

# VANDERMONDE-LAGRANGE MUTUALLY ORTHOGONAL FLEXIBLE TRANSCEIVERS FOR BLIND CDMA IN UNKNOWN MULTIPATH

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## ABSTRACT

A Mutually-Orthogonal Usercode-Receiver (AMOUR) system was recently proposed to guarantee identifiability of transmitted symbols irrespective of channel nulls in addition to offering deterministic MUI elimination and low complexity transceivers. Motivated by the desire to equip the AMOUR with added flexibility, we develop in this paper a Vandermonde-Lagrange AMOUR system that achieves the same MUI eliminating property and offers additional advantages in the code assignment procedure. We also derive blind adaptive equalizers within the VL-AMOUR framework and illustrate via simulations that they perform very close to the theoretical bound derived with perfect channel information.

## 1. INTRODUCTION

Suppression of multiuser interference (MUI) and mitigation of multipath effects constitute major challenges in the design of third-generation wireless mobile systems such as UMTS and IMT-2000. Most wideband and multicarrier uplink CDMA schemes suppress MUI statistically in the presence of unknown multipath and often impose restrictive and difficult to check conditions on the FIR channel nulls (see e.g., [7]). Generalizing the LV-CDMA system [5], A Mutually-Orthogonal Usercode-Receiver (AMOUR) system was proposed recently [1], to guarantee identifiability of transmitted symbols irrespective of channel nulls in addition to offering deterministic MUI elimination and low complexity transceivers. In the AMOUR system however, whenever one user changes code or a new user enters the system, other users also need to adjust their codes to maintain the mutual orthogonality between the usercodes and receivers. To offer the system more flexibility in the code assignment procedure, we design in this paper a VL-AMOUR system in which only the receiver (typically the base station) needs to make changes when one user changes code or when new users join the system. Based on the system modeling in Section 2, we develop the VL-AMOUR System in Section 3. In Section 4, we derive a criterion based direct blind equalizer and test its performance via simulations.

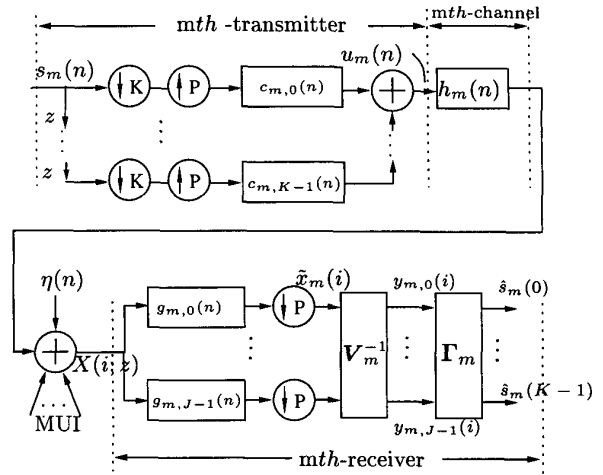


Figure 1: Discrete-time baseband AMOUR system

## 2. SYSTEM MODELING

Generalizing the filterbank model proposed in [1], Fig. 1 represents the uplink channel of a CDMA system, described in terms of its discrete-time equivalent baseband model, where signals, codes, and channels are represented by samples of their complex envelopes taken at the chip rate (only transmitter and receiver filters for one, the  $m$ th, user are shown). Advance elements and down/up-samplers serve the purpose of blocking and inserting zeros, so that each of the  $M$  users maps successive blocks of  $K$  symbols  $\mathbf{s}_m(i) := [s_m(iK), \dots, s_m(iK + K - 1)]^T$  to blocks of  $P > K$  chips  $\mathbf{u}_m(i) := [u_m(iP), \dots, u_m(iP + P - 1)]^T$  through a matrix  $\mathbf{C}_m$ , whose  $(n, k)$ th entry is  $c_{m,k}(n)$ :  $\mathbf{u}_m(i) = \mathbf{C}_m \mathbf{s}_m(i)$  (the system considered in [1] is a special case of Fig. 1 corresponding to  $c_{m,k}(n) = c_{m,0}(n - k)$ ). The coded chip sequence  $u_m(n)$  then passes through the discrete-time equivalent baseband channel, denoted by its impulse response  $h_m(n)$ . The channels  $h_m(n)$  or their  $\mathcal{Z}$ -transforms  $H_m(z)$ ,  $m = 0, \dots, M - 1$ , are assumed to be of order  $\leq L$ , a common assumption in quasi-synchronous CDMA systems. In addition to multipath and the transmit-receive filters,  $h_m(n)$  includes the  $m$ th user's asynchronism in the form of delay factors. The discrete-time

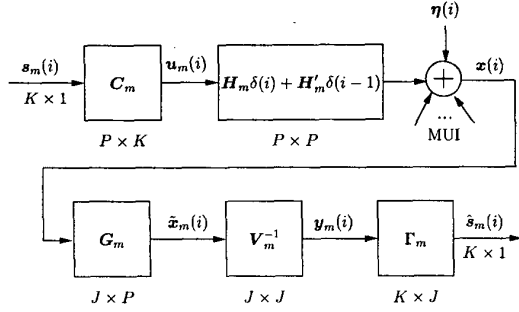


Figure 2: Block model of Fig. 1

received sequence is  $x(n) = \sum_{m=0}^{M-1} x_m(n) + \eta(n)$ , where:  $x_m(n) = \sum_{j=-\infty}^{\infty} u_m(j)h_m(n-j)$ , and  $\eta(n)$  is the AGN. In matrix form, introducing  $\mathbf{x}(i) := [x(iP) \cdots x(iP+P-1)]^T$  ( $T$  denotes transpose) and we obtain

$$\mathbf{x}(i) = \sum_{\mu=0}^{M-1} (\mathbf{H}_\mu \mathbf{C}_\mu \mathbf{s}_\mu(i) + \mathbf{H}'_\mu \mathbf{C}_\mu \mathbf{s}_\mu(i-1)) + \boldsymbol{\eta}(i), \quad (1)$$

where  $\boldsymbol{\eta}(i)$  is defined similar to  $\mathbf{x}(i)$ ,  $\mathbf{H}_\mu$  is a  $P \times P$  Toeplitz matrix with the first row  $[h_\mu(0) \ 0 \ \dots \ 0]$  and first column  $[h_\mu(0) \ \dots \ h_\mu(L) \ 0 \ \dots \ 0]^T$ , and  $\mathbf{H}'_\mu$  is a  $P \times P$  upper triangular Toeplitz matrix with first row  $[0 \ \dots \ 0 \ h_\mu(L) \ \dots \ h_\mu(1)]$ , which accounts for interblock interference (IBI).

The receive-filterbank consists of  $J$  parallel filters each of length  $P$  and will be described by the  $J \times P$  matrix  $\mathbf{G}_m$  whose  $(j, p)$ th (starting from  $(0,0)$ ) entry is  $g_{m,j}(P-1-p)$ . Matrix  $\mathbf{G}_m$  and downsamplers map the block  $\mathbf{x}(i)$  to an MUI-free block  $\tilde{\mathbf{x}}_m(i) := \mathbf{G}_m \mathbf{x}(i)$  in a transformed domain:

$$\tilde{\mathbf{x}}_m(i) = \sum_{\mu=0}^{M-1} (\mathbf{G}_m \mathbf{H}_\mu \mathbf{C}_\mu \mathbf{s}_\mu(i) + \mathbf{G}_m \mathbf{H}'_\mu \mathbf{C}_\mu \mathbf{s}_\mu(i-1)) + \mathbf{G}_m \boldsymbol{\eta}(i). \quad (2)$$

The  $J \times J$  matrix  $\mathbf{V}_m^{-1}$  performs the inverse transform on  $\tilde{\mathbf{x}}(i)$ , to yield the  $J \times 1$  vector  $\mathbf{y}_m(i)$ . Finally, symbol estimates  $\hat{\mathbf{s}}_m(i)$  are obtained by multiplying  $\mathbf{y}_m(i)$  with the equalizing matrix  $\mathbf{\Gamma}_m$  (matrices  $\mathbf{G}_m$ ,  $\mathbf{V}_m^{-1}$ , and  $\mathbf{\Gamma}_m$  will be specified in Section 3 and 4). Fig. 1 can be redrawn compactly in matrix form as in Fig. 2.

### 3. VANDERMONDE-LAGRANGE AMOUR

In order for  $\tilde{\mathbf{x}}_m(i)$  to be free of MUI irrespective of the signal constellation adopted, we need that  $\forall m \neq \mu$

$$\begin{aligned} \text{(a)} \quad & \mathbf{G}_m \mathbf{H}_\mu \mathbf{C}_\mu = \mathbf{0}_{J \times P} \quad \text{and} \\ \text{(b)} \quad & \mathbf{G}_m \mathbf{H}'_\mu \mathbf{C}_\mu = \mathbf{0}_{J \times P}. \end{aligned} \quad (3)$$

Eq. (3b) can be satisfied if we choose either

$$\begin{aligned} \text{(a)} \quad & c_{m,k}(p) = 0, \quad \forall m, k, \forall p \geq P-L, \quad \text{or} \\ \text{(b)} \quad & g_{m,j}(P-1-p) = 0, \quad \forall m, j, \forall p \leq L-1, \end{aligned} \quad (4)$$

where (4a) corresponds to precoders with trailing zeros, while (4b) corresponds to receive-filters with leading zeros.

Eq. (3a) can be satisfied irrespective of  $h_\mu(n)$ ,  $\forall \mu$ , if and only if it is satisfied for  $h_\mu(n) = \delta(n-l)$ ,  $\forall \mu, \forall l \in [0, L]$ . Or equivalently,  $\forall l \in [0, L]$ , the following must hold:

$$\sum_{p=0}^{P-1-l} g_{m,j}(P-1-p-l)c_{\mu,k}(p) = 0, \quad \forall m \neq \mu, \forall j, k \quad (5)$$

In [1], (4a) is used and the receive-filters are chosen to be Vandermonde,  $g_{m,j}(p) = \rho_{m,j}^{-P+1+p}$  (where  $\rho_{m,j}$  are termed the ‘‘signature points’’ of user  $m$ ), and the precoder  $c_{\mu,k}(p)$  is designed so that  $C_{\mu,k}(z) := \sum_{p=0}^{P-1} c_{\mu,k}(p)z^{-p}$  satisfies  $C_{\mu,k}(\rho_{m,j}) = 0 \quad \forall m \neq \mu, \forall j, k$ . With such precoders and receive-filters, (5) is satisfied because  $\sum_{p=0}^{P-1-l} g_{m,j}(p+l)c_{\mu,k}(p) = \rho_{m,j}^{-l} C_{\mu,k}(\rho_{m,j}) = 0, \quad \forall m \neq \mu, \forall j, k$ . Note that  $\{\rho_{m,j}\}_{j=0}^{J-1}$  are roots common to all (except the  $m$ th) users’ codes  $C_{\mu,k}(z)$ . Therefore when user  $m$  changes code or joins the system, all other users must change their codes accordingly.

To simplify the code assignment procedure, we pursue an idea also used in VL-CDMA [4]: namely, we adopt Vandermonde precoders

$$c_{\mu,k}(p) = \mathcal{K}_\mu \sum_{j=0}^{J-1} \rho_{\mu,j}^{-P+1+p-k}, \quad p = 0 \cdots P-1, \quad (6)$$

where  $\mathcal{K}_\mu$  is a constant controlling transmitted power [1], and Lagrange receive-filters  $G_{m,j}(z) := \sum_{p=0}^{P-1} g_{m,j}(p)z^{-p}$  that satisfy  $\forall j, \lambda \in [0, J-1]$

$$G_{m,j}(\rho_{\mu,\lambda}) = \delta(m-\mu)\delta(j-\lambda), \quad \forall m, \mu. \quad (7)$$

Specifically,  $G_{m,j}(z)$  is a polynomial obtained by Lagrange interpolation through the points  $\rho_{\mu,\lambda}$ ,  $(\mu, \lambda) \neq (m, j)$ :

$$G_{m,j}(z) = \prod_{\substack{\mu=0 \\ (\mu,\lambda) \neq (m,j)}}^{M-1} \prod_{\lambda=0}^{J-1} \frac{1 - \rho_{\mu,\lambda} z^{-1}}{1 - \rho_{\mu,\lambda} \rho_{m,j}^{-1}}. \quad (8)$$

Because  $\{G_{m,j}(z)\}_{j=0}^{J-1}$  are of order  $MJ-1$ , if we choose  $P = MJ + L$ , then the receive-vectors

$$\mathbf{g}_{m,j}^T := [0 \cdots 0 \ g_{m,j}(MJ-1) \cdots g_{m,j}(0)]^T \quad (9)$$

will be equipped with  $P - MJ = L$  leading zeros and (4b) will be satisfied.

With the designed Vandermonde precoders (6), we have

$$\begin{aligned} \mathbf{G}_m \mathbf{H}_\mu \mathbf{C}_\mu \mathbf{s}_\mu(i) &= \mathcal{K}_\mu \mathbf{G}_m \mathbf{H}_\mu \begin{pmatrix} \rho_{\mu,0}^{-P+1} & \cdots & \rho_{\mu,J-1}^{-P+1} \\ \vdots & & \vdots \\ \rho_{\mu,0}^0 & \cdots & \rho_{\mu,J-1}^0 \end{pmatrix} \begin{pmatrix} S_\mu(i; \rho_{\mu,0}) \\ \vdots \\ S_\mu(i; \rho_{\mu,J-1}) \end{pmatrix} \\ &= \mathcal{K}_\mu \begin{pmatrix} G_{m,0}(\rho_{\mu,0}) & \cdots & G_{m,0}(\rho_{\mu,J-1}) \\ \vdots & & \vdots \\ G_{m,J-1}(\rho_{\mu,0}) & \cdots & G_{m,J-1}(\rho_{\mu,J-1}) \end{pmatrix} \begin{pmatrix} H_\mu(\rho_{\mu,0}) S_\mu(i; \rho_{\mu,0}) \\ \vdots \\ H_\mu(\rho_{\mu,J-1}) S_\mu(i; \rho_{\mu,J-1}) \end{pmatrix} \end{aligned}$$

where  $S_m(i; z) := \sum_{k=0}^K s_m(iK + k)z^{-k}$ ,  $H_m(z) := \sum_{l=0}^L H_m(l)z^{-l}$ , and because of (7), the square matrix in the second equality is an identity matrix  $\mathbf{I}$ , if  $m = \mu$ , and  $\mathbf{0}$  otherwise. Therefore, we can write  $\tilde{\mathbf{x}}_m(i)$  as [c.f. (2)]

$$\tilde{\mathbf{x}}_m(i) = \mathcal{K}_m \begin{pmatrix} H_m(\rho_{m,0})S_m(i; \rho_{m,0}) \\ \vdots \\ H_m(\rho_{m,J-1})S_m(i; \rho_{m,J-1}) \end{pmatrix} + \mathbf{G}_m \boldsymbol{\eta}(i). \quad (10)$$

In order to recover the ‘‘information bearing’’ polynomial  $H_m(z)S_m(i; z)$  of order  $L + K - 1$  from its  $J$  (noisy) samples  $\tilde{\mathbf{x}}_m(i)$ , we choose  $J = L + K$ . Although choosing  $J > L + K$  is also possible, the bandwidth efficiency

$$\mathcal{E} := \frac{KM}{P} = \frac{KM}{MJ + L} \quad (11)$$

will decrease. From (10), we can see that the designed VL-AMOUR transmit- and receive- filters achieve the same deterministic MUI elimination property as the Lagrange-Vandermonde AMOUR does in [1]. In addition, since the transmit-filters are linear combinations of the Vandermonde vectors derived from the users’ own signature points only, when one user  $\mu$  changes code (or equivalently signature points), no other user needs to adjust and only the receiver, typically the base station, needs to alter its filters  $\{G_{\mu,j}(z)\}_{j=0}^{J-1}$  accordingly. This substantially reduces the need for cooperation required among users and thus agrees with the CDMA philosophy.

In steps, we design the VL-AMOUR system as follows:

- s1) Choose  $K \gg L$ ,  $J = L + K$ , and  $P = MJ + L$  so that  $\mathcal{E}$  in (11) is close to one; choose  $MJ$  distinct complex points  $\rho_{m,j}$  and assign  $J$  of them  $\{\rho_{m,j}\}_{j=0}^{J-1}$  to be signature points of the  $m$ th user,  $\forall m$ ;
- s2) Design the Vandermonde precoders according to (6);
- s3) Design the Lagrange receive filters according to (8).

After eliminating MUI in (10), the same linear mappings  $\mathbf{V}_m^{-1}$ , where  $\mathbf{V}_m$  is a  $J \times J$  matrix with its  $(i, j)$ th entry  $\rho_{m,j}^{-i}$ , as in [1], can be applied to  $\tilde{\mathbf{x}}_m(i)$  to obtain the single input single output (SISO) vector model

$$\mathbf{y}_m(i) = \tilde{\mathbf{H}}_m \tilde{\mathbf{s}}_m(i) + \boldsymbol{\eta}_m(i), \quad (12)$$

where,

$$\tilde{\mathbf{H}}_m := \begin{bmatrix} h_m(i; 0) & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ h_m(i; K-1) & & h_m(i; 0) & & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & h_m(i; K-1) & \cdots & h_m(i; 0) \end{bmatrix} \quad (13)$$

is an  $(L + K) \times (L + K)$  lower triangular Toeplitz matrix,  $\tilde{\mathbf{s}}_m(i) := [s_m(i; 0), \dots, s_m(i; K-1), 0, \dots, 0]^T$  has  $L$  trailing zeros appended to  $\mathbf{s}_m(i)$ , and  $\boldsymbol{\eta}_m(i) := \mathbf{V}_m^{-1} \mathbf{G}_m \boldsymbol{\eta}(i)$ .

Equation (12) also shows that the designed VL-AMOUR system converts the multiuser CDMA system into  $M$  parallel single-user systems irrespective of the multipath as in [1]. Any single user equalizer  $\boldsymbol{\Gamma}_m$  can then be applied to  $\mathbf{y}_m(i)$  in order to recover the symbols  $s_m(i; k)$ ,  $\forall k \in [0, K - 1]$ .

#### 4. ADAPTIVE EQUALIZATION

In this section, we develop a direct adaptive blind equalizer within the VL-AMOUR framework and test its performance via simulations. By direct we mean that the equalizer can be constructed without explicit estimation of the channel.

Since we know that the last  $L$  symbols of  $\tilde{\mathbf{s}}_m(i)$  are zeros, a good criterion for selecting the equalizer is to minimize the total energy (sample averaged or ensemble averaged) in the last  $L$  symbols of the equalized data. Notice that decorrelating solution  $\boldsymbol{\Gamma}_m^{(zf)} = \tilde{\mathbf{H}}_m^{-1}$  forces the last  $L$  symbols to be zeros when there is no noise. Because  $\tilde{\mathbf{H}}_m$  is a lower triangular Toeplitz matrix, so is  $\boldsymbol{\Gamma}_m^{(zf)}$ . But we know that at high SNR, MMSE solutions boil down to zero-forcing ones, so it is reasonable if we pose a similar condition on the direct equalizer  $\boldsymbol{\Gamma}_m$ ; i.e., constrain the equalizer to be lower triangular Toeplitz matrix. Subject to this constraint, our criterion becomes

$$\hat{\boldsymbol{\Gamma}}_m = \arg \min_{\boldsymbol{\Gamma}_m} (\epsilon_L(\boldsymbol{\Gamma}_m)), \quad (14)$$

where  $\epsilon_L(\boldsymbol{\Gamma}_m)$  is the total energy in the equalized last  $L$  symbols. The energy can be defined either as a sample average or as an ensemble average. Here, we adopt the latter for simplicity. Let  $\boldsymbol{\Gamma}_m := [\gamma_0, \gamma_1, \dots, \gamma_{L+K-1}]^T$ . Then the energy in (14) is given by

$$\epsilon_L(\boldsymbol{\Gamma}_m) = \sum_{l=0}^{L-1} E\{|\gamma_{L+K-1-j}^H \mathbf{y}_m(i)|^2\}. \quad (15)$$

Since  $\boldsymbol{\Gamma}_m$  is a lower triangular Toeplitz matrix, all rows  $\{\gamma_i\}_{i=0}^{L+K-1}$  can be obtained from the last row  $\gamma_{L+K-1}^T$ . So design of the direct equalizer based on (14) entails finding the vector  $\gamma_{L+K-1}^T$  only. To exclude the trivial solution of (14), we further constrain  $\|\gamma_{L+K-1}\| = \text{const}$ .

Let  $\mathbf{J}$  denote a  $P \times P$  shift matrix having all ones in the first sub-diagonal. Using  $\mathbf{J}$ , the rows of  $\boldsymbol{\Gamma}_m$  can be obtained from  $\gamma_{L+K-1}^T$  as  $\gamma_l^T = \gamma_{L+K-1}^T \mathbf{J}^{L+K-1-l}$ . Therefore, we can write (15) as

$$\begin{aligned} \epsilon_L(\boldsymbol{\Gamma}_m) &= \sum_{l=0}^{L-1} \gamma_{L+K-1}^H \mathbf{J}^l \mathbf{R}_{\mathbf{y}_m} (\mathbf{J}^l)^H \gamma_{L+K-1} \\ &= \gamma_{L+K-1}^H \mathcal{R}_{\mathbf{y}_m} \gamma_{L+K-1}, \end{aligned} \quad (16)$$

where  $\mathcal{R}_{\mathbf{y}_m} := \sum_{l=0}^{L-1} \mathbf{J}^l \mathbf{R}_{\mathbf{y}_m} (\mathbf{J}^l)^H$  and  $\mathbf{R}_{\mathbf{y}_m}$  is the auto-correlation matrix of  $\mathbf{y}_m(i)$ . Vector  $\gamma_{L+K-1}$  can be found as

the eigenvector corresponding to the minimum eigenvalue of  $\mathcal{R}_{y_m}$ .

Similar direct equalization methods are dealt with in [6] and [3]. It can be shown that if the channel is minimum phase, the three equalizers turn out to be the same, except that the one in [6] also takes into account of the noise covariance. The proposed equalizer herein can be viewed as a re-derivation or interpretation of those in [6, 3] with the optimality criterion (14). If the channel is non-minimum phase, an optimum delay must be taken into account in the equalizer design (see [3] for details). Here we will only test minimum phase channels.

To adaptively evaluate the last row  $\gamma_{L+K-1}$  of  $\mathbf{T}_m$ , we adopt a modified version of the algorithm proposed in [2] as follows:

$$\begin{aligned}\tilde{\mathcal{R}}_{y_m}^{(i)} &= \sum_{l=0}^{L-1} \mathbf{J}^l \mathbf{y}_m(i) \mathbf{y}_m^H(i) (\mathbf{J}^l)^H, \\ \tilde{\gamma}_{L+K-1}^{(i)} &= \gamma_{L+K-1}^{(i-1)} - \delta_i \tilde{\mathcal{R}}_{y_m}^{(i)} \gamma_{L+K-1}^{(i-1)}, \\ \gamma_{L+K-1}^{(i)} &= \tilde{\gamma}_{L+K-1}^{(i)} / \|\tilde{\gamma}_{L+K-1}^{(i)}\|,\end{aligned}\quad (17)$$

where the Euclidean vector norm is used, and  $\delta_i$  is the usual gain of stochastic approximation. As the iteration  $i \rightarrow \infty$ ,  $\gamma_i$  will converge almost surely [2] to the minimum eigenvector of  $E[\tilde{\mathcal{R}}_{y_m}^{(i)}] = \mathcal{R}_{y_m}$ .

To test convergence of the proposed direct adaptive equalizer, we simulated a system (the  $m$ th user's equivalent system in (12)) with parameters  $K = 16$  and  $L = 2$ . The channel was chosen to be  $H_m(z) = 1 + 0.6z^{-1} + 0.3z^{-2}$  (minimum phase). QPSK modulation was used. We implemented (17) with  $\delta_i = 10/i$ . Fig. 3 shows the Mean Square Error (MSE) of  $\gamma_{L+K-1}^{(i)}$  normalized with its true value, i.e., the one evaluated with true autocorrelation matrix, versus the number of iterations  $i$  used. After 500 iterations (8000 symbols) the algorithm essentially converges to the desired value for reasonable SNRs ( $\geq 5$  dB).

For a VL-AMOUR system with  $(M, K, L) = (4, 16, 1)$ , we compared the performance of the proposed equalizer with the theoretical bound derived in [1, Eq. (11)] with perfect CSI (see Fig. 4). For the proposed equalizer, the BER was computed after the algorithm had reached steady state. One hundred Rayleigh fading channels of order  $L = 1$  were simulated. The BER was averaged over all users and all 100 channels. We can see that the proposed equalizer performs very close to the theoretical bound even when CSI is not available.

## 5. REFERENCES

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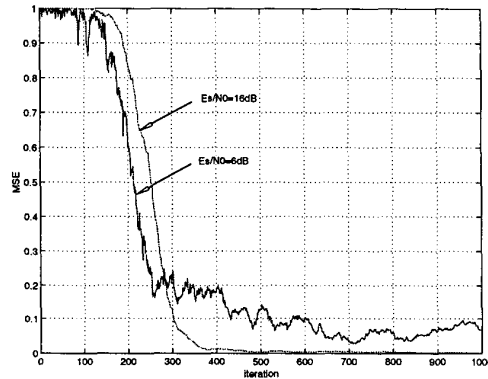


Figure 3: MSE of normalized  $\gamma_{L+K-1}$  vs. iterations

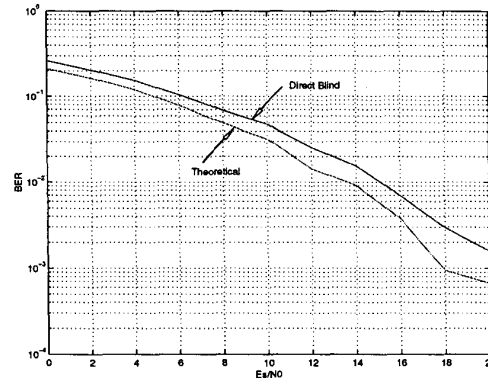


Figure 4: Performance of the proposed equalizer