AN ALGORITHM FOR JOINT SYMBOL TIMING AND CHANNEL ESTIMATION FOR OFDM SYSTEMS

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ABSTRACT

In this paper, we consider the problem of joint synchronization and channel estimation for orthogonal frequency division multiplexing (OFDM) systems. A new algorithm is proposed that estimates the channel and symbol timing simultaneously by using a technique based on maximum-likelihood (ML) theory and the generalized Akaike information criterion (GAIC). Finally, we demonstrate the performance of our algorithm by simulation results.

1. OFDM

The OFDM access technology is based on the transmission of data packets, each of which consists of a number of consecutive *OFDM symbols.* Each OFDM symbol x has a length of N samples and carries a certain number of information bits or training data (that is, known data that are used to assist the demodulator). An OFDM symbol is created by taking the discrete Fourier transform (DFT) of N data symbols (taken from a finite constellation $\mathcal{A}$, such as BPSK, QPSK or QAM). Furthermore, each OFDM symbol is preceded by a *cyclic prefix* (CP) (also called *guard interval* (GI)) of length M that is an exact replica of the M last samples of the OFDM symbol. The reason for this (as will become apparent below) is that demodulation in the presence of frequency-selective fading can be carried out very easily. Before proceeding, let us remark on the fact that in the case that two (or more) *identical* OFDM symbols are transmitted directly subsequent to each other, the tail of the first symbol can serve as the CP for the second.

In the WLAN standard recently adopted by the IEEE 802.11 standardization group [1], each data packet consists of a *preamble* and a data carrying part. The preamble consists of 10 "short" identical known OFDM symbols of length $N_s = 16$ concatenated with 2 "long" identical and known OFDM symbols of length $N_l = 64$ which are all utilized for carrier frequency offset (CFO) correction, channel estimation and synchronization. The data carrying part consists of a variable number of OFDM symbols of length $N_d = 64$, where each OFDM symbol contains useful information plus some known *pilot* bits, which are typically used for updating the phase of the channel estimates. An OFDM packet for the IEEE 802.11 standard is depicted in Figure 1. Note that t_1 serves as a CP for t_2, t_2 is the CP for t_3, and so on. For the long symbols in the preamble, GI2 is the CP for T_1 and it contains the 32 last samples of T_1.

Let $\boldsymbol{y} = [y_1 \ \cdots \ y_N]^T$ be a vector of N data symbols taken from $\mathcal{A}$ (the elements of $\boldsymbol{y}$ are sometimes referred to as *sub-carriers*) and let $\boldsymbol{W}$ be a DFT matrix of size $N \times N$, that is, element k,l of $\boldsymbol{W}$ is equal to $\boldsymbol{W}_{k,l} = e^{-j2\pi\frac{(k-1)(l-1)}{N}}$. Then the OFDM symbol $\boldsymbol{x}$ corresponding to the data $\boldsymbol{y}$ is computed by taking the inverse DFT of $\boldsymbol{y}$, viz. $\boldsymbol{x} = \boldsymbol{W}^*\boldsymbol{y}$, where $(\cdot)^*$ denotes the conjugate transpose. The CP $\tilde{\boldsymbol{x}}$ corresponding to $\boldsymbol{x}$ contains the $M = 16$ last samples of $\boldsymbol{x}$, a relation that can be expressed as $\tilde{\boldsymbol{x}} = \boldsymbol{T}_M\boldsymbol{W}^*\boldsymbol{y}$ where $\boldsymbol{T}_M$ consists of the last M rows of the $N \times N$ identity matrix.

Assume that the effect of the propagation channel can be described by a finite impulse response (FIR) filter with an effective length $L \leq M + 1$ and impulse response $\{h_0, \ldots, h_{L-1}\}$. For reasons that will be apparent later, we augment the channel impulse response with $M - L$ zeros and define

$$\boldsymbol{h} = [h_0 \ \cdots \ h_{M-1}]^T = [h_0 \ \cdots \ h_{L-1} \ 0 \ \cdots \ 0]^T$$

To illustrate the demodulation procedure, we write the

THIS WORK WAS PARTLY SUPPORTED BY THE SWEDISH FOUNDATION FOR STRATEGIC RESEARCH (SSF) AND THE NATIONAL SCIENCE FOUNDATION GRANT MIP-9457388.

received signal as (neglecting receiver noise)

$$
\boldsymbol{r} = \begin{bmatrix} r_0 \\ \vdots \\ r_{N-1} \end{bmatrix}
= \begin{bmatrix} h_{M-1} & \cdots & h_0 & & & & \\ & h_{M-1} & \cdots & h_0 & & & \\ & & \ddots & \cdots & \ddots & & \\ & & & h_{M-1} & \cdots & h_0 \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_{M-1}\boldsymbol{W}^*\boldsymbol{y} \\ \boldsymbol{W}^*\boldsymbol{y} \end{bmatrix}
$$

$$
= \underbrace{\begin{bmatrix} h_0 & & & h_{M-1} & & \cdots & h_1 \\ h_1 & h_0 & & & h_{M-1} & \cdots & h_2 \\ \vdots & \ddots & \ddots & & & \ddots & \vdots \\ & & \ddots & \ddots & & & h_{M-1} \\ h_{M-1} & & & \ddots & \ddots & & \\ & \ddots & & & \ddots & \ddots & \\ & & h_{M-1} & & \cdots & h_1 & h_0 \end{bmatrix}}_{\boldsymbol{H}} \boldsymbol{W}^*\boldsymbol{y}
$$

The matrix $\boldsymbol{H}$ is of dimension $N \times N$ and *circulant*, so the DFT of the received data vector $\boldsymbol{r}$ can be written [9]

$$\boldsymbol{Wr} = \boldsymbol{WHW}^*\boldsymbol{y} + \text{noise} = \boldsymbol{\Delta y} + \text{noise} \qquad (1)$$

where $\boldsymbol{\Delta} = \text{diag}\{\delta_1, \ldots, \delta_N\}$ is a *diagonal* matrix containing the DFT of the channel impulse response $\boldsymbol{h}$, that is, $[\delta_1 \ \cdots \ \delta_N]^T = N\boldsymbol{Wh}$. We remark on the fact that another way to see this is that the cyclic prefix gives the effect of the propagation channel an interpretation in terms of circular convolution; however we prefer to remain in the matrix algebra framework. From (1) we see that, provided the channel $\boldsymbol{\Delta}$ is known, each data bit y_n in the OFDM symbol under consideration can be estimated as $\hat{y}_n = P_{\mathcal{A}}\left(\frac{[\boldsymbol{Wr}]_n}{\delta_n}\right)$ where $P_{\mathcal{A}}(\cdot)$ denotes projection onto the alphabet $\mathcal{A}$, and $[\cdot]_n$ denotes the nth element of a vector. This common and simple demodulator can be implemented by one single FFT.

2. ML CHANNEL ESTIMATION

Channel estimation for OFDM is discussed in some detail in [2, 3, 6], so we merely summarize some results using our notation and framework. Assume that the received data $\boldsymbol{r}$ has been adjusted to compensate for a possible CFO [4], that a proper timing T is obtained and that the effective channel length L is known. Consider first the estimation of the channel $\boldsymbol{h}(L)$ (we use the index L to emphasize that the last $M - L$ elements of $\boldsymbol{h}$ are zero) based on a least-squares (LS) criterion using received data corresponding to the first (known) long OFDM symbol in the preamble. Denote the $N \times 1$ vector of the known data symbols in the long OFDM preamble symbol with $\boldsymbol{p}$. Then LS channel estimation (conditioned on the timing T) amounts to (cf. (1))

$$\min \|\boldsymbol{Wr} - \boldsymbol{\Delta}(L)\boldsymbol{p}\|^2$$

subject to the constraint that the effective channel length is L, i.e., $\boldsymbol{T}_{M-L}\boldsymbol{h} = \boldsymbol{0}$. This is equivalent to

$$\min \|\boldsymbol{Wr} - \text{diag}\{\boldsymbol{p}\}\boldsymbol{Wh}(L)\|^2 \qquad (2)$$

subject to $\boldsymbol{T}_{M-L}\boldsymbol{h} = \boldsymbol{0}$. For the symbols in the IEEE 802.11 WLAN standard, 12 of the elements of $\boldsymbol{p}$ are equal to zero, and the rest belong to the (unitary) BPSK constellation. Using this fact it is not difficult to see that (2) has the solution (see, e.g., [7])

$$[h_1 \ \cdots \ h_L]^T = (\tilde{\boldsymbol{W}}^*\tilde{\boldsymbol{W}})^{-1}\tilde{\boldsymbol{W}}^*\bar{\boldsymbol{W}}\boldsymbol{r} \qquad (3)$$

where $\bar{\boldsymbol{W}}$ equals the matrix $\boldsymbol{W}$ with all rows removed for which the corresponding element of $\boldsymbol{p}$ is zero, and $\tilde{\boldsymbol{W}}$ equals the first L columns of $\bar{\boldsymbol{W}}$. Note that $(\tilde{\boldsymbol{W}}^*\tilde{\boldsymbol{W}})^{-1}\tilde{\boldsymbol{W}}^*\bar{\boldsymbol{W}}$ can be precomputed (for different L) and further that in case the noise in (1) is Gaussian and white, (3) gives the ML estimate of the channel. Having established this, it is straightforward to show that the LS (or ML) channel estimate based on *both* long OFDM symbols in the preamble is nothing but the average of the estimate based on the first and the second symbol, respectively.

3. JOINT TIMING AND CHANNEL ESTIMATION

Once an initial timing T_1 is obtained that is less (earlier) than the true timing, the channel impulse response will contain leading zeros (due to the too early timing) and tailing zeros (provided that the effective channel length plus the synchronization error is less than M). If the number of leading and tailing zeros (or equivalently, the correct channel length and timing) can be estimated, the number of unknown channel coefficients will decrease. Hence a more accurate channel estimate can be obtained, which will reduce the bit error rate (BER) in the system (this is known as the *parsimonious principle* in the system identification literature [7]). This is exactly the idea behind our joint timing, channel length and channel coefficient vector estimation algorithm.

To obtain the initial timing estimate, we use a simple correlation approach (see, e.g., [2]) that exploits the fact that the two long OFDM symbols in the preamble are identical. The initial timing estimate T_1 is determined such that it is (with a very large probability) less than the true timing (unless this is ensured, the channel impulse response will not contain leading zeros). Following this, we refine the timing estimate at the same time as the channel estimation is performed. The details of the procedure are as follows:

1. Let T_1 denote the sample number corresponding to the initial timing (based on a correlation approach [2]).

2. Fix $L = 16$ and increment the timing T starting from $T = T_1$ until the criterion in (2) is minimized. Let the so-obtained T be denoted by T_2.

3. Decrease L starting from $L = 16$ until the following generalized Akaike information criterion (GAIC) is minimized:

$$\ln \|\boldsymbol{Wr} - \mathrm{diag}\{\boldsymbol{p}\}\boldsymbol{Wh}(L)\|^2 + \gamma L \tag{4}$$

where $\gamma = 0.08$ (the rationale behind GAIC are discussed in some detail in, e.g., [7]). Denote by L_1 the L that minimizes (4).

4. Increment T (starting from $T = T_2$) and simultaneously decrease L (starting from $L = L_1$) until (4) is minimized. Let the so-obtained final timing and channel length estimates be denoted by $\hat{T}$ and $\hat{L}$, respectively.

Note that the algorithm is iterative but terminates within a finite number of steps.

4. PHASE CORRECTION BASED ON PILOT SYMBOLS

The received signal will inevitably suffer from a CFO, which can be estimated and corrected for using methods such as those in [3, 2, 4]. These methods estimate the CFO based on the received data in the preamble only, and despite being statistically sound, they will never be perfectly accurate. The remaining CFO error results in a phase error that increases linearly with time. As a remedy to this problem, we perform an additional phase correction for each OFDM symbol to compensate for the (small) remaining CFO error.

Each OFDM symbol contains 4 known pilot symbols. Let $\boldsymbol{q}$ be a 4×1 vector of these pilot symbols, and let $\boldsymbol{z}$ be the corresponding 4 elements of the DFT of the received data, i.e., of $\boldsymbol{Wr}$. For each OFDM symbol, we estimate a channel phase correction ϕ by minimizing the LS criterion $\|\boldsymbol{z} - \boldsymbol{q}e^{j\phi}\|^2$ which has the solution $\phi = \arg(\boldsymbol{q}^*\boldsymbol{z})$. This phase correction is used to obtain a compensated received signal $\hat{\boldsymbol{r}} = e^{-j\phi}\boldsymbol{r}$, upon which the detection of the data symbols is based. As we illustrate below, this phase correction can have a significant influence on the performance.

5. NUMERICAL EXAMPLES

We provide a few Monte-Carlo simulation results to illustrate the effectiveness of our new algorithm. In all simulations, we consider a Rayleigh fading channel according to [8], with $L = 6$ Gaussian distributed coefficients h_l having a mean power of $\sigma_l^2 = E[|h_l|^2] = \sigma_0^2 e^{-\alpha l}$ for $l = 1, \ldots, L$ and where σ_0 is such that $\sum_{l=1}^{L} \sigma_l^2 = 1$ and $\alpha = 5/3$. The channel is fixed during the transmission of one packet but independent from one packet to another. A CFO of 0.025 Hz is introduced in the simulation and a simple algorithm based on the phase of the correlation of two subsequent OFDM symbols in the preamble is applied to estimate and remove the CFO error (see, e.g., [4]). White Gaussian noise is added to the data to simulate a received signal with a certain ratio of energy per information bit to the spectral density of the noise (E_b/N_0).

Example 1: Timing estimation. Figure 2 shows the distribution of the different timing estimates T_1 (initial coarse timing), T_2 (refined timing estimate from Step 2) and $\hat{T}$ (final timing estimate). The true timing is $T = 194$ and E_b/N_0 is 14 dB. It is clear from the figure that our algorithm succeeded to recover the true timing exactly in more than 90% of the realization, and to within a few sample intervals in virtually all test cases.

Example 2: Estimation of the effective channel length. In Figure 3 we show the distribution of the channel length estimates L_1 (after Step 3) and $\hat{L}$ (the final channel length estimate). Note that the channel length is underestimated in most realizations since the last elements of the impulse response are usually very small.

Example 3: Bit error rate (BER) for QPSK data symbols. We illustrate the BER obtained by simulation of an IEEE 801.11 OFDM system using (a) using our algorithm without the additional channel phase correction; (b) using our algorithm together with the additional phase correction based on the pilot symbols; and (c) perfect knowledge of the timing, channel and CFO. The results are shown in Figure 4. We observe from the figure that our synchronization and channel estimation algorithm achieves a performance close to the bound provided by the exact knowledge of the timing and the transmission channel. Furthermore, it is evident that the usage of the pilot symbols is necessary to fully compensate for the CFO.

6. CONCLUDING REMARK

We have presented a novel and conceptually simple algorithm for joint synchronization and channel estimation for the IEEE 801.11 WLAN standard. The algorithm is based on ML estimation and the GAIC information theoretic criterion. Numerical examples show that as far as the BER is concerned, our algorithm achieves a performance close to the ultimate bound provided by the exact knowledge of the transmission channel; and therefore eliminates the need for more complicated approaches to the CFO, timing and channel estimation.

7. REFERENCES

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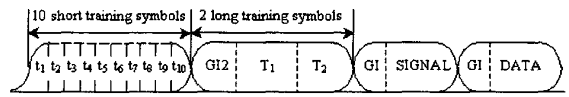

Figure 1: The structure of an OFDM packet.

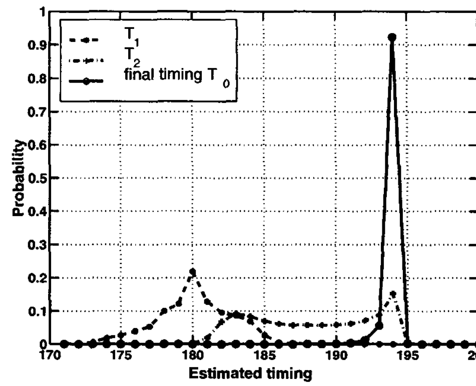

Figure 2: Distribution of the timing estimates.

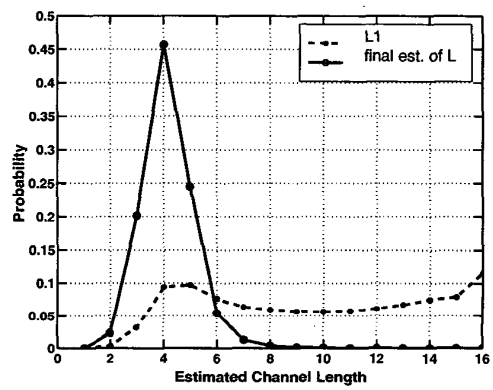

Figure 3: Distribution of the channel length estimates.

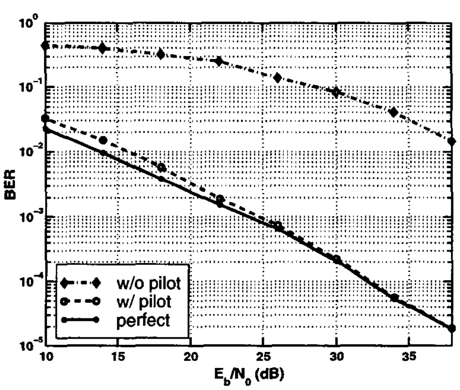

Figure 4: Simulated BER for QPSK data.