Block Differentially Encoded OFDM With Maximum Multipath Diversity

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***Abstract*—This letter proposes a novel block differentially encoded orthogonal frequency-division multiplexing for multicarrier transmissions over frequency-selective fading channels. Choosing appropriate system parameters, we divide the set of correlated subchannels into subsets of independent subchannels. Within each subset, differential unitary space–time modulation is performed by treating each subchannel as a transmit antenna. In addition to low complexity, the proposed system enjoys maximum multipath diversity and high coding advantages. Analytic evaluation and corroborating simulations reveal its performance merits.**

***Index Terms*—Differential encoding, diversity methods, orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) transforms a frequency-selective channel into a set of flat subchannels, thereby avoiding complex channel equalization. In particular, differentially encoded OFDM (DOFDM) (e.g., [1]) is practically attractive by further forgoing channel estimation that is costly or even impossible in rapidly changing mobile environments. Because each information data symbol is transmitted over a single flat subchannel that may undergo fading, however, DOFDM suffers from loss of multipath diversity. In order to robustify its performance, DOFDM usually resorts to channel coding and interleaving, which not only sacrifices bandwidth efficiency, but also induces extra decoding delay.

This letter develops a novel block differentially encoded OFDM (B-DOFDM) system. Designing system parameters properly, we split the set of generally correlated subchannels into subsets of independent subchannels to which the differential unitary space–time modulation (DUSTM) of [5] and [6] is then applied by treating each subchannel as a transmit antenna. In addition to low complexity, the resulting B-DOFDM achieves maximum multipath diversity and high coding advantages. Analytical evaluation and corroborating simulations confirm its performance merits.

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II. System Description

Consider OFDM transmissions with P subcarriers over an Lth-order frequency-selective fading channel with discrete-time equivalent impulse response vector $\mathbf{h} := [h(0), \ldots, h(L)]^T$. Let $t(n;\, p)$ be the transmitted data symbol on the pth subcarrier of the nth OFDM block; and let $y(n;\, p)$ be the corresponding received data sample after fast Fourier transform (FFT) processing at the receiver. OFDM's input–output relationship can be expressed as

$$y(n;\, p) = H(p)t(n;\, p) + w(n;\, p), \qquad p = 0, 1, \ldots, P-1 \tag{1}$$

where the pth subchannel gain $H(p)$ is defined as $H(p) := \sum_{l=0}^{L} h(l)\exp(-j2\pi lp/P)$; and the additive noise $w(n;\, p)$ is zero-mean complex Gaussian distributed, with variance $N_0/2$ per dimension, that is statistically independent with respect to both n and p.

Depending on how $t(n;\, p)$s are generated from information symbols, the model in (1) can be used to represent different OFDM systems. In B-DOFDM, the generation of $t(n;\, p)$s will rely on what we term subchannel grouping that was also suggested in [7] and [9] for linearly precoded OFDM systems. Let us choose the block length $P = MK$ for positive integers M and K, and denote by $\mathcal{I} := \{0, 1, \ldots, P-1\}$ the index set of the P subchannels. Mathematically, subchannel grouping is represented by partitioning $\mathcal{I}$ into M nonintersecting index subsets $\{\mathcal{I}_\mu\}_{\mu=0}^{M-1}$ of equal size K. With $\mathcal{I}_\mu := \{p_{\mu,1}, \ldots, p_{\mu,K}\}$, we infer from (1) that the K subchannels indexed by $\mathcal{I}_\mu$ are uniquely associated with K equations that can be written in a compact matrix-vector form as

$$\mathbf{y}_\mu(n) = \boldsymbol{\mathcal{S}}_\mu(n)\tilde{\mathbf{h}}_\mu + \mathbf{w}_\mu(n), \qquad \mu = 0, \ldots, M-1 \tag{2}$$

where $\mathbf{y}_\mu(n) := [y(n;\, p_{\mu,1}), \ldots, y(n;\, p_{\mu,K})]^T$, $\tilde{\mathbf{h}}_\mu := [H(p_{\mu,1}), \ldots, H(p_{\mu,K})]^T$, $\mathbf{w}_\mu(n) := [w(n;\, p_{\mu,1}), \ldots, w(n;\, p_{\mu,K})]^T$, and $\boldsymbol{\mathcal{S}}_\mu(n) := \mathrm{diag}[t(n;\, p_{\mu,1}), \ldots, t(n;\, p_{\mu,K})]$ denotes a diagonal matrix.

If we consider each subchannel as a transmit antenna, the system modeled in (2) can be thought of as a multiantenna system with K transmit antennas and one receive antenna considered for DUSTM in [5, eq. (23)]. Targeting differential reception with diversity, we thus choose DUSTM to generate $\boldsymbol{\mathcal{S}}_\mu(n)$ as a whole. Specifically, letting $\mathcal{V}$ be a finite group of $K \times K$ unitary and diagonal matrices, the generation of $\boldsymbol{\mathcal{S}}_\mu(n)$ follows the recursion:

$$\boldsymbol{\mathcal{S}}_\mu(n) = \begin{cases} \mathbf{V}_\mu(n)\boldsymbol{\mathcal{S}}_\mu(n-1), & \text{if } n \geq 1 \\ \mathbf{I}_K, & \text{if } n = 0 \end{cases} \tag{3}$$

where the $K \times K$ matrix $\mathbf{V}_\mu(n) \in \mathcal{V}$ conveys the information, and is chosen to correspond one-to-one with the nth information symbol; and $\mathbf{I}_K$ is the $K \times K$ identity matrix. Let R be the transmission rate defined as the number of bits transmitted per subchannel use. In order to support rate R, we need to design $\mathcal{V}$ with cardinality $|\mathcal{V}| = 2^{RK}$.

Note that (2) and (3) are the counterparts of [5, eqs. (18) and (19)], respectively. Following similar steps used to derive [5, eq. (22)], the decision statistics for $\mathbf{V}_\mu(n)$ are, thus, given by

$$\mathbf{y}_\mu(n) = \mathbf{V}_\mu(n)\mathbf{y}_\mu(n-1) + \tilde{\mathbf{w}}_\mu(n) \tag{4}$$

where $\tilde{\mathbf{w}}_\mu(n) := \mathbf{w}_\mu(n) - \mathbf{V}_\mu(n)\mathbf{w}_\mu(n-1)$ is the counterpart of the noise term in [5, eq. (22)], and is complex Gaussian distributed with correlation matrix $2N_0\mathbf{I}_K$, as proved in [5]. Using (4), the maximum-likelihood (ML) detector can be represented by

$$\hat{\mathbf{V}}_\mu(n) = \arg\min_{\mathbf{V}\in\mathcal{V}} \|\mathbf{y}_\mu(n) - \mathbf{V}\mathbf{y}_\mu(n-1)\| \tag{5}$$

where $\|\cdot\|$ denotes the Frobenius norm. Clearly, $\mathbf{V}_\mu(n)$ can be decoded from $\mathbf{y}_\mu(n)$ and $\mathbf{y}_\mu(n-1)$ without channel state information. Since $|\mathcal{V}| = 2^{RK}$, the complexity of optimal ML decoding is exponential in K. However, based on a lattice reduction, suboptimal decoding is possible with polynomial complexity in K, using the algorithm in [2].

Having described B-DOFDM and having linked it to DUSTM, we proceed to specify its parameters.

III. Parameter Specification

Targeting maximum diversity advantage and minimum decoding complexity, while improving coding advantage as much as possible, we design the parameters K and $\mathcal{I}_\mu$s, as well as the matrix group $\mathcal{V}$. Here are the following assumptions.

A1) the channel taps $h(l)$s are independent and identically distributed, zero-mean complex Gaussian variables with variance $\sigma_h^2 := 1/(L+1)$;

A2) the signal-to-noise ratio (SNR) is high.

We first analyze the pairwise error probability (PEP) $P[\mathbf{V}_\mu(n) \rightarrow \mathbf{V}'_\mu(n)]$ that is defined as the probability that the ML detector (5) incorrectly decodes $\mathbf{V}'_\mu(n)$ as $\mathbf{V}_\mu(n)$, when $\mathcal{V}$ consists of only these two matrices. Recalling (4), the conditional PEP is upper bounded by [8]

$$P[\mathbf{V}_\mu(n) \rightarrow \mathbf{V}'_\mu(n)|\mathbf{y}_\mu(n-1)] \le \exp\left[-\frac{\|[\mathbf{V}'_\mu(n) - \mathbf{V}_\mu(n)]\mathbf{y}_\mu(n-1)\|^2}{8N_0}\right]. \tag{6}$$

Defining the $K\times(L+1)$ Vandermonde matrix $\mathbf{U}_\mu$ with (k, q)th entry $[\mathbf{U}_\mu]_{k,q} = \exp[-j2\pi p_{\mu,k}(q-1)/P]$, we can express $\tilde{\mathbf{h}}_\mu = \mathbf{U}_\mu\mathbf{h}$. Based on A2), we ignore the noise term in (2) to relate $\mathbf{y}_\mu(n-1)$ to $\mathbf{h}$, via $\mathbf{y}_\mu(n-1) = \boldsymbol{\mathcal{S}}_\mu(n-1)\mathbf{U}_\mu\mathbf{h}$. Therefore, (6) can be re-rewritten as

$$P[\mathbf{V}_\mu(n) \rightarrow \mathbf{V}'_\mu(n)|\mathbf{h}] \le \exp\left[-\frac{\|[\mathbf{V}'_\mu(n) - \mathbf{V}_\mu(n)]\boldsymbol{\mathcal{S}}_\mu(n-1)\mathbf{U}_\mu\mathbf{h}\|^2}{8N_0}\right]. \tag{7}$$

We are interested in the average PEP over all possible channel realizations, which is then used to obtain two important performance metrics: the diversity advantage, and the coding advantage [8]. Starting from (7) and based on A1), the derivation of the average PEP has been well documented in [8], [9], and will be omitted due to the lack of space. Instead, we present the resulting expressions of diversity advantage and coding advantage as follows.

Let us define the $(L+1)\times(L+1)$ matrix $\mathbf{A}_e := \mathbf{U}_\mu^{\mathcal{H}}[\mathbf{V}'_\mu(n) - \mathbf{V}_\mu(n)]^{\mathcal{H}}[\mathbf{V}'_\mu(n) - \mathbf{V}_\mu(n)]\mathbf{U}_\mu$, and denote its nonzero eigenvalues as $\lambda_{e,l}$, $l = 0, \ldots, \mathrm{rank}(\mathbf{A}_e) - 1$, where $\mathrm{rank}(\mathbf{A}_e)$ stands for the rank of $\mathbf{A}_e$. The diversity and coding advantages for B-DOFDM are respectively given by [9]

$$G_d = \min_{\forall \mathbf{V}_\mu(n)\neq\mathbf{V}'_\mu(n)} \mathrm{rank}(\mathbf{A}_e),$$

$$G_c = \min_{\forall \mathbf{V}_\mu(n)\neq\mathbf{V}'_\mu(n)} \left[\prod_{l=0}^{\mathrm{rank}(\mathbf{A}_e)-1} \sigma_h^2\lambda_{e,l}\right]^{1/\mathrm{rank}(\mathbf{A}_e)} \tag{8}$$

where the minimization is taken over all possible pairwise errors.

We deduce from (8) the maximum diversity advantage $G_d = L+1$ that is achieved if and only if $\mathbf{A}_e$ has full rank $L+1$, for all possible pairwise errors. To ensure $\mathrm{rank}(\mathbf{A}_e) = L+1$, checking the dimensionality of $\mathbf{U}_\mu$ reveals that one necessary condition is $K \ge L+1$. On the other hand, as mentioned in Section II, the decoding complexity of B-OFDM grows as K increases. Therefore, it is meaningful to design D1) $K = L+1$ for minimum decoding complexity. Define $\boldsymbol{\Delta}_\mu(n) := \mathbf{V}'_\mu(n) - \mathbf{V}_\mu(n)$. Because $\mathbf{U}_\mu$ is Vandermonde, and $\boldsymbol{\Delta}_\mu(n)$ is diagonal, ensuring $\mathrm{rank}(\mathbf{A}_e) = L+1$ is, thus, equivalent to designing $\mathcal{V}$ such that $\boldsymbol{\Delta}_\mu(n)$ has full rank for all possible pairwise errors.

Suppose now that $\mathbf{A}_e$ has full rank $L+1$, and $K = L+1$ has been chosen. Examining (8) reveals that G_c depends on the minimum determinant of $\mathbf{A}_e$ over all possible pairwise errors. By the definition of $\mathbf{A}_e$, the determinant of $\mathbf{A}_e$ is given by

$$\det(\mathbf{A}_e) = \det(\mathbf{U}_\mu^{\mathcal{H}}\mathbf{U}_\mu) \cdot \det[\boldsymbol{\Delta}_\mu^{\mathcal{H}}(n)\boldsymbol{\Delta}_\mu(n)]$$

where it is implied that maximizing G_c is equivalent to maximizing both $\det(\mathbf{U}_\mu^{\mathcal{H}}\mathbf{U}_\mu)$ and $\det[\boldsymbol{\Delta}_\mu^{\mathcal{H}}(n)\boldsymbol{\Delta}_\mu(n)]$, independently.

By the definition of $\mathbf{U}_\mu$, direct calculation yields the matrix trace $\mathrm{tr}(\mathbf{U}_\mu^{\mathcal{H}}\mathbf{U}_\mu) = (L+1)^2$. Because $\mathbf{U}_\mu$ has full rank, $\mathbf{U}_\mu^{\mathcal{H}}\mathbf{U}_\mu$ is positive definitive. Thus, it follows that $\det(\mathbf{U}_\mu^{\mathcal{H}}\mathbf{U}_\mu) \le (L+1)^{L+1}$, where the equality is achieved if we design D2)

$$\mathcal{I}_\mu = \{\mu, M+\mu, 2M+\mu, \ldots, LM+\mu\}$$
$$\mu = 0, \ldots, M-1. \tag{9}$$

In order to improve coding advantage as much as possible, we henceforth adopt (9) as our subchannel grouping scheme. It is important to point out that with (9), $\mathbf{U}_\mu$ becomes (scaled) unitary. Under A1), the subchannels within each $\mathcal{I}_\mu$ are thus statistically independent, which, intuitively thinking, should yield the highest coding advantage. Note that the design (9) corresponds to assigning P subchannels periodically interleaved to M OFDM subsystems modeled as in (2).

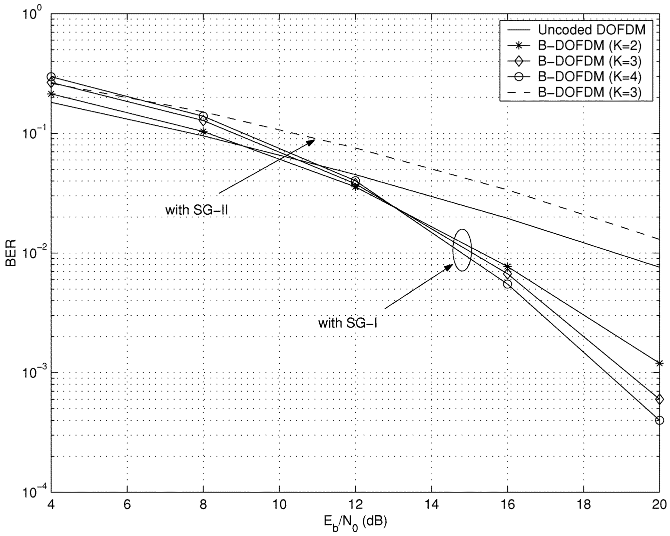

Fig. 1. B-DOFDM versus uncoded DOFDM ($R = 2$).

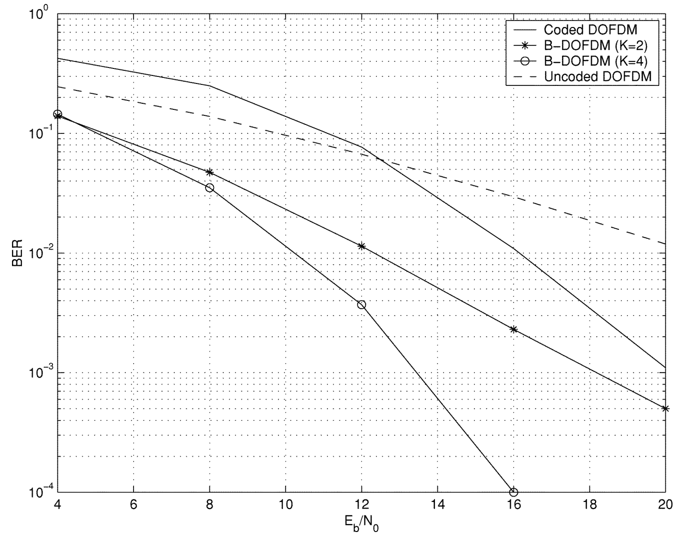

Fig. 2. B-DOFDM versus coded DOFDM ($R = 1$).

So far, we have specified the system parameters K and $\mathcal{I}_\mu$s in D1) and D2). In addition, our analysis suggested the following design criteria for $\mathcal{V}$:

C1) (Diversity advantage criterion) Design $\mathcal{V}$ such that: $\forall\, \mathbf{V}_\mu(n) \neq \mathbf{V}'_\mu(n) \in \mathcal{V}$, $\mathbf{\Delta}_\mu(n)$ has full rank;

C2) (Coding advantage criterion) Design $\mathcal{V}$ such that: $\forall\, \mathbf{V}_\mu(n) \neq \mathbf{V}'_\mu(n) \in \mathcal{V}$, the minimum value of $\det[\mathbf{\Delta}_\mu^{\mathcal{H}}(n)\mathbf{\Delta}_\mu(n)]$ is maximized.

Because the matrix $\mathbf{\Delta}_\mu(n)$ is diagonal, we note that if the criterion C2) is satisfied, then C1) will be automatically satisfied. Thus, we only need to consider design criterion C2). Based on C2), the construction of the unitary and diagonal matrix group $\mathcal{V}$ has been detailed in [5], as summarized in [5, Tab. I]. This letter will not pursue the design of $\mathcal{V}$; but instead, we will use the results from [5, Tab. I].

IV. Simulations

In addition to theoretical analysis, we will now use two examples to simulate the performance of our design. All curves are averaged over 250 random channel realizations.

Example 1: We compare our B-DOFDM to the uncoded DOFDM with parameters: $P = 48$, $R = 2$, and $L = 2$. The $L + 1 = 3$ channel taps are generated under A1) in Section III. Differential QPSK (DQPSK) modulation is employed in DOFDM. In practice, the exact value of the channel order L is usually unknown at the transmitter. Instead, one may know an upper-bound $\overline{L}$, and a lower bound $\underline{L}$ of L; i.e., $\underline{L} \leq L \leq \overline{L}$. Supposing $\underline{L} = 1$ and $\overline{L} = 3$, we simulate three B-OFDM systems by choosing parameter triplets: $(L, K, M) = (1, 2, 24)$, $(2, 3, 16)$, $(3, 4, 12)$, with the subchannel grouping $\mathcal{I}_\mu$ of (9). In addition, we simulate B-DOFDM with $(L, K, M) = (2, 3, 16)$ using a subchannel grouping scheme specified by $\overline{\mathcal{I}}_\mu = \{\mu M, \mu M+1, \ldots, \mu M+M-1\}$. It is observed in Fig. 1 that the three B-DOFDM systems with the subchannel grouping $\mathcal{I}_\mu$ (denoted as SG-I in Fig. 1) outperform the uncoded DOFDM considerably, especially at high SNR. However, the performance of B-DOFDM with the subchannel grouping $\overline{\mathcal{I}}_\mu$ (denoted as SG-II in Fig. 1) is the worst among all, thereby justifying our subchannel grouping design in Section III. It is also observed that the slope of the bit-error-rate (BER) curve of B-DOFDM with $K = 3$ is almost the same as that with $K = 4$, which confirms our argument that the maximum diversity advantage in OFDM is $L + 1 = 3$ and does not depend on the choice of K as long as $K \geq (L + 1)$. The results in Fig. 1 also suggest that if the exact L is unknown, one should design B-DOFDM for its upper bound.

It is important to point out that diversity and coding advantages only provide a good approximation of the BER performance at sufficiently high SNR. In other words, the BER curves in Fig. 1 may not exactly reflect the diversity and coding advantages predicted by (8).

Example 2: We equip DOFDM with channel coding and compare it to B-DOFDM in HIPERLAN/2 [3]. As specified in [3], $P = 48$ is chosen. The channels are generated according to Channel Model A ($L = 8$) specified in [4], where each channel tap varies according to the Jakes model with Doppler shifts corresponding to a typical terminal speed 3 m/s, and a carrier frequency of 5.2 GHz. The coded DOFDM employs DQPSK as modulation, together with conventional coding with rate 1/2, followed by interleaving as specified in [3]. Thus, the overall transmission rate is $R = 1$. Two B-DOFDM systems with parameter pairs $(K, M) = (2, 24)$ and (4, 12) are simulated using the subchannel grouping $\mathcal{I}_\mu$ of (9). The DUSTM designs with $R = 1$ in [5, Table I] are used. To investigate the performance gain from channel coding, the performance of the uncoded DOFDM is also simulated. From the results in Fig. 2, we see that the coded DOFDM improves performance over the uncoded one, but it is outperformed by B-DOFDM with $K = 4$. It is noted that the channel model in this example does not satisfy A1), and our chosen parameters do not follow the specifications in Section III exactly. However, our design still yields good performance, which speaks for its robustness.

V. Conclusion

This letter developed a novel block differentially encoded OFDM system. Without channel knowledge, the proposed system was shown capable of achieving maximum diversity advantage and high coding advantage, with low decoding complexity. The merits of our scheme were confirmed by simulations.

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