A Robust High-Throughput Tree Algorithm Using Successive Interference Cancellation

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***Abstract*—A novel random access protocol combining a tree algorithm (TA) with successive interference cancellation (SIC) has been introduced recently. To mitigate the deadlock problem of SICTA arising in error-prone wireless networks, we put forth a SICTA with first success (SICTA/FS) protocol, which is capable of high throughput while requiring limited-sensing and gaining robustness to errors relative to SICTA.**

***Index Terms*—Random access, successive interference cancellation (SIC), tree algorithm.**

I. INTRODUCTION

CONVENTIONAL random access protocols are designed based on a noiseless collision channel model, where the collided packets are discarded. Exploiting the fact that collided packets contain information, Tsatsanis *et al.* proposed network diversity multiple access (NDMA) protocols [2], which rely on proper retransmissions and signal separation principles to resolve collisions. Although the NDMA protocols achieve high throughput, they may suffer from channel-induced ill-conditioning, difficulties with order determination and relatively high computational complexity. Inspired by the NDMA, a novel protocol combining a tree algorithm (TA) with successive interference cancellation (SIC), called the SICTA, was developed in [4] and [5]. Relying on the nice property of TAs where all packets are retained in an orderly one-by-one fashion according to the underlying tree structure, the SICTA employs SIC techniques to separate collided packets. As shown in [5], the SICTA can afford a 0.693 maximum stable throughput (MST), the highest among all TAs to date. However, the SICTA may suffer from the deadlock problem in error-prone wireless networks. This motivates the protocol of this paper that we call the SICTA with first success (SICTA/FS), which offers a truncated version of the SICTA, rendering it applicable to wireless random access channels. Analysis and simulations reveal that besides its robustness to errors, the SICTA/FS can provide high MST around 0.6, which is close to the 0.693 MST of SICTA, and markedly higher than those of other existing TAs.

II. MODELING

We consider a wireless link under the following modeling conditions:

C1) *Finite number of users with infinite buffer:* A finite number of users are transmitting over a single slotted channel to a common access point (AP) packets of length equal to one slot, and each user has a large buffer, which can store an infinite number of packets, as in [2].

C2) *Immediate* $0/1/e$ *feedback:* By the end of each slot, users are informed of the feedback from the AP immediately and errorlessly, as in [2] and [5]. The feedback here is one of: a) idle, i.e., 0, when no packet transmission occurs; b) success, i.e., 1, upon successful packet reception; and c) failure, i.e., e, upon erroneous packet reception.

C3) *Poisson arrivals and head-of-line (HOL) access:* Each user's packets are generated according to a Poisson source, as in [2] and [5]. In SICTA/FS, the time interval from the slot where an initial collision occurs up to and including the slot at the end of which a success, i.e., 1, feedback is sent by the AP, is called a collision resolution interval (CRI). At the beginning of each CRI, each user transmits the first packet in its buffer, i.e., its HOL packet, as in [2].

C4) *Noisy collision channel:* Unlike the noiseless collision channel model, where collisions are the only sources of erroneous packet receptions, packets can be corrupted also by noise even when collisions are absent. All erroneously received packets are saved for future reuse.

In the noisy collision channel, even if only one user transmits during a slot, its packet can still be corrupted by noise, which leads to a nonzero packet-error rate (PER). Suppose that BPSK is used to modulate the information bits per packet, and let $\rho = E_b/N_0$ denote the SNR per bit, where E_b and N_0 are the bit energy and one-sided noise power density, respectively. For the additive white Gaussian noise (AWGN) channel in [1], the bit-error rate (BER) is given by $P_b^{(0)} = Q\left(\sqrt{2\rho}\right) = Q\left(\sqrt{2E_b/N_0}\right)$, where $Q(x) := \int_x^{\infty} (1/\sqrt{2\pi})\, e^{-y^2/2}\, dy$ is the Marcum's Q-function. Moreover, we assume that a packet comprising L_p bits can be successfully recovered only if all its bits are correctly received. The corresponding PER is, then, given by

$$P_e^{(0)} = 1 - (1 - P_b)^{L_p}. \tag{1}$$

Similar to the SICTA [5], the SIC is also employed in the SICTA/FS to recover a packet from a collision provided that

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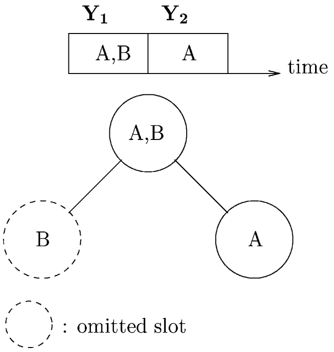

Fig. 1. SICTA example: Possible error propagation.

all other packets involved in this collision have been decoded. However, since a noisy channel is considered, the performance of the SIC is no longer perfect. Supposing that cancellation errors are also Gaussian, the PER associated with decoding a packet after SIC is similar to $P_e^{(0)}$ in (1), but with reduced SNR. For analysis purposes only, we, subsequently, assume that power control is, in effect, to ensure identical E_b for each user at the AP. Moreover, let N_i denote the induced noise density per cancellation. Then, given that all the other n_c collided packets have been correctly decoded, the PER of a packet after SIC is given by

$$P_e^{(n_c)} = 1 - \left(1 - P_b^{(n_c)}\right)^{L_p} \tag{2}$$

where $P_b^{(n_c)} = Q(\sqrt{2E_b/(N_0 + n_c N_i)})$. For completeness, we assume that once a packet is falsely decoded, no other packets can be separated via SIC from the reserved collided packets.

III. SICTA/FS PROTOCOL

We refer the readers to [4] and [5] for a detailed implementation of the SICTA. Although the SICTA can achieve very high MST when errors are not prevailing [5], its performance may be hampered by the deadlock problem arising due to error propagation and/or channel fading. Consider the simple example of Fig. 1 where two packets collide at the first slot of a CRI, and the received signal vector is: $\mathbf{y}_1 = \mathbf{x}_A + \mathbf{x}_B + \mathbf{n}$, where $\mathbf{x}_A$ and $\mathbf{x}_B$ denote packets A and B, and $\mathbf{n}$ denotes the noise vector. At the end of the second slot, packet A is decoded and, then, cancelled to obtain $\widetilde{\mathbf{y}}_1 = \mathbf{x}_B + \mathbf{n}_A + \mathbf{n}$. If $\mathbf{n}_A + \mathbf{n}$ is sufficiently large to render the receiver unable to recover packet B, then $\widetilde{\mathbf{y}}_1$ is incorrectly assumed to be the superposition of collided packets and the AP requires the remaining fictitious "multiple users" to split. In the following slot, packet B is decoded and cancelled; but the receiver cannot discern the remaining noise terms from a real collision, and, for this reason, it forces nonexistent users to further split, which gives rise to idle subsequent slots. Therefore, splitting will continue during the CRI, until an external criterion terminates it. Similarly, channel fading may also lead to deadlock.

To avoid the deadlock problem, our idea in what we term the SICTA/FS protocol is to adopt the first success concept, which, in a different context, was also used in [6]. The SICTA/FS follows the SICTA until it arrives at its first success. At that point, a single packet is decoded and the SIC is employed to extract as many extra packets as possible from reserved previous collisions. However, even though packets may remain unresolved, the SICTA/FS terminates the CRI at this point and starts another CRI. Clearly, the SICTA/FS is actually a truncated version of the SICTA, which only utilizes the rightmost branch of the underlying tree structure of the SICTA. The motivating reasons behind the SICTA/FS are: 1) The error propagation affects the SICTA when it turns back from a leaf (success point) of its underlying tree to resolve the remaining packets. Since the SICTA/FS never turns back, error propagation is avoided. 2) We observe that when the number of initially collided packets is small in a given CRI, the SICTA/FS protocol will capture most of the SICTA gains. We expect the SICTA/FS to perform well for the most part of SICTA's stable throughput range, where the number of initially collided packets is relatively small. To better appreciate the SICTA/FS, we outline its pros and cons against the SICTA as follows:

1) SICTA/FS cannot achieve as high an MST as the SICTA, due to truncation. Nonetheless, the SICTA/FS can still afford higher MST than the existing TAs, thanks to the SIC.
2) SICTA/FS avoids error propagation, which may cause SICTA failure due to infinite splitting.
3) SICTA may suffer from counter mismatch in the presence of feedback errors. In contrast, the SICTA/FS is robust to feedback errors since it employs the success feedback as a clear ending sign of a CRI, which helps users reset their mismatched counters.
4) SICTA is a full-sensing protocol and requires each user to observe the channel outcomes continuously. SICTA/FS is a limited-sensing protocol, that is, users do not need to monitor the channel all the time and can drop in and out at any time. This nice property renders SICTA/FS a perfect fit for random access, which should facilitate new users join-in.

IV. PERFORMANCE ANALYSIS

We first derive the CRI statistics. Let l_n and s_n denote the conditional length and number of successes in a CRI, respectively, given that n packets initially collide. Next, we derive the probability-generating functions (PGFs) for l_n and s_n, which are, then, used to recursively compute the moments: $\overline{L_n} := E\{l_n\}$, $\overline{L_n^2} := E\{l_n^2\}$ and $\overline{S_n} := E\{s_n\}$. Supposing that power control is in effect, we impose that the idle slots can be errorlessly identified in SICTA/FS, which, in turn, yields $l_0 = 1$, and, thus, $\overline{L_0^2} = \overline{L_0} = 1$. In a noisy channel, single-packet transmission per slot does not guarantee a success. According to the specification of the SICTA/FS, we have (w.p. stands for with probability)

$$l_1 = \begin{cases} 1, & \text{w.p. } 1 - P_e^{(0)} \\ m + l_1, & \text{w.p. } P_e^{(0)} p(1-p)^{m-1} \end{cases} \tag{3}$$

where $m-1 \in [0, \infty)$ is the number of idle slots caused by the tree splitting before the next transmission, p denotes the splitting probability, and PER $P_e^{(0)}$ is given by (1). Define $B(i,n,p)$ as the probability mass at the value i of a binomial random variable with total n trials and a success probability p, given by $B(i,n,p) := \binom{n}{i} p^i (1-p)^{n-i}$. Then, for $n \geq 2$, we have for l_n that

$$l_n = \begin{cases} 1 + l_i, & 1 \leq i \leq n-1, \text{ w.p. } B(i,n,p) \\ 1 + l_n, & i = 0, n, \text{ w.p. } B(0,n,p) + B(n,n,p) \end{cases} \quad (4)$$

where i is the number of users joining the right subset of the tree structure. Upon defining the conditional PGF as $Q_n(z) := \sum_{k=0}^{\infty} \Pr\{l_n = k\} z^k = E\{z^{l_n}\}$, we have from (3) and (4) that

$$Q_1(z) = (1 - P_e^{(0)})z + \sum_{m=1}^{\infty} P_e^{(0)} p (1-p)^{m-1} z^m Q_1(z) \quad (5)$$

$$Q_n(z) = (p^n + (1-p)^n) z Q_n(z) + \sum_{i=1}^{n-1} B(i,n,p) z Q_i(z). \quad (6)$$

Let $Q_n'(1)$ and $Q_n''(1)$ denote the first and second derivatives of $Q_n(z)$ w.r.t. z, evaluated at $z = 1$. Using the fact that $Q_n'(1) = \overline{L_n}$ and $Q_n''(1) = \overline{L_n^2} - \overline{L_n}$, we obtain

$$\overline{L_1} = \frac{1 - P_e^{(0)} + P_e^{(0)}/p}{1 - P_e^{(0)}}$$

$$\overline{L_1^2} = \frac{2(1-p)P_e^{(0)}}{p^2(1 - P_e^{(0)})} + \left(1 + \frac{2P_e^{(0)}}{p(1 - P_e^{(0)})}\right) \overline{L_1}$$

and for $n \geq 2$,

$$\overline{L_n} = \frac{1 + \sum_{i=1}^{n-1} B(i,n,p) \overline{L_i}}{1 - p^n - (1-p)^n}$$

$$\overline{L_n^2} = \frac{(1 + p^n + (1-p)^n)\overline{L_n} + \sum_{i=1}^{n-1} B(i,n,p)(\overline{L_i^2} + \overline{L_i})}{1 - p^n - (1-p)^n}.$$

We define the number of successes in a CRI as the average number of successfully decoded packets at the end of the CRI. In the SICTA/FS, it is clear that $s_0 = 0$ and $s_1 = 1$. Hence

$$\overline{S_0} = 0; \quad \overline{S_1} = 1.$$

For $n \geq 2$, the specification of the SCITA/FS implies that

$$s_n = \begin{cases} s_n, & i = 0, n, \text{ w.p. } B(0,n,p) + B(n,n,p) \\ s_i, & 1 \leq i \leq n-2, \text{ w.p. } B(i,n,p) \\ s_{n-1} + 1, & i = n-1, \text{ w.p. } B(n-1,n,p) P_a \\ s_{n-1}, & i = n-1, \text{ w.p. } B(n-1,n,p)(1 - P_a) \end{cases} \quad (7)$$

where $P_a = (1 - P_e^{(n-1)}) \prod_{m=2}^{n-1} B(m-1,m,p)/1 - p^m - (1-p)^m (1 - P_e^{(m-1)})$ with $P_e^{(n_c)}$ given by (2). Specifically, P_a stands for the probability that with $n-1$ users falling into the right subset, the SICTA/FS can possibly decode the only packet in the left subset provided that the other $n-1$ packets are correctly recovered. It, thus, follows from (7) that the conditional PGF for s_n is given by $R_n(z) = (p^n + (1-p)^n) R_n(z) + \sum_{i=1}^{n-2} B(i,n,p)\ R_i(z) + B(n-1,n,p) R_{n-1}(z)(P_a z + 1 - P_a)$. Subsequently, for $n \geq 2$, the first moment of s_n is given by (8) shown at the bottom of the page.

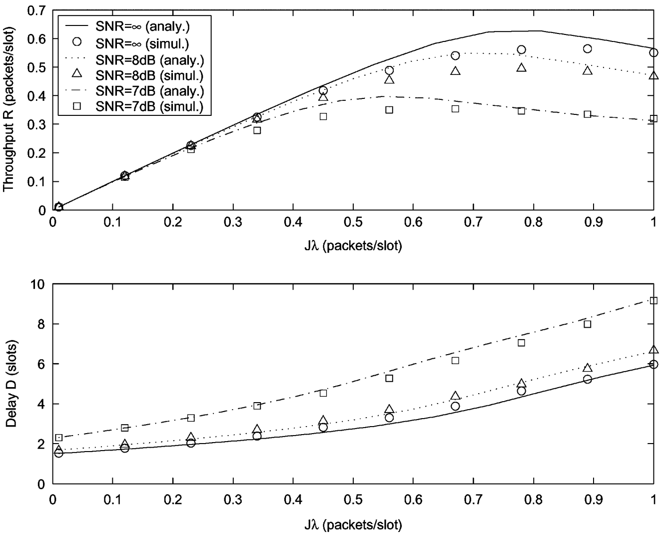

Fig. 2. Comparison between analytical and simulated results.

Per C1)–C4), we consider a system with a number of users, each having an infinite-length buffer. Each user's buffer is fed with a Poisson source having intensity λ packets/slot. Following [2] and [3], we view the traffic in the channel as a flow of CRIs. For a particular user, there are two types of CRIs: relevant CRIs and irrelevant CRIs. In a relevant CRI, the user is active; i.e., it is involved in the initial collision, whereas in the irrelevant CRI, this user is not active. Queueing of each user's buffer can, then, be approximately modeled as an M/G/1 queue with vocation, in which relevant and irrelevant CRIs play the roles of service time and vocation time, respectively. Given the CRI statistics $\overline{L_n}$, $\overline{L_n^2}$, and $\overline{S_n}$, the throughput and the average packet delay of the SICTA/FS can, then, be calculated with the standard M/G/1 queueing analysis, along the lines of [2] and [3].

V. SIMULATION RESULTS

We consider a slotted data packet communication system with J users, each having a buffer that can contain as many as 5000 packets. The user packets have fixed length and consist of L_p bits. BPSK is used to modulate the information bits of each packet with bit energy E_b. The system operates in AWGN with one-sided noise spectral density N_0. The SIC in use is not perfect and the induced (Gaussian) noise spectral density per

$$\overline{S_n} = \frac{\sum_{i=1}^{n-1} B(i,n,p)\overline{S_i} + B(n-1,n,p)(1 - P_e^{(n-1)}) \prod_{m=2}^{n-1} \frac{B(m-1,m,p)}{1-p^m-(1-p)^m} (1 - P_e^{(m-1)})}{1 - p^n - (1-p)^n}. \quad (8)$$

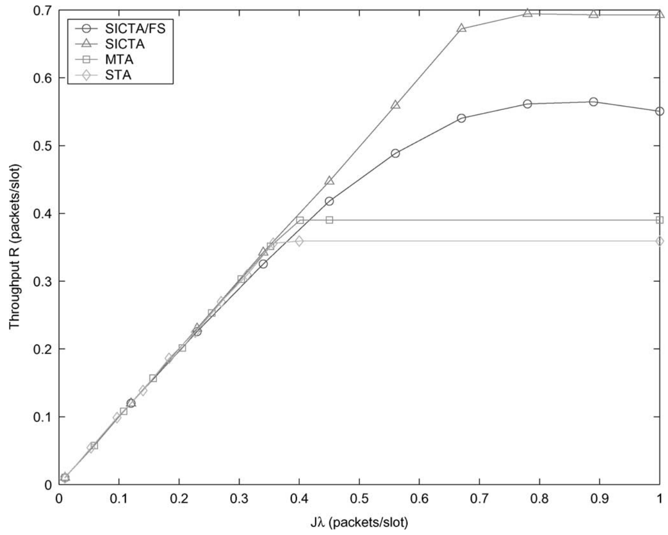

Fig. 3. Throughput comparison among the SICTA/FS, SICTA, MTA, and the STA under infinite SNR.

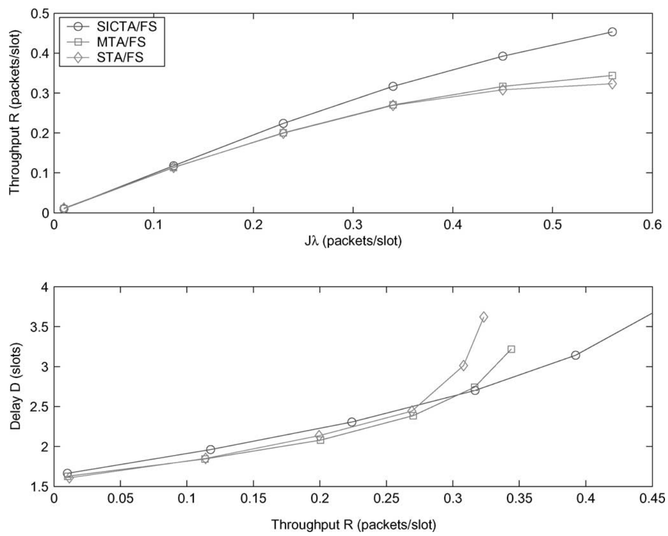

Fig. 4. Performance comparison among the SICTA/FS, MTA/FS, and the STA/FS for an AWGN channel with SNR $= 8$ dB.

cancellation is $N_i = 0.1N_0$. For the SICTA/FS, binary splitting is used with splitting probability p.

We tested the proposed SICTA/FS protocol on a simulated system with parameters: $J = 10$, $L_p = 424$ bits, and $p = 0.5$. The simulations were carried out for an AWGN channel with three $\mathrm{SNRs} = E_b/N_0$ values (in decibells): ∞, 8, and 7. Per SNR value, ten cases were tested, where, in each case, the user's buffer was fed with a different arrival rate λ packets/slot. For each case, the result is obtained as the average of ten independent runs, where the system was run for a time period equivalent to 10,000 CRIs per run. Fig. 2 compares analytical with simulation results, where the solid or dotted lines are obtained through our analytical expressions, while each point depicts the corresponding simulation result. The slight difference between analytical results and simulation is caused by the approximate M/G/1 queue modeling used in the analysis.

We also compared the SICTA/FS with other TAs. Fig. 3 compares the throughput of the SICTA/FS with the SICTA, the standard TA (STA) [7], and the modified TA (MTA) [8] under infinite SNR (error-free case). Although inferior to the SICTA, as expected, the SICTA/FS produces markedly higher maximum throughput than the MTA and the STA. For an AWGN channel with SNR $= 8$ dB (erroneous case), we implemented the robust MTA/FS and STA/FS versions with similar first-success concepts for a fair comparison with the SICTA/FS. As depicted in Fig. 4, besides providing a higher maximum throughput, the SICTA/FS has comparable (or slightly worse) delay performance to the MTA/FS and the STA/FS for low throughput, while outperforming them in delay for high-throughput settings.

VI. CONCLUSION

In this paper, we developed a practical variant of the SICTA random access protocol and evaluated its performance through extensive analysis and simulations. Besides providing high throughput, the SICTA/FS remains robust to errors, facilitates new user's join-in, and it is simple to implement.

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