

# High-Performance Cooperative Demodulation With Decode-and-Forward Relays

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**Abstract**—Cooperative communication systems using various relay strategies can achieve spatial diversity gains, enhance coverage, and potentially increase capacity. For the practically attractive decode-and-forward (DF) relay strategy, we derive a high-performance low-complexity coherent demodulator at the destination in the form of a weighted combiner. The weights are selected adaptively to account for the quality of both source-relay-destination and source-destination links. Analysis proves that the novel coherent demodulator can achieve the maximum possible diversity, regardless of the underlying constellation. Its error performance tightly bounds that of maximum-likelihood (ML) demodulation, which provably quantifies the diversity gain of ML detection with DF relaying. Simulations corroborate the analysis and compare the performance of the novel decoder with existing diversity-achieving strategies including analog amplify-and-forward and selective-relaying.

**Index Terms**—Diversity gain, decode-and-forward (DF), full diversity, relay channel, relaying protocol, user cooperation.

## I. INTRODUCTION

THE proliferation of wireless terminals has naturally led to cooperative (also known as relay) links whereby communicators benefit from their neighbors [4], [8], [9], [11], [13], [14]. As with multi-input multi-output (MIMO) systems, where multiple collocated antennas are deployed at the transmit- or receive-ends, a main objective with single-antenna cooperating terminals is also to enable spatial diversity. Beyond MIMO with collocated antennas, relay transmissions offer resilience against shadowing and enhanced coverage. In such cooperative links, the message sent by the source arrives at the destination through diverse paths: one directly from the source node and others through *relay* nodes. Performance will, thus, depend on the number of cooperating relay nodes as well as the processing operations at both relays and destination. If properly designed, cooperative networks can achieve diversity order up

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to the number of diverse paths (what we, henceforth, refer to as full diversity). To put our contribution in context, we next review the existing relay strategies, detectors, and their error performance in terms of diversity.

If relays can afford analog processing, they can amplify-and-forward (AF) the source waveform to the destination. Unfortunately, analog AF transceivers require expensive RF chains to mitigate the existing coupling effects. This motivates digital processing at relay nodes to sample and store the source waveform digitally before retransmission. Because such relays forward the decoded message to the destination, they are known as decode-and-forward (DF) relays. A third option is to have relays forward only those correctly decoded messages, in which case we say that they implement selective-relaying (SR). Use of SR presumes incorporation of, e.g., cyclic redundancy check (CRC) codes from a higher layer in order to detect errors.

When it comes to performance analysis, the symbol-error-probability (SEP) for general AF links has been reported in [12], where it is also proved that full diversity is achievable with AF; see also [10]. In SR-links, a weighted superposition of the source and relay signals arriving at the destination can be formed using maximum-ratio-combining (MRC), which also collects the maximum available diversity order. Unfortunately, with DF relays, MRC does not offer a full diversity achieving receiver [2]. To the best of our knowledge, no efficient demodulation technique is available, which at affordable complexity, can provably collect full diversity, regardless of the constellation, when using the most practical relay strategy, i.e., DF. And this is the gap which this paper aspires to fill.

### A. Assumptions and Related Work

Being the most practical relay strategy, DF has drawn a great deal of interest recently. Although it has been shown that relay transmissions equipped with error control codes can improve error performance considerably [6], for simplicity and due to space limitations, we will focus on uncoded DF-streams and symbol-by-symbol demodulators; that is, in the remainder of this paper, DF will refer to uncoded-DF.

Our goal is to show that, through a suitable demodulator, full diversity can be achieved with DF links. This result was also alluded to in [13], where a maximum-likelihood (ML) optimal detector has been presented only for binary phase shift keying (BPSK). As recognized in [13], performance analysis of such a detector is quite complicated, which prevents one from any quantitative diversity assessment, especially for general constellations. For this reason, a suboptimum combiner termed as  $\lambda$ -MRC was derived in [13]. Such a combiner

facilitates performance analysis and leads to closed-form expressions for the probability of error. Simulations illustrate that  $\lambda$ -MRC performs close to ML, but since the combiner parameter  $\lambda$  has not been specified analytically, full-diversity claims cannot be proved. In [4], a piece-wise linear (PL) near-ML decoder has been derived for coherent and noncoherent demodulation of binary modulations only. Exploiting the *average* bit-error probability (BEP) of the source-relay link that can be made available to the destination, this PL approximation leads to closed-form bounds on error performance, concluding that with  $M - 1$  parallel relays, the diversity order in coherent operation is at the most  $(M/2) + 1$  for  $M$  even, and  $(M + 1)/2$  for  $M$  odd.

Our idea in this paper is to exploit the knowledge of the *instantaneous* (per fading realization) BEP of the source-relay link at the destination, and derive a novel combiner capable of collecting full diversity with DF for any coherent modulation and in any general (possibly multibranch and multihop) relay links. Such a combiner that we term *cooperative* MRC (C-MRC), will be shown to provide a tight lower bound on the performance of ML detection; meaning that full diversity claims we will establish for this combiner carry over to the ML case too. Unlike ML and PL demodulators, it will turn out that C-MRC offers a high-performance demodulator with low-complexity, *regardless* of the underlying constellation.

The rest of this paper is organized as follows. In Section II, the relay model will be stated for a single cooperating terminal, the C-MRC detector will be derived, and the diversity gain of its asymptotic error probability will be analyzed. Section III will extend these results to general cooperation setups with multiple cooperating branches and multiple cooperating hops per branch. In Section IV, simulations for several settings will corroborate performance claims. Finally, conclusions will be drawn in Section V.

Notation:  $(\cdot)^*$  denotes conjugation;  $\mathcal{CN}(0, \sigma^2)$  denotes the circular symmetric complex Gaussian distribution with zero mean and variance  $\sigma^2$ ;  $\text{Re}\{z\}$  denotes the real part of a complex number  $z$ ; for a random variable  $\gamma$ ,  $p(\gamma)$  is its probability density function; and  $\bar{\gamma} = E\{\gamma\}$  is its mean.

## II. SINGLE RELAY COOPERATION

### A. System Model

With reference to Fig. 1, let us first consider the simplest model in which only one relay ( $R$ ) helps the source ( $S$ ) to communicate with the destination ( $D$ ). The source  $S$  broadcasts the symbols to  $D$  that are also received by  $R$ , which forwards the decoded symbols to  $D$ . Because signals from  $S$  and  $R$  arrive through two different paths, if fading is present, one can (at least in principle) design a detector capable of collecting diversity up to the order of two.

In practice, terminals cannot transmit and receive at the same time and over the same frequency band; however,  $S$  and  $R$  can transmit over orthogonal channels. In this paper, we suppose a time division duplex (TDD) mode, where data transmission consists of two slots. In *Slot I*,  $S$  broadcasts modulated symbol

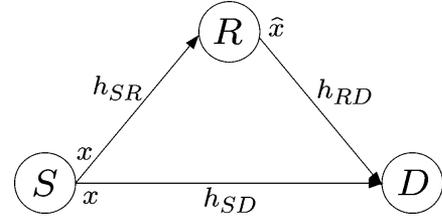


Fig. 1. Block model for a single-relay cooperative system.

$x$  with average power  $P_x$ . The received symbols at  $R$  and  $D$  are

$$y_{SR} = h_{SR}x + z_{SR} \quad (1)$$

$$y_{SD} = h_{SD}x + z_{SD} \quad (2)$$

where  $h_{SR}$  and  $h_{SD}$  denote the fading coefficients from  $S$  to  $R$  and  $D$ , modeled as  $h_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ ,  $h_{SD} \sim \mathcal{CN}(0, \sigma_{SD}^2)$ , with  $\sigma_{SR}^2 := E\{|h_{SR}|^2\}$  and  $\sigma_{SD}^2 := E\{|h_{SD}|^2\}$ , respectively. Without loss of generality, we assume that the noise terms  $z_{SR}$  and  $z_{SD}$  have equal variances  $N_0$  and are modeled as  $z_{SR} \sim \mathcal{CN}(0, N_0)$ ,  $z_{SD} \sim \mathcal{CN}(0, N_0)$ . In the uncoded DF protocol,  $R$  performs coherent ML demodulation

$$\hat{x}_R = \arg \min_{x \in \mathcal{A}_x} |y_{SR} - h_{SR}x|^2 \quad (3)$$

where  $|\mathcal{A}_x| = \Theta$  denotes the cardinality (size) of the  $\Theta$ -ary constellation. Then, the detected symbol  $\hat{x}_R$  is re-modulated and subsequently transmitted during *Slot II* with the same average power  $P_x$ . The received symbol at  $D$  is

$$y_{RD} = h_{RD}\hat{x}_R + z_{RD} \quad (4)$$

where  $\hat{x}_R$  is the remodulated symbol at  $R$ ,  $h_{RD}$  denotes the channel coefficient from  $R$  to  $D$ ,  $h_{RD} \sim \mathcal{CN}(0, \sigma_{RD}^2)$  with  $\sigma_{RD}^2 := E\{|h_{RD}|^2\}$ , and  $z_{RD} \sim \mathcal{CN}(0, N_0)$  denotes the noise at  $D$ .

### B. Relay Link Analysis

Define the instantaneous signal-to-noise-ratio (SNR) at links  $S - R$ ,  $R - D$  and  $S - D$  as  $\gamma_{SR} := |h_{SR}|^2\bar{\gamma}$ ,  $\gamma_{RD} := |h_{RD}|^2\bar{\gamma}$ , and  $\gamma_{SD} := |h_{SD}|^2\bar{\gamma}$ , respectively, with  $\bar{\gamma} = P_x/N_0$  denoting average SNR. Notice also that errors at the destination occur either when the  $S - R$  transmission is received correctly and the  $R - D$  transmission is received in error, or when the  $S - R$  transmission is received in error and the  $R - D$  transmission is received correctly. Hence, for any given modulation, the two-hop  $S - R - D$  channel has end-to-end BEP given by

$$P_{\text{eq}}^b(\gamma_{SR}, \gamma_{RD}) = [1 - P_{SR}^b(\gamma_{SR})]P_{RD}^b(\gamma_{RD}) + [1 - P_{RD}^b(\gamma_{RD})]P_{SR}^b(\gamma_{SR}) \quad (5)$$

where  $P_{SR}^b(\gamma_{SR})$  and  $P_{RD}^b(\gamma_{RD})$  are the conditional BEPs at both hops which we assume available at  $D$  [5]. Due to possible errors at the relay, the  $S - R - D$  channel is clearly nonlinear and non-Gaussian. However, one can think of the BEP in (5) as the error probability at the receiver of an *equivalent* one-hop AWGN link whose output SNR  $\gamma_{\text{eq}}$  is

$$\gamma_{\text{eq}} := \frac{1}{\alpha} \{Q^{-1}[P_{\text{eq}}^b(\gamma_{SR}, \gamma_{RD})]\}^2 \quad (6)$$

where  $Q(x) := (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2)dt$ , and  $\alpha$  is a constant that depends on the underlying constellation; e.g.,  $\alpha = 2$  for BPSK. This equivalent one-hop SNR  $\gamma_{\text{eq}}$  jointly accounts for the quality of both  $S - R$  and  $R - D$  links, and will guide the design of our novel demodulator. To this end, we establish the following property for  $\gamma_{\text{eq}}$  (see Appendix A for the proof).

*Property 1: Upon defining  $\gamma_{\min} := \min\{\gamma_{RD}, \gamma_{SR}\}$ , it holds that  $\gamma_{\text{eq}}$  in (6) is bounded by*

$$\gamma_{\min} - \frac{3.24}{\alpha} < \gamma_{\text{eq}} \leq \gamma_{\min}. \quad (7)$$

Property 1 upper-bounds the end-to-end equivalent SNR by the minimum of its single-hop SNRs. This is intuitively expected, because the BEP over the aggregate  $S - R - D$  link cannot exceed that of  $R - D$  or  $S - R$ . On the other hand, the lower bound in (7) implies that for a relatively large  $\gamma_{\min}$ , the constant  $3.24/\alpha$  can be negligible, showing that indeed  $\gamma_{\min}$  can offer a tight approximation to  $\gamma_{\text{eq}}$ . Property 1 will come handy in our subsequent asymptotic analyses.

As recognized by [12] and [17] in the context of AF, knowledge of the  $S - R$  link quality ( $\gamma_{SR}$ ) at  $D$  is possible by sending pilot symbols through  $R$ . In regenerative schemes such as DF, one can acquire the  $S - R$  link quality by sending a pilot from  $S$  for the relay to estimate the  $S - R$  channel, and forward it to the destination via a second pilot whose power is scaled according to the estimated channel coefficient. At the receiver, as in AF, one again recovers the product of the two  $S - R$  and  $R - D$  fading coefficients. Without knowledge of the  $S - R$  link at  $D$ , SR requires bandwidth consuming CRC codes to ensure perfect error detection at the relay.

### C. Cooperative MRC

Consider combining the received  $y_{SD}$  and  $y_{RD}$  at the destination to obtain

$$\hat{x}_D = \arg \min_{x \in \mathcal{A}_x} |w_{SD}y_{SD} + w_{RD}y_{RD}(w_{SD}h_{SD} + w_{RD}h_{RD})x|^2 \quad (8)$$

where weights  $w_{SD}$  and  $w_{RD}$  are functions of  $h_{SD}$ ,  $h_{SR}$ , and  $h_{RD}$  to be specified later. In a collocated multiantenna setup, MRC employs weights  $w_{SD} = h_{SD}^*$  and  $w_{RD} = h_{RD}^*$ , and is known to maximize the SNR at the combiner output. This would also be the optimal choice in our context if  $\hat{x}_R = x$ . However, since the fading link  $S - R$  causes detection errors at the relay, performance of the standard MRC is far from being optimal.

Motivated by this, we fix  $w_{SD} = h_{SD}^*$  to maximize  $\gamma_{SD}$ , and seek a weight  $w_{RD}$  to maximize the equivalent SNR  $\gamma_{\text{eq}}$  in the link  $S - R - D$ , instead of  $R - D$  alone. These considerations lead to the choice

$$w_{RD}(h_{SR}, h_{RD}) = \frac{\gamma_{\text{eq}}}{\gamma_{RD}} h_{RD}^*. \quad (9)$$

Combiner (8) with weights  $w_{SD} = h_{SD}^*$  and  $w_{RD}$  as in (9) constitutes what we term cooperative MRC (C-MRC). Consider the product  $w_{RD}h_{RD}$ , and define  $|h_{\text{eq}}|^2 := \gamma_{\text{eq}}/\bar{\gamma}$ . If one opts for MRC,  $w_{RD} = h_{RD}^*$  and thus  $w_{RD}h_{RD} = |h_{RD}|^2$ . However,

the weight (9) modifies the last product to  $w_{RD}h_{RD} = |h_{\text{eq}}|^2$ , which now jointly considers the  $S - R$  and  $R - D$  links. We also note from Property 1 that  $\gamma_{\text{eq}} \approx \gamma_{\min}$  at sufficiently high SNR. From this approximation,  $w_{RD}$  can be seen as either part of a conventional (one-hop) MRC (when  $\gamma_{SR} > \gamma_{RD}$ ) or as a *two-hop* weighted combiner (when  $\gamma_{SR} < \gamma_{RD}$ ). In the first case, because the link  $S - R$  is better than  $R - D$ , the combiner places full confidence to the arriving symbols from  $R$ . In the second case,  $S - R$  is a weak link and thus the confidence is placed on the link  $S - R - D$  by  $|h_{\text{eq}}|^2$ , instead of the link  $R - D$  alone by  $|h_{RD}|^2$ .

Our C-MRC is reminiscent of the  $\lambda$ -MRC in [14], if we select  $w_{RD} = \lambda h_{RD}^*$  with  $0 \leq \lambda \leq 1$ . Recall though that in lieu of a closed-form, the optimum  $\lambda$  in [14] is found through numerical search. Our closed-form in (9) will allow for analytical BEP evaluation. Interestingly, our simulations will show that (9) is intimately close to the optimum  $\lambda$  obtained by [14].

Relative to ML, the C-MRC defined by (8) and (9) is sub-optimum. To confirm the latter, it suffices to write down the ML coherent detector, which for BPSK takes a relatively simple form

$$\hat{x}_D^{ML} = \arg \max_{x \in \mathcal{A}_x} \left\{ \frac{1 - P_{SR}(\gamma_{SR})}{2\pi N_0} \times \exp \left[ -\frac{|y_{SD} - h_{SD}x|^2 + |y_{RD} - h_{RD}x|^2}{2N_0} \right] + \frac{P_{SR}(\gamma_{SR})}{2\pi N_0} \exp \left[ -\frac{|y_{SD} - h_{SD}x|^2 + |y_{RD} + h_{RD}x|^2}{2N_0} \right] \right\} \quad (10)$$

where  $|\mathcal{A}_x| = 2$ . An implementation of the BPSK demodulator in (10) can be found in [14]. Clearly, implementing for higher order constellations becomes prohibitively complex. Moreover, asymptotic analysis of (10) is very complicated, which prevents one from assessing ML performance [4], [14]. Assuming that the average SNR of the  $S - R$  link is available at the destination, the PL-ML approximation was advocated in [4] to overcome this problem.

### D. BEP of DF Using BPSK

Although the C-MRC in (8) is suitable for any  $\Theta$ -ary constellation, for clarity in exposition, we will first analyze its performance for BPSK. Extensions to any general constellation will be discussed in Section II-F. With BPSK,  $\hat{x}_R$  at the relay can only take one of two values:  $\hat{x}_R = x$  or  $\hat{x}_R = -x$ . For each value, the C-MRC output is

$$y_D = w_{RD}y_{RD} + w_{SD}y_{SD} = \begin{cases} (w_{SD}h_{SD} + w_{RD}h_{RD})x + w_{RD}z_{RD} + w_{SD}z_{SD}, & \text{if } \hat{x}_R = x, \\ (w_{SD}h_{SD} - w_{RD}h_{RD})x + w_{RD}z_{RD} + w_{SD}z_{SD}, & \text{if } \hat{x}_R = -x. \end{cases}$$

Since BPSK is real-valued, it suffices to consider only the real part  $y := \text{Re}\{y_D\}$ , which is a real Gaussian random variable with zero mean and variance  $\sigma^2 = (|w_{RD}|^2 + |w_{SD}|^2)N_0/2$ .

The BEP can be expressed in terms of  $\gamma_{SR}, h_{SD}, h_{RD}$  as

$$P^b(\gamma_{SR}, h_{SD}, h_{RD}) = [1 - P_{SR}^b(\gamma_{SR})] \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left\{-\frac{[y + (w_{SD}h_{SD} + w_{RD}h_{RD})\sqrt{P_x}]^2}{2\sigma^2}\right\} dy + P_{SR}^b(\gamma_{SR}) \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left\{-\frac{[y + (w_{SD}h_{SD} - w_{RD}h_{RD})\sqrt{P_x}]^2}{2\sigma^2}\right\} dy. \quad (11)$$

The suboptimality of C-MRC relative to ML shows up in the second integral of (11), in which the integration region may be different from that of ML (with probability  $P_{SR}^b(\gamma_{SR})$ ), whenever  $(w_{SD}h_{SD} - w_{RD}h_{RD}) < 0$ . The effect of this will be pronounced when both  $w_{RD}h_{RD}$  and  $P_{SR}^b(\gamma_{SR})$  are large, which happens if the link  $R - D$  is strong compared to  $S - R$  and  $S - D$ . It is to be noted that this suboptimality becomes negligible as error probability in the link  $S - R$  decreases. We will see that by judiciously selecting  $w_{RD}$  it will become possible to reduce error probability in this second integral. Plugging  $w_{SD} = h_{SD}^*$  and (9) into (11), we can rewrite compactly (11) in terms of the  $Q$ -function as

$$P^b(\gamma_{SR}, \gamma_{SD}, \gamma_{RD}) = [1 - P_{SR}^b(\gamma_{SR})] Q\left[\frac{\sqrt{2}(\gamma_{SD} + \gamma_{eq})}{\sqrt{\gamma_{SD} + \gamma_{eq}^2/\gamma_{RD}}}\right] + P_{SR}^b(\gamma_{SR}) Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{eq})}{\sqrt{\gamma_{SD} + \gamma_{eq}^2/\gamma_{RD}}}\right] \quad (12)$$

Taking the expectation over the instantaneous SNRs, (12) yields the average BEP, which we will analyze in Section II-E for sufficiently large SNR values.

### E. Diversity Analysis of DF Relaying

Diversity gain (diversity order)  $G_d$  is defined as the negative exponent of the average BEP plotted in a log-log scale when the average SNR tends to infinity; that is

$$P^b \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} (G_c \bar{\gamma})^{-G_d} \quad (13)$$

where  $G_c$  denotes the coding gain. As mentioned in the introduction, we assume uncoded DF-streams and symbol-by-symbol demodulation; so, in this case,  $G_c$  depends solely on the constellation distances and the weights in (8). However, our goal in this section is to assess the diversity order of C-MRC by establishing bounds on the average BEP obtained from (12). To this end, we first notice that  $\gamma_{eq} \leq \gamma_{RD}$ , which upper-bounds (12) as

$$P^b(\gamma_{SR}, \gamma_{SD}, \gamma_{RD}) \leq Q\left[\sqrt{2(\gamma_{SD} + \gamma_{eq})}\right] + Q\left[\sqrt{2\gamma_{SR}}\right] Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{eq})}{\sqrt{\gamma_{SD} + \gamma_{eq}}}\right]. \quad (14)$$

Because a sum is dominated by the term with the lowest diversity exponent, we need to prove that both terms in the right-hand side of (14) decay with the same exponent (diversity order), which here equals two. Because these two terms affect diversity through distinct means, we will analyze them separately. Let us start by defining  $P_1^b(\gamma_{SR}, \gamma_{SD}, \gamma_{RD}) := Q\left[\sqrt{2(\gamma_{SD} + \gamma_{eq})}\right]$  and  $(\bar{\gamma}_{SR}, \bar{\gamma}_{SD}, \bar{\gamma}_{RD}) := (\sigma_{SR}^2 \bar{\gamma}, \sigma_{SD}^2 \bar{\gamma}, \sigma_{RD}^2 \bar{\gamma})$ . Next, we invoke Property 1, use the Chernoff bound, and take expectation over the three instantaneous SNRs to bound  $P_1^b$  as

$$P_1^b \leq \frac{\exp(1.62)(\bar{\gamma}_{SR} + \bar{\gamma}_{RD})}{2(\bar{\gamma}_{SD} + 1)(\bar{\gamma}_{SR}\bar{\gamma}_{RD} + \bar{\gamma}_{SR} + \bar{\gamma}_{RD})} \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} (k_1 \bar{\gamma})^{-2} \quad (15)$$

where  $k_1$  is a constant, which depends on  $\sigma_{SR}^2, \sigma_{SD}^2, \sigma_{RD}^2$ . To appreciate (15), recall that  $(k_1 \bar{\gamma})^{-2}$  is present whenever the relay is forwarding the correct symbol ( $\hat{x}_R = x$ ), which corresponds to the colocated multiantenna scenario, that is capable of collecting full diversity.

Turning back our attention to (14), let us define the second summand in the bound as

$$P_2^b(\gamma_{SR}, \gamma_{SD}, \gamma_{RD}) := Q\left[\sqrt{2\gamma_{SR}}\right] Q\left[\frac{\sqrt{2}(\gamma_{SD} - \gamma_{eq})}{\sqrt{\gamma_{SD} + \gamma_{eq}}}\right]. \quad (16)$$

The next proposition upper bounds  $P_2^b$  (see Appendix B for the proof).

*Proposition 1: The expectation  $E\{P_2^b(\gamma_{SR}, \gamma_{SD}, \gamma_{RD})\} := \tilde{P}_2^b$  can be bounded by a term  $\tilde{P}_2^b$ , which decays with exponent equal to two; that is, with  $k_2$  denoting a constant, we have*

$$P_2^b \leq \tilde{P}_2^b \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} (k_2 \bar{\gamma})^{-2}. \quad (17)$$

Equation (17) together with (15) establish that C-MRC achieves full diversity with DF, when BPSK is used.

One can gain insight about Proposition 1 by inspecting (16), where  $\gamma_{eq}$  is always smaller than  $\gamma_{SR}$ . An increase in  $\gamma_{eq}$  implies an increase of the second Q-function factor, which is mitigated by a decrease in the first Q-function factor in (16). This tradeoff suggests that the use of  $\gamma_{eq}$  may be optimal in C-MRC to jointly account for the quality of both  $S - R$  and  $R - D$  links.

### F. Performance Bounds for General Constellations

Following the steps similar to Section II-E, we analyze here the SEP of C-MRC for higher order constellations. Using the same notational conventions, the SEP can be expressed as the superposition of two terms, now denoted as  $P_1^s$  and  $P_2^s$ . Defining the SEP from  $S$  to  $R$  as  $P_{SR}^s(\gamma_{SR})$ ,  $P_1^s$  corresponds to the case where, with probability  $(1 - P_{SR}^s(\gamma_{SR}))$ , we are combining identical symbols arriving from  $S$  and  $R$ . Error performance of the latter is the same as in a colocated antenna system, and the associated error probability is known to decay with an exponent of the order of two. To ensure full diversity, we also need to show that  $P_2^s$  decays with the same exponent when  $R$  is sending an erroneously decoded symbol. To prove this, we again rely

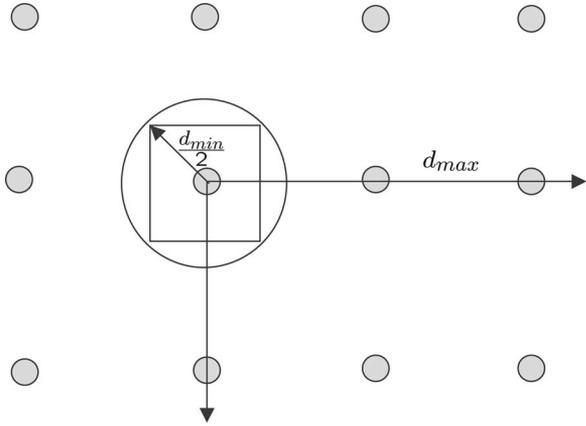


Fig. 2. Worst case in general modulation.

on bounds of  $P_2^s$ , which happens with probability  $P_{SR}^s(\gamma_{SR})$ . With  $d_{\min}$  ( $d_{\max}$ ) denoting the minimum (maximum) Euclidean distance between points in the constellation,  $P_2^s$  can be bounded by the worst case, which corresponds to the decoded symbol at  $R$  being at distance  $d_{\max}$  from the actual symbol sent from  $S$ . In such a case, integration of the associated probability density functions (pdfs) can be simplified by reducing the decision region to a square inscribed within the circle of radius  $d_{\min}$ , as shown in Fig. 2. Finally, invoking the union bound, we can write  $P_2^s \leq 2P_{2,1D}^s$ , where  $P_{2,1D}^s$  is the error probability across one-dimension, and arrive at

$$\begin{aligned}
 & P_2^s(\gamma_{SR}, \gamma_{SD}, \gamma_{RD}) \\
 & \leq 2P_{SR}^s(\gamma_{SR}) \left[ \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \right. \\
 & \times \exp \left\{ -\frac{[y + (w_{SD}h_{SD}d_{\min} - w_{RD}h_{RD}d_{\max})]^2}{2\sigma^2} \right\} dy \\
 & + \int_{-\infty}^{2w_{SD}h_{SD}d_{\min}} \frac{1}{\sqrt{2\pi\sigma^2}} \\
 & \times \exp \left\{ -\frac{[y + (w_{SD}h_{SD}d_{\min} - w_{RD}h_{RD}d_{\max})]^2}{2\sigma^2} \right\} dy \Big] \quad (18)
 \end{aligned}$$

where  $\tilde{d}_{\min} := d_{\min}/(2\sqrt{2})$ ,  $\tilde{d}_{\max} := d_{\max} - \tilde{d}_{\min}$ , and  $\sigma^2 = (|w_{RD}|^2 + |w_{SD}|^2)N_0/2$ .

Knowing the BER conditioned on the instantaneous  $\gamma_{SR}$  for a general  $\Theta$ -ary modulation, call it  $P_{SR}^b$ , we can readily upper bound the corresponding SEP as:  $P_{SR}^s(\gamma_{SR}) \leq (\log_2 \Theta)P_{SR}^b(\gamma_{SR}) = (\log_2 \Theta)Q[\sqrt{\alpha\gamma_{SR}}]$ . Using the latter, we can rewrite (18) compactly as

$$\begin{aligned}
 & P_2^s(\gamma_{SR}, \gamma_{SD}, \gamma_{RD}) \\
 & \leq 4(\log_2 \Theta)Q[\sqrt{\alpha\gamma_{SR}}]Q \left[ \frac{\sqrt{2}(\gamma_{SD} - \beta\gamma_{eq})}{\sqrt{(\gamma_{SD} + \beta\gamma_{eq})}} \right] \quad (19)
 \end{aligned}$$

where  $\beta := \tilde{d}_{\max}/\tilde{d}_{\min} \geq 1$  and the average transmit-SNR has also been bounded by  $\bar{\gamma} \geq \tilde{d}_{\min}^2/N_0$ .

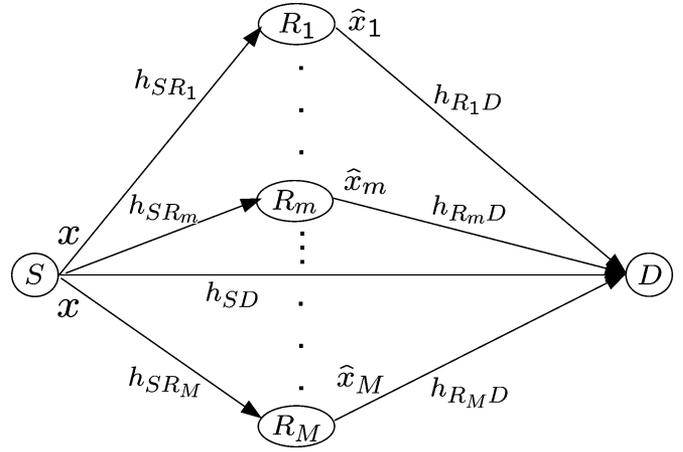


Fig. 3. Block model for a multibranch cooperative system.

We now observe that (19) has a form identical (within a scale) to  $P_2^b(\gamma_{SR}, \gamma_{SD}, \gamma_{RD})$  in Proposition 1. Because high SNR behavior is not affected by the constant  $\beta$ , our performance claims in Proposition 1 are, thus, still valid for (19), which establishes that our C-MRC demodulator enjoys full diversity in DF-based cooperation, regardless of the underlying constellation.

### III. GENERAL COOPERATION SCENARIOS

In this section, we generalize the results of Section II in several directions. For simplicity in exposition, we confine ourselves to BPSK, keeping in mind that as shown in Section II-F, any general constellation can be bounded using worst case distances among points in the constellation, and diversity claims can, thus, be mapped to those stated here for BPSK.

#### A. Multibranch Cooperative Diversity

First, we generalize our model to the multibranch cooperation setup depicted in Fig. 3. We have  $M$  relays  $R_1, \dots, R_M$ , forming  $M$  branches besides the direct link  $S - D$ . Each  $R_m$ ,  $m = 1, \dots, M$ , processes symbols as  $R$  does in the single-relay case. We assume that all relays transmit over mutually orthogonal channels ( $M + 1$  time slots, for example). Letting  $\hat{x}_m$  denote the transmitted symbol from  $R_m$  in Slot  $m + 1$ , the corresponding received symbol at  $D$  is  $y_{R_m D} = h_{R_m D}\hat{x}_m + z_{R_m D}$ . Once again, we use C-MRC to obtain

$$\begin{aligned}
 \hat{x}_D = \arg \min_x & \left| w_{SD}y_{SD} + \sum_{m=1}^M w_{R_m D}y_{R_m D} \right. \\
 & \left. - \left( w_{SD}h_{SD} + \sum_{m=1}^M w_{R_m D}h_{R_m D} \right) x \right|^2 \quad (20)
 \end{aligned}$$

where  $y_{SD} = h_{SD}x + z_{SD}$  is the symbol received directly from  $S$ . We choose  $w_{SD} = h_{SD}^*$  and  $w_{R_m D} = \gamma_{eq_m} h_{R_m D}^*/\gamma_{R_m D}$ , with  $\gamma_{eq_m} := (1/\alpha)\{Q^{-1}[P_{eq}(\gamma_{SR_m}, \gamma_{R_m D})]\}^2$ , and  $P_{eq}(\gamma_{SR_m}, \gamma_{R_m D}) = [1 - P_{SR_m}(\gamma_{SR_m})]P_{R_m D}(\gamma_{R_m D}) + [1 - P_{R_m D}(\gamma_{R_m D})]P_{SR_m}(\gamma_{SR_m})$ .

As with (14), the conditional BEP can be expressed in terms of  $\gamma_{SR_m}$ ,  $\gamma_{SD}$ , and  $\gamma_{R_m D}$  as

$$P^b(\underline{\gamma}) = \sum_{\epsilon=0}^M \sum_{j=1}^{\binom{M}{\epsilon}} \left\{ \prod_{k=1}^{\epsilon} P_{SR_{E_k^j}}^b(\gamma_{SR_{E_k^j}}) \prod_{l=1}^{M-\epsilon} \left[ 1 - P_{SR_{C_l^j}}^b(\gamma_{SR_{C_l^j}}) \right] \right. \\ \left. \times Q \left[ \frac{\sqrt{2}(\gamma_{SD} + \sum_{l=1}^{M-\epsilon} \gamma_{eq_{C_l^j}} - \sum_{k=1}^{\epsilon} \gamma_{eq_{E_k^j}})}{\sqrt{\gamma_{SD} + \sum_{m=1}^M (\gamma_{eq_m})^2 / \gamma_{R_m D}}} \right] \right\}. \quad (21)$$

where  $\epsilon$  is used to index the nodes forwarding erroneously detected symbols  $\hat{x}_R$  to  $D$ ;  $E^j$  ( $C^j$ ) is the set of  $\epsilon$  ( $M - \epsilon$ ) distinct elements from the set  $\{1, 2, \dots, M\}$ ;  $E_k^j$  ( $C_l^j$ ) is the  $k$ th ( $l$ th) element of  $E^j$  ( $C^j$ ),  $E^j \cup C^j = \{1, 2, \dots, M\}$ ,  $E^j \cap C^j = \emptyset$  and for any  $j \neq j'$ ,  $E^j \neq E^{j'}$ ,  $C^j \neq C^{j'}$ ; finally,  $\underline{\gamma} := [\gamma_{SD}, \gamma_{SR_1}, \dots, \gamma_{SR_M}, \gamma_{R_1 D}, \dots, \gamma_{R_M D}]$ .

It is to be noted that  $\epsilon = 0$  corresponds to all relays having decoded correctly the symbol  $x$ , which is again tantamount to a MIMO system with  $(M + 1)$ -collocated antennas where full diversity of order  $M + 1$  is ensured. For this reason, we only need to prove our full diversity claims for  $\epsilon \geq 1$ . Taking expectation over  $\underline{\gamma}$  on both sides of (21), we obtain the average BEP

$$P^b = \int P^b(\underline{\gamma}) p(\underline{\gamma}) d\underline{\gamma} \leq \sum_{\epsilon=0}^M \sum_{j=1}^{\binom{M}{\epsilon}} I(\epsilon, j).$$

Considering for simplicity but without loss of generality (wlog)  $\gamma_{eq_m} \xrightarrow{\bar{\gamma} \rightarrow \infty} \min(\gamma_{SR_m}, \gamma_{R_m D}) := \gamma_{\min_m}$ , we can express  $I(\epsilon, j)$  as

$$I(\epsilon, j) = \int \frac{1}{2^\epsilon} \exp(-\gamma_{SR}^\epsilon) \\ \times Q \left[ \frac{\sqrt{2}(\gamma_{SD} + \gamma_{\min}^c - \gamma_{\min}^\epsilon)}{\sqrt{\gamma_{SD} + \gamma_{\min}^c + \gamma_{\min}^\epsilon}} \right] p(\underline{\gamma}) d\underline{\gamma} \quad (22)$$

where  $\gamma_{SR}^\epsilon := \sum_{k=1}^{\epsilon} \gamma_{SR_{E_k^j}}$ ,  $\gamma_{\min}^c := \sum_{l=1}^{M-\epsilon} \gamma_{\min_{C_l^j}}$  and  $\gamma_{\min}^\epsilon := \sum_{k=1}^{\epsilon} \gamma_{\min_{E_k^j}}$ . At this point, we should clarify that at high SNR, the variance  $\sigma_{SR_m}^2$  does not affect the slope of  $I(\epsilon, j)$  that we are looking for. Let us now assume wlog that  $\bar{\gamma}_{SR_m} = \bar{\gamma}_{R_m D} = \bar{\gamma} \forall m$ ,  $\bar{\gamma}_{SD} = \bar{\gamma}/2$ , and define  $\gamma_{RD}^\epsilon := \sum_{k=1}^{\epsilon} \gamma_{R_{E_k^j} D}$ . One can easily verify that  $\gamma_{\min}^\epsilon \leq \min(\gamma_{SR}^\epsilon, \gamma_{RD}^\epsilon) := \gamma_m$ , which implies that  $I(\epsilon, j) \leq I(\epsilon)$ , with

$$I(\epsilon) := \int_0^\infty \int_0^\infty \int_0^\infty \frac{1}{2^\epsilon} \exp(-\gamma_{SR}^\epsilon) \\ \times Q \left[ \frac{\sqrt{2}(\gamma_s - \gamma_m)}{\sqrt{\gamma_s + \gamma_m}} \right] p(\gamma_s) p(\gamma_{SR}^\epsilon) p(\gamma_{RD}^\epsilon) d\gamma_s d\gamma_{SR}^\epsilon d\gamma_{RD}^\epsilon \quad (23)$$

where we further simplified our notation by using  $\gamma_s := \gamma_{SD} + \gamma_{\min}^c$ . The innermost integral in (23) has the same structure encountered in Proposition 1 with the expectation taken over

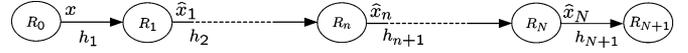


Fig. 4. Block model for a multihop cooperative system.

different pdfs. For the given SNR forms, the corresponding pdfs are gamma distributed as

$$p(\gamma_s) = \frac{2^{M-\epsilon+1} (\gamma_s)^{M-\epsilon}}{(M-\epsilon)! \bar{\gamma}^{M-\epsilon+1}} \exp\left(-\frac{2\gamma_s}{\bar{\gamma}}\right) \quad (24)$$

$$p(\gamma_{SR}^\epsilon) = \frac{(\gamma_{SR}^\epsilon)^{\epsilon-1}}{(\epsilon-1)! \bar{\gamma}^\epsilon} \exp\left(-\frac{\gamma_{SR}^\epsilon}{\bar{\gamma}}\right) \quad (25)$$

$$p(\gamma_{RD}^\epsilon) = \frac{(\gamma_{RD}^\epsilon)^{\epsilon-1}}{(\epsilon-1)! \bar{\gamma}^\epsilon} \exp\left(-\frac{\gamma_{RD}^\epsilon}{\bar{\gamma}}\right) \quad (26)$$

Based on (24)–(26), we prove in Appendix C the following result, which bounds the performance of  $I(\epsilon)$  in (23).

*Proposition 2:* With the pdfs in (24)–(26),  $I(\epsilon)$  with  $\epsilon \geq 1$  can be bounded by a term  $\tilde{I}(\epsilon)$ , which decays with exponent equal to  $M + 1$ ; that is, with  $k(\epsilon)$  denoting a constant, we have

$$I(\epsilon) \leq \tilde{I}(\epsilon) \approx [k(\epsilon) \bar{\gamma}]^{-M-1}. \quad (27)$$

Together with the case  $\epsilon = 0$ , (27) proves that C-MRC achieves full diversity  $(M + 1)$  in the multibranch cooperation scenario, where all relays utilize the DF strategy.

## B. Multihop Cooperative Diversity

Before tackling the most general cooperative scenario with one source and one destination, we will analyze the case of  $N$  relays  $R_1, \dots, R_N$  in cascade, as depicted in Fig. 4; see also [3], [5], and [12]. We will also view this system from our end-to-end equivalent SNR perspective.

For uniformity, in notation, we rename  $S$  as  $R_0$ , and  $D$  as  $R_{N+1}$ . Each  $R_n$ ,  $n = 1, \dots, N$ , transmits its decoded symbol  $\hat{x}_n$ , while  $R_0$  transmits  $\hat{x}_0 = x$ . The received symbol at each  $R_n$ ,  $n = 1, \dots, N + 1$  is now renamed as  $y_n = h_n \hat{x}_{n-1} + z_n$ , where  $h_n$  ( $\gamma_n$ ) denotes the fading coefficient (output SNR) of the link between  $R_n$  and  $R_{n-1}$ . Again, we assume relays transmitting over mutually orthogonal channels ( $N + 1$  time slots, for example).

For each node  $R_n$ , we express the BEP from  $R_{n-1}$  to  $R_n$  as  $P_{n-1,n}^b = Q[\sqrt{2\gamma_n}]$ . The BEP  $P_n^b$  at node  $R_n$  is affected by all previous  $n - 1$  hops and can be iteratively calculated according to the recursion  $P_n^b = (1 - P_{n-1,n}^b) P_{n-1,n}^b + (1 - P_{n-1,n}^b) P_{n-1,n}^b$ , with  $P_0^b = 0$ . Finally, the end-to-end BEP at the destination takes the form  $P_{N+1}^b = (1 - P_N^b) P_{N,N+1}^b + (1 - P_{N,N+1}^b) P_N^b$ . Using this recursion, it is not necessary for the destination ( $R_{N+1}$ ) to have centralized knowledge of all per-hop BEPs. Let us now define  $\gamma_{eq_n}$  to be the solution of the equation  $P_n^b = Q[\sqrt{2\gamma_{eq_n}}]$ . We can again simplify notation by setting  $\gamma_{eq} := \gamma_{eq_{N+1}}$ . Then,

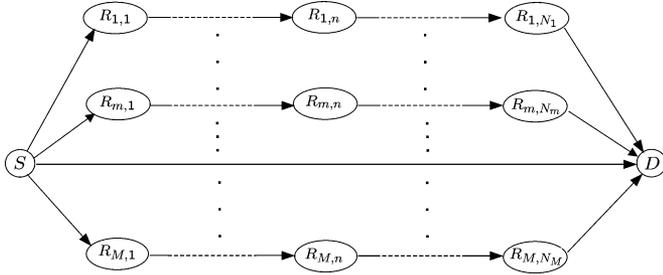


Fig. 5 Block model for multihop and multibranch cooperation.

the average BEP is just

$$\begin{aligned} P^b &= \int_0^\infty Q[\sqrt{2\gamma_{\text{eq}}}] p(\gamma_{\text{eq}}) d\gamma_{\text{eq}} \\ &\leq \int_0^\infty \frac{1}{2} \exp(-\gamma_{\text{eq}}) p(\gamma_{\text{eq}}) d\gamma_{\text{eq}}. \end{aligned} \quad (29)$$

Letting  $\gamma_{\min_n} := \min(\gamma_1, \gamma_2, \dots, \gamma_n)$ , we can easily prove by induction that at high SNR  $\gamma_{\text{eq}_n} \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \gamma_{\min_n}$ , since  $\gamma_{\text{eq}_n} \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \min(\gamma_{\text{eq}_{n-1}}, \gamma_n)$ , and the case  $n = 2$  is already proven in Property 1. Clearly, we have  $\gamma_{\text{eq}} \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \gamma_{\min_{N+1}} := \gamma_{\min}$ . If  $\tilde{P}^b$  denotes the right hand side of (29), we can write

$$\tilde{P}^b \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \int_0^\infty \frac{1}{2} \exp(-\gamma_{\min}) \frac{1}{\bar{\gamma}_{\min}} \exp\left(-\frac{\gamma_{\min}}{\bar{\gamma}_{\min}}\right) d\gamma_{\min}.$$

Let us further consider wlog that all average SNRs are identical; that is, we set  $\bar{\gamma}_n = \bar{\gamma}$ ,  $n = 1, \dots, N + 1$  and thus  $\bar{\gamma}_{\min} = \bar{\gamma}/(N + 1)$ . Under these conventions, we obtain

$$P^b \leq \tilde{P}^b \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \frac{N}{2(\bar{\gamma} + N)}. \quad (30)$$

Equation (30) shows that the diversity order of  $P^b$  is 1. We, therefore, confirm that a multihop cooperative system can only achieve diversity of order one.

### C. Multihop, Multibranch Cooperative Diversity

Combining results from Sections III-A and III-B, we are ready to derive the diversity gain in the BEP of multihop and multibranch systems using C-MRC with DF relays. Let us consider a cooperative system with  $M + 1$  branches,  $B_0, B_1, \dots, B_M$ , as depicted in Fig. 5. Without loss of generality, let  $B_0$  denote the direct link. Branch  $B_m$  consists of  $N_m$  relays and  $R_{m,n}$  denotes the  $n$ th node in branch  $B_m \forall n = 1, \dots, N_m$  and  $N_0 = 0$ . As before,  $R_{m,0}$  denotes  $S$ , and  $R_{m,N_m+1}$  denotes  $D$ . The received symbol at node  $R_{m,n}$  is  $y_{m,n} = h_{m,n}\hat{x}_{m,n-1} + z_{m,n}$ , where  $h_{m,n}$  ( $\gamma_{m,n}$ ) denotes the fading coefficient (output SNR) of the link between  $R_{m,n}$  and  $R_{m,n-1}$ . Relaxing the notation

for the direct link, we write  $y_0 = h_0x_0 + z_0$ , where  $h_0 = h_{0,1}$  and  $\gamma_0 = \gamma_{0,1}$ . The C-MRC detector is now

$$\begin{aligned} \hat{x}_D = \arg \min_{x \in \mathcal{A}_x} & \left| w_0 y_0 + \sum_{m=1}^M w_m y_{m,N_m+1} \right. \\ & \left. - \left( w_0 h_0 + \sum_{m=1}^M w_m h_{m,N_m+1} \right) x \right|^2 \end{aligned} \quad (31)$$

where  $w_m$  weighs the symbol arriving from the  $m$ th branch. Letting  $P_{m,n}^b$  denote the BEP at node  $R_{m,n}$ , one can focus on branch  $m$  and calculate the equivalent end-to-end fading coefficient  $\gamma_{\text{eq}_{m,n}}$  from  $P_{m,n}^b = Q[\sqrt{2\gamma_{\text{eq}_{m,n}}}]$ , and set again  $\gamma_{\text{eq}_m} := \gamma_{\text{eq}_{m,N_m+1}}$ . The C-MRC weights here are chosen as  $w_0 = h_0^*$  and  $w_m = \gamma_{\text{eq}_m} h_{m,N_m+1}^* / \gamma_{m,N_m+1}$ .

For given  $\gamma_{\text{eq}_{m,N_m}}, \gamma_0$  and  $\gamma_{m,N_m+1}$ , the conditional BEP is given by (28), at the bottom of this page, where  $\underline{\gamma} := [\gamma_0, \gamma_{1,1}, \dots, \gamma_{1,N_1+1}, \gamma_{2,1}, \dots, \gamma_{m,n}, \dots, \gamma_{M,N_M+1}]$ .

For the same reason as in Subsection III-A, full diversity of order  $M + 1$  will be collected when  $\epsilon = 0$ . For  $\epsilon \geq 1$ , taking expectations on both sides of (28), we obtain

$$P^b = \int P^b(\underline{\gamma}) p(\underline{\gamma}) d\underline{\gamma} \leq \sum_{\epsilon=0}^M \sum_{j=1}^{\binom{M}{\epsilon}} J(\epsilon, j). \quad (32)$$

Given  $m$ , and since  $\gamma_{\text{eq}_{m,n}} \stackrel{\bar{\gamma} \rightarrow \infty}{\approx} \min(\gamma_{m,1}, \dots, \gamma_{m,n}) := \gamma_{\min_{m,n}} \forall n = 2, \dots, N_m + 1$ , we find that  $J(\epsilon, j)$  for  $\bar{\gamma} \rightarrow \infty$  is given by

$$J(\epsilon, j) = \int \frac{1}{2^\epsilon} \exp(-\gamma_{SR}^\epsilon) Q\left[\frac{\sqrt{2}(\gamma_0 + \gamma_{\min}^c - \gamma_{\min}^\epsilon)}{\sqrt{\gamma_0 + \gamma_{\min}^c + \gamma_{\min}^\epsilon}}\right] p(\underline{\gamma}) d\underline{\gamma}, \quad (33)$$

where  $\gamma_{SR}^\epsilon := \sum_{k=1}^\epsilon \gamma_{\min_{E_k^j, N_{E_k^j}}}$ ,  $\gamma_{RD}^\epsilon := \sum_{k=1}^\epsilon \gamma_{E_k^j, N_{E_k^j}}$ ,  $\gamma_{\min}^\epsilon := \sum_{k=1}^\epsilon \gamma_{\min_{E_k^j, N_{E_k^j}+1}}$ ,  $\gamma_{\min}^c := \sum_{l=1}^{M-\epsilon} \gamma_{\min_{C_l^j, N_{C_l^j}+1}}$  and  $\gamma_{\min}^\epsilon \leq \min(\gamma_{SR}^\epsilon, \gamma_{RD}^\epsilon)$ .

Once arrived to (33), the remaining steps to establish the full diversity of order  $M + 1$  mimic those after (22) and will be thus omitted. In a nutshell, we have proved that with DF-based relays, C-MRC is full diversity achieving in general cooperation scenarios.

## IV. NUMERICAL RESULTS AND SIMULATIONS

This section presents simulated BEP tests in single-relay and multihop/branch scenarios for practical SNR values. Unless otherwise stated, the adopted modulation is BPSK;  $\gamma_{\text{eq}}$  is defined as in (6) and nodes transmit with the same power  $P_x$ , resulting in an average input SNR  $\bar{\gamma} = P_x/N_0$ .

$$P^b(\underline{\gamma}) = \sum_{\epsilon=0}^M \sum_{j=1}^{\binom{M}{\epsilon}} \left\{ \prod_{k=1}^{\epsilon} P_{E_k^j, N_{E_k^j}}^b \left( \gamma_{\text{eq}_{E_k^j, N_{E_k^j}}} \right) \prod_{l=1}^{M-\epsilon} \left[ 1 - P_{C_l^j, N_{C_l^j}}^b \left( \gamma_{\text{eq}_{C_l^j, N_{C_l^j}}} \right) \right] \right\} Q\left[\frac{\sqrt{2} \left( \gamma_0 + \sum_{l=1}^{M-\epsilon} \gamma_{\text{eq}_{C_l^j}} - \sum_{k=1}^{\epsilon} \gamma_{\text{eq}_{E_k^j}} \right)}{\sqrt{\gamma_0 + \sum_{m=1}^M (\gamma_{\text{eq}_m})^2 / \gamma_{m, N_m+1}}}\right] \quad (28)$$

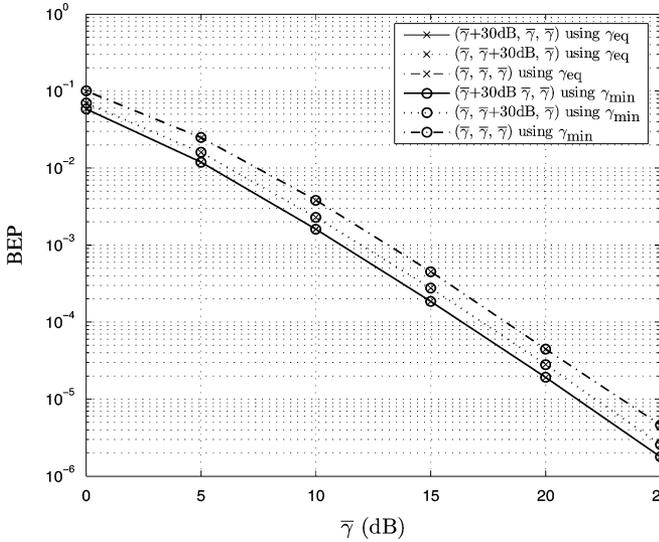


Fig. 6 Performance comparison for a single-relay system using C-MRC with BPSK:  $\gamma_{\text{eq}}$  versus  $\gamma_{\text{min}}$ .

### A. Single Relay Cooperation

With reference to Fig. 1, we consider representative attenuation levels that correspond to those in which  $R$  is located either close to the source, close to the destination, or, equidistant from both; the corresponding average output SNRs ( $\bar{\gamma}_{SR}$ ,  $\bar{\gamma}_{RD}$ ,  $\bar{\gamma}_{SD}$ ) in logarithmic-scale are  $(\bar{\gamma} + 30 \text{ dB}, \bar{\gamma}, \bar{\gamma})$ ,  $(\bar{\gamma}, \bar{\gamma} + 30 \text{ dB}, \bar{\gamma})$ , and  $(\bar{\gamma}, \bar{\gamma}, \bar{\gamma})$ , respectively.

The validity of Property (1) is tested in Fig. 6, where the  $\gamma_{\text{eq}}$  in (6) is substituted by  $\gamma_{\text{min}}$  defined in Property 1. Performance is seen to be virtually identical, which implies that the bounds we derived are tight. The SNR setup of  $(\bar{\gamma} + 30 \text{ dB}, \bar{\gamma}, \bar{\gamma})$  performs better than  $(\bar{\gamma}, \bar{\gamma} + 30 \text{ dB}, \bar{\gamma})$  because with the  $S - R$  link having higher SNR, error propagation to  $D$  is mitigated. Same explanation applies also to the case  $(\bar{\gamma}, \bar{\gamma}, \bar{\gamma})$ , for which error performance is even worse because both  $S - R$  and  $R - D$  links have low SNR at the same time. Nevertheless, full diversity is achieved in all the three settings. Fig. 7 compares the bounds associated with C-MRC,  $\lambda$ -MRC (with numerically optimized  $\lambda$  as in [14]), and the ML detector in (10). As expected, C-MRC tightly bounds error performance of the rest, which as a by-product demonstrates the performance claims not proven in [14].

We next compare DF with alternative diversity-achieving strategies, namely, AF and SR for block length = 100 and CRC bits = 16, which is assumed to be perfect. We reiterate that the focus here is on diversity gain, which is not affected by the block length, i.e., the diversity gain of different schemes will not change when the block length changes. Based on analog processing, AF slightly outperforms DF and SR, as shown in Fig. 8. When it comes to regenerative protocols, the proposed DF decoder outperforms SR. Increasing the block length in SR will increase this gap. However, if one decreases the block length, the spectral efficiency of SR decreases. Moreover, this can also be explained if one considers that although both DF based on C-MRC and SR are adaptive protocols achieving full diversity,

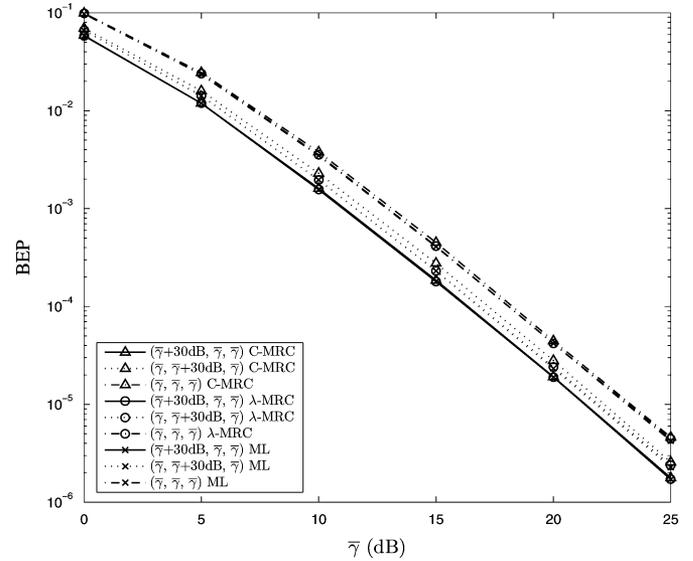


Fig. 7 Performance comparison for a single-relay system using C-MRC,  $\lambda$ -MRC, or ML demodulation with BPSK.

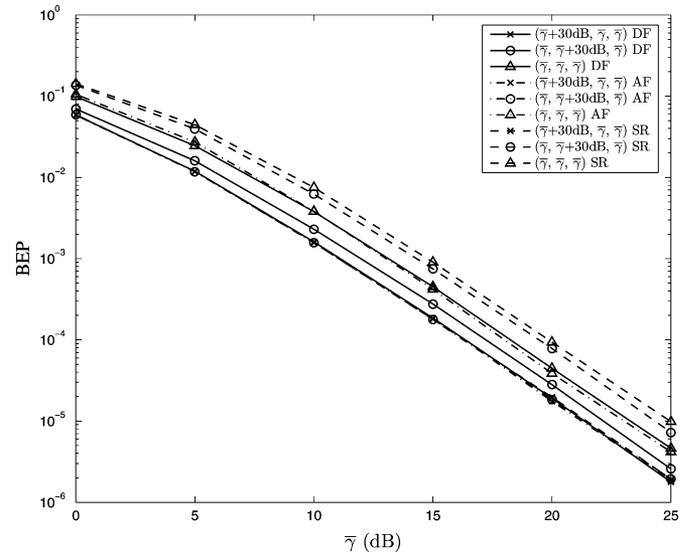


Fig. 8 Performance comparison for a single-relay system using DF-based C-MRC versus AF versus SR (block length = 100, 16 bit-CRC is assumed to be perfect), with BPSK.

the SR one is based on *hard* decisions at the relay, while DF with C-MRC exploits *soft* reliability of the  $S - R - D$  link, measured by  $\gamma_{\text{eq}}$  at the destination.

To check modulations higher than BPSK, we also tested QPSK and compared C-MRC with traditional MRC. As shown in Fig. 9, diversity loss is particularly evident for low-quality  $S - R$  links, which justifies the importance of link quality information, which C-MRC exploits to collect full diversity.

### B. Multibranch Multihop Cooperation

Here, we validate full-diversity claims for an arbitrary number of branches and relaying nodes per branch. Diversity claims are simulated when considering multiple hops per path. A fair

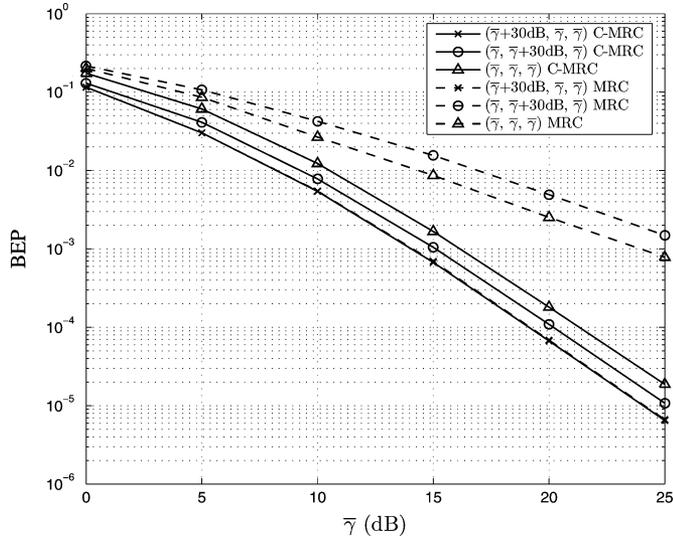


Fig. 9 Performance comparison for a single-relay system using C-MRC *versus* MRC (QPSK modulation).

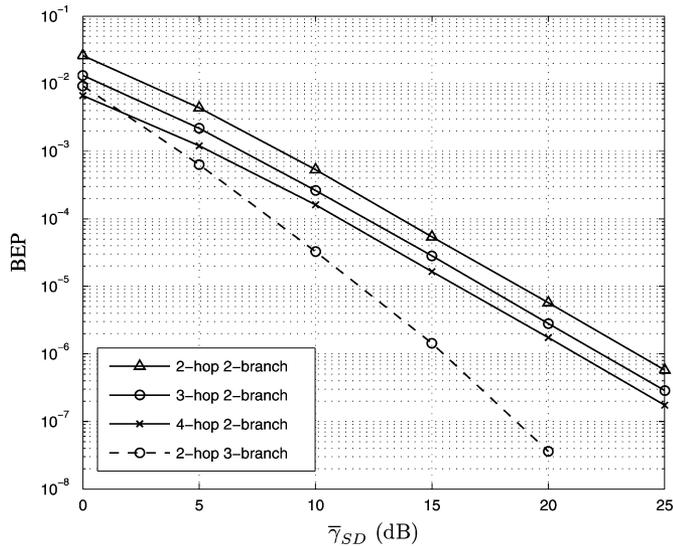


Fig. 10 Performance comparison for multibranch and multihop cooperation using C-MRC with BPSK.

comparison requires hops  $R_1, \dots, R_N$  to be equally spaced between  $S$  and  $D$ , which may be realistic when  $D$  is far from  $S$  [7], [12]. Assuming path-loss exponent equal to 3, the average output SNR per hop is  $\bar{\gamma}_n = \bar{\gamma}_{SD}(d_0/d_{n,n-1})^3$ , where  $d_0$  stands for the distance between  $S$  and  $D$ , and  $d_{n,n-1}$  denotes the distance between  $R_n$  and  $R_{n-1}$ . Fig. 10 shows that the full diversity is achieved regardless of the number of hops; that is, with the same number of branches, the BEP curves have identical slopes. On the other hand, more cooperating nodes per branch improve performance in parallel BEP shifts, which is the well-known coverage enhancement effect achieved by multihop transmissions [5].

Nevertheless, we may compare the performance if we allow these same terminals to directly communicate with the destination. The resulting BEP curve is also depicted in the same figure

for comparison. We see that in this scenario, the number of arriving paths (diversity order) increases to 3, with no loss in coding gain. In view of this result, we infer that when fading is present, having all relay nodes communicate with the destination offers a desirable tradeoff between diversity and coding gains. By inspecting the slope of the BEP curves, we can also verify that the achieved full diversity order increases as  $M$  increases.

## V. CONCLUSION

We developed a high-performance C-MRC demodulator when cooperating relays utilize the practical DF strategy. We proved that full diversity gains can be achieved with C-MRC in general cooperative links entailing any number of hops and branches, and regardless of the underlying constellation. Simulations illustrated that our C-MRC performs surprisingly close to ML, while its computational simplicity relative to ML is irrespective of the constellation used. C-MRC adapts its structure depending on the knowledge of the  $S-R$  link quality, which is feasible to acquire through training. Relative to competing alternatives, C-MRC with DF relays outperforms existing link-adaptive regenerative strategies and comes very close to AF, which is certainly more expensive to implement in practice.<sup>1</sup>

## APPENDIX A

### PROOF OF PROPERTY 1

We first derive an upper bound on  $\gamma_{eq}$ . Since  $Q[x]$  is monotonically decreasing, we have  $P_{RD}^b(\gamma_{RD}) = Q[\sqrt{\alpha\gamma_{RD}}] \leq Q[0] = 1/2$ , and  $P_{SR}^b(\gamma_{SR}) = Q[\sqrt{\alpha\gamma_{SR}}] \leq 1/2$ . It, thus, follows readily from that  $P_{eq}^b(\gamma_{SR}, \gamma_{RD}) \geq P_{RD}^b(\gamma_{RD}) = Q[\sqrt{\alpha\gamma_{RD}}]$  and  $P_{eq}^b(\gamma_{SR}, \gamma_{RD}) \geq P_{SR}^b(\gamma_{SR}) = Q[\sqrt{\alpha\gamma_{SR}}]$ . Because  $Q^{-1}[y]$  is also monotonically decreasing, we deduce that

$$\begin{aligned} \gamma_{eq} &\leq \frac{1}{\alpha} \{\min\{\sqrt{\alpha\gamma_{RD}}, \sqrt{\alpha\gamma_{SR}}\}\}^2 \\ &= \min\{\gamma_{RD}, \gamma_{SR}\} := \gamma_{\min}. \end{aligned} \quad (34)$$

We will next lower bound  $\gamma_{eq}$  by

$$(1/\alpha) \{Q^{-1}\{Q(\sqrt{\alpha\gamma_{RD}}) + Q(\sqrt{\alpha\gamma_{SR}})\}\}^2$$

using (5) and (6). As

$$\frac{1}{\mu\sqrt{2\pi}x} \exp\left(-\frac{x^2}{2}\right) \leq Q[x] \leq \frac{1}{\sqrt{2\pi}x} \exp\left(-\frac{x^2}{2}\right)$$

when  $x \geq \sqrt{\frac{\mu}{\mu-1}}$ , we infer that  $Q^{-1}[y] \geq \sqrt{W_0(\frac{1}{2\mu^2 y^2 \pi^2})}$ , when

$$Q^{-1}[y] \geq \sqrt{\frac{\mu}{\mu-1}}$$

where  $W_0(x)$  is the principal branch of the Lambert W-function [16]. It, thus, follows that  $\gamma_{eq} \geq \frac{1}{\alpha} W_0[\frac{1}{2\mu^2 y^2 \pi^2}]$

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where

$$y = Q(\sqrt{\alpha\gamma_{RD}}) + Q(\sqrt{\alpha\gamma_{SR}}) \leq \frac{\exp(-\alpha\gamma_{SR}/2)}{\sqrt{2\pi}\sqrt{\alpha\gamma_{SR}}} + \frac{\exp(-\alpha\gamma_{RD}/2)}{\sqrt{2\pi}\sqrt{\alpha\gamma_{RD}}}$$

when  $y \leq Q(\sqrt{\frac{\mu}{\mu-1}})$ . Using the property of  $W_0[z]$ , we have  $W_0[z] \geq \ln[z] - \ln[\ln[z]]$ , when  $z \geq \exp(1)$  [16].

Then, we have  $\gamma_{\text{eq}} \geq \frac{1}{\alpha} \{\ln[\gamma_z] - \ln[\ln[\gamma_z]]\}$ , when  $\gamma_{\min} \geq d/\alpha$ , where

$$\gamma_z = \frac{\alpha^2\gamma_{SR}\gamma_{RD}}{\mu^2[\sqrt{\alpha\gamma_{SR}}\exp(-\alpha\gamma_{RD}/2) + \sqrt{\alpha\gamma_{RD}}\exp(-\alpha\gamma_{SR}/2)]^2}$$

$$d := \left\{ Q^{-1} \left[ \frac{1}{2} \min \left\{ Q \left( \sqrt{\frac{\mu}{\mu-1}} \right), \sqrt{\frac{1}{2\mu^2\pi^2\exp(1)}} \right\} \right] \right\}^2.$$

With some additional manipulations, we find that

$$\gamma_{\text{eq}} \geq \gamma_{\min} - (1/\alpha) \left\{ 2\ln(2\mu) + \ln \left( \frac{d + \nu \ln 2}{d} \right) \right\}$$

for any  $\mu > 1$  and  $\nu > 1$ . Using numerical search to find  $\mu$  and  $\nu$ , we obtain a tight lower bound when  $\mu = 1.89$ ,  $\nu = 3.6$ ,  $d = 3.23157$ , from which we deduce that  $\gamma_{\text{eq}} \geq \gamma_{\min} - 3.23165/\alpha > \gamma_{\min} - 3.24/\alpha$ , if  $\gamma_{\min} \geq d/\alpha$ . Combining this lower bound with the upper bound in (34), it follows that for any  $\gamma_{\text{eq}} \geq 0$

$$\gamma_{\min} - \frac{3.24}{\alpha} < \gamma_{\text{eq}} \leq \gamma_{\min}. \quad (35)$$

## APPENDIX B

### PROOF OF PROPOSITION 1

From (16) and Property 1, we obtain  $P_2^b \leq A + B$ , where

$$A := \int_0^\infty \int_0^\infty \int_{\gamma_{\min}}^\infty \frac{1}{2} \exp(-\gamma_{SR}) \frac{1}{2} \exp \left[ -\frac{(\gamma_{SD} - \gamma_{\min})^2}{\gamma_{SD} + \gamma_{\min}} \right] \\ \times p(\gamma_{SD})p(\gamma_{SR})p(\gamma_{RD})d\gamma_{SD}d\gamma_{SR}d\gamma_{RD}$$

$$B := \int_0^\infty \int_0^\infty \int_0^{\gamma_{\min}} \frac{1}{2} \exp(-\gamma_{SR}) \\ \times p(\gamma_{SD})p(\gamma_{SR})p(\gamma_{RD})d\gamma_{SD}d\gamma_{SR}d\gamma_{RD}.$$

For  $A$ , we have

$$A = \int_0^\infty \int_0^\infty \frac{1}{4} \exp(-\gamma_{SR}) \frac{1}{\gamma_{SR}} \exp \left( -\frac{\gamma_{SR}}{\gamma_{SR}} \right) \\ \times \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{RD}}{\gamma_{RD}} \right) \frac{1}{\gamma_{SD}} f_A(\gamma_{\min})d\gamma_{SR}d\gamma_{RD}$$

where

$$f_A(\gamma_{\min}) \\ = \int_{\gamma_{\min}}^\infty \exp \left[ -\left( \gamma_{SD} + \gamma_{\min} - \frac{4\gamma_{SD}\gamma_{\min}}{\gamma_{SD} + \gamma_{\min}} \right) \right]$$

$$\times \exp \left( -\frac{\gamma_{SD}}{\gamma_{SD}} \right) d\gamma_{SD} \leq \int_{\gamma_{\min}}^\infty \\ \times \exp \left[ -(\gamma_{SD} + \gamma_{\min} - 2\sqrt{\gamma_{SD}\gamma_{\min}}) \right] \\ \times \exp \left( -\frac{\gamma_{SD}}{\gamma_{SD}} \right) d\gamma_{SD} \leq \frac{\bar{\gamma}_{SD}}{(1 + \bar{\gamma}_{SD})^2} \exp \left( -\frac{\gamma_{\min}}{\bar{\gamma}_{SD}} \right) \\ \times \left[ 1 + \bar{\gamma}_{SD} + \sqrt{(1 + \bar{\gamma}_{SD})\pi\bar{\gamma}_{SD}\gamma_{\min}} \right].$$

After integrating over different regions and recalling that  $\gamma_{\min} = \min\{\gamma_{SR}, \gamma_{RD}\}$ , we arrive at

$$A \leq \tilde{A} := \frac{\bar{\gamma}_{SD}\gamma_1}{4(1 + \bar{\gamma}_{SD})(1 + \bar{\gamma}_{SR})(\bar{\gamma}_{SR}\bar{\gamma}_{RD} + \bar{\gamma}_{SD}\gamma_1)} \\ + \frac{\pi\bar{\gamma}_{SD}(\frac{\bar{\gamma}_{SD}}{1 + \bar{\gamma}_{SD}})^{3/2}\bar{\gamma}_{SR}^3\bar{\gamma}_{RD}\sqrt{1 + \frac{1}{\bar{\gamma}_{SD}} + \frac{1}{\bar{\gamma}_{SR}} + \frac{1}{\bar{\gamma}_{RD}}}}{8(1 + \bar{\gamma}_{SR})\gamma_2(\bar{\gamma}_{SR}\bar{\gamma}_{RD} + \bar{\gamma}_{SD}\gamma_1)^2} \\ + \frac{\bar{\gamma}_{SD}^2\bar{\gamma}_{RD}\sqrt{\bar{\gamma}_{SR}}\sqrt{1 + \frac{\bar{\gamma}_{SD}\bar{\gamma}_{SR}}{\gamma_2\bar{\gamma}_{RD}}}\pi}{8(1 + \bar{\gamma}_{SD})^{3/2}\sqrt{\gamma_2}(\bar{\gamma}_{SR}\bar{\gamma}_{RD} + \bar{\gamma}_{SD}\gamma_1)} \quad (36)$$

where  $\gamma_1 := \bar{\gamma}_{SR} + \bar{\gamma}_{RD} + \bar{\gamma}_{SR}\bar{\gamma}_{RD}$  and  $\gamma_2 := \bar{\gamma}_{SD} + \bar{\gamma}_{SR} + \bar{\gamma}_{SD}\bar{\gamma}_{SR}$ .

Likewise for  $B$ , we have

$$B = \int_0^\infty \int_0^\infty \frac{1}{2} \exp(-\gamma_{SR}) \frac{1}{\gamma_{SR}} \exp \left( -\frac{\gamma_{SR}}{\gamma_{SR}} \right) \\ \frac{1}{\gamma_{RD}} \exp \left( -\frac{\gamma_{RD}}{\gamma_{RD}} \right) \left[ 1 - \exp \left( -\frac{\gamma_{\min}}{\bar{\gamma}_{SD}} \right) \right] d\gamma_{SR}d\gamma_{RD} \\ = \frac{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}{2(1 + \bar{\gamma}_{SR})(\bar{\gamma}_{SR}\bar{\gamma}_{RD} + \bar{\gamma}_{SD}(\bar{\gamma}_{SR} + \bar{\gamma}_{RD} + \bar{\gamma}_{SR}\bar{\gamma}_{RD}))}.$$

Upon defining  $(\bar{\gamma}_{SR}, \bar{\gamma}_{SD}, \bar{\gamma}_{RD}) := (\sigma_{SR}^2\bar{\gamma}, \sigma_{SD}^2\bar{\gamma}, \sigma_{RD}^2\bar{\gamma})$ , where  $\sigma_{SR}^2$ ,  $\sigma_{SD}^2$ , and  $\sigma_{RD}^2$  are three finite constants depending on the practical design, one can easily verify that  $P_2^b \leq \tilde{A} + B \approx (k_2\bar{\gamma})^{-2}$ , where  $k_2$  is a constant, which depends on  $\sigma_{SR}^2$ ,  $\sigma_{SD}^2$ , and  $\sigma_{RD}^2$ .

## APPENDIX C

### PROOF OF PROPOSITION 2

Expanding (23), we obtain  $I(\epsilon) = C + D$  where

$$C := \int_0^\infty \int_0^\infty \int_{\gamma_m}^\infty \frac{1}{2^\epsilon} \exp(-\gamma_{SR}^\epsilon) \frac{1}{2} \exp \left[ -\frac{(\gamma_s - \gamma_m)^2}{\gamma_s + \gamma_m} \right] \\ \times p(\gamma_s)p(\gamma_{SR}^\epsilon)p(\gamma_{RD}^\epsilon)d\gamma_s d\gamma_{SR}^\epsilon d\gamma_{RD}^\epsilon,$$

$$D := \int_0^\infty \int_0^\infty \int_0^{\gamma_m} \frac{1}{2^\epsilon} \exp(-\gamma_{SR}^\epsilon) \\ \times p(\gamma_s)p(\gamma_{SR}^\epsilon)p(\gamma_{RD}^\epsilon)d\gamma_s d\gamma_{SR}^\epsilon d\gamma_{RD}^\epsilon.$$

Following the steps similar to those in Appendix B, we find  $C \leq \tilde{C}$ , with  $\tilde{C}$  defined as

$$\tilde{C} := \sum_{k=0}^{2M-2\epsilon+1} \left\{ 2^{M-2\epsilon}(\bar{\gamma})^{-\epsilon} \left( \frac{\sqrt{\bar{\gamma}}}{2 + \bar{\gamma}} \right)^k \right. \\ \left. \times (2 + \bar{\gamma})^{-1+\epsilon+k/2-M} \binom{2M-2\epsilon+1}{k} \frac{\Gamma[1 - \epsilon - k/2 + M]}{\Gamma[\epsilon]\Gamma[M - \epsilon + 1]} \right\}$$

$$\left[ \Gamma[\epsilon+k/2] \left(1 + \frac{1}{\bar{\gamma}} + \frac{2}{2+\bar{\gamma}}\right)^{-\epsilon-k/2} + (\bar{\gamma})^{-\epsilon} \sum_{i=0}^{\epsilon-1} \left(1 + \frac{1}{\bar{\gamma}}\right)^{i-\epsilon} \times \left(1 + \frac{2}{\bar{\gamma}} + \frac{2}{2+\bar{\gamma}}\right)^{-\epsilon-k/2-i} \frac{\Gamma[\epsilon+k/2+i]}{\Gamma[i+1]} \right] \Bigg\}$$

where  $\Gamma[a, x] := \int_x^\infty t^{a-1} \exp(-t) dt$  is the upper incomplete gamma function, and  $\Gamma[a] := \Gamma[a, 0]$  [15].

Using  $\Gamma[n, x] := (n-1)! \exp(-x) \sum_{k=0}^{n-1} \frac{x^k}{k!}$  for any integer  $n$ , we obtain (after integrating and manipulating) that  $D \leq \tilde{D} := D_1 + D_2$ , where

$$D_1 = 2^{M-2\epsilon+2} \left(\frac{1}{3+\bar{\gamma}}\right)^{M+1} \times {}_2F_1\left(1, 1+M; 2-\epsilon+M; \frac{2}{3+\bar{\gamma}}\right) \frac{\Gamma[M+1]}{\Gamma[M-\epsilon+2]\Gamma[\epsilon]}$$

$$D_2 = \frac{(\bar{\gamma})^{-\epsilon}}{\Gamma[\epsilon]\Gamma[M-\epsilon+2]} \left(\frac{2}{4+\bar{\gamma}}\right)^{M+1} \sum_{i=0}^{\epsilon-1} \left\{ \left(1 + \frac{4}{\bar{\gamma}}\right)^{-\epsilon} \times {}_2F_1\left(1, 1+i+M; 2-\epsilon+M; \frac{2}{4+\bar{\gamma}}\right) \frac{\Gamma[1+i+M]}{\Gamma[1+i]} \right\}$$

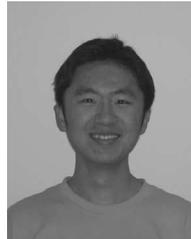
where  ${}_2F_1(a, b; c; z)$  denotes the Gauss hypergeometric function defined in e.g., [1, eq. (15.1.1)]. Given  $\epsilon \geq 1$ , one can verify that  $I(\epsilon) \leq \tilde{C} + \tilde{D} \approx \tilde{\gamma}^{-\infty} [k(\epsilon)\bar{\gamma}]^{-M-1}$ , where  $k(\epsilon)$  is a constant, which does not depend on  $\bar{\gamma}$ .

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