Multi-Tier Cooperative Broadcasting with Hierarchical Modulations

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***Abstract*— We consider broadcasting to multiple destinations with uneven quality receivers. Based on their quality of reception, we group destinations in *tiers* and transmit using hierarchical modulations. These modulations are known to offer a practical means of achieving variable error protection of the broadcasted information to receivers of variable quality. After the initial broadcasting step, tiers successively re-broadcast part of the information they received from tiers of higher-quality to tiers with lower reception capabilities. This multi-tier cooperative broadcasting strategy can accommodate variable rate and error performance for different tiers but requires complex demodulation steps. To cope with this complexity in demodulation, we derive simplified per-tier detection schemes with performance close to maximum-likelihood and ability to collect the diversity provided as symbols propagate through diversified channels across successive broadcastings. Error performance is analyzed and compared to (non)-cooperative broadcasting strategies. Simulations corroborate our theoretical findings.**

***Index Terms*— User cooperation, broadcast channel, diversity gain, relaying protocol, hierarchical modulation.**

I. Introduction

IN standard broadcasting scenarios, information is broadcasted by a single source and decoded by different receivers independently [4]. With the proliferation of wireless terminals in sparse broadcast networks, there has been a growing interest towards modalities where besides decoding their own information, certain receiving ends are willing to cooperate with other destinations. With these receivers acting as relays, well-appreciated benefits emerge in terms of resilience against shadowing, enhanced coverage, diversity and rates [14], [7], [16], [17], [12]. For these reasons, cooperative communications have attracted research attention recently from several perspectives. From an information-theoretic point of view, capacity improvements offered by user cooperation and relaying in the degraded broadcast channel (BC) motivate the idea of exploiting user cooperation in broadcasting scenarios [5], [9]. The advantages of cooperative broadcasting (CBC) from a practical perspective, have been also demonstrated in the context of network lifetime maximization and coverage improvements [11], [19].

Pairing well with intended receivers of hierarchically unequal quality, hierarchical constellations can achieve variable error protection of the broadcasted information [15], [4], [13]. These modulations map information bits according to their importance onto non-uniformly spaced constellation points. Non-uniformity allows important (a.k.a. basic) bits to be decoded with fewer errors than less important (a.k.a. enhancement) bits. As such, hierarchical modulations have found applications in broadcasting networks with destinations having unequal quality requirements. These include commercial for e.g., multimedia delivery as well as tactical broadcasting networks [1], [10]. Recently, the related notion of superposition coding has been analyzed in a half-duplex cooperative multiple-access channel [2], where the diversity-multiplexing tradeoff has been investigated using the amplify-and-forward (AF) protocol. A similar coded scheme has been considered also in [8] using decode-and-forward (DF) relaying and superposition modulation, with the goal of effecting cooperative *transmit* diversity.

Motivated by these considerations, the present paper deals with cooperative broadcasting based on hierarchical (superposition) modulation but with the focus placed on cooperative *receive* diversity. Consider a single source broadcasting information bits mapped to a hierarchical modulation, and suppose that destinations are classified into *tiers* according to their specific reception capabilities.[1] The lower the tier lies, the less (i.e., more basic) information it can only receive. This is because lower tiers have lower reception quality and thus can only acquire the most critical part of the received information. A tier closer to the source is able to reliably detect most of the transmitted bits, re-encode part of the bits (the basic information) using a reduced size hierarchical constellation, and act as a relay broadcasting it to other tiers. A tier located farther away from the source with poorer reception conditions can combine the symbol received from the original broadcasting with the symbols received from cooperating tiers. This strategy will be

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[1] Although superimposed transmissions are used here too, we prefer the term "hierarchical modulation" because it matches well with the multi-tier setting and the uncoded transmissions considered in the present paper.

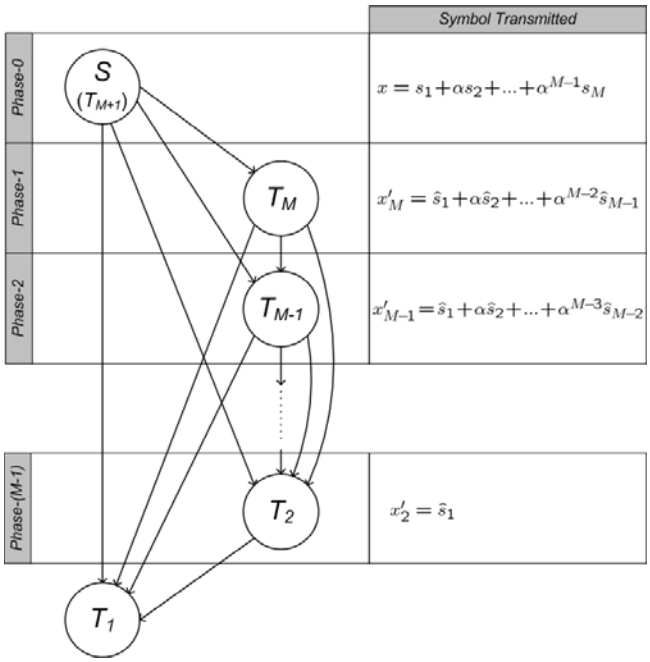

Fig. 1. An M-tier CBC setup.

shown to offer a convenient means of increasing reliability of the basic information successively broadcasted through the network. We will have cooperating tiers implement modified versions of the DF protocol adapted to multi-tier CBC, which we will henceforth abbreviate as DFb. The main difference with the classical DF is that relative to received symbols, constellation points of re-encoded symbols are carrying less (only *basic*) information. When information bits are coded, successive interference cancelation (SIC) or joint decoding can improve error performance of the multi-tier CBC system. But since at medium to high SNR, error performance mainly depends on the spatial diversity order, which is the same for coded and uncoded transmissions, this paper focuses the uncoded case and aims at establishing the benefit of simplified detection with hierarchical modulation in CBC with one node per tier. If there is more than one node per tier, each node in a tier first estimates the information bits coming from nodes in higher tiers independently, and then forwards only estimates of the basic bits to the nodes in lower tiers using time division multiple access (TDMA).

To match variable QoS requirements, we will pursue detectors with affordable complexity which efficiently combine heterogeneous constellations broadcasted from the source and tiers. To this end, we advocate combiners with adaptive-weights that account for the unequal error probabilities of the received symbols; see also [21] where similar combiners are proposed but not for the CBC setup with hierarchical modulations. If properly designed, such detectors can collect diversity order up to the number of diversified replicas of the broadcasted signal.

The rest of this paper is organized as follows: in Section II, we describe the transmission model; we analyze error performance in Section III; and simulate various broadcasting options in Section IV; finally, we summarize our conclusions in Section V.

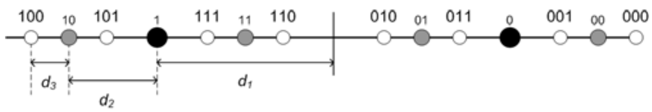

Fig. 2. Hierarchical 2/4/8-PAM constellation.

Notation: Lower case bold letters will be used to denote column vectors; $(\cdot)^*$ will denote conjugation; $(\cdot)^T$ transpose; $(\cdot)^{\mathcal{H}}$ Hermitian transpose; $\mathcal{CN}(0, \sigma^2)$ the circular symmetric complex Gaussian distribution with zero mean and variance σ^2; $Re\{\}$ the real part of a complex number; $\overline{\gamma} = E\{\gamma\}$ the mean of the random variable γ; and $\hat{x}$ will denote the estimate of x.

II. Hierarchical transmissions

With reference to Fig. 1, we consider a source terminal S, and M tiers $\{T_m\}_{m=1}^M$. We map information-bearing bits at S to a hierarchical constellation. For simplicity, we will confine ourselves to real-valued hierarchical Pulse-Amplitude-Modulation (PAM) constellations; such constellations can also be seen as one-dimensional counterparts of complex-valued hierarchical Quadrature-Amplitude-Modulation (QAM) constellations. Let us consider a block of M bits $i_1, \ldots, i_M$. As depicted in Fig. 2, bit mappings in hierarchical constellations assign the highest priority bit (bit i_1) to the most significant bit (MSB) position. The bit with second highest priority (bit i_2) is assigned the second most significant position, and so on, until the least priority bit (bit i_M) is assigned the least significant bit (LSB) position. Such construction can be viewed as a nested $\{2/4/\cdots/2^M\}$-PAM constellation [20]. As an example, for $M = 3$, a nested $\{2/4/8\}$-PAM constellation is depicted in Fig. 2.

A hierarchically modulated symbol x at S, can be generically written as

$$x = \pm d_1 \pm d_2 \pm \cdots \pm d_M, \tag{1}$$

where $\pm d_m$ is mapped from bit i_m, and represents the distance of the constellation point (from the origin/center) at this level of the hierarchy [c.f. Fig. 2]. In order to reduce the amount of parameters that model a hierarchical constellation, we can define a parameter $\alpha \in [0, 1/2]$ and constrain x to have the following structure:

$$x = s_1 + \alpha s_2 + \cdots + \alpha^{M-1} s_M, \tag{2}$$

where $s_m = \pm d_1$ is the symbol corresponding to bit i_m with weight α^{m-1}. Notice in general, the power allocation in (2) may be suboptimal for a general M-tier network. But since we are interested in achieving the available spatial diversity - a task not dependent on whether power allocation is optimal - we adopt this specific power allocation in order to reduce the number of parameters and simplify the ensuing diversity analysis. A better power allocation can improve error performance by increasing the coding gain, but as far as maximizing diversity, optimization of power allocation is not necessary.

Clearly, what motivates hierarchical modulation and reception is a setup entailing e.g., high-definition television and conventional (i.e., not high-definition) television receivers. As symbols $s_1, \ldots, s_M$ are ordered according to their significance (s_1 is most significant), $x_m := s_1 + \alpha s_2 + \cdots + \alpha^{m-1} s_m$ bears the most significant m symbols (or bits) in the source transmission. Tier T_m focuses on the reception of x_m corresponding to the most significant m bits.

A. Broadcasting phases

Similar to [7], [14], [21], we adopt a transmission protocol based on successive broadcasting phases implemented in a TDM fashion. In *phase*-0, S transmits x and the received symbol at any tier, e.g., the m^{th} one, can be written as

$$y_{S,m} = h_{S,m} x + n_{S,m}, \tag{3}$$

where x is given by (2), $h_{S,m}$ is the flat fading channel coefficient between S and T_m, modeled as $\mathcal{CN}(0, E\{|h_{S,m}|^2\})$; term $n_{S,m}$ denotes additive white Gaussian noise (AWGN), distributed according to $\mathcal{CN}(0, N_0)$, and N_0 is the noise energy, which here is supposed to be the same for all $m = 1, \ldots, M$.

The first tier T_M, detects source bits $i_1, \ldots, i_M$ and constructs a new hierarchically modulated symbol x'_M given by [c.f. Fig. 1]

$$x'_M = \hat{s}_1 + \alpha \hat{s}_2 + \cdots + \alpha^{M-2} \hat{s}_{M-1}. \tag{4}$$

Notice that relative to x in (2), x'_M in (4) contains one bit less, because different tiers can decode (and hence aim at) bits of different significance. In the ensuing *phase*-1, x'_M is broadcasted from T_M to the remaining tiers $T_{M-1}, T_{M-2}, \ldots, T_1$. The next tier T_{M-1}, receives information pertaining to x twice; once from S during *phase*-0, in the form $y_{S,M-1} = h_{S,M-1} x + n_{S,M-1}$; and second time from T_M during *phase*-1, in the form $y_{M,M-1} = h_{M,M-1} x'_M + n_{M,M-1}$. From $y_{S,M-1}$ and $y_{M,M-1}$, T_{M-1} decodes bits $i_1, \ldots, i_{M-1}$ and constructs a new hierarchically modulated symbol, $x'_{M-1} = \hat{s}_1 + \alpha \hat{s}_2 + \cdots + \alpha^{M-3} \hat{s}_{M-2}$ which is broadcasted[2] in *phase*-2.

We can clearly generalize this successive broadcasting process to more than two tiers. To this end, let us define $T_{M+1} := S$ for uniformity in notation. Tier T_m receives a set of $M - m + 1$ symbols $\{x'_n\}_{n=m+1}^{M+1}$ in the presence of AWGN, each corresponding to $M - m + 1$ transmission phases from $T_{M+1}, T_M, \ldots, T_{m+1}$. We can concatenate all these symbols received by T_m to form the $(M - m + 1) \times 1$ vector

$$\mathbf{x}_m = \begin{bmatrix} s_1 + \alpha s_2 + \cdots + \alpha^{M-1} s_M \\ \hat{s}_1 + \alpha \hat{s}_2 + \cdots + \alpha^{M-2} \hat{s}_{M-1} \\ \vdots \\ \hat{s}_1 + \alpha \hat{s}_2 + \cdots + \alpha^{m-1} \hat{s}_m \end{bmatrix}. \tag{5}$$

The received symbols can then be correspondingly collected in an $(M - m + 1) \times 1$ vector $\mathbf{y}_m$, which can be written as

$$\mathbf{y}_m = \text{diag}(\mathbf{h}_m) \mathbf{x}_m + \mathbf{n}_m, \tag{6}$$

where $\mathbf{h}_m := [h_{M+1,m}, h_{M,m}, \ldots, h_{m+1,m}]^T$ and $h_{n,m}$ is the fading coefficient between T_n and T_m, for $n = m + 1, \ldots, M + 1$, distributed according to $\mathcal{CN}(0, E\{|h_{n,m}|^2\})$; and $\mathbf{n}_m := [n_{M+1,m}, n_{M,m}, \ldots, n_{m+1,m}]^T$ collects all AWGN terms at T_m, with each entry adhering to $\mathcal{CN}(0, N_0)$.

[2]Notice that in general, the $\hat{s}$'s in x'_{M-1} are different from those in (4). But to reduce the number of parameters, we will not differentiate $\hat{s}$'s in different tiers.

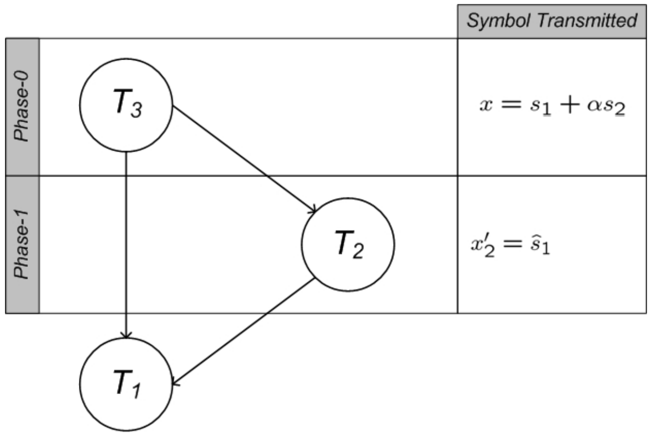

Fig. 3. A 2-tier CBC setup.

From $\mathbf{y}_m$, tier T_m detects bits $i_1, \ldots, i_m$ and broadcasts a new constellation point

$$x'_m = \hat{s}_1 + \alpha \hat{s}_2 + \cdots + \alpha^{m-2} \hat{s}_{m-1} \tag{7}$$

to $T_{m-1}, T_{m-2}, \ldots, T_1$ in *phase*-$(M - m + 1)$. With this simple broadcasting protocol, we guarantee that information is adaptively broadcasted in accordance with the different detection capabilities of each tier.

Vector $\mathbf{y}_m$ in (6) contains $M - m + 1$ renditions of the broadcasted signal x which undergo uncorrelated fading realizations; thus, the maximum diversity that can be collected by tier T_m is of order $M - m + 1$. Challenged by this benchmark, we will next propose optimum and simplified cooperative detection strategies at T_m which can collect this order of diversity for all $m = 1, \ldots, M$.

B. ML detection

Recall that tier T_m, only aims at detecting symbol $x_m := s_1 + \alpha s_2 + \cdots + \alpha^{m-1} s_m$. Thus, the maximum-likelihood (ML) optimal coherent detector at any tier T_m has the general form:

$$\hat{x}_m^{ML} = \arg \max_{x_m \in \mathcal{A}_{x_m}} \{ f_{\mathbf{y}_m|x_m}(\mathbf{y}_m|x_m) \}, \tag{8}$$

where $f_{\mathbf{y}_m|x_m}(\mathbf{y}_m|x_m)$ is the probability density function (pdf) of the received vector $\mathbf{y}_m$ in (6), conditioned on the transmitted x_m, and $|\mathcal{A}_{x_m}| = 2^m$ denotes the cardinality of the hierarchical PAM constellation to be detected at T_m. When evaluating the pdf in (8), one has to account for the bit-error-probability (BEP) of all possible previously-estimated symbols sent from tiers $T_{M+1}, \ldots, T_{m+1}$.

As recognized by [17], when using non-hierarchical constellations, the likelihood and its maximization in cooperative links using DF relay strategies can be quite complex, and available expressions are tractable only for BPSK constellations. Our multi-tier cooperative scenario is further complicated by the fact that symbols arriving from different tiers are mapped to different constellations. As an illustrative example, let us consider the ML detector at T_1 when $M = 2$. Here, $x_1 = s_1$,

s_1 is carrying bit i_1 to be detected by both tiers, and s_2 is carrying bit i_2 to be detected only by tier T_2. Tier T_1, on the other hand, receives two signals: $y_{3,1}$ from T_3 and $y_{2,1}$ from T_2; see also Fig. 3. Received symbol $y_{3,1}$ contains the transmitted $s_1 + \alpha s_2$ and the likelihood function conditioned on s_1 becomes

$$\begin{aligned} & f_{y_{3,1}|s_1}(y_{3,1}|s_1) \\ = \; & \frac{1}{2\sqrt{2\pi N_0}} \exp\left[-\frac{|y_{3,1} - h_{3,1}(s_1 + \alpha|s_2|)|^2}{2N_0}\right] \\ + \; & \frac{1}{2\sqrt{2\pi N_0}} \exp\left[-\frac{|y_{3,1} - h_{3,1}(s_1 - \alpha|s_2|)|^2}{2N_0}\right]. \quad (9) \end{aligned}$$

Similarly, the transmitted symbol from tier T_2 is $\hat{s}_1$ and the corresponding likelihood function is

$$\begin{aligned} f_{y_{2,1}|s_1}(y_{2,1}|s_1) \;=\; & \frac{1 - P_2(i_1)}{\sqrt{2\pi N_0}} \exp\left[-\frac{|y_{2,1} - h_{2,1}s_1|^2}{2N_0}\right] \\ + \; & \frac{P_2(i_1)}{\sqrt{2\pi N_0}} \exp\left[-\frac{|y_{2,1} + h_{2,1}s_1|^2}{2N_0}\right], \quad (10) \end{aligned}$$

where $P_2(i_1)$ is the BEP of bit i_1 at T_2, which is assumed to be known at T_1. Based on (9) and (10), the ML decoder (after considering both possibilities $s_2 = \pm|s_2|$) is given in (11) at the bottom of this page, where $|\mathcal{A}_{s_1}| = 2$ and the maximization is only over s_1.

It is important to stress at this point that the complexity of ML increases exponentially with the number of tiers, M. Hence, as M increases the complexity of ML detection becomes increasingly intractable and so is its performance analysis. For this reason, the next subsection is devoted to deriving near-ML detectors which can afford simpler implementation and their error performance can be quantified in terms of the achievable diversity order.

C. Cooperative Demodulation

The last example illustrates that the detector in (8) needs all *per-tier* BEPs to be known at T_m. Furthermore, its inherent complexity prevents one from any general conclusive performance assessment. In [3], a piece-wise linear (PL) decoder has been derived for binary modulations. Exploiting the average bit-error probability (BEP) of the source-relay link that can be made available to the destination, this PL approximation leads to closed-form bounds on error performance, concluding that with $M-1$ parallel relays, the diversity order in coherent operation is at most $(M/2)+1$ for M even, and $(M+1)/2$ for M odd. In general, the PL detector cannot achieve the full diversity order M, which plays a critical role in the error performance especially at medium to high SNR values.

Aiming at full diversity gain, our idea is to simplify such a detector using properly weighted signal combiners. In this context, one may be tempted to rely on maximum-ratio-combining (MRC), which yields

$$\hat{x}_m^{MRC} = \arg\min_{x_m \in \mathcal{A}_{x_m}} |\mathbf{h}_m^{\mathcal{H}}\mathbf{y}_m - \|\mathbf{h}_m\|^2 x_m|^2. \quad (12)$$

Unfortunately, MRC is not generally equivalent to ML when at least one copy comes from a cooperating relay because regenerative relay strategies are prone to errors. In fact, MRC maximizes the output signal-to-noise ratio (SNR) of the $T_n \to T_m$ links without accounting for the errors that may occur when re-encoding symbols at T_n.

Instead of MRC, our approach will be to seek combiners that maximize the output SNRs of the *equivalent* end-to-end paths $T_{M+1} \to \cdots \to T_n \to T_m, \forall\, n \in [m+1, M+1]$ whose receive-SNRs account for the per-hop errors. With this objective, we advocate the following general weighted combiner, which we term cooperative (C) MRC. C-MRC detects bits recursively starting with i_1. For bit i_b, it takes the general form

$$\hat{x}_m^{C-MRC}(i_b) = \arg\min_{x'_b \in \mathcal{A}_{x'_b}} \left|(\mathbf{w}_m^b)^{\mathcal{H}}\mathbf{y}_m - (\mathbf{w}_m^b)^{\mathcal{H}}\mathbf{h}_m x'_b\right|^2 \quad (13)$$

where $b = 1, \ldots, m$, $x'_b = \hat{x}_m^{C-MRC}(i_{b-1}) + \alpha^{b-1}s_b$ and $x'_1 = s_1$. The search now is performed over the set $\mathcal{A}_{x'_b}$ with cardinality $|\mathcal{A}_{x'_b}| = 2$. If we define the instantaneous receive-SNR of link $T_n \to T_m$ as $\gamma_{n,m} := |h|^2_{n,m}P_x/N_0$, $\forall\ n = m+1, \ldots, M+1$, then vector $\mathbf{w}_m^b$ is given by

$$\mathbf{w}_m^b = [h_{M+1,m}, \frac{\gamma^b_{eq_{M,m}}}{\gamma_{M,m}} h_{M,m}, \ldots, \frac{\gamma^b_{eq_{m+1,m}}}{\gamma_{m+1,m}} h_{m+1,m}]^T \quad (14)$$

where $\gamma^b_{eq_{n,m}}$ is what we term *equivalent SNR* and can be calculated as follows. Define $P_{n,m}(i_b)$ as the BEP for transmitting i_b from T_n to T_m. When $P_n(i_b)$ is known at T_m, one can calculate the overall BEP of bits i_b at T_m sent from T_n, as [6]

$$P_m(i_b, T_n) = [1 - P_n(i_b)]P_{n,m}(i_b) + [1 - P_{n,m}(i_b)]P_n(i_b). \quad (15)$$

Now, $\gamma^b_{eq_{n,m}}$ can be understood as representing the SNR of a virtual BPSK-based equivalent link for bit i_b, which can be calculated by inverting the function

$$P_m(i_b, T_n) \;=\; Q\left[\sqrt{2\gamma^b_{eq_{n,m}}}\right], \quad (16)$$

where $Q[x] := (1/\sqrt{2\pi})\int_x^{\infty} \exp\left(-t^2/2\right) dt$ and $Q[x] := (1/\pi)\int_0^{\pi/2} \exp[-x^2/(2\sin^2\Phi)]d\Phi$ for $x \geq 0$. This last expression is just the standard BEP of a BPSK transmission through a channel with instantaneous SNR $\gamma^b_{eq_{n,m}}$. Returning

$$\begin{aligned} \hat{x}_1^{ML} = \hat{s}_1^{ML} = \arg\max_{s_1 \in \mathcal{A}_{s_1}} \Bigg\{ & \frac{1 - P_2(i_1)}{4\pi N_0} \exp\left[-\frac{|y_{3,1} - h_{3,1}(s_1 + \alpha|s_2|)|^2 + |y_{2,1} - h_{2,1}s_1|^2}{2N_0}\right] \\ & + \frac{1 - P_2(i_1)}{4\pi N_0} \exp\left[-\frac{|y_{3,1} - h_{3,1}(s_1 - \alpha|s_2|)|^2 + |y_{2,1} - h_{2,1}s_1|^2}{2N_0}\right] \\ & + \frac{P_2(i_1)}{4\pi N_0} \exp\left[-\frac{|y_{3,1} - h_{3,1}(s_1 + \alpha|s_2|)|^2 + |y_{2,1} + h_{2,1}s_1|^2}{2N_0}\right] \\ & + \frac{P_2(i_1)}{4\pi N_0} \exp\left[-\frac{|y_{3,1} - h_{3,1}(s_1 - \alpha|s_2|)|^2 + |y_{2,1} + h_{2,1}s_1|^2}{2N_0}\right] \Bigg\} \end{aligned} \quad (11)$$

to (14), one can now recognize that vector $\mathbf{w}_m^b$ weighs each entry of $\mathbf{y}_m$ in order to maximize $\gamma_{eq_{n,m}}^b$ instead of $\gamma_{n,m}$ as in (12).

The decoder in (13) is very simple and has general applicability regardless of the underlying constellation. Nevertheless, the calculation of $P_n(i_b)$, as in the ML case, becomes complicated when there are multiple heterogeneous constellations arriving at T_n. To handle those, we look for a simple means of calculating (14) by propagating SNRs in an M-tier CBC network. As we treated $\gamma_{eq_{n,m}}^b$ to be an equivalent SNR, we can further approximate it to be independent of b. This is established in the following lemma (see Appendix A for the proof) :

Lemma 1: *At high SNR, $\gamma_{eq_{n,m}}$ can be approximated by*

$$\gamma_{eq_{n,m}} \approx \min\left\{\gamma_{eq_n}\frac{1+\alpha^2+\cdots+\alpha^{2(n-2)}}{1+\alpha^2+\cdots+\alpha^{2(n-1)}}, \gamma_{n,m}\right\}, \quad (17)$$

where $m = 1, \ldots, M-1$, $n = m+1, \ldots, M$, and

$$\gamma_{eq_n} := \sum_{l=n+1}^{M} \gamma_{eq_{l,n}} + \gamma_{M+1,n}. \quad (18)$$

Based on (17) and (18), we can iteratively update $\gamma_{eq_{n,m}}$, $m = 1, \ldots, M-1$, starting with $\gamma_{eq_M} := \gamma_{M+1,M}$ at tier M, without being necessary to calculate any BEP. Analysis in the ensuing section will show that this approximation incurs negligible performance loss.

It is worth mentioning that the simple structure of our weighted combiner comes at the price of sub-optimality relative to the ML demodulator. For this reason, BEP performance of C-MRC receivers will be lower bounded by the ML scheme. Unfortunately, the complex structure of ML detection in cooperative settings prevents one from quantifying its performance [17].

III. Performance Analysis

We will first derive BEP metrics for $M = 2$ with 2/4-PAM constellations [c.f. Fig. 3]; pertinent performance for higher M will be commented later on. The received signals at T_2 and T_1 from T_3 are denoted as

$$y_{3,1} = h_{3,1}(s_1 + \alpha s_2) + n_{3,1}, \quad (19)$$

$$y_{3,2} = h_{3,2}(s_1 + \alpha s_2) + n_{3,2}. \quad (20)$$

Using the results in [20] and after straightforward changes of variables, we find that the average BEP per fading realization for i_1 and i_2 at T_2 is given, respectively, by

$$P_2(i_1) = \frac{1}{2}\left\{Q\left[\frac{(1+\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right] + Q\left[\frac{(1-\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right]\right\}, (21)$$

$$P_2(i_2) = \frac{1}{4}\left\{4Q\left[\frac{\alpha\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right] - 2Q\left[\frac{(2+\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right] + 2Q\left[\frac{(2-\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right]\right\}. \quad (22)$$

The BEP for i_1 at T_1 results from the C-MRC of signals sent by T_3 and T_2.

To delineate the advantages of the C-MRC based detector, we will first provide closed-form expressions of the BEP when relaying nodes are forwarding the received signal in an analog fashion. In non-broadcasting scenarios, this strategy has been shown to be optimum in terms of BEP, thus benchmarking performance in our CBC setup. Later on, we will also compare it with the DFb strategy tailored for our broadcasting setting.

As usual, we define the diversity gain (diversity order) G_d, as the negative exponent in the expression of average BEP when the average SNR tends to infinity, that is $P^b \overset{\overline{\gamma}\to\infty}{\approx} (G_c\overline{\gamma})^{-G_d}$, where G_c denotes the coding gain. Although channel coding does make a difference on error performance, it affects only the factor G_c. As SNR $\overline{\gamma}$ increases, the diversity order G_d which affects the exponent in the expression of P^b, becomes the critical factor. And this is the scenario we focus on in this paper.

A. Performance of AF

The amplify-and-forward (AF) relay strategy, where e.g., T_2 amplifies before re-transmitting the analog received signal $y_{3,2}$, will benchmark performance of our DF strategies. To serve this purpose, we will derive closed-form BEP expressions for multi-tier AF broadcasting. In this case, the transmitted signal from T_2 is

$$x' = Ay_{3,2} = A(h_{3,2}x + n_{3,2}), \quad (23)$$

where A is the amplification factor. Then the received signal at T_1 from T_2 is

$$y_{2,1} = h_{2,1}A(h_{3,2}x + n_{3,2}) + n_{2,1} = h_{2,1}Ah_{3,2}x + n_1, \quad (24)$$

where $n_1 := h_{2,1}An_{3,2} + n_{2,1}$. A convenient choice for A is [14]

$$A^2 = \frac{P_x}{P_x|h_{3,2}|^2 + N_0} \quad (25)$$

because it maintains constant average power at the relay, equal to the originally transmitted power. At the receiver, we combine $y_{3,1}$ and $y_{2,1}$ using MRC [14], which is the optimal detector for the AF protocol. As the powers of $n_{3,1}$ and n_1, namely $\sigma_{3,1}^2$ and σ_1^2, are not identical, the MRC should be preceded by a noise normalization step. The resulting SNR at the MRC output is

$$\gamma_{AF} = |h_{3,1}|^2\frac{P_x}{\sigma_{3,1}^2} + |Ah_{2,1}h_{3,2}|^2\frac{P_x}{\sigma_1^2} = \frac{\gamma_{3,2}\gamma_{2,1}}{1+\gamma_{3,2}+\gamma_{2,1}} + \gamma_{3,1}. \quad (26)$$

At sufficiently high SNR, the 1 in the denominator can be ignored and (26) reduces to

$$\gamma_{AF} = \frac{\gamma_{3,2}\gamma_{2,1}}{\gamma_{3,2}+\gamma_{2,1}} + \gamma_{3,1}. \quad (27)$$

The following proposition establishes the performance of this MRC detector for AF relaying in CBC settings (see Appendix B for the proof):

Proposition 1: *Given the average SNRs of all tier-to-tier links, say $\overline{\gamma}_{3,2}$, $\overline{\gamma}_{2,1}$ and $\overline{\gamma}_{3,1}$, the asymptotic average error probabilities for bits i_1 and i_2 at T_1 are given respectively by*

$$P_1^{AF}(i_1) \to \frac{3(1+6\alpha^2+\alpha^4)(1+\alpha^2)^2}{16(1+\alpha)^4(1-\alpha)^4\overline{\gamma}^{AF}}, \quad (28)$$

$$P_1^{AF}(i_2) \to \frac{3(1+\alpha^2)^2[(4-\alpha^2)^4 + 8\alpha^5(4+\alpha^2)]}{16\alpha^4(4-\alpha^2)^4\overline{\gamma}^{AF}}, \quad (29)$$

where $(\overline{\gamma}^{AF})^{-1} := [(\overline{\gamma}_{3,2})^{-1} + (\overline{\gamma}_{2,1})^{-1}](\overline{\gamma}_{3,1})^{-1}$.

Eqs. (28) and (29) reveal that diversity of order 2 is achieved from the product of two independent SNRs, that of the direct path and the one of the relay path.

B. Performance of DFb

Recall that T_1 aims at detecting only bit i_1; so T_2 only forwards one BPSK symbol $x' = \hat{s}_1$. At T_1, the entries of the vector $\mathbf{y}_1 := [y_{3,1}, y_{2,1}]^T$ are:

$$y_{3,1} = h_{3,1}(s_1 + \alpha s_2) + n_{3,1}, \quad (30)$$

$$y_{2,1} = h_{2,1}\hat{s}_1 + n_{2,1}. \quad (31)$$

Any receiver at tier T_1 combines two received signals in the same constellation with weights $w_{2,1}$ and $w_{3,1}$ to obtain

$$y_1 = w_{3,1}y_{3,1} + w_{2,1}y_{2,1} = \begin{cases}(w_{3,1}h_{3,1}+w_{2,1}h_{2,1})s_1+w_{3,1}h_{3,1}\alpha s_2+n_1, \text{if } \hat{s}_1=s_1,\\(w_{3,1}h_{3,1}-w_{2,1}h_{2,1})s_1+w_{3,1}h_{3,1}\alpha s_2+n_1, \text{if } \hat{s}_1=-s_1,\end{cases} \quad (32)$$

where $n_1 := w_{2,1}n_{2,1} + w_{3,1}n_{3,1}$. The combiner output y_1 in (32) is a complex Gaussian random variable whose mean has only nonzero real part, since 2/4-PAM is a one-dimensional constellation. Because the complex Gaussian distribution is circularly symmetric, we can take the real part $y = Re\{y_1\}$ before detection, which is a real Gaussian random variable with zero mean and variance $N_0/2$. Then, given $\gamma_{3,2}$, $\gamma_{3,1}$, and $\gamma_{2,1}$, the BEP for bit i_1 at T_1, denoted as $P_1^{DFb}(i_1|\gamma_{3,2},\gamma_{3,1},\gamma_{2,1})$, can be found as

$$\begin{aligned}&P_1^{DFb}(i_1|\gamma_{3,2},\gamma_{3,1},\gamma_{2,1})\\&= \frac{1}{2}[1-P_2(i_1|\gamma_{3,2})]Q\left[\frac{w_{3,1}h_{3,1}(1-\alpha)+w_{2,1}h_{2,1}}{\sqrt{|w_{3,1}|^2+|w_{2,1}|^2}}\sqrt{\frac{2E_s}{N_0}}\right]\\&+ \frac{1}{2}[1-P_2(i_1|\gamma_{3,2})]Q\left[\frac{w_{3,1}h_{3,1}(1+\alpha)+w_{2,1}h_{2,1}}{\sqrt{|w_{3,1}|^2+|w_{2,1}|^2}}\sqrt{\frac{2E_s}{N_0}}\right]\\&+ \frac{1}{2}P_2(i_1|\gamma_{3,2})Q\left[\frac{w_{3,1}h_{3,1}(1-\alpha)-w_{2,1}h_{2,1}}{\sqrt{|w_{3,1}|^2+|w_{2,1}|^2}}\sqrt{\frac{2E_s}{N_0}}\right]\\&+ \frac{1}{2}P_2(i_1|\gamma_{3,2})Q\left[\frac{w_{3,1}h_{3,1}(1+\alpha)-w_{2,1}h_{2,1}}{\sqrt{|w_{3,1}|^2+|w_{2,1}|^2}}\sqrt{\frac{2E_s}{N_0}}\right], \quad (33)\end{aligned}$$

where $E_s := d_1^2 = P_x/(1+\alpha^2)$. Following the choice in (14), we have $w_{3,2} = h_{3,2}^*$ and $w_{2,1} = \frac{\gamma_{eq_{2,1}}}{\gamma_{2,1}}h_{2,1}^*$. To simplify notation, let us define $\gamma_{eq} := \gamma_{eq_{2,1}}$. Substituting now into (33), we obtain

$$\begin{aligned}&P_1^{DFb}(i_1|\gamma_{3,2},\gamma_{3,1},\gamma_{2,1})\\&= \frac{1}{2}[1-P_2(i_1|\gamma_{3,2})]Q\left[\frac{\gamma_{3,1}(1-\alpha)+\gamma_{eq}}{\sqrt{\gamma_{3,1}+\gamma_{eq}^2/\gamma_{2,1}}}\sqrt{\frac{2}{1+\alpha^2}}\right]\\&+ \frac{1}{2}[1-P_2(i_1|\gamma_{3,2})]Q\left[\frac{\gamma_{3,1}(1+\alpha)+\gamma_{eq}}{\sqrt{\gamma_{3,1}+\gamma_{eq}^2/\gamma_{2,1}}}\sqrt{\frac{2}{1+\alpha^2}}\right]\\&+ \frac{1}{2}P_2(i_1|\gamma_{3,2})Q\left[\frac{\gamma_{3,1}(1-\alpha)-\gamma_{eq}}{\sqrt{\gamma_{3,1}+\gamma_{eq}^2/\gamma_{2,1}}}\sqrt{\frac{2}{1+\alpha^2}}\right]\\&+ \frac{1}{2}P_2(i_1|\gamma_{3,2})Q\left[\frac{\gamma_{3,1}(1+\alpha)-\gamma_{eq}}{\sqrt{\gamma_{3,1}+\gamma_{eq}^2/\gamma_{2,1}}}\sqrt{\frac{2}{1+\alpha^2}}\right]. \quad (34)\end{aligned}$$

The BEP for i_1 at T_1 using DFb is given by $P_1^{DFb}(i_1) = E\{P_1^{DFb}(i_1|\gamma_{3,2},\gamma_{3,1},\gamma_{2,1})\}$, and the next proposition provides an upper bound to the BEP (see Appendix C for the proof).

Proposition 2: *For multi-tier DFb relaying, where T_2 only forwards the basic information to T_1, full diversity can be achieved for the basic information using C-MRC with 2/4-PAM constellation; i.e., $P_1^{DFb}(i_1) \le \tilde{P}_1^{DFb}(i_1) \overset{\bar{\gamma}\to\infty}{\approx} (k_2\bar{\gamma})^{-2}$, with k_2 denoting a constant.*

When T_2 forwards the entire symbol estimate $\hat{x} = \hat{s}_1 + \alpha\hat{s}_2$ to T_1, DFb reduces to the conventional DF protocol. Error performance of the DF strategy with an equivalent C-MRC detector has been analyzed in [21] where C-MRC was shown to enjoy full diversity gain regardless of the underlying constellation. Treating $(s_1 + \alpha s_2)$ as a symbol, we know that the average symbol-error-probability (SEP) as well as the BEP curves exhibit full diversity; therefore, full diversity can also be achieved for the basic information in DFb.

One more thing to stress is that DFb prevents T_2 from sending the enhancement information to T_1 and hence it saves power that is used to transmit the basic information bits. For this reason, our simulations will also confirm that DFb exhibits better error performance than DF at T_1.

Further capitalizing on the results of [21], one can extend the analysis in Proposition 2 to any M-tier network to establish that full diversity of order $M - m + 1$ can be achieved at T_m, for all $m = 1, \ldots, M$. But detailed analysis for the M-tier network will be tedious and goes beyond the scope of this work since (33) and (34) are already very complicated for the 2-tier case, and the number of terms will increase exponentially in the number of tiers.

IV. Simulations and Numerical Results

In this section, we compare various schemes on the basis of BEP using numerical results and Monte-Carlo simulations. We assume that the path-loss exponent in all links involved is 3, so the average output SNR of the link $T_n \to T_m$ with respect to the link, $T_{M+1} \to T_m$, is

$$\overline{\gamma}_{n,m} = \overline{\gamma}_{M+1,m}\left(\frac{d_{M+1,m}}{d_{n,m}}\right)^3, \quad (35)$$

where $d_{M+1,m}$ and $d_{n,m}$ denote the corresponding distances. Unless otherwise stated, we will rely on a two-tier model where T_2 is equally spaced from T_3 and T_1. In this case, the average SNR setting in (35) becomes $\overline{\gamma}_{3,2} = \overline{\gamma}_{2,1} = \overline{\gamma}_{3,1} \times 2^3$. Transmission power will be set to be the same across all terminals.

A. Probability of error at T_2

In Fig. 4, we plot the BEP of bits i_1 and i_2 at T_2, denoted as $P_2(i_1)$ and $P_2(i_2)$ respectively, for different values of α. Because $T_3 \to T_2$ is a point-to-point link, the error probabilities are the ones in (21) and (22), respectively. In any case, the expected diversity in detecting bits i_1 and i_2 is of order 1. As α goes to zero, $P_2(i_1)$ approaches the BEP of BPSK transmissions, also shown in the figure, while the BEP in detecting the enhancement information bit i_2 approaches 0.5. For increasing values of α, $P_2(i_2)$ will also decrease with

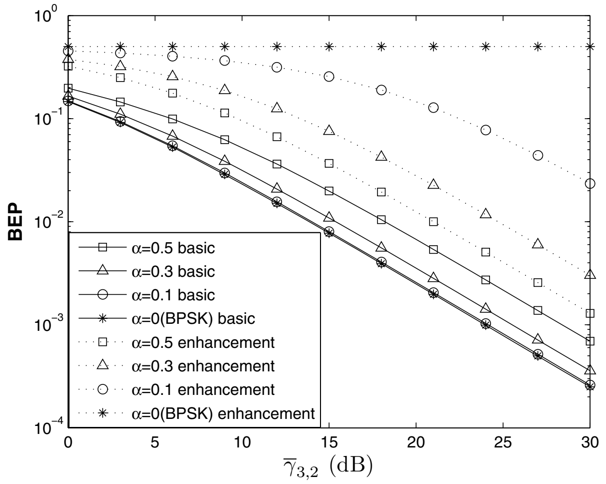

Fig. 4. BEP comparison for bits i_1 and i_2 at T_2 (point-to-point link).

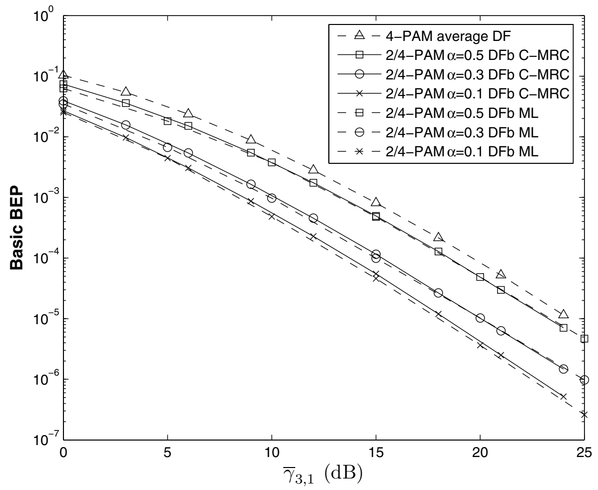

Fig. 6. BEP comparison for 2-tier CBC using 2/4-PAM with DFb (C-MRC and ML) vs. 4-PAM with DF.

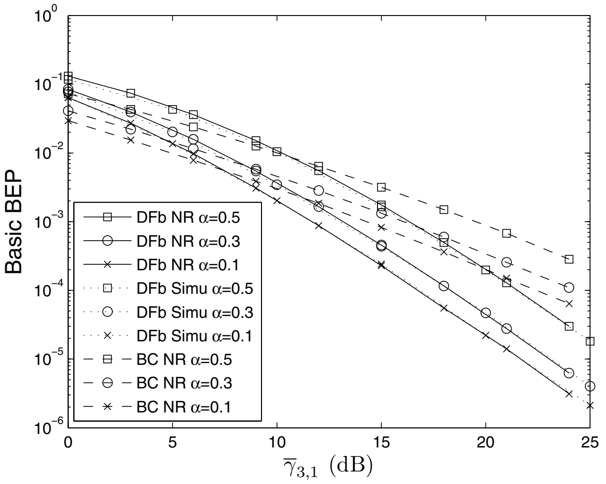

Fig. 5. BEP comparison for BC using 2/4-PAM vs. 2-tier CBC using 4/16-QAM with DFb.

diversity gain 1. BEP curves in this subsection further suggest that a reasonable choice for α in hierarchical constellations seems to be in the range $[0.1, 0.5]$.

B. Probability of error at T_1

In Figs. 5-7, we compare the probabilities of error at T_1 of different (re-) transmission strategies. Since T_1 can only detect the basic information in our multi-tier setting, all error probabilities are henceforth with respect to the MSB i_1.

If no relaying strategy were employed, i_1 in T_1 would be detected with diversity 1 with a 9dB penalty due to the distance. In Fig. 5, we compare the BEP of such a *conventional* broadcasting (BC) scenario using 2/4-PAM with a 2-tier cooperative broadcasting (CBC) setup using DFb for successive broadcastings with both simulations (Simu) and numerical results (NRs) and for different values of α. To compensate for the $1/2$-rate loss due to the time-phases of CBC, we use 4/16-QAM for CBC transmissions and thus all strategies have identical rate $R = 1$ bit per symbol per channel use (bps-pcu). We can see that the simulations accurately match numerical results, which corroborates the accuracy of our performance analysis. For any α, because of its higher diversity, CBC outperforms BC at sufficiently high SNR values. As α becomes smaller, both BC and CBC improve their performance, as expected.

In Fig. 6, we numerically compare the BEP of 2/4-PAM with DFb against 4-PAM with DF and Gray mapping. We observe that they all achieve the full diversity gain while 2/4-PAM outperforms 4-PAM for all values of α. Notice that when $\alpha = 0.5$, we know from Fig. 2 that constellation points coincide with the conventional 4-PAM constellation with Gray mapping. Different from hierarchical modulations, 4-PAM treats bits i_1 and i_2 with equal importance and this shows also in the average BEP. For various values of α, ML detectors in equation (11) are also depicted in the same figure. Now we can verify that DFb using the C-MRC scheme in (13) yields error performance very close to ML.

Since previous simulations employed DFb protocols, we also need to check whether the proposed protocol offers advantages relative to conventional DF and AF alternatives. In Fig. 7, we plot the simulated BEP of different relaying protocols using 2/4-PAM. All three protocols achieve the full diversity gain, which is 2 here. When $\alpha = 0.1$, the three protocols perform almost identically. As α increases, DFb outperforms DF and AF, since DFb avoids sending the enhancement information bit and thereby saves in transmit-power. Since the comparison is based on the same transmit power, DFb yields a higher SNR at the relay (for s_1, as T_1 does not detect s_2) than AF and DF because DFb uses all the power to transmit s_1 only. Notice that the performance gap between DFb and other protocols will become more pronounced when the number of tiers becomes larger. This holds because transmitting s_1 instead of $s_1 + \alpha s_2 + \alpha^2 s_3 + \ldots + \alpha^{(M-1)} s_M$ reduces power requirements for DFb as M increases. Moreover, using the DFb strategy, the relays (higher tier receivers) bypass the need for storing analog waveforms which is required by the AF protocol.

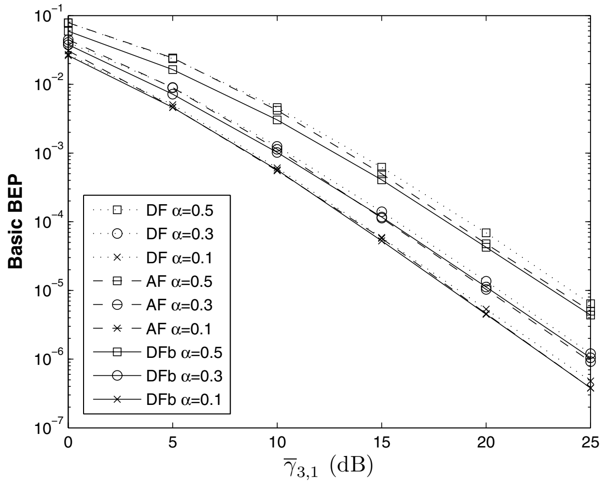

Fig. 7. BEP comparison for 2-tier CBC using 2/4-PAM with DFb vs. DF vs. AF.

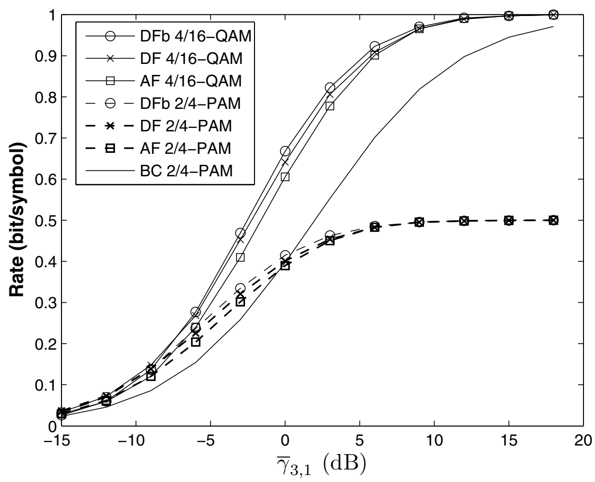

Fig. 8. Achievable rate comparison of basic information at T_1 for different broadcasting strategies with $\alpha = 0.3$.

C. Rate comparison

We also tested the achievable rates of our CBC protocol to assess its effective capacity compared to other non-hierarchical or non-cooperative strategies. As usual, the achievable rate is defined as the maximum number of bits that can be decoded with arbitrarily low error probability. The final $T_{M+1} \to T_m$ link, with or without the relay link, can always be made equivalent to a binary symmetric channel with cross-over probability depending on the specific cooperative (or non-cooperative) scheme adopted. Notice that the achievable rates discussed here are actually lower bounds since we rely on hard decisions which is certainly suboptimum relative to soft decoding. We let $P_m(i_b|\mathbf{h})$ denote this probability which is here conditioned on the channel realization, $\mathbf{h} := [\mathbf{h}_M, \mathbf{h}_{M-1}, \ldots, \mathbf{h}_m]^T$. The capacity of such an equivalent channel is $C_m(i_b) = E\{C_m(i_b|\mathbf{h})\}$, where

$$C_m(i_b|\mathbf{h}) := 1 + P_m(i_b|\mathbf{h}) \log_2[P_m(i_b|\mathbf{h})] + [1 - P_m(i_b|\mathbf{h})] \log_2[1 - P_m(i_b|\mathbf{h})],$$

and $P_m(i_b|\mathbf{h})$ can be obtained for the specific detector used. The maximum rate $R_m(i_b)$ for i_b to be reliably transmitted from T_{M+1} to T_m, incurs a penalty depending on the number of transmission phases considered in the protocol; hence,

$$R_m(i_b) = \frac{1}{M} C_m(i_b) \text{ bps-pcu.} \quad (36)$$

In Figs. 8 and 9, we compare the achievable rates at T_1 and T_2, respectively, for different schemes with $\alpha = 0.3$ and $M = 2$. Fig. 8 shows the achievable rates of different constellations and relaying protocols for the basic information bit at T_1. Even though this cooperative scheme was intended to improve BEP performance rather than rate, we can see that DFb also brings advantages in rate compared to DF and AF. This figure also demonstrates that DF outperforms AF in the low SNR regime. BC with 2/4-PAM outperforms a 2-tier CBC with 2/4-PAM only in the medium-high SNR range. However, this rate is higher when using 4/16-QAM. The achievable rates for both basic and enhancement information bits at T_2 using 2/4-PAM in BC are depicted in Fig. 9. For a comparative reference, we also draw the average achievable rate at T_2 using 4-PAM in BC, which outperforms the rate of the enhancement information while being outperformed by the basic information.

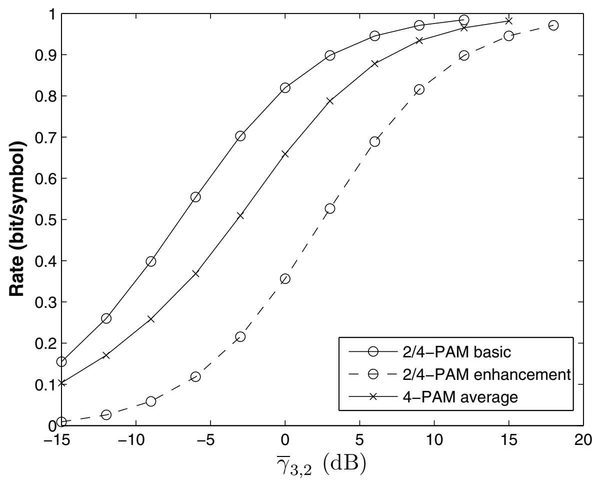

Fig. 9. Achievable rate comparison of both basic and enhancement information at T_2 for BC using 2/4-PAM with $\alpha = 0.3$ vs. 4-PAM with average rate.

D. BEP comparison for M-tier CBC

In this test case, we validate our full-diversity claims for DFb in an M-tier CBC with constellation parameter $\alpha = 0.3$. Fig. 10 shows that the BEP slope changes with the number of tiers, which corroborates that diversity of order M is achieved at T_1 for any M-tier CBC network, as our analysis asserted in Section III. Moreover, this figure confirms that successive broadcasting strategies bring major performance improvements quantified by both coding and diversity gains for terminals at the edge of a sparse network.

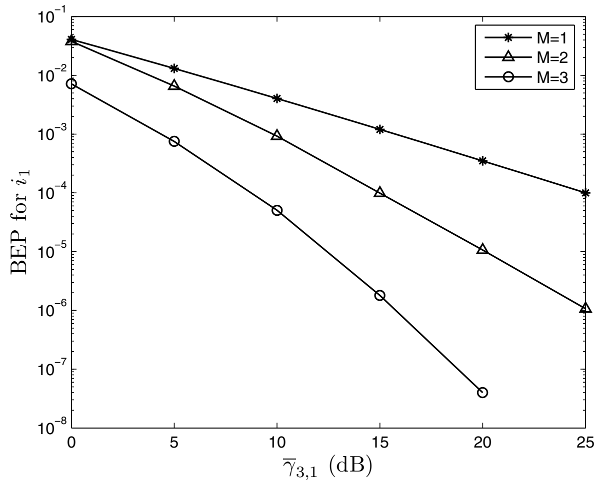

Fig. 10. BEP comparison for bit i_1 at T_1 in M-tier CBC using 2/4-PAM and DFb with $\alpha = 0.3$.

V. Conclusions

This work advocated wedding hierarchical modulations with cooperative broadcasting in multi-tier networks. In the envisioned cooperative scenario, we order terminals in tiers according to their reception capabilities. Thus, different tiers collect information and successively broadcast part of the information to other terminals. Specifically, some tiers collect all broadcasted information (basic and enhancement) and broadcast only the basic information to tiers with worse reception conditions. This offers an adaptive DF protocol, which we termed DFb, where successive broadcastings aim to further protect the basic information. We incorporated simple weighted combiners for demodulation in order to adaptively account for the heterogeneous signals involved in each phase. The simplicity of these demodulators irrespective of the underlying constellation allowed us to assess performance based on the diversity order which had not so far been quantified even for the ML detectors in cooperative broadcasting.[3]

Appendix

A. Proof of Lemma 1

First, we need to establish the following property for $\gamma_{\text{eq}} := \gamma_{\text{eq}_{2,1}} = \gamma^1_{\text{eq}_{2,1}}$ in a 2-tier CBC:

Property 1: *It holds that γ_{eq} is bounded by*

$$\min\left\{\frac{(1-\alpha)^2\gamma_{3,2}}{1+\alpha^2}, \gamma_{2,1}\right\} - 1.62 < \gamma_{eq} \le \min\left\{\frac{\gamma_{3,2}}{1+\alpha^2}, \gamma_{2,1}\right\}. \quad (37)$$

Proof. To establish the bound on γ_{eq}, let us first define $\gamma_{3,2}^{BPSK} := \frac{1}{2}\left\{Q^{-1}\left[P_2(i_1|\gamma_{3,2})\right]\right\}^2$ to be the equivalent SNR when S (T_3) transmits bit i_1 to T_2 using BPSK. Then, using [21, Property 1], we deduce that

$$\min\{\gamma_{3,2}^{BPSK}, \gamma_{2,1}\} - 1.62 < \gamma_{\text{eq}} \le \min\{\gamma_{3,2}^{BPSK}, \gamma_{2,1}\}. \quad (38)$$

[3]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

Since $Q[x]$ is a convex function, when $x \ge 0$, we have

$$P_2(i_1|\gamma_{3,2}) = \frac{1}{2}\left\{Q\left[\frac{(1+\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right] + Q\left[\frac{(1-\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right]\right\}$$
$$\ge Q\left[\sqrt{\frac{2\gamma_{3,2}}{1+\alpha^2}}\right]. \quad (39)$$

As $0 \le \alpha \le 1$, we obtain $P_2(i_1|\gamma_{3,2}) \le Q\left[\sqrt{\frac{2(1-\alpha)^2\gamma_{3,2}}{1+\alpha^2}}\right]$. While $P_2(i_1|\gamma_{3,2}) = Q\left[\sqrt{2\gamma_{3,2}^{BPSK}}\right]$, which implies that $\frac{(1-\alpha)^2\gamma_{3,2}}{1+\alpha^2} \le \gamma_{3,2}^{BPSK} \le \frac{\gamma_{3,2}}{1+\alpha^2}$, with both equalities holding true when $\alpha = 0$. Implementing the above inequality of $\gamma_{3,2}^{BPSK}$, we arrive at equation (37). ■

From [21, Property 1], we know that the right-hand-side (RHS) of (38) approximates well γ_{eq} at high SNR. Hence, the RHS of (37) is also a good approximation for γ_{eq} when SNRs are high, since in this case $Q[x]$ approaches a linear function and the equality in (39) holds. So, in a 2-tier CBC, we have $\gamma_{eq_{2,1}} \approx \min\{\frac{\gamma_{3,2}}{1+\alpha^2}, \gamma_{2,1}\}$. We next use the recursive algorithm of [20] to calculate the BEP of bit i_b, $P_b(M, \mathbf{d}, i_b)$, $b = 1, 2, \ldots, M-1$ for a $\{2/4/\cdots/2^M\}$-PAM constellation, as $P_b(M, \mathbf{d}, i_b) = \frac{1}{2}[P_b(M-1, \mathbf{d}_+, i_b) + P_b(M-1, \mathbf{d}_-, i_b)]$, $P_b(2, \mathbf{d}, i_1) = \frac{1}{2}\left\{Q\left[\frac{(d_1+d_2)\sqrt{2\gamma}}{\sqrt{d_1^2+d_2^2}}\right] + Q\left[\frac{(d_1-d_2)\sqrt{2\gamma}}{\sqrt{d_1^2+d_2^2}}\right]\right\}$, where $\mathbf{d} := [d_1, d_2, \ldots, d_M]$, $\mathbf{d}_\pm := [d_1, d_2, \ldots, d_{M-1} \pm d_M]$, and $\gamma := P_x/N_0$ is the receive SNR in an AWGN channel. Using this algorithm and the convex inequality of Q[x] recursively, we can arrive at the conclusion that in an M-tier CBC with general hierarchical modulation, $\gamma_{eq_{M,m}} \approx \min\left\{\gamma_{S,D_M}\frac{1+\alpha^2+\cdots+\alpha^{2(M-2)}}{1+\alpha^2+\cdots+\alpha^{2(M-1)}}, \gamma_{M,m}\right\}$, provided that all SNRs involved are sufficiently high. The same conclusion can then be easily extended to the link $S \to \cdots \to T_n \to T_m$ as

$$\gamma_{eq_{n,m}} \approx \min\left\{\gamma_{eq_n}\frac{1+\alpha^2+\cdots+\alpha^{2(n-2)}}{1+\alpha^2+\cdots+\alpha^{2(n-1)}}, \gamma_{n,m}\right\}, \quad (40)$$

in the high SNR regime. Notice that when $n = M$, we have $\gamma_{eq_n} = \gamma_{S,M}$.

B. Proof of Proposition 1

Defining $\gamma_\Gamma := \frac{\gamma_{3,2}\gamma_{2,1}}{\gamma_{3,2}+\gamma_{2,1}}$, we obtain $\gamma_{AF} = \gamma_\Gamma + \gamma_{3,1}$ from (27). When the channels from S to T_2 and from T_2 to T_1 are both flat Rayleigh faded, the moment generating function (MGF) of γ_Γ, can be expressed as $\mathcal{M}_\Gamma(s) = {}_2F_1\left(1, 2; \frac{3}{2}; -\frac{\overline{\gamma}}{4}s\right)$, when $\overline{\gamma}_{3,2} = \overline{\gamma}_{2,1} = \overline{\gamma}$ [6]. In the presence of Rayleigh fading, the MGF of $\gamma_{3,1}$ is given by $\mathcal{M}_{\gamma_{3,1}}(s) = (1 - s\overline{\gamma}_{3,1})^{-1}$.

Using the MGF-based approach in [18] for independent fading across the L diversity paths, we have $P(\gamma_1, \gamma_2, \ldots, \gamma_L) = \prod_{l=1}^{L} P(\gamma_l)$. As a consequence, the conditional BEP is given by $P_b(E) = \frac{1}{\pi}\int_0^{\pi/2}\left[\prod_{l=1}^{L}\mathcal{M}_{\gamma_l}\left(-\frac{c^2}{\sin^2\Phi}\right)\right]d\Phi \triangleq I(L, c, \boldsymbol{\gamma})$, where $\boldsymbol{\gamma} := [\gamma_1, \gamma_2, \ldots, \gamma_L]$. When $c = 1$, this is the average BEP for 2-PAM. Relying on (21) and (22), we can derive the average BEPs for i_1 and i_2 with 2/4-PAM in a similar way.

The final expressions are given by

$$P_b(L,\mathbf{d},\boldsymbol{\gamma},i_1)=\frac{1}{2}\left[I\left(L,\frac{d_1+d_2}{|d|},\boldsymbol{\gamma}\right)+I\left(L,\frac{d_1-d_2}{|d|},\boldsymbol{\gamma}\right)\right], \quad (41)$$

$$P_b(L,\mathbf{d},\boldsymbol{\gamma},i_2)=$$
$$\frac{1}{4}\left[4I\left(L,\frac{d_2}{|d|},\boldsymbol{\gamma}\right)-2I\left(L,\frac{2d_1+d_2}{|d|},\boldsymbol{\gamma}\right)+2I\left(L,\frac{2d_1-d_2}{|d|},\boldsymbol{\gamma}\right)\right] (42)$$

where $|d| := \sqrt{d_1^2+d_2^2}$. With the MGF of $\gamma_{3,1}$ and γ_Γ available, we can obtain the average BEPs for i_1 and i_2 at T_1 when cooperative broadcasting relies on the AF strategy, $P_1^{AF}(i_1) = P_b(2,\mathbf{d},[\gamma_{3,1},\gamma_\Gamma],i_1)$, $P_1^{AF}(i_2) = P_b(2,\mathbf{d},[\gamma_{3,1},\gamma_\Gamma],i_2)$, where the RHSs have been defined in (41) and (42), respectively.

To assess the diversity gain of this scheme, we can use the results of [14]. When the channels are Rayleigh fading, the asymptotic average symbol error probability (SEP) with AF is

$$P_e \to \frac{3}{4k^2}[(\overline{\gamma}_{3,2})^{-1}+(\overline{\gamma}_{2,1})^{-1}](\overline{\gamma}_{3,1})^{-1}, \quad (43)$$

where k is a modulation-dependent constant. It is given in the expression of SEP conditioned on the instantaneous SNR γ_{AF} as: $P_e = Q[\sqrt{k\gamma_{AF}}]$. Proposition 1 then follows readily by checking the expressions in (21) and (22) and making use of (43).

C. Proof of Proposition 2

We can upper bound the expression in (34) by $P_1^{DFb}(i_1|\gamma_{3,2},\gamma_{3,1},\gamma_{2,1}) \le A + B$, where $A := Q\left[\frac{\gamma_{3,1}(1-\alpha)+\gamma_{eq}}{\sqrt{\gamma_{3,1}+\gamma_{eq}^2/\gamma_{2,1}}}\sqrt{\frac{2}{1+\alpha^2}}\right]$, $B := P_2(i_1|\gamma_{3,2})Q\left[\frac{\gamma_{3,1}(1-\alpha)-\gamma_{eq}}{\sqrt{\gamma_{3,1}+\gamma_{eq}^2/\gamma_{2,1}}}\sqrt{\frac{2}{1+\alpha^2}}\right]$. Because a sum is dominated by the term with the lowest diversity exponent, we need to prove that both A and B decay with the same exponent (diversity order), which here equals 2 since we have only one cooperative broadcasting node.

From Property 1 in Appendix A, we deduce that $\frac{(1-\alpha)^2}{1+\alpha^2}\min\{\gamma_{3,2},\gamma_{2,1}\} - 1.62 < \gamma_{\rm eq} \le \min\{\frac{\gamma_{3,2}}{1+\alpha^2},\gamma_{2,1}\}$. Using this inequality, it is easy to find that:

$$A \le \tilde{A} := Q\left[\sqrt{2(\gamma_{3,1}+\gamma_{eq})}\frac{(1-\alpha)}{\sqrt{1+\alpha^2}}\right],$$

$$B \le \tilde{B} := Q\left[\frac{(1-\alpha)\sqrt{2\gamma_{3,2}}}{\sqrt{1+\alpha^2}}\right]Q\left[\frac{\gamma_{3,1}(1-\alpha)-\gamma_{eq}}{\sqrt{\gamma_{3,1}(1-\alpha)+\gamma_{eq}}}\sqrt{\frac{2(1-\alpha)}{1+\alpha^2}}\right].$$

Comparing Property 1, $\tilde{A}$, $\tilde{B}$ here with [21, Property 1], $P_1^b(\gamma_{S,R},\gamma_{S,D},\gamma_{R,D})$, $P_2^b(\gamma_{S,R},\gamma_{S,D},\gamma_{R,D})$ in [21], respectively, we find that they all have identical forms within some constant scaling factors. Because the high SNR behavior is not affected by these constants, the full diversity claim in [21, Proposition 1] also holds true here.

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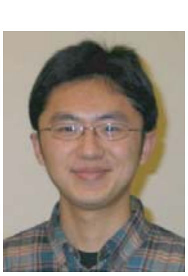

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