Space-time Coded Transmissions with Maximum Diversity Gains over Frequency-Selective Multipath Fading Channels *

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***Abstract*— In this paper, we study space-time block coding for single carrier block transmissions through frequency selective multipath channels of order L. We prove that maximum diversity of order $2(L+1)$ can be achieved with two transmit and one receive antennae. We show that simple linear receiver processing can achieve full antenna diversity gains, and investigate the performance versus complexity tradeoffs that emerge when one exploits also the embedded multipath diversity. Simulation results demonstrate that joint exploitation of multi-antenna and multipath diversities yields significantly enhanced performance in frequency selective multipath channels.**

I. Introduction

Space-time (ST) coding has by now been well documented as an attractive means of achieving high data rate transmissions with diversity and coding gains in wireless applications; see e.g., [4, 5] for tutorial treatments. Existing space-time codes are mainly designed for frequency flat channels. In broadband wireless applications however, the symbol duration becomes smaller than the channel delay spread and frequency selectivity arises. It is thus important to design ST coded transmissions through frequency selective multipath channels.

Optimal design of ST codes for dispersive multipath channels is complex since signals are mixed both in space and in time. In order to maintain decoding simplicity and take advantage of existing ST codes, most works on ST coding for frequency selective channels employ orthogonal frequency division multiplexing (OFDM) modulation to convert the frequency selective channels to a set of flat fading subchannels (see e.g., [4] and references therein). However, ST coded OFDM transmissions essentially deploy ST codes designed for flat fading channels and do not exploit the maximum achievable multipath diversity. Furthermore, an inherent drawback of multicarrier (OFDM) schemes is their non-constant modulus transmissions, that incur considerable power efficiency loss.

In this paper, we study space-time block codes design for *single carrier block* transmissions in the presence of frequency-selective multipath channels. ST codes designed for single carrier systems in the presence of frequency selective channels have been reported in [2] for serial transmissions and in [8] for block transmissions. Unlike [8] that mainly focuses on mitigating rapidly time varying channels with suboptimum receivers, we here deal with block quasi static channels and prove that the maximum diversity gain is the product of the number of transmit-antennas, the number of receive-antennas, and the length of the channel.

We investigate two different block transmission formats, one inserting cyclic prefix (CP) in the front of each symbol block (termed as CP-only), and another one padding zeros after each symbol block (termed as ZP-only). We establish here that CP-only with ST coding achieves only full multi-antenna diversity, while ZP-only exploits both multi-antenna and multipath diversities. A maximum diversity of order $2(L+1)$ can be gained in rich scattering environments with two transmit and one receive antennae, where L is the order of underlying FIR multipath channels. We further show that simple frequency domain linear processing achieves full multi-antenna diversity, while tradeoffs need to be made when it comes to exploiting the embedded multipath diversity. Simulation results demonstrate the superior performance when both multiantenna and multipath diversities are exploited.

Notation: Bold upper (lower) letters denote matrices (column vectors); $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^{\mathcal{H}}$ denote conjugate, transpose and Hermitian transpose; $\mathbf{I}_K$ denotes the identity matrix of size K, $\mathbf{0}_{M\times N}$ ($\mathbf{1}_{M\times N}$) denotes an all-zero (all-one) matrix with size $M\times N$, and $\mathbf{F}_N$ denotes an $N\times N$ FFT matrix with its (p,q)th entry to be $(1/\sqrt{N})e^{-j\frac{2\pi}{N}(p-1)(q-1)}$, $\forall p,q\in[1,N]$; diag$(\mathbf{x})$ stands for a diagonal matrix with $\mathbf{x}$ on its diagonal. $[\cdot]_p$ denotes the pth entry of a vector, and $[\cdot]_{p,q}$ denotes the (p,q)th entry of a matrix; Matlab notation $\mathbf{A}(:,p:q)$ is used to denote a submatrix of $\mathbf{A}$ constructed from columns p to q.

II. Single Carrier Block Transmissions

Fig. 1 depicts the discrete-time equivalent model of a communication system with $N_t=2$ transmit antennas and $N_r=1$ receive antenna. At the transmitter, the information data symbols $s(n)$ belonging to the constellation set $\mathcal{A}$ are first parsed to form $K\times 1$ blocks $\mathbf{s}(i):=[s(iK),s(iK+1),\ldots,s(iK+K-1)]^T$. The blocks $\mathbf{s}(i)$ are precoded by a $J\times K$ matrix $\mathbf{\Theta}_1$ to yield $J\times 1$ symbol blocks: $\tilde{\mathbf{s}}(i):=\mathbf{\Theta}_1\mathbf{s}(i)$. We will here restrict $\mathbf{\Theta}_1=\mathbf{I}_K$ and leave the general case for [11]; i.e., we have $J=K$ and $\tilde{\mathbf{s}}(i)=\mathbf{s}(i)$ in this paper.

Our ST encoder takes two consecutive blocks $\tilde{\mathbf{s}}(2i)$ and $\tilde{\mathbf{s}}(2i+1)$ to output the following $2J\times 2$ matrix of ST coded blocks:

$$\begin{bmatrix}\bar{\mathbf{s}}_1(2i) & \bar{\mathbf{s}}_1(2i+1)\\ \bar{\mathbf{s}}_2(2i) & \bar{\mathbf{s}}_2(2i+1)\end{bmatrix}=\begin{bmatrix}\tilde{\mathbf{s}}(2i) & -\mathbf{P}\tilde{\mathbf{s}}^*(2i+1)\\ \tilde{\mathbf{s}}(2i+1) & \mathbf{P}\tilde{\mathbf{s}}^*(2i)\end{bmatrix}, \quad (1)$$

where $\mathbf{P}$ is a permutation matrix that will be defined soon. At each block transmission time interval i, the blocks $\bar{\mathbf{s}}_1(i)$ and $\bar{\mathbf{s}}_2(i)$ are forwarded to the first and the second antenna, respectively. The important relationship from (1) is:

$$\bar{\mathbf{s}}_1(2i+1)=-\mathbf{P}\bar{\mathbf{s}}_2^*(2i), \qquad \bar{\mathbf{s}}_2(2i+1)=\mathbf{P}\bar{\mathbf{s}}_1^*(2i), \quad (2)$$

which indicates that the transmitted block at time slot $2i+1$ from one antenna is a conjugated and permuted version of the transmitted block at time slot $2i$ from the other antenna (with a possible sign change). For flat fading channels, symbol blocking is unnecessary and the design of (1) reduces to the well known Alamouti ST Code with $K=1$ and $\mathbf{P}=1$ [1]. However, it

* This work was supported by NSF Wireless Initiative grant no. 99-79443.

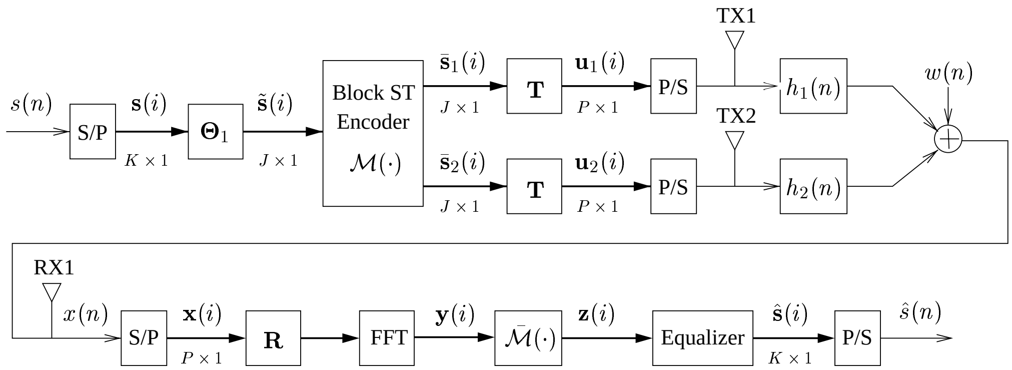

Fig. 1. Single Carrier Space-Time Transceiver model

will turn out that for frequency selective multipath channels, the permutation matrix $\mathbf{P}$ is necessary.

The $J \times J$ matrix $\mathbf{P}$ can be drawn from the following set $\{\mathbf{P}_J^{(n)}\}_{n=0}^{J-1}$, where J signifies the dimensionality. Each permutation matrix $\mathbf{P}_J^{(n)}$ actually performs a reversed cyclic shift, i.e., for a $J \times 1$ vector $\mathbf{a} := [a(0), a(1), \ldots, a(J-1)]^T$, we have $[\mathbf{P}_J^{(n)}\mathbf{a}]_i = a((J - i + n) \bmod J)$. Two important special cases are $\mathbf{P}_J^{(0)}$ and $\mathbf{P}_J^{(1)}$. The permuted block $\mathbf{P}_J^{(0)}\mathbf{a} = [a(J-1), a(J-2), \ldots, a(0)]^T$ is actually a time-reversed version of $\mathbf{a}$, which has been also utilized by [2, 8]. On the other hand, $\mathbf{P}_J^{(1)}\mathbf{a} = [a(0), a(J-1), a(J-2), \ldots, a(1)]^T = \mathbf{F}_J^{-1}\mathbf{F}_J^{\mathcal{H}}\mathbf{a} = \mathbf{F}_J^{\mathcal{H}}\mathbf{F}_J^{\mathcal{H}}\mathbf{a}$ corresponds to taking IFFT twice on the vector $\mathbf{a}$; thus, it is a special case of the Z-transform approach proposed in [3], when the Z-domain points are chosen equally spaced on the unit circle as: $\{e^{j\frac{2\pi}{J}n}\}_{n=0}^{J-1}$.

At each antenna $\mu \in [1, 2]$, a tall $P \times J$ transmit-matrix $\mathbf{T}$ is applied on $\bar{\mathbf{s}}_\mu(i)$ to obtain $P \times 1$ blocks: $\mathbf{u}_\mu(i) = \mathbf{T}\bar{\mathbf{s}}_\mu(i)$, before transmission through the frequency selective multipath channels; the role of $\mathbf{T}$ will be detailed soon.

With chip rate sampling at the receiver, we model the baseband equivalent channel between the μth transmit antenna and the receive antenna as a finite impulse response (FIR) filter with coefficients $h_\mu(0), \ldots, h_\mu(L)$, where L is the channel order; this discrete time equivalent model includes the physical multipath channel as well as transmit and receive pulse shaping filters. Furthermore, in this paper, we consider a rich scattering environment for which $h_\mu(l)$'s are well modeled as i.i.d. Gaussian distributed random variable with variance $1/(L+1)$ (correlated cases are studied in [11]). Therefore, defining the channel vector as $\mathbf{h}_\mu := [h_\mu(0), \ldots, h_\mu(L)]^T$, we denote the channel covariance matrix as $\mathbf{R}_{h_\mu} = \mathrm{E}\{\mathbf{h}_\mu\mathbf{h}_\mu^{\mathcal{H}}\} = \mathbf{I}_{L+1}/(L+1)$.

The received samples $x(n)$ are serial to parallel converted to form $P \times 1$ vectors: $\mathbf{x}(i) := [x(iP), x(iP+1), \ldots, x(iP+P-1)]^T$, and likewise for the noise vector $\mathbf{w}(i)$. Let $\mathbf{H}_\mu$ be the $P \times P$ lower triangular Toeplitz channel matrix with first column $[h_\mu(0), \ldots, h_\mu(L), 0, \ldots, 0]^T$, and $\mathbf{H}_\mu^{(\text{ibi})}$ be the $P \times P$ upper triangular Toeplitz matrix with first row $[0, \ldots, 0, h_\mu(L), \ldots, h_\mu(1)]$. The block input-output relationship of the FIR channels can be described by (see e.g., [9]):

$$\mathbf{x}(i) = \textstyle\sum_{\mu=1}^{2}[\mathbf{H}_\mu\mathbf{T}\bar{\mathbf{s}}_\mu(i) + \mathbf{H}_\mu^{(\text{ibi})}\mathbf{T}\bar{\mathbf{s}}_\mu(i-1)] + \mathbf{w}(i), \quad (3)$$

where the second term in the sum accounts for the so-called Inter Block Interference (IBI).

To allow for low-complexity block by block processing, a receive-matrix $\mathbf{R}$ with appropriate dimensions is applied to $\mathbf{x}(i)$ to get rid of the IBI. The latter is accomplished by choosing $(\mathbf{T}, \mathbf{R})$ such that:

$$\mathbf{R}\mathbf{H}_\mu^{(\text{ibi})}\mathbf{T} = \mathbf{0}. \quad (4)$$

Among many other possibilities [11], two choices are popular and will be discussed here. The first is the cyclic prefix (CP) approach that is also adopted by conventional OFDM systems. It corresponds to setting $P = J + L$, and selecting

$$\mathbf{T} = \mathbf{T}_{cp} := [\mathbf{I}_{cp}^T, \mathbf{I}_J^T]^T, \qquad \mathbf{R} = \mathbf{R}_{cp} := [\mathbf{0}_{L \times J}, \mathbf{I}_J], \quad (5)$$

where $\mathbf{I}_{cp}$ is a matrix formed by the last L rows of $\mathbf{I}_J$. Indeed, multiplying $\mathbf{T}_{cp}$ with $\bar{\mathbf{s}}_\mu(i)$ replicates the last L entries of $\bar{\mathbf{s}}_\mu(i)$ in its front. Unlike OFDM, our term CP-only reinforces the fact that we here deal with single-carrier transmissions.

The second approach to removing IBI is zero padding (ZP) [9], which amounts to choosing $P = J + L$ and

$$\mathbf{T} = \mathbf{T}_{zp} := [\mathbf{I}_J^T, \mathbf{0}_{J \times L}^T]^T, \qquad \mathbf{R} = \mathbf{I}_P. \quad (6)$$

By substituting (5) or (6), into (4), we can verify that both CP and ZP achieve IBI elimination. With either CP or ZP transceiver pairs $(\mathbf{T}, \mathbf{R})$, the resulting IBI-free output becomes:

$$\mathbf{R}\mathbf{x}(i) = \sum_{\mu=1}^{2}\mathbf{R}\mathbf{H}_\mu\mathbf{T}\bar{\mathbf{s}}_\mu(i) + \mathbf{R}\mathbf{w}(i). \quad (7)$$

We next specify CP-only and ZP-only transmissions, and analyze their performance from a diversity perspective.

A. Cyclic Prefixing (CP)-only design

Here, the $(\mathbf{T}, \mathbf{R})$ pair is chosen to be $(\mathbf{T}_{cp}, \mathbf{R}_{cp})$ as in (5) and we can thus re-write (7) as:

$$\begin{aligned}\tilde{\mathbf{x}}(i) := \mathbf{R}_{cp}\mathbf{x}(i) &= \textstyle\sum_{\mu=1}^{2}\mathbf{R}_{cp}\mathbf{H}_\mu\mathbf{T}_{cp}\bar{\mathbf{s}}_\mu(i) + \tilde{\mathbf{w}}(i) \\ &:= \textstyle\sum_{\mu=1}^{2}\tilde{\mathbf{H}}_\mu\bar{\mathbf{s}}_\mu(i) + \tilde{\mathbf{w}}(i),\end{aligned} \quad (8)$$

where $\tilde{\mathbf{w}}(i) = \mathbf{R}_{cp}\mathbf{w}(i)$ denotes the truncated noise vector and $\tilde{\mathbf{H}}_\mu := \mathbf{R}_{cp}\mathbf{H}_\mu\mathbf{T}_{cp}$ denotes the resulting equivalent channel matrix. From the definition of $\mathbf{T}_{cp}, \mathbf{H}_\mu$, and $\mathbf{R}_{cp}$, we

can easily verify that $\tilde{\mathbf{H}}_\mu$ is circulant with entries $[\tilde{\mathbf{H}}_\mu]_{p,q} = h_\mu((p-q) \bmod J)$ [9]. Circulant matrices have two nice properties that are exploited in this paper:
i) circulant matrices can be diagonalized by FFT operations:

$$\tilde{\mathbf{H}}_\mu = \mathbf{F}_J^{\mathcal{H}} \mathbf{D}(\tilde{\mathbf{h}}_\mu) \mathbf{F}_J \quad \text{and} \quad \tilde{\mathbf{H}}_\mu^{\mathcal{H}} = \mathbf{F}_J^{\mathcal{H}} \mathbf{D}(\tilde{\mathbf{h}}_\mu^*) \mathbf{F}_J, \tag{9}$$

where $\mathbf{D}(\tilde{\mathbf{h}}_\mu) := \text{diag}(\tilde{\mathbf{h}}_\mu)$ is a diagonal matrix having $\tilde{\mathbf{h}}_\mu := [H_\mu(e^{j0}), H_\mu(e^{j\frac{2\pi}{J}}), \ldots, H_\mu(e^{j\frac{2\pi}{J}(J-1)})]$ as its main diagonal, with $[\tilde{\mathbf{h}}_\mu]_p$ being the channel frequency response $H_\mu(z) := \sum_{l=0}^{L} h_\mu(l) z^{-l}$ evaluated at the frequency $z = e^{j\frac{2\pi}{J}(p-1)}$.
ii) pre- and post- multiplying the circulant matrix $\tilde{\mathbf{H}}_\mu$ by $\mathbf{P}$ yields $\tilde{\mathbf{H}}_\mu^T$:

$$\mathbf{P}\tilde{\mathbf{H}}_\mu \mathbf{P} = \tilde{\mathbf{H}}_\mu^T \quad \text{and} \quad \mathbf{P}\tilde{\mathbf{H}}_\mu^* \mathbf{P} = \tilde{\mathbf{H}}_\mu^{\mathcal{H}}. \tag{10}$$

To verify (10), notice that:

$$[\mathbf{P}_J^{(n)} \tilde{\mathbf{H}}_\mu \mathbf{P}_J^{(n)}]_{p,q} = h_\mu(q-p \bmod J) = [\tilde{\mathbf{H}}_\mu]_{q,p}.$$

With the ST design in (1) and (2), we can thus write the two consecutive blocks as:

$$\tilde{\mathbf{x}}(2i) = \tilde{\mathbf{H}}_1 \bar{\mathbf{s}}_1(2i) + \tilde{\mathbf{H}}_2 \bar{\mathbf{s}}_2(2i) + \tilde{\mathbf{w}}(i) \tag{11}$$

$$\tilde{\mathbf{x}}(2i+1) = -\tilde{\mathbf{H}}_1 \mathbf{P} \bar{\mathbf{s}}_2^*(2i) + \tilde{\mathbf{H}}_2 \mathbf{P} \bar{\mathbf{s}}_1^*(2i) + \tilde{\mathbf{w}}(2i+1). \tag{12}$$

We next conjugate the permuted block $\mathbf{P}\tilde{\mathbf{x}}(2i+1)$ and use (10) to arrive at:

$$\mathbf{P}\tilde{\mathbf{x}}^*(2i+1) = -\tilde{\mathbf{H}}_1^{\mathcal{H}} \bar{\mathbf{s}}_2(2i) + \tilde{\mathbf{H}}_2^{\mathcal{H}} \bar{\mathbf{s}}_1(2i) + \mathbf{P}\tilde{\mathbf{w}}^*(2i+1). \tag{13}$$

Note that without the permutation matrix $\mathbf{P}$ inserted at the transmitter, it would have been impossible to have the Hermitian of the channel matrices that will prove instrumental for enabling multiantenna diversity gains with linear receiver processing.

We will next pursue frequency domain processing by forming $\mathbf{y}(2i) := \mathbf{F}_J \tilde{\mathbf{x}}(2i)$, $\mathbf{y}^*(2i+1) := \mathbf{F}_J \mathbf{P} \tilde{\mathbf{x}}^*(2i+1)$, and likewise $\tilde{\boldsymbol{\eta}}(2i) := \mathbf{F}_J \tilde{\mathbf{w}}(2i)$, $\tilde{\boldsymbol{\eta}}^*(2i+1) := \mathbf{F}_J \mathbf{P} \tilde{\mathbf{w}}^*(2i+1)$. For notational simplicity, we also define $\boldsymbol{\mathcal{D}}_1 := \mathbf{D}(\tilde{\mathbf{h}}_1)$ and $\boldsymbol{\mathcal{D}}_2 := \mathbf{D}(\tilde{\mathbf{h}}_2)$. Using (9) in (11) and (13), we obtain the FFT-processed successive blocks as:

$$\mathbf{y}(2i) = \boldsymbol{\mathcal{D}}_1 \mathbf{F}_J \bar{\mathbf{s}}_1(2i) + \boldsymbol{\mathcal{D}}_2 \mathbf{F}_J \bar{\mathbf{s}}_2(2i) + \tilde{\boldsymbol{\eta}}(2i), \tag{14}$$

$$\mathbf{y}^*(2i+1) = -\boldsymbol{\mathcal{D}}_1^* \mathbf{F}_J \bar{\mathbf{s}}_2(2i) + \boldsymbol{\mathcal{D}}_2^* \mathbf{F}_J \bar{\mathbf{s}}_1(2i) + \tilde{\boldsymbol{\eta}}(2i+1). \tag{15}$$

Permutation, conjugation, and FFT operation on the received blocks $\tilde{\mathbf{x}}(i)$ do not incur any information loss and the additive noise in (14) and (15) remains white. Therefore, relying on $\mathbf{y}(2i)$ and $\mathbf{y}^*(2i+1)$ to perform symbol detection entails no loss of optimality.

Defining $\check{\mathbf{y}}(i) := [\mathbf{y}^T(2i), \mathbf{y}^{\mathcal{H}}(2i+1)]^T$, we obtain from (14) and (15) the matrix-vector representation:

$$\check{\mathbf{y}}(i) = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{D}}_1 & \boldsymbol{\mathcal{D}}_2 \\ \boldsymbol{\mathcal{D}}_2^* & -\boldsymbol{\mathcal{D}}_1^* \end{bmatrix}}_{:=\boldsymbol{\mathcal{D}}} \begin{bmatrix} \mathbf{F}_J \mathbf{s}(2i) \\ \mathbf{F}_J \mathbf{s}(2i+1) \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{\eta}}(2i) \\ \tilde{\boldsymbol{\eta}}^*(2i+1) \end{bmatrix}, \tag{16}$$

where the identities $\bar{\mathbf{s}}_1(2i) = \mathbf{s}(2i)$ and $\bar{\mathbf{s}}_2(2i) = \mathbf{s}(2i+1)$ have been used following the design in (1).

We will further suppose here that $\mathbf{h}_1$ and $\mathbf{h}_2$ do not share common roots, so that the diagonal matrix $\bar{\boldsymbol{\mathcal{D}}}_{12} := [\boldsymbol{\mathcal{D}}_1^* \boldsymbol{\mathcal{D}}_1 + \boldsymbol{\mathcal{D}}_2^* \boldsymbol{\mathcal{D}}_2]^{1/2}$ is invertible (see [11] for means of bypassing this assumption). Based on the fact that $\boldsymbol{\mathcal{D}}^{\mathcal{H}} \boldsymbol{\mathcal{D}} = \mathbf{I}_2 \otimes \bar{\boldsymbol{\mathcal{D}}}_{12}^2$, where $\otimes$ stands for Kronecker product, we can construct the unitary matrix $\mathbf{U} := \boldsymbol{\mathcal{D}}(\mathbf{I}_2 \otimes \bar{\boldsymbol{\mathcal{D}}}_{12}^{-1})$ which satisfies: $\mathbf{U}^{\mathcal{H}}\mathbf{U} = \mathbf{I}_{2J}$ and $\mathbf{U}^{\mathcal{H}} \boldsymbol{\mathcal{D}} = \mathbf{I}_2 \otimes \bar{\boldsymbol{\mathcal{D}}}_{12}$. Multiplying $\check{\mathbf{y}}(i)$ by $\mathbf{U}^{\mathcal{H}}$ does not incur any loss of decoding optimality. Thus, forming $\check{\mathbf{z}}(i) := [\mathbf{z}^T(2i), \mathbf{z}^T(2i+1)]^T$, we arrive at:

$$\check{\mathbf{z}}(i) = \mathbf{U}^{\mathcal{H}} \check{\mathbf{y}}(i) = \begin{bmatrix} \bar{\boldsymbol{\mathcal{D}}}_{12} \mathbf{F}_J \mathbf{s}(2i) \\ \bar{\boldsymbol{\mathcal{D}}}_{12} \mathbf{F}_J \mathbf{s}(2i+1) \end{bmatrix} + \mathbf{U}^{\mathcal{H}} \begin{bmatrix} \tilde{\boldsymbol{\eta}}(2i) \\ \tilde{\boldsymbol{\eta}}^*(2i+1) \end{bmatrix} \tag{17}$$

where the noise $\check{\boldsymbol{\eta}}(i) := [\boldsymbol{\eta}^T(2i), \boldsymbol{\eta}^T(2i+1)]^T$ is still white.

We deduce from (17) that the blocks $\mathbf{s}(2i)$ and $\mathbf{s}(2i+1)$ can be demodulated separately. Equivalently, we need to demodulate $\mathbf{s}(i)$ from the following blocks:

$$\mathbf{z}(i) = \bar{\boldsymbol{\mathcal{D}}}_{12} \mathbf{F}_J \mathbf{s}(i) + \boldsymbol{\eta}(i). \tag{18}$$

Deferring specific equalizer designs to Section III, we will now focus on the benchmark performance of ML decoding to examine what is the best achievable performance.

Dropping the block index i for brevity, we will henceforth denote e.g., $\mathbf{s}(i)$ by $\mathbf{s}$. With perfect Channel State Information (CSI) at the receiver, we consider the pairwise error probability (PEP) $P(\mathbf{s} \to \mathbf{s}' | \mathbf{h}_1, \mathbf{h}_2)$ that the symbol block $\mathbf{s}$ is transmitted but is erroneously decoded as $\mathbf{s}' \neq \mathbf{s}$. The PEP can be approximated using the Chernoff bound as

$$P(\mathbf{s} \to \mathbf{s}' | \mathbf{h}_1, \mathbf{h}_2) \leq \exp(-d^2(\mathbf{z}, \mathbf{z}')/4N_0), \tag{19}$$

where $d(\mathbf{z}, \mathbf{z}')$ is the Euclidean distance between $\mathbf{z}$ and $\mathbf{z}'$.

Define the error vector as $\mathbf{e} := \mathbf{s} - \mathbf{s}'$. Starting with (18), we can express the distance as:

$$\begin{aligned} d^2(\mathbf{z}, \mathbf{z}') &= |\bar{\boldsymbol{\mathcal{D}}}_{12} \mathbf{F}_J \mathbf{e}|^2 = \mathbf{e}^{\mathcal{H}} \mathbf{F}_J^{\mathcal{H}} \bar{\boldsymbol{\mathcal{D}}}_{12}^2 \mathbf{F}_J \mathbf{e} \\ &= \mathbf{e}^{\mathcal{H}} \mathbf{F}^{\mathcal{H}} (\boldsymbol{\mathcal{D}}_1^* \boldsymbol{\mathcal{D}} + \boldsymbol{\mathcal{D}}_2^* \boldsymbol{\mathcal{D}}_2) \mathbf{F}_J \mathbf{e} = |\boldsymbol{\mathcal{D}}_1 \mathbf{F}_J \mathbf{e}|^2 + |\boldsymbol{\mathcal{D}}_2 \mathbf{F}_J \mathbf{e}|^2. \end{aligned} \tag{20}$$

Defining $\mathbf{D}_e := \text{diag}(\mathbf{F}_J \mathbf{e})$, we have for each $\mu \in [1, 2]$ that $\boldsymbol{\mathcal{D}}_\mu \mathbf{F}_J \mathbf{e} = \mathbf{D}_e \tilde{\mathbf{h}}_\mu = \mathbf{D}_e \mathbf{V} \mathbf{h}_\mu$, where $\mathbf{V}$ is the first $L+1$ columns of $\sqrt{J}\mathbf{F}_J$; i.e., $\mathbf{V} := \sqrt{J}\mathbf{F}_J(:, 1 : L+1)$. With $|\boldsymbol{\mathcal{D}}_\mu \mathbf{F}_J \mathbf{e}|^2 = |\mathbf{D}_e \mathbf{V} \mathbf{h}_\mu|^2$, we can rewrite (20) as:

$$d^2(\mathbf{z}, \mathbf{z}') = |\mathbf{D}_e \mathbf{V} \mathbf{h}_1|^2 + |\mathbf{D}_e \mathbf{V} \mathbf{h}_2|^2. \tag{21}$$

Defining $\mathbf{A}_e = \mathbf{D}_e \mathbf{V}$, we note that $\mathbf{A}_e^{\mathcal{H}} \mathbf{A}_e$ is symmetric and non-negative definite; therefore, there exists a unitary matrix $\mathbf{U}_e$ such that $\mathbf{U}_e^{\mathcal{H}} \mathbf{A}_e^{\mathcal{H}} \mathbf{A}_e \mathbf{U}_e = (L+1)\boldsymbol{\Lambda}_e$, where $\boldsymbol{\Lambda}_e$ is a diagonal matrix with non-increasing diagonal entries collected in the vector $\boldsymbol{\lambda}_e := [\lambda_e(0), \lambda_e(1), \lambda_e(L)]^T$.

Let $\mathbf{h}'_\mu := \sqrt{L+1}\mathbf{U}_e^{\mathcal{H}} \mathbf{h}_\mu$ have correlation matrix $\mathbf{R}_{\mathbf{h}'_\mu} = (L+1)\mathbf{U}_e^{\mathcal{H}} \mathbf{R}_{h_\mu} \mathbf{U}_e = \mathbf{I}_{L+1}$. Therefore, $\mathbf{h}'_\mu$ is a zero mean complex Gaussian vector with unit variance i.i.d entries. With $\mathbf{h}'_\mu$, we can rewrite (21) as:

$$\begin{aligned} d^2(\mathbf{z}, \mathbf{z}') &= \sum_{\mu=1}^{2} (\mathbf{h}'_\mu)^{\mathcal{H}} \mathbf{U}_e^{\mathcal{H}} \mathbf{A}_e^{\mathcal{H}} \mathbf{A}_e \mathbf{U}_e \mathbf{h}'_\mu / (L+1) \\ &= \sum_{l=0}^{L} \lambda_e(l) |h'_1(l)|^2 + \sum_{l=0}^{L} \lambda_e(l) |h'_2(l)|^2. \end{aligned} \tag{22}$$

Using (22), we average (19) with respect to the i.i.d. Rayleigh random variables $|\bar{h}'_1(l)|, |\bar{h}'_2(l)|$, to obtain the following upper bound on the average PEP:

$$P(\mathbf{s} \rightarrow \mathbf{s}') \leq \prod_{l=0}^{L} \frac{1}{(1+\lambda_e(l)/(4N_0))^2}. \qquad (23)$$

If r_e is the rank of $\mathbf{A}_e$ (and thus $\mathbf{A}_e^{\mathcal{H}}\mathbf{A}_e$), then $\lambda_e(l) \neq 0$ if and only if $l \in [0, r_e - 1]$. It thus follows from (23) that

$$P(\mathbf{s} \rightarrow \mathbf{s}') \leq (1/N_0)^{-2r_e} \left(\prod_{l=0}^{r_e} \lambda_e(l)/4 \right)^{-2}. \qquad (24)$$

As in [6], we call $2r_e$ the diversity advantage $G_{d,e}$ and $[\prod_{l=0}^{r_e} \lambda_e(l)/4]^{1/r_e}$ the coding advantage $G_{c,e}$ of our system for a given symbol error vector $\mathbf{e}$. The diversity advantage $G_{d,e}$ determines the slope of the averaged (w.r.t. the random channel) PEP (between $\mathbf{s}$ and $\mathbf{s}'$) as a function of the signal to noise ratio (SNR) at high SNR ($N_0 \rightarrow 0$). Correspondingly, $G_{c,e}$ determines the shift of this PEP curve in SNR relative to a benchmark error rate curve of $(1/N_0)^{-2r_e}$. Different from [6] that relied on PEP to design (nonlinear) ST codes for flat fading channels, we here invoke PEP bounds to prove diversity properties of linear CP (or ZP)-only block transmissions over frequency selective channels.

Since both $G_{d,e}$ and $G_{c,e}$ depend on the choice of $\mathbf{e}$ (thus on $\mathbf{s}$ and $\mathbf{s}'$), we define the diversity and coding diversity advantages for our system respectively as:

$$G_d := \min_{\mathbf{e} \neq 0} G_{d,e}, \quad \text{and} \quad G_c := \min_{\mathbf{e} \neq 0} G_{c,e}. \qquad (25)$$

Because the matrix $\mathbf{A}_e^{\mathcal{H}}\mathbf{A}_e$ has dimensionality of $L+1$, we first have the following result: *The maximum achievable diversity for CP-only is $G_d = 2(L+1)$.*

Now let us check the diversity order actually achieved with CP-only transmissions. The worst case is to select $\mathbf{s} = a\mathbf{1}_{J\times 1}$ and $\mathbf{s}' = a'\mathbf{1}_{J\times 1}$ such that $\mathbf{e} = (a - a')\mathbf{1}_{J\times 1}$ where $a, a' \in \mathcal{A}$. For these error events, the matrix $\mathbf{D}_e = \text{diag}(\mathbf{F}_J\mathbf{e})$ has only one non-zero entry, so that $r_e = 1$. Therefore, the system diversity order is

$$G_d = 2, \quad \text{for CP-only.} \qquad (26)$$

This order 2 diversity is primarily coming from the two transmit antennas and not from the multipath channels.

In the next subsection, we show that ZP-only achieves maximum diversity order provided by both multiantenna transmissions and multipath channels.

B. Zero Padding (ZP)-only design

The transceiver pair $(\mathbf{T}, \mathbf{R})$ is now changed to $\mathbf{T} = \mathbf{T}_{zp}$ and $\mathbf{R} = \mathbf{I}_P$ as defined in (6). The transmitted blocks are thus $\mathbf{u}_\mu(i) = \mathbf{T}_{zp}\bar{\mathbf{s}}_\mu(i)$ with L zeros padded at the end of each block. These L guard zeros are instrumental since we can now replace the $P \times P$ Toeplitz matrix $\mathbf{H}_\mu$ in (7) by $P \times P$ circulant matrix $\acute{\mathbf{H}}_\mu$ with $[\acute{\mathbf{H}}_\mu]_{p,q} = h_\mu(p - q \bmod P)$, thanks to the equality $\mathbf{H}_\mu\mathbf{T}_{zp} = \acute{\mathbf{H}}_\mu\mathbf{T}_{zp}$. Therefore, we have

$$\mathbf{x}(i) = \sum_{\mu=1}^{2} \acute{\mathbf{H}}_\mu\mathbf{T}_{zp}\bar{\mathbf{s}}_\mu(i) + \mathbf{w}(i). \qquad (27)$$

Compared with (8), we can view ZP-only as a CP-only transmission but with information blocks precoded to yield $\mathbf{T}_{zp}\bar{\mathbf{s}}_\mu(i)$. To follow the frequency domain linear processing of the CP-only case, we need an equation like (2) but for the equivalent symbol blocks $\mathbf{T}_{zp}\bar{\mathbf{s}}_\mu(i)$. Unlike CP-only, where we had multiple choices for $\mathbf{P}$, here we need to fix $\mathbf{P} = \mathbf{P}_J^{(0)}$ in (1) to achieve this goal. We can then verify that:

$$\begin{aligned} \mathbf{T}_{zp}\bar{\mathbf{s}}_1(2i+1) &= -\mathbf{P}_P^{(K)}\mathbf{T}_{zp}\bar{\mathbf{s}}_2^*(2i), \\ \mathbf{T}_{zp}\bar{\mathbf{s}}_2(2i+1) &= \mathbf{P}_P^{(K)}\mathbf{T}_{zp}\bar{\mathbf{s}}_1^*(2i), \end{aligned} \qquad (28)$$

only when $\mathbf{P} = \mathbf{P}_J^{(0)}$ is used in (1). Recognizing the resemblance between (27) and (8), and together with (28), we obtain the following system output by taking similar steps to derive (18) in CP-only:

$$\acute{\mathbf{z}}(i) = \bar{\boldsymbol{\mathcal{D}}}'_{12}\mathbf{F}_P\mathbf{T}_{zp}\mathbf{s}(i) + \acute{\boldsymbol{\eta}}(i), \qquad (29)$$

where $\bar{\boldsymbol{\mathcal{D}}}'_{12} := [\acute{\boldsymbol{\mathcal{D}}}_1^*\acute{\boldsymbol{\mathcal{D}}}_1 + \acute{\boldsymbol{\mathcal{D}}}_2^*\acute{\boldsymbol{\mathcal{D}}}_2]^{1/2}$ with the diagonal matrix defined as $\acute{\boldsymbol{\mathcal{D}}}_\mu := \text{diag}(H_\mu(e^{j0}), \ldots, H_\mu(e^{j\frac{2\pi}{P}(P-1)}))$; and $\acute{\mathbf{z}}(i)$, $\acute{\boldsymbol{\eta}}(i)$ defined accordingly.

Let us now check the diversity gain achieved by ZP-only. Define $\acute{\boldsymbol{\Theta}} := \mathbf{F}_P\mathbf{T}_{zp}$ and $\acute{\mathbf{V}} := \sqrt{P}\mathbf{F}_P(:, 1 : L+1)$. We thus obtain correspondingly $\acute{\mathbf{D}}_e = \text{diag}(\acute{\boldsymbol{\Theta}}\mathbf{e})$ and $\acute{\mathbf{A}}_e = \acute{\mathbf{D}}_e\acute{\mathbf{V}}$. Similarly, with $r_e = \text{rank}(\acute{\mathbf{A}}_e)$, the system diversity gain for ZP-only is $2r_e$. Since $\acute{\boldsymbol{\Theta}}$ here is a Vandermonde matrix and any of its K rows are linearly independent, $\acute{\boldsymbol{\Theta}}\mathbf{e}$ has at least $(L+1)$ nonzero entries for any $\mathbf{e}$. Indeed, if $\acute{\boldsymbol{\Theta}}\mathbf{e}$ has only L nonzero entries for some $\mathbf{e}$, then it has K zero entries. Picking the corresponding K rows of $\acute{\boldsymbol{\Theta}}$ to form the truncated matrix $\bar{\boldsymbol{\Theta}}$, we have $\bar{\boldsymbol{\Theta}}\mathbf{e} = \mathbf{0}$. The latter shows that these K rows are linearly dependent, which is impossible. With $\acute{\mathbf{D}}_e = \text{diag}(\acute{\boldsymbol{\Theta}}\mathbf{e})$ having at least $L+1$ nonzero diagonal entries, we have that $\acute{\mathbf{A}}_e = \acute{\mathbf{D}}_e\acute{\mathbf{V}}$ has full rank because any $L+1$ rows of $\acute{\mathbf{V}}$ are linearly independent. Thus, the maximum achievable diversity gain is indeed achieved by ZP-only; i.e.,

$$G_d = 2(L+1) \qquad \text{for ZP-only.} \qquad (30)$$

More general, it is shown in [11] that $G_d = N_tN_r(L+1)$ when N_t transmit- and N_r receive- antennas are used.

Compared with CP-only, the advantage of ZP-only is that full multipath diversity is gained with the same redundancy used to get rid of IBI. Furthermore, we emphasize here that the processing for ZP-only in (29) is *overall ML optimal*, which is never the case for CP-only because the cyclic prefix that is discarded at the receiver contains information about the symbols $s(n)$ we want to detect.

III. Discussion on Equalization Options

To unify (18) and (29), let us express the system output after frequency domain linear ML processing as:

$$\mathbf{z}(i) = \mathbf{A}\mathbf{s}(i) + \boldsymbol{\eta}(i), \qquad (31)$$

with the corresponding $\mathbf{A}$'s being

$$\mathbf{A} = \bar{\boldsymbol{\mathcal{D}}}_{12}\mathbf{F}_J \qquad \text{CP-only} \tag{32}$$

$$\mathbf{A} = \bar{\boldsymbol{\mathcal{D}}}'_{12}\mathbf{F}_P\mathbf{T}_{zp} \qquad \text{ZP-only} \tag{33}$$

$$\mathbf{A} = \bar{\boldsymbol{\mathcal{D}}}_{12} \qquad \text{CP-OFDM} \tag{34}$$

where (34) is copied from [4] for the ST encoded OFDM.

For our information blocks $\mathbf{s}(i)$ with length K, ML decoding based on (31) needs $|\mathcal{A}|^K$ enumerations ($|\mathcal{A}|$ is the cardinality of the alphabet), which is certainly prohibitive when the constellation size and/or the block length increases. A relatively faster ML search is possible with the sphere decoding (SD) algorithm, which only searches for vectors that are within a sphere centered at the received symbols [7]. The theoretical complexity of SD is polynomial in K, which is better than exponential but still too high for large block size K.

Linear Zero-Forcing (ZF) and MMSE equalizers certainly offer low complexity alternatives, but they may not be able to collect the full diversity. An encouraging observation from our simulations indicates that linear equalization comes close to the ML performance when the multi-antenna diversity increases.

We refer the reader to [10] for detailed complexity comparisons of various equalizers. We underscore here that the multiantenna diversity is first achieved by simple linear processing in the frequency domain, and the overall receiver complexity mainly depends on multipath channel equalization; thus, space time coded transmissions have almost the same receiver complexity as with the single antenna system in [10].

IV. SIMULATIONS

To test the system performance, we set $L = 2$ (3-ray channels) and the block size $K = 8$. We assume that the channels between each transmit and each receive antenna are i.i.d. Gaussian distributed with covariance matrix $\mathbf{I}_{L+1}/(L+1)$. We use QPSK constellations and use E_b/N_0 to denote the average received bit energy to noise ratio on each receive antenna. With $N_t = 2$ and $N_r = 1$, we infer from Fig. 2 that ZP-only outperforms CP-only and CP-OFDM significantly, especially with linear ZF equalizers because $\mathbf{A}$ in (31) is guaranteed to have full rank for ZP-only which is not the case for CP-only [11]. However, SD equalization yields similar performance for both CP-only and ZP-only in the considered SNR range; explanations are given in [11].

In Fig. 3, we increase our multiantenna diversity by deploying $N_r = 2, 4$ receive antennas. For CP-only and CP-OFDM transmissions, the multiantenna diversity goes up to $2N_r = 4, 8$, while the total diversity offered by ZP-only now increases to $2N_r(L+1) = 12, 24$. Similar to Fig. 2, ZP-only outperforms CP-only and CP-OFDM. However, an interesting observation is that the difference between linear ZF and near-optimal SD equalization becomes smaller as multi-antenna diversity increases. With $N_r = 4$, the difference between ZF and SD equalizers for ZP-only and CP-only is only within several tenths of a dB, which is very encouraging from an overall performance-complexity perspective.

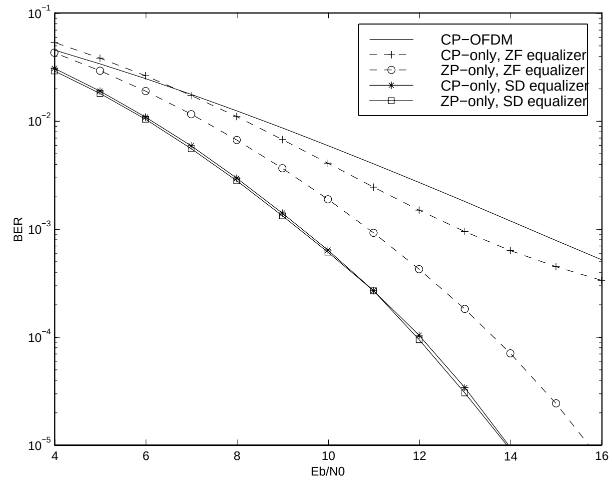

Fig. 2. Comparisons between CP-OFDM, CP-only and ZP-only, $N_r = 1$

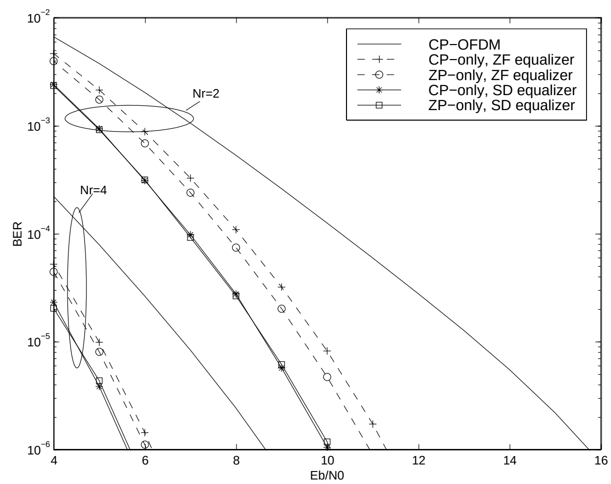

Fig. 3. Comparisons between CP-OFDM, CP-only and ZP-only, $N_r = 2, 4$

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