Cross-Layer Congestion and Contention Control for Wireless Ad Hoc Networks

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***Abstract*— We consider joint congestion and contention control for multihop wireless ad hoc networks, where the goal is to find optimal end-to-end source rates at the transport layer and per-link persistence probabilities at the medium access control (MAC) layer to maximize the aggregate source utility. The primal formulation of this problem is non-convex and non-separable. Under certain conditions, by applying appropriate transformations and introducing new variables, we obtain a decoupled and dual-decomposable convex formulation. For general non-logarithmic concave utilities, we develop a novel dual-based distributed algorithm using the subgradient method. In this algorithm, sources at the transport layer adjust their log rates to maximize their net benefits, while links at the MAC layer select transmission probabilities proportional to their conceived contribution to the system reward. The two layers are connected and coordinated by link prices. Our solutions enjoy the benefits of cross-layer optimization while maintaining the simplicity and modularity of the traditional layered architecture.**

***Index Terms*— Ad-hoc networks, cross-layer design, optimization, random access, wireless networks.**

I. Introduction

AD-HOC wireless networks are usually defined as an autonomous system of nodes connected by wireless links and communicating in a multi-hop fashion. The benefits of ad-hoc networks are many, but the most important one is their ease of deployment without centralized administration or fixed infrastructure, thereby enabling an inexpensive way to achieve the goal of ubiquitous communications. One of the fundamental tasks that an ad hoc network should often perform is congestion control. Congestion control is the mechanism by which the network bandwidth is distributed across multiple end-to-end connections.[1] Its main objective is to limit the delay and buffer overflow caused by network congestion and provide tradeoffs between efficient and fair resource allocation [2].

In wireline networks, congestion control is implemented at the transport layer and is often designed separately from functions of other layers. Since wired links have fixed capacities and are independent, this methodology is well justified and has been studied extensively. In recent years, useful mathematical models and tools based on convex optimization and control theory have been developed, which cast congestion control algorithms as decentralized primal-dual schemes to solve network utility maximization problems; see e.g., [8], [13] and [15].

However, these results do not apply directly to wireless networks because unlike their wireline counterparts, capacities of wireless links are not fixed but dependent on channel conditions as well as the specific medium access control (MAC) protocol used. Since the wireless channel is a shared and thus interference-limited medium, concurrent link transmissions in a local area may fail if interference is very strong. This effect complicates congestion control due to the fact that transport layer flows that do not even share a wireless link in their path can compete if they are located sufficiently close in space. For this reason, a MAC protocol defining rules for orderly or random access to the physical shared medium plays a vital role, and should be optimized jointly with congestion control to ensure efficient utilization and fair sharing of network resources.

In this paper, we consider a contention-based (namely, random access) MAC and study joint end-to-end congestion control and per-link contention control over a multihop wireless ad hoc network. Since our primary goal is to study the interaction between the transport layer and the MAC layer, we do not consider issues induced by channel fading and node mobility. The feasible rate region depends on link transmission probabilities and is often a non-convex and non-separable set. In [16], this problem is formulated as a nonlinear and non-convex aggregate utility maximization problem, which should be optimized over both MAC layer transmission probabilities and transport layer source rates.

We introduce a set of new variables interpreted as fractions of traffic contributed to each link by sources that use this link, and under some sufficient conditions, provide a convex formulation, which now becomes decoupled and dual-decomposable. This novel technique allows generalization from single-hop flows [9] to general multiple-hop end-to-end flows. Using a dual-based approach, we show that the joint problem can be vertically decomposed to the congestion control subproblem at the transport layer and the contention control subproblem at the MAC layer. Our solution enjoys benefits of cross-layer optimization while maintaining the simplicity and modularity of the traditional layered architecture. Also, our solution has proven convergence and optimality and nice economic interpretations, as its wireline counterpart in [13].

Extensions of NUM to ad hoc wireless networks with cross-layer designs have also been studied; see *e.g.*, [4]–

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[1]In this paper, the terms sources, users, flows, and connections are used interchangeably.

[7], [10]–[12], [16]–[18]. For scheduling-based MACs like TDMA, deterministic allocation can be achieved via coordinated collision-free transmissions, and the corresponding joint congestion control and MAC problem has been studied in [4]–[7], [11], [12], [17], [18]. In [6], Chen *et. al.* consider the joint design of congestion control and scheduling-based MAC; however, no efficient distributed link scheduling algorithms have been developed. In [5], Chen *et. al.* study the optimal cross-layer congestion control, routing and scheduling for both time-invariant and time-varying channel models. Although scheduling-based schemes can exhibit high throughput, they often require centralized knowledge and are difficult to implement in a distributed setting. Contention-based MACs are considered in [16], [10], which are closest to our work. In [16], Wang *et. al.* first formulate the aggregate logarithmic utility maximization problem for random access and develop both primal-based and dual-based algorithms to achieve end-to-end proportional fairness. In [10], Lee *et. al.* generalize the dual-based algorithm in [16] to more general utilities.

The rest of the paper is organized as follows. Section II describes the problem formulation and some preliminaries. Section III presents the dual-based solution for general concave utilities. Section IV evaluates our algorithms with simulations, and Section V concludes the paper.

II. Modeling and Preliminaries

We consider a single-channel wireless network modeled as an unidirected graph $G = (N, L)$, where N is the set of nodes and L is the set of links. We assume that the network topology is static and each link $l \in L$ has a fixed positive capacity c_l expressed in bits-per-second (bps). Let $L_{out}(n)$ denote the set of outgoing links from node n, and $L_{in}(n)$ the set of incoming links to node n. Suppose that the network is shared by a set S of sources. Each source is characterized by a utility function $U_s(x_s)$, which is an increasing function of the source rate[2] $x_s \in [0, \infty)$. Let $L(s) \subseteq L$ be the set of links that s uses, and $S(l) \subseteq S$ the set of sources that use link l. Note that $l \in L(s)$ if and only if $s \in S(l)$.

We assume that each node can not transmit or receive simultaneously, and can transmit to or receive from at most one adjacent node at a time. Since each node has a limited transmission range, contention among links for the shared medium is location-dependent. Spatial reuse is possible only when links are sufficiently far apart.[3] As in [9], we define two types of sets, $L^I(n)$ and $N^I(l)$, to capture the location-dependent contention relations, where $L^I(n)$ is the set of links whose receptions are affected adversely by the transmission of node n, excluding outgoing links from node n, and $N^I(l)$ is the set of nodes whose transmission fail the reception of link l, excluding the transmit node of link l. Also note that $l \in L^I(n)$ if and only if $n \in N^I(l)$.

Time is slotted in intervals of equal unit length and the i-th slot refers to the time interval $[i, i+1)$, where $i = 0, 1, \ldots$; i.e., transmission attempts of each node occur at discrete time instances i. We assume a MAC protocol based on random access with probabilistic (re-)transmissions. At the beginning of a slot, each node n transmits data with probability q_n. When it determines to transmit data, it selects one of its outgoing links $l \in L_{out}(n)$ with probability p_l/q_n, where p_l is the link persistence probability; therefore, $\sum_{l \in L_{out}(n)} p_l = q_n, \forall n \in N$.

Our objective is to choose source rates $\mathbf{x} = \{x_s | s \in S\}$ and node persistence probabilities $(\mathbf{p}, \mathbf{q}) = \{(p_l, q_n) | n \in N, l \in L_{out}(n)\}$ so as to maximize the aggregate source utility of all users in the network. This resource allocation problem can be formulated as the following nonlinear program [16]

$$\begin{aligned} \max\ & \sum\nolimits_s U_s(x_s) \\ \text{s.t.}\ & \sum\nolimits_{s \in S(l)} x_s \le \eta_l := c_l p_l \prod\nolimits_{k \in N^I(l)} (1 - q_k), \forall l \\ & \sum\nolimits_{l \in L_{out}(n)} p_l = q_n \le 1, \forall n \\ & 0 \le p_l \le 1, \forall l, \\ & x_s \ge 0, \forall s, \end{aligned} \tag{1}$$

where $\boldsymbol{\eta} = \{\eta_l | l \in L\}$ are link throughputs given $\mathbf{p}$ and $\mathbf{q}$, since the term $p_l \prod_{k \in N^I(l)} (1 - q_k)$ is the probability that a packet is transmitted over link l and successfully received by its receiver.

The problem formulation in (1) entails congestion control at the network layer (finding $\mathbf{x}$), and contention control at the MAC layer (finding $\mathbf{p}$ and $\mathbf{q}$). The two layers are coupled through the first constraint in (1), which asserts that for each link l, the aggregate source rate $\sum_{s \in S(l)} x_s$ does not exceed the link throughput. The transport layer source rates and the MAC layer transmission probabilities should be jointly optimized to maximize the aggregate source utility. Due to the first constraint, (1) is in general a non-convex and non-separable problem, which is difficult to optimize over both $\mathbf{x}$ and $\mathbf{p}, \mathbf{q}$ in a distributed way directly. Under certain conditions, however, it can be transformed to a problem which is both convex and dual-decomposable, as we will discuss in the next section.

III. General Concave Utilities

In this section, we first reformulate problem (1) by introducing auxiliary variables, and then derive a dual-based distributed algorithm for solving it.

A. Reformulation and Dual Problem

Due to the first constraint, (1) is in general a non-convex and non-separable optimization problem. Under certain conditions, however, it can be transformed into a convex one by taking the logarithm on both sides of the first constraint and replacing the rate variables by their logarithmic counterparts, i.e., $z_s = \log(x_s)$ [16], [9]. This yields a new constraint

$$\begin{aligned} \log\Big(\sum\nolimits_{s \in S(l)} e^{z_s}\Big) - \log(c_l) \\ - \log p_l - \sum\nolimits_{k \in N^I(l)} \log(1 - q_k) \le 0, \forall l. \end{aligned} \tag{2}$$

However, the difficulty arises due to the non-separability of the term $\log(\sum_{s \in S(l)} e^{z_s})$, although it is a convex function. To overcome this challenge, we introduce a set of new variables $\boldsymbol{\alpha} = \{\alpha_{ls} | 0 \le \alpha_{ls} \le 1, \sum_{s \in S(l)} \alpha_{ls} = 1, l \in L, s \in S(l)\}$,

[2]The results of this paper can be easily extended to the case where the source rate x_s is limited to an interval $[x_s^{\min}, x_s^{\max}]$.

[3]We do not consider capture effect in this paper.

where each α_{ls} can be interpreted as the fraction of the overall traffic on link l contributed by source $s \in S(l)$. We observe that the first constraint in (1) for each link l is equivalent to a number[4] $|S(l)|$ of separable constraints:

$$x_s \leq \alpha_{ls} c_l p_l \prod\nolimits_{k \in N^I(l)} (1 - q_k), \forall s \in S(l). \tag{3}$$

Similarly, we take $\log(\cdot)$ on both sides of (3) and obtain the following program:

$$\begin{aligned} \max\ & \sum\nolimits_s U'_s(z_s) \\ \text{s.t.}\ & z_s - \log \alpha_{ls} - \log c_l - \log p_l \\ & - \sum\nolimits_{k \in N^I(l)} \log(1 - q_k) \leq 0, \forall l, \forall s \in S(l) \\ & 0 \leq \alpha_{ls} \leq 1, \sum\nolimits_{s \in S(l)} \alpha_{ls} = 1, \forall l, \forall s \in S(l) \\ & \sum\nolimits_{l \in L_{out}(n)} p_l = q_n \leq 1, \forall n \\ & 0 \leq p_l \leq 1, \forall l, \end{aligned} \tag{4}$$

where $U'_s(z_s) := U_s(e^{z_s})$ is the transformed utility, which is a function of the log-rate z_s.

In (4), the constraint set is convex since $-\log(\cdot)$ functions are convex. We still need to check the concavity of $U'(z_s)$. Upon defining

$$g_s(x_s) = \frac{d^2 U_s(x_s)}{dx_s^2} x_s + \frac{U_s(x_s)}{dx_s}, \tag{5}$$

it is easy to prove the following Lemma [9].

Lemma 1: If $g_s(x_s) < 0$, then $U'_s(z_s)$ is a strictly concave function of z_s.

Given that the condition of Lemma 1 is satisfied, problem (4) is indeed a convex problem, and all log rates are decoupled, enabling the dual decomposition. To proceed, let us define the Lagrangian function

$$\begin{aligned} L(\boldsymbol{\lambda}, \mathbf{z}, \mathbf{p}, \mathbf{q}, \boldsymbol{\alpha}) &= \sum_{s \in S} U'_s(z_s) \\ &- \sum_{l \in L} \sum_{s \in S(l)} \lambda_{ls} \left(z_s - \log\left[\alpha_{ls} c_l p_l \prod_{k \in N^I(l)} (1 - q_k)\right] \right) \\ &= \sum_{s \in S} \left(U'_s(z_s) - \lambda^s z_s \right) + \sum_{l \in L} \lambda^l \log c_l \\ &+ \sum_n \sum_{l \in L_{out}(n)} \sum_{s \in S(l)} \lambda_{ls} \log \alpha_{ls} \\ &+ \sum_n \left(\sum_{l \in L_{out}(n)} \lambda^l \log p_l + \sum_{l \in L^I(n)} \lambda^l \log(1 - q_n) \right), \end{aligned} \tag{6}$$

where $\lambda^s := \sum_{l \in L(s)} \lambda_{ls}$, $\lambda^l := \sum_{s \in S(l)} \lambda_{ls}$, $\boldsymbol{\lambda} := \{\lambda_{ls} | s \in S, l \in L(s)\}$, and $\mathbf{z} := \{z_s | s \in S\}$.

The Lagrangian dual function is

$$D(\boldsymbol{\lambda}) = \max_{\substack{0 \leq \alpha_{ls} \leq 1,\ \sum_{s \in S(l)} \alpha_{ls} = 1, \forall l, \forall s \in S(l) \\ \sum_{l \in L_{out}(n)} p_l = q_n \leq 1, \forall n \\ 0 \leq p_l \leq 1, \forall l}} L(\boldsymbol{\lambda}, \mathbf{z}, \mathbf{p}, \mathbf{q}, \boldsymbol{\alpha}), \tag{7}$$

[4]The notation $|\cdot|$ represents the cardinality of a set.

and the dual problem to (4) is

$$\mathbf{D} : \min_{\boldsymbol{\lambda} \geq 0} D(\boldsymbol{\lambda}). \tag{8}$$

It is straightforward to prove the following result using standard convex optimization tools (e.g., see [3]).

Proposition 1: The minimal solution to (8) is equal to the maximal solution to (4), i.e., there is no duality gap.

We notice that the only complexity involved with the constraint reformulation Eq. (3) arises from the fact that each link has to store an individual price information per flow going through it, which incurs only a linearly increasing memory usage. However, the communication complexity remains identical to that with a common link price, since the source only requires the sum-price information.

B. Distributed Algorithms

The maximization in (7) for a given $\boldsymbol{\lambda}$ can be decomposed into three subproblems: one at each source and the other two at each node. The source subproblem is

$$\max_{z_s} \big(U'_s(z_s) - \lambda^s z_s \big), \forall s \in S. \tag{9}$$

If we interpret the Lagrange multiplier λ_{ls} as the price per unit of log bandwidth charged by link l to source s, then the source strategy is to maximize its net benefit $U'_s(z_s) - \lambda^s z_s$, since $\lambda^s z_s$ is just the sum bandwidth cost charged by all links on its path if source s transmits at log rate z_s. Since $U'_s(z_s)$ is strictly concave over z_s, a unique maximizer exists.

The other two subproblems at each node n for every outgoing link $l \in L_{out}(n)$ are, respectively

$$\max_{0 \leq \alpha_{ls} \leq 1, \sum_{s \in S(l)} \alpha_{ls} = 1} \sum_{s \in S(l)} \lambda_{ls} \log \alpha_{ls}, \forall l \in L_{out}(n), \tag{10}$$

and

$$\max_{\substack{\sum_{l \in L_{out}(n)} p_l = q_n \leq 1 \\ 0 \leq p_l \leq 1}} \sum_{l \in L_{out}(n)} \lambda^l \log p_l + \sum_{l \in L^I(n)} \lambda^l \log(1 - q_n). \tag{11}$$

Both (10) and (11) are convex programs which can be solved in closed form. Notice also that subproblem (11) has form identical to [9, eq. (21)] for which we can derive a similar solution.

Proposition 2: Given $\boldsymbol{\lambda}$, the $\boldsymbol{\alpha}(\boldsymbol{\lambda})$ solving problem (10) is (for node n and link $l \in L_{out}(n)$)

$$\alpha_{ls}(\boldsymbol{\lambda}) = \begin{cases} \frac{\lambda_{ls}}{\sum_{s \in S(l)} \lambda_{ls}}, & \text{if } \sum_{s \in S(l)} \lambda_{ls} \neq 0 \\ \frac{1}{|S(l)|}, & \text{if } \sum_{s \in S(l)} \lambda_{ls} = 0 \end{cases}, \tag{12}$$

and the $\mathbf{p}(\boldsymbol{\lambda}), \mathbf{q}(\boldsymbol{\lambda})$ solving problem (11) are

$$p_l(\boldsymbol{\lambda}) = \begin{cases} \frac{\lambda^l}{\sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}}, & \text{if } k(n) \neq 0 \\ \frac{1}{|L_{out}(n)| + |L^I(n)|}, & \text{if } k(n) = 0 \end{cases}, \tag{13}$$

and

$$q_n(\boldsymbol{\lambda}) = \begin{cases} \frac{\sum_{l' \in L_{out}(n)} \lambda^l}{\sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}}, & \text{if } k(n) \neq 0 \\ \frac{|L_{out}(n)|}{|L_{out}(n)| + |L^I(n)|}, & \text{if } k(n) = 0 \end{cases}, \tag{14}$$

where $k(n) := \sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}$.

Proof: Problem (10) can be easily solved using the Lagrange multiplier method. The solutions (13) and (14) can be proved from the first-order condition, and similar results are given in [9]. Therefore, the proof is omitted here. ■

Solutions in Proposition 2 have nice economic interpretations. As mentioned before, α_{ls} represents the fraction of the overall traffic on link l contributed by source $s \in S(l)$. The solution (12) asserts that α_{ls} is equal to the normalized price that user s pays for link l; the higher the price that the user s is willing to pay at, the more traffic the link allows to pass through. For the solution in (13), let us consider the case when $k(n) \neq 0$. If a packet is successfully transmitted over link $l \in L_{out}(n)$, this link contributes a reward λ^l to the system in a local area. However, all other links $l' \in L_{out}(n) \bigcup L^I(n), l' \neq l$ must remain silent during the transmission of link l. If those links can transmit simultaneously without collisions, then the total reward would be $\sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}$. But this cannot happen due to interference. For this reason, the true fraction of the reward in a local area contributed by link l should be λ^l normalized by the total $\sum_{l' \in L_{out}(n)} \lambda^{l'} + \sum_{l' \in L^I(n)} \lambda^{l'}$, which is equal to p_l as asserted by Proposition 2. In other words, the attempt probability of a link is equal to the fraction of the reward it can generate locally with a successful transmission.

Now we are ready to solve the dual problem (8) using a projected subgradient method [1]. At each node n for $\forall l \in L_{out}(n)$ and $\forall s \in S(l)$, the outgoing link prices for sources involved are adjusted as follows

$$\lambda_{ls}(t+1) = \left[\lambda_{ls}(t) - \gamma(t)\frac{\partial D}{\partial \lambda_{ls}}(\boldsymbol{\lambda}(t))\right]^+, \qquad (15)$$

where $[a]^+ := \max\{0, a\}$ and $\gamma(t) > 0$ is a step size. According to Danskin's theorem [1, p. 737], we have

$$\frac{\partial D}{\partial \lambda_{ls}} = \log c_l + \log \alpha_{ls} + \log p_l + \sum_{k \in N^I(l)} \log(1 - q_k) - z_s. \qquad (16)$$

Substituting (16) into (15), we obtain the following adjustment rule for link $l \in L_{out}(n)$ at each node n

$$\lambda_{ls}(t+1) = \Bigg[\lambda_{ls}(t) + \gamma(t)\Bigg(z_s(\boldsymbol{\lambda}(t)) - \log c_l - \log \alpha_{ls}(\boldsymbol{\lambda}(t)) - \log p_l(\boldsymbol{\lambda}(t)) - \sum_{k \in N^I(l)} \log(1 - q_k(\boldsymbol{\lambda}(t)))\Bigg)\Bigg]^+. \qquad (17)$$

This link price adjustment rule follows the law of supply and demand in a fashion similar to that observed by [13] for congestion control in wireline networks. One apparent difference is that all sources $s \in S(l)$ sharing the same link l are charged with an individual price, λ_{ls}, which may be different from others, while in wireline networks all sources using the same link share a common link price.

In the following, we summarize the source and link algorithms.

Algorithm at Source s:

1) Receives from the network the sum $\lambda^s(t) = \sum_{l \in L(s)} \lambda_{ls}(t)$ of link prices in s's path;

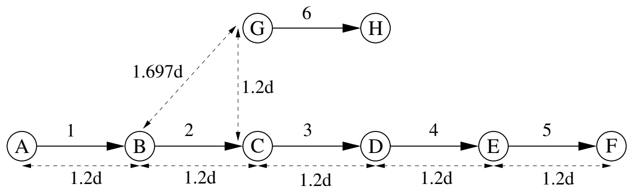


Fig. 1. Network topology.

2) Computes the new log rate using

$$z_s(t+1) = \arg\max_{z_s}(U'_s(z_s) - \lambda^s(t) z_s); \qquad (18)$$

3) Communicates the new log rate $z_s(t+1)$ to all links $l \in L(s)$ on s's path.

Algorithm at Node n:

1) Receives log rates $z_s(t)$ from all sources $s \in \cup_{l \in L_{out}(n)} S(l)$ that go through the outgoing links of node n;
2) Receives prices $\lambda^{l'}(t), \forall l' \in L^I(n)$ from the neighboring nodes n', where $l' \in L_{out}(n')$;
3) Calculates $\alpha_{ls}(t), p_l(t), q_n(t), \forall l \in L_{out}(n), \forall s \in L(s)$, according to Proposition 2;
4) Computes new prices using Eq. (17);
5) For each outgoing link $l \in L_{out}(n)$, communicates new prices $\lambda_{ls}(t+1)$ to all sources $s \in S(l)$ that use link l and $\lambda^l(t+1)$ to all nodes in $N^I(l)$.

For the convergence and optimality of this distributed algorithm, we have the following result.

Theorem 1: If the following condition is satisfied at the optimal dual solution $\boldsymbol{\lambda}^*$

$$k^*(n) = \sum_{l \in L_{out}(n)} \sum_{s \in S(l)} \lambda^*_{ls} + \sum_{l \in L^I(n)} \sum_{s \in S(l)} \lambda^*_{ls} \neq 0, \forall n \in N \qquad (19)$$

and $\boldsymbol{\lambda}^*$ denotes a minimizer of the dual problem (8), then step sizes $\{\gamma(t)\}_{t=0}^{\infty}$ exist to guarantee $\lim_{t \to \infty} \boldsymbol{\lambda}(t) = \boldsymbol{\lambda}^*$. In addition, at $\boldsymbol{\lambda}^*$, solutions to (9), (10) and (11) denoted as $\mathbf{z}^*, \mathbf{p}^*, \mathbf{q}^*$ optimize problem (4).

Proof: The proof can be deduced easily from Theorem 2.2 of [14, p.25] and is similar to that of Theorem 4 in [9]. And for this reason, it is omitted. ■

IV. Numerical Results

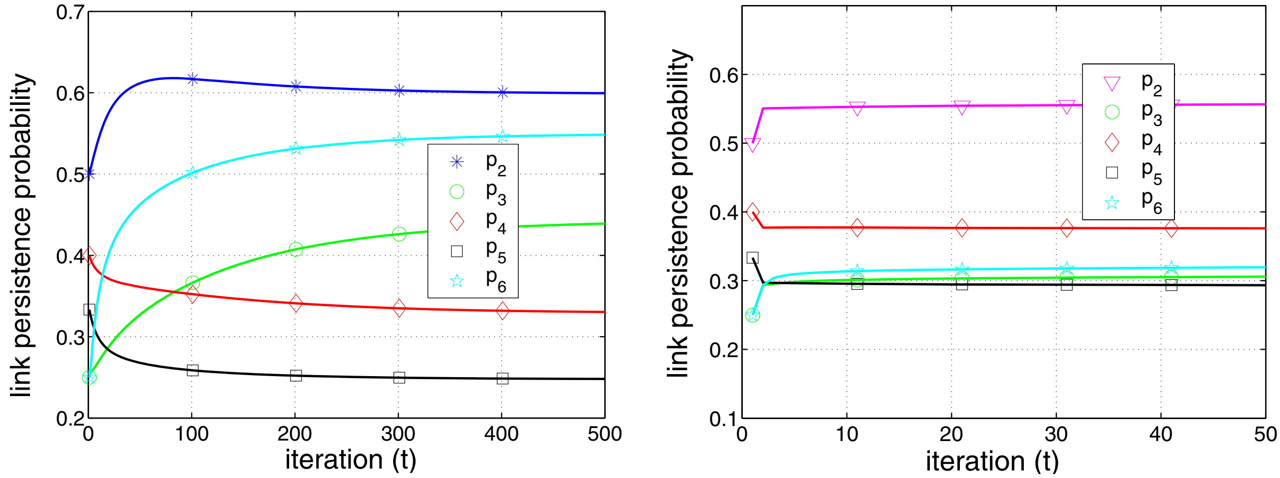
In this section, we provide numerical examples for a network topology shown in Fig. 1. We assume that all links have identical capacity $c_l = 1$ and that only if the distance between the transmitter of one link and the receiver of the other link is less than $2d$, transmission of the first link will cause interference strong enough to influence reception of the second link. The two sets, $N^I(l)$ and $L^I(n)$, can thus be obtained from the given network. In all simulations, we use constant step sizes.

Suppose that there are three end-to-end flows $f1 : G \to H$, $f2 : A \to B \to C \to D \to E \to F$, and $f3 : D \to E \to F$. We test utilities $U_s(x_s) = (1-\theta)^{-1} x_s^{1-\theta}$ with $\theta = 2, 10$, both of which lead to strictly concave transformed utilities $U'_s(z_s) = (1-\theta)^{-1} \exp[z_s(1-\theta)]$. We set all initial link prices to 1. Step size γ should be chosen appropriately to avoid

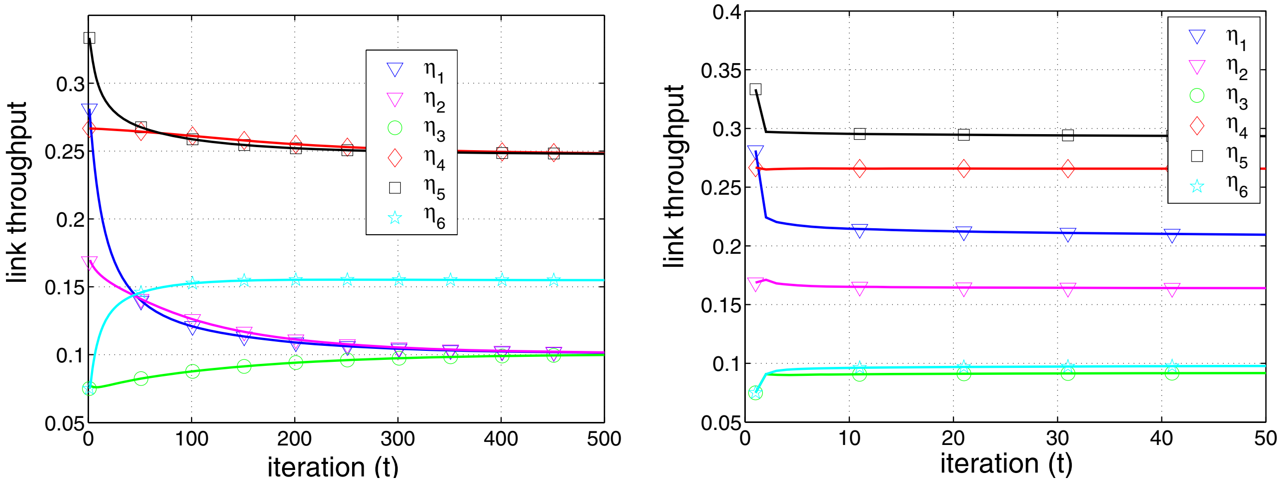
TABLE I

SOLUTIONS FOR $\theta = 1.1, 2$ AND 10 AND THE NETWORK IN FIG. 1(A)

θ	γ	itr.	p_1	p_2	p_3	p_4	p_5	p_6	x_1	x_2	x_3
2	0.05	500	1.0000	0.5994	0.4391	0.3304	0.2480	0.5484	0.1549	0.0998	0.1472
10	1.0	50	1.0000	0.5565	0.3058	0.3761	0.2934	0.3194	0.0978	0.0918	0.1354



(a) Link Persistence Probabilities for $\theta = 2$ and 10 (from left to right)



(b) Link throughputs for $\theta = 2$ and 10 (from left to right)

Fig. 2. The evolution of link persistence probabilities and link throughputs.

big oscillations as well as ensure fast convergence, which in general are two conflicting objectives. From experiments, we found that the smaller the value of θ is, the smaller the step size should be. Since the step size determines rate of convergence, we expect that the algorithm converges faster for larger θ, which is also confirmed by simulations. Fig. 2 shows the evolution of link persistence probabilities and link throughputs with step sizes 0.05 and 1.0 for $\theta = 2$ and 10, respectively.

V. CONCLUSIONS

We studied the joint design of congestion and contention control for wireless ad hoc networks. While the original problem is non-convex and coupled, we provided a decoupled and dual-decomposable convex formulation, based on which subgradient-based cross-layer algorithms were derived to solve the dual problem in a distributed fashion for non-logarithmic utilities. Our algorithms decompose vertically in two layers, the network layer where sources adjust their end-to-end rates, and the MAC layer where links update persistence probabilities. These two layers interact and are coordinated through link prices. In the future, we plan to study the joint congestion control and contention control problem in a hybrid wireline and wireless network.[5]

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