamma}_i}{m}|\Phi_i|^2\right)^{-m}. \quad (9)$$

Assuming that the instantaneous $P_e(n, f_i) \ll 1$, the error probability in our communication link setup is: $\mathcal{P}_e(n) = 1 - \prod_i (1 - P_e(n, f_i)) \approx \sum_i P_e(n, f_i)$ and

$$\bar{\mathcal{P}}_e = E\{1 - \prod_i (1 - P_e(n, f_i))\} \approx \sum_i \bar{P}_e(f_i). \quad (10)$$

Therefore, it is meaningful to design our adaptive precoding based on the following optimization criterion:

$$\min_{\Phi_i} \sum_i \bar{P}_e(f_i) \qquad \text{subject to} \qquad \sum_i |\Phi_i|^2 = \mathcal{A}, \quad (11)$$

where the constraint limits the average transmit power per block $E_s \mathcal{A}$. The optimization can be solved using the method of Lagrange multipliers by minimizing

$$\mathcal{F}(|\Phi_i|^2, \lambda) := \sum_i \frac{1}{2}\left(1 + \frac{\bar{\gamma}_i}{m}|\Phi_i|^2\right)^{-m} + \lambda(\sum_i |\Phi_i|^2 - \mathcal{A}). \quad (12)$$

We obtain two solutions for optimal loading:

$$\Phi_i = 0 \ , \qquad |\Phi_i|^2 = \left[\left(\frac{\bar{\gamma}_i}{2\lambda}\right)^{\frac{1}{m+1}} - 1\right]\left(\frac{m}{\bar{\gamma}_i}\right), \quad (13)$$

where $\Phi_i = 0$ is reached whenever the average transmit power is insufficient to guarantee a positive solution for all $|\Phi_i|^2$, and some sub-channels must be discarded. Substituting the second solution in (13) back to the constraint, we obtain

$$\{|\Phi_i|^2\}_{opt.} = \left(\varphi\, \bar{\gamma}_i^{\frac{1}{m+1}} - 1\right)^+ \left(\frac{m}{\bar{\gamma}_i}\right) \quad (14)$$

where

$$\varphi := (2\lambda)^{-\frac{1}{m+1}} = \frac{\left(\frac{\mathcal{A}}{m} + \sum_k \bar{\gamma}_k^{-1}\right)}{\sum_k \bar{\gamma}_k^{-\frac{m}{m+1}}}, \quad (15)$$

where $^+$ indicates that only the positive values have to be retained, while the negative values have to be set to zero. Excluding sub-channels with $\Phi_i = 0$, the resulting minimun average probability of error over the remaining subchannels is [c.f. (9)]

$$\min(\bar{\mathcal{P}}_e) = \frac{1}{2}\sum_i \left(\varphi\, \bar{\gamma}_i^{\frac{1}{m+1}}\right)^{-m}. \quad (16)$$

From (16) we infer that in order to minimize $\bar{\mathcal{P}}_e$, the indices for which $\Phi_i = 0$ must correspond to the frequency bins i that have the smaller $\bar{\gamma}_i$: these sub-channels according to (14) consume more power and, at the same time, have larger average probability of error.

3.1 Optimal Power Loading as $m \to \infty$: deterministic channel

It is particularly interesting to derive the asymptotic value of the optimal $|\Phi_i|^2$ in (14) as $m \to \infty$. This would provide the optimal design when there is no fading, or in our setup, for the case where the channel amplitudes are exactly known at both receiver and transmitter. As $m \to \infty$ the Nagakami-m distribution tends to a delta function centered around $\bar{\gamma}$ and the probability of error approaches the BER performance of DPSK over AWGN; i.e.,

$$\lim_{m\to\infty} \bar{P}_e(f_i) = \lim_{m\to\infty} \frac{1}{2}\left(1+\frac{\bar{\gamma}_i}{m}|\Phi_i|^2\right)^{-m}, \qquad (17)$$

is

$$\lim_{m\to\infty} \bar{P}_e(f_i) = \frac{1}{2}\exp(-\bar{\gamma}_i|\Phi_i|^2). \qquad (18)$$

The limit of $|\Phi_i|^2$ as $m \to \infty$ is [c.f. (13)]:

$$\lim_{m\to\infty} \{|\Phi_i|^2\}_{opt} = \frac{(\log\bar{\gamma}_i - \log(2\lambda))^+}{\gamma_i}. \qquad (19)$$

Substituting (19) in the constraint $\sum_k |\Phi_k|^2 = \mathcal{A}$ we obtain

$$-\log(2\lambda) = \frac{\mathcal{A} + \sum_k \frac{\log\bar{\gamma}_k}{\bar{\gamma}_k}}{\sum_k \bar{\gamma}_k^{-1}}, \qquad (20)$$

which when plugged in (19) provides the optimal power loading for a *deterministic channel*. Notice that direct minimization of the Lagrangian for the deterministic case would have required the solution of non linear equations. Our approach was instrumental to solving the analytical problem and finding the elegant closed form solution in (19). From (19) and (20) we can also obtain the following expression for the minimum average probability of error

$$\begin{aligned}\lim_{m\to\infty} \min(\bar{\mathcal{P}}_e) &= \frac{1}{2}\sum_i \exp(-\bar{\gamma}_i \lim_{m\to\infty}\{|\Phi_i|^2\}_{opt}) \\ &= \sum_i \frac{2\lambda}{\bar{\gamma}_i} \qquad (21) \\ &= \sum_i \frac{1}{\bar{\gamma}_i}\exp\left(-\frac{\mathcal{A}+\sum_k \frac{\log\bar{\gamma}_k}{\bar{\gamma}_k}}{\sum_k \bar{\gamma}_k^{-1}}\right).\end{aligned}$$

4 Numerical test

This numerical example is aimed at illustrating the possible advantages of our optimal loading strategy in terms of BER compared to an OFDM scheme without power loading, i.e. with $|\Phi_i|^2 = \mathcal{A}/I \;\forall\, i$ and with average probability of error:

$$\bar{\mathcal{P}}_e = \sum_i \left(1+\frac{\bar{\gamma}_i\mathcal{A}}{mI}\right)^m. \qquad (22)$$

High Performance Radio LAN (HIPERLAN) standard targets short distance, high speed radio links between computer systems using the 5.2 GHz and the 17.1 GHz frequency bands. We chose $I = 1024$ and generated the parameter $\bar{\gamma}_i$ from (4) - (8) assuming that $\boldsymbol{R}_{hh}(0)$ is exponential, i.e.:

$$\{\boldsymbol{R}_{hh}(0)\}_{l+1,k+1} = \sigma_l\sigma_k\rho^{|l-k|} \qquad (23)$$

with $\rho = 0.5$ and using for σ_l^2, $l = 0,\ldots,L$ a power delay profile named "Channel A" (Fig.1), chosen as a typical indoor multipath scenario for HIPERLAN/2 in [9], operating at 5.2 GHz, with $B = 200\ MHz$ ($T_s = 10\ nsec$). The channel order is $L \approx 19$, because the impulse response samples beyond the 19th are strongly attenuated.

In Fig. 2 we compare $\min(\mathcal{P}_e)/I$ and $\bar{\mathcal{P}}_e/I$ (which are the average BER per block of transmitted bits) [c.f. (16) and (22) respectively] for different values of the fading parameter m, versus the average received SNR in dB per bit for uniform loading $\overline{SNR} := 1/I\sum_i \bar{\gamma}_i(\mathcal{A}/I)$. We observe that the optimal loading provides significant improvement for higher values of the parameter m that correspond to less severe fading. In all cases no subchannels have been discarded by the optimal design.

The improvement is more pronounced in Fig. 3 where we compare the asymptotic $\min(\bar{\mathcal{P}}_e)$ in (21) with the corresponding value of $\bar{\mathcal{P}}_e$ of the uniform loading that can be obtained by substituting $|\Phi|^2 = \mathcal{A}/I$ in (18).

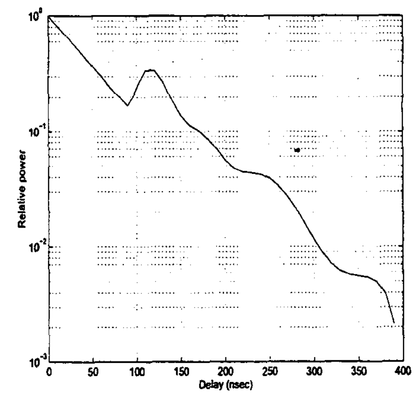

Figure 1. Channel power-delay profile.

5 Conclusion

In this paper we derived the optimal power loading strategy in for a multicarrier transmission operating in correlated Nagakami-m multipath. The design criterion is aimed minimizing the average BER under the constraint of a limited average transmit power, assuming non-coherent detection

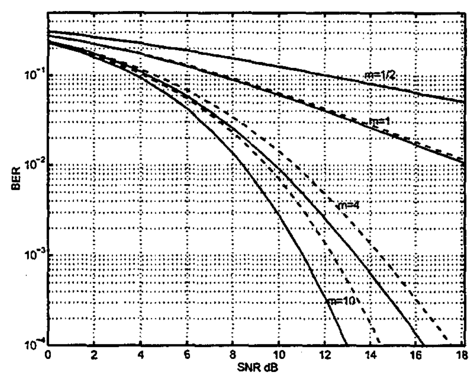

Figure 2. BER vs. average SNR for $m = 1/2, 1, 4, 10$; optimal (—) and uniform (- -) loading.

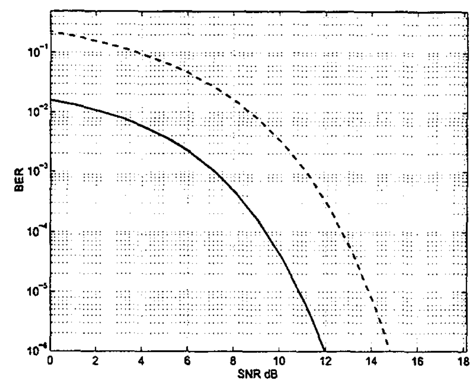

Figure 3. BER vs. average SNR for $m \to \infty$; optimal (—) and uniform (- -) loading.

and DPSK modulation. The derivations provide a simple closed form expression that indicates, as a function of the fading parameters, the portion of the total average transmit power which should be assigned to each subcarrier to achieve the minimum average BER. From the solution corresponding to a finite value of the fading parameter m we derived an asymptotic loading algorithm for the fading parameter $m \to \infty$ which is valid for the deterministic channel case. This solution is particularly intriguing because a direct solution for the deterministic problem is difficult to find since it involves the solution of a nonlinear set of equations. Interestingly, the solution operates approximately a zero forcing pre-equalization but discards the most attenuated subchannels, where such operation would drain most of the available power.

References

[1] M.S. Alouini A.J. Goldsmith, "Adaptive modulation for Nagakami fading channels", *Proc. of IEEE GLOBECOM Conf.*, Phoenix, November 1997.

[2] S. Barbarossa, A. Scaglione, "Optimal precoding for transmissions over linear time-varying channels", *Proc. of IEEE GLOBECOM Conf.*, Rio de Janeiro, Dec. 1999.

[3] W. R. Braun and U. Dersch, "A physical mobile radio channel model," *IEEE Trans. Veh. Technol.*, Vol. 40, pp. 472482, May 1991.

[4] J.K. Cavers, "Variable-rate transmission for Rayleigh fading channels", *IEEE Trans. on Comm.*, Vol. 20, pp. 15-22, February 1972.

[5] J.K. Cavers, "Optimized Use of Diversity Modes in Transmitter Diversity Systems", *IEEE Veh. Technol. Conf.*, Houston, May 1999, pp. 1768-1772.

[6] J. S. Chow, J. C. Tu, J. M. Cioffi, "A discrete multitone transceiver system for HDSL applications," *IEEE Journal on Selected Areas in Commun.*, pp. 895-908, Aug. 1991.

[7] R. Gallager, *Elements of Information Theory*, Section 8, New York: Wiley, 1968.

[8] T. Eyceoz, A. Duel-Hallen, H. Allen, "Deterministic channel modeling and long range prediction of fast fading mobile radio channels", *IEEE Comm. Letters*, Vol. 2, pp. 254-256, September 1998.

[9] ETSI Normalization Commitee, Norme ETSI, doc. 3ERI085B, Sophia-Antipolis, Valbonne, France 1998.

[10] A.J. Goldsmith and S.C. Chua, "Variable-rate variable power M-QAM for fading channels", *IEEE Trans. on Comm.*, Vol. 45, pp. 1218-1230, October 1997.

[11] W.C. Jakes, *Microwave Mobile Communication.* Pitaskaway, NJ; *IEEE Press*, second ed., 1994.

[12] I. Kalet, "The multitone channel", *IEEE Trans. on Commun.*, vol. 37, pp.119-124, Febr. 1989.

[13] M. K. Simon and M. S. Alouini, "A unified approach to the probability of error for noncoherent and differentially coherent modulations over generalized fading channels," *IEEE Trans. Commun.*, vol. COM-46, pp. 1625–1638, December 1998.

[14] A. U. Sheikh, M. Handforth, and M. Abdi, "Indoor mobile radio channel at 946 MHz: Measurements and modeling," in Proc. IEEE Vehicular Technology Conf., Secaucus, NJ, May 1993, pp. 7376.

[15] W.T. Webb and R. Steele, "Variable rate QAM for mobile radio", *IEEE Trans. on Comm.*, Vol. 43, pp. 2223-2230, July 1995.

[16] C. Y. Wong, R. S. Cheng, K. B. Letaief, R. D. Ross,"Multiuser OFDM with adaptive subcarrier, bit and power allocation", *IEEE Trans. on Sel. Areas in Comm.*, Vol. 17, pp. 1747-1757, October 1999.