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Research Article

Energy-Constrained Optimal Quantization for Wireless Sensor Networks

Xiliang Luo[1] and Georgios B. Giannakis[2]

[1] *Qualcomm Inc., San Diego, CA 92121, USA*
[2] *Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455, USA*

Correspondence should be addressed to Georgios B. Giannakis, georgios@umn.edu

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As low power, low cost, and longevity of transceivers are major requirements in wireless sensor networks, optimizing their design under energy constraints is of paramount importance. To this end, we develop quantizers under strict energy constraints to effect optimal reconstruction at the fusion center. Propagation, modulation, as well as transmitter and receiver structures are jointly accounted for using a binary symmetric channel model. We first optimize quantization for reconstructing a single sensor's measurement, and deriving the optimal number of quantization levels as well as the optimal energy allocation across bits. The constraints take into account not only the transmission energy but also the energy consumed by the transceiver's circuitry. Furthermore, we consider multiple sensors collaborating to estimate a deterministic parameter in noise. Similarly, optimum energy allocation and optimum number of quantization bits are derived and tested with simulated examples. Finally, we study the effect of channel coding on the reconstruction performance under strict energy constraints and jointly optimize the number of quantization levels as well as the number of channel uses.

1. INTRODUCTION

Wireless sensor networks (WSN) are gaining increasing research interest for their emerging potential in both consumer and national security applications. Sensor networks are envisioned to be used for surveillance, identification, and tracking of targets. They can also serve as the first line of detection for various types of biological hazards such as toxic gas attacks. In civilian applications, WSN can be used to monitor the environment and measure quantities such as temperature and pollution levels.

In most application scenarios, WSN nodes are powered by small batteries, which are practically nonrechargeable, either due to cost limitations or because they are deployed in hostile environments with high temperature, high pollution levels, or high nuclear radiation levels. These considerations motivate well energy-saving and energy-efficient WSN designs. One approach to prolong battery lifetime is the use of energy-harvesting radios as the ones in [1] with power dissipation levels below 100 μW. A lot of research has been carried out to devise energy efficient algorithms in each layer of WSN [2]. Optimal modulation with minimum energy requirements to transmit a given number of bits with a prescribed bit error rate (BER) bound is considered in [3]. Energy efficient medium access control (MAC) and routing protocols are studied in [4, 5], respectively.

In this paper, we consider a WSN with a fusion center which collects data from sensor nodes and performs the final information extraction task. A common goal in most WSN applications is to reconstruct the underlying physical phenomenon (e.g., temperature) based on sensor measurements. Energy as well as bandwidth limitations prevent sensor nodes from transmitting real valued (analog-amplitude) data to the fusion center. This motivates the goal of this paper which is to derive optimal quantization schemes at sensor nodes under strict energy constraints. Optimality here is in the sense of minimizing a bound on the mean-absolute reconstruction error at the fusion center. The problem setup originates from the following considerations. Suppose we deploy a WSN powered by nonrechargeable batteries and expect it to operate a given number of times, which bounds the energy allowed per time of operation. One operation could be, for example, one time transmitting a burst of temperature measurements to the fusion center with bounded energy allowed per burst. The problem of designing quantizers to optimize pertinent reconstruction performance metrics under a given energy budget emerges naturally.

Most of the prior works on optimal quantization deal with optimization of the quantization rules for detecting a signal in dependent or independent noise [6–9]. Other related works include [10–15]. Assuming error-free transmission, [10, 11] focus on the impact of bandwidth/rate constraints in WSN on the distributed estimation performance. Optimal quantization thresholds, given the number of quantization levels and channel coding for binary symmetric channels (BSC), are jointly designed in [13] to minimize the mean-square error of reconstruction. In [14], scaling of the reconstruction error with the number of quantization bits per Nyquist-period is studied. The rate-distortion region, when taking into account the possible failure of communication links and sensor nodes, is presented in [12]. Possibly the most closely related to our present work, [15] minimizes the total transmission energy for a given target estimation error performance. Different from these works, our objective is to optimize the quantization per node (including the number of quantization bits and the transmission energy allocation across bits) under a fixed total energy per measurement in order to minimize the reconstruction error at the fusion center. We account for both transmission energy as well as circuit energy consumption, while we (i) incorporate the noisy channel between each sensor and the fusion center by modeling it as a BSC with cross-over probability controlled by the transmitted bit energy, and (ii) allow different quantization bits to be allocated different energy and, thus, effect different cross-over probabilities.

The rest of the paper is organized as follows. In Section 2, we consider optimal quantization in a point-to-point (single-hop) link to recover a single sensor's measurement with uncoded transmission schemes. In Section 3, optimal quantization is addressed in a multisensor (star topology) setting. The role of channel coding is then examined in Section 4. Section 5 provides some illustrative numerical results and Section 6 concludes the paper.

2. POINT-TO-POINT LINK

Let us consider the system depicted in Figure 1, where a single sensor acquires a local measurement A, which is properly scaled so that $A \in [0,1]$, and wishes to transmit it to the fusion center. For digital transmission, the sensor first quantizes the real valued measurement A to A_Q. Letting $A := \sum_{i=1}^{\infty} b_i 2^{-i}$, throughout this paper, we consider N-bit uniform quantization so that

$$A_Q = \sum_{i=1}^{N} b_i 2^{-i}. \quad (1)$$

The quantization bits $\{b_i\}_{i=1}^{N}$ are transmitted through the wireless channel to the fusion center, and are demodulated as

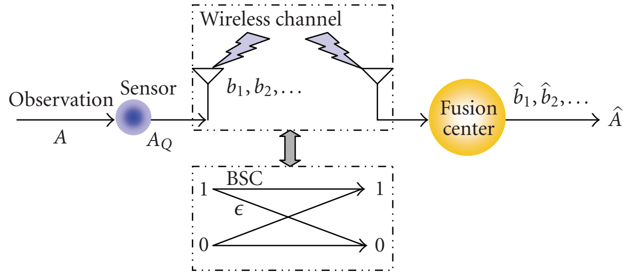


Figure 1: System model of a single sensor's quantized measurement received through a BSC at the fusion center.

$\{\hat{b}_i\}_{i=1}^{N}$. At the fusion center, the sensor's local measurement is reconstructed as

$$\hat{A} = \sum_{i=1}^{N} \hat{b}_i 2^{-i}. \quad (2)$$

Here, for simplicity, we consider only uncoded transmissions. The underlying channel is assumed to be memoryless with different raw bits experiencing independent detection errors. Under this condition, we can model the wireless air interface between the sensor and the fusion center as a binary symmetric channel (BSC) with cross-over probability ϵ. In fact, the BSC model can be used to characterize a more general class of channels including multipath fading and multiaccess ones. Even for a channel with memory, BSC is still applicable provided that a suitable equalizer is incorporated, and $\{\hat{b}_i\}$ denote the bits at the output of the slicer that follows the equalizer.

Because one of the key issues in optimizing the design of sensor networks is the energy constraint, we are interested in the following problem.

If the allowable energy each time we transmit a measurement is fixed to $\mathcal{E}$, what is the optimal number of quantization bits and how can the energy per bit be allocated optimally in order to minimize the reconstruction error at the fusion center?

In the following subsections, we will first address this question under the assumption that the total energy budget used for RF transmission of the measurement A is fixed and equal to $\mathcal{E}$. The energy consumed by the circuit electronics will be taken into account afterwards.

2.1. Optimizing the number of quantization bits

Let us consider a simple scenario where all quantization bits are allocated equal energy. We wish to find the optimal value of N in (1) which minimizes a meaningful metric of the reconstruction error. When using an N-bit quantizer, with the total transmission energy of all bits fixed to $\mathcal{E}$, the energy per bit depends clearly on N, since $\mathcal{E}_b = \mathcal{E}/N$. Noticing that the BSC model's cross-over probability ϵ will generally be a function of the bit energy-to-noise ratio, and letting N_0 denote the channel noise level, we can write ϵ as $\epsilon(\mathcal{E}_b/N_0)$ to make this functional relationship explicit. With $\mathcal{E}_b = \mathcal{E}/N$, we find that the cross-over probability is actually a function of N with $\epsilon = \epsilon(\mathcal{E}/(NN_0))$.

The reconstruction error, which is defined to be $A - \widehat{A}$, can be expressed as

$$A - \widehat{A} = A - A_Q + A_Q - \widehat{A} = \sum_{i=N+1}^{\infty} b_i 2^{-i} + \sum_{i=1}^{N} (b_i - \widehat{b}_i) 2^{-i}. \quad (3)$$

Using the triangle inequality, we can readily bound the absolute value of the reconstruction error as

$$|A - \widehat{A}| \le \sum_{i=N+1}^{\infty} 2^{-i} + \sum_{i=1}^{N} |b_i - \widehat{b}_i| 2^{-i}. \quad (4)$$

Taking expectation on both sides of (4), we have

$$\begin{aligned} \mathsf{E}[|A - \widehat{A}|] &\le 2^{-N} + \sum_{i=1}^{N} \mathsf{E}[|b_i - \widehat{b}_i|] 2^{-i} \\ &= 2^{-N} + \epsilon\left(\frac{\mathcal{E}}{NN_0}\right) \sum_{i=1}^{N} 2^{-i} \\ &= 2^{-N} + (1 - 2^{-N})\epsilon\left(\frac{\mathcal{E}}{NN_0}\right) := f(N), \end{aligned} \quad (5)$$

where in deriving the first equality we used the fact that $|b_i - \widehat{b}_i|$ is a $\{0,1\}$ Bernoulli random variable with mean $\epsilon(\mathcal{E}/(NN_0))$.

In order to minimize the mean-absolute reconstruction error, it suffices to minimize the bound $f(N)$ in (5) with respect to N, which corresponds to optimizing the worst-case performance. Under this criterion, the optimal number of quantization bits will be chosen as follows:

$$\begin{aligned} N_{\text{opt}} &= \arg\min_N f(N) \\ &= \arg\min_N \left[2^{-N} + (1 - 2^{-N})\epsilon\left(\frac{\mathcal{E}}{NN_0}\right)\right]. \end{aligned} \quad (6)$$

Clearly, the first summand in $f(N)$, namely, 2^{-N}, decreases as N becomes larger. With $\epsilon(\cdot)$ being a monotonically decreasing function of its argument, the second summand, $(1 - 2^{-N})\epsilon(\mathcal{E}/(NN_0))$, will be an increasing function of N. Intuition suggests that there should exist an optimal N such that $f(N)$, that is, the sum of the two terms, will reach a minimum. If the latter is unique, a simple one-dimensional numerical search will reveal N_{opt}, as long as $\epsilon(\cdot)$ is specified. In Section 5, we will give examples of the optimal number of quantization bits when $\epsilon(\gamma)$ is specified for different modulations and receiver formats, with $\gamma := \mathcal{E}_b/N_0$ denoting the bit energy-to-noise ratio.

2.2. Optimizing the energy allocation per bit

In the previous subsection, we assumed that each bit is allocated identical energy. However, observing that each bit in (4) has a different weight suggests that there is room to optimize the energy per bit. This motivates us to look for an optimal energy allocation scheme when the total number of bits N is fixed. Let us suppose that bit i is allocated a fraction x_i of the total energy $\mathcal{E}$ for $i = 1, \dots, N$. Then, following the derivation of (5), we have

$$\begin{aligned} \mathsf{E}[|A - \widehat{A}|] &\le 2^{-N} + \sum_{i=1}^{N} \mathsf{E}[|b_i - \widehat{b}_i|] 2^{-i} \\ &= 2^{-N} + \sum_{i=1}^{N} \epsilon\left(\frac{\mathcal{E}x_i}{N_0}\right) 2^{-i}. \end{aligned} \quad (7)$$

In order to account for the mean-absolute reconstruction error with respect to $\mathbf{x} := [x_1, \dots, x_N]^T$, we can formulate the following optimization problem:

$$\begin{aligned} &\text{minimize}_{\mathbf{x}} \ f_0(\mathbf{x}; N) := 2^{-N} + \sum_{i=1}^{N} \epsilon\left(\frac{\mathcal{E}x_i}{N_0}\right) 2^{-i} \\ &\text{subject to } f_i(\mathbf{x}) := -x_i \le 0, \quad i = 1, \dots, N, \\ &\qquad h(\mathbf{x}) := \sum_{i=1}^{N} x_i = 1. \end{aligned} \quad (8)$$

It is easily seen that the optimal solution and the minimum value of $f_0(\mathbf{x}; N)$ are actually functions of N. To make this functional relationship explicit, we denote the optimal solution by $\mathbf{x}_N^* := [x_1^*, x_2^*, \dots, x_N^*]^T$, and accordingly, the minimum of the objective function $f_0(\mathbf{x}_N^*; N)$. Interestingly, as long as $N > \widetilde{N}$, we find $f_0(\mathbf{x}_N^*; N) < f_0(\mathbf{x}_{\widetilde{N}}^*; \widetilde{N})$, see the appendix for details.

With $\mathbf{x}_N^* := [x_1^*, x_2^*, \dots, x_N^*]^T$ denoting the optimal solution, the well-known Karush-Kuhn-Tucker (KKT) conditions [16, page 243] dictate that there must exist $\{\lambda_i^*\}_{i=1}^N$ and ν^* such that

$$x_i^* \ge 0, \quad \lambda_i^* \ge 0, \quad \lambda_i^* x_i^* = 0, \quad i = 1, 2, \dots, N, \quad (9)$$

$$\sum_{i=1}^{N} x_i^* = 1, \quad (10)$$

$$\nabla f_0(\mathbf{x}_N^*; N) + \sum_{i=1}^{N} \lambda_i^* \nabla f_i(\mathbf{x}_N^*) + \nu^* \nabla h(\mathbf{x}_N^*) = 0, \quad (11)$$

where ∇ denotes the gradient. It follows from (11) that the $\{x_i^*\}_{i=1}^N$ must satisfy

$$2^{-i} \frac{\mathcal{E}}{N_0} \frac{d\epsilon(\gamma)}{d\gamma}\bigg|_{\gamma = (\mathcal{E}/N_0)x_i^*} - \lambda_i^* + \nu^* = 0, \quad i = 1, \dots, N. \quad (12)$$

In order to gain further insight from (12), let us take a closer look at the optimal energy allocation in two special cases.

2.2.1. BPSK over AWGN channel

The cross-over probability ϵ expressed in terms of bit energy-to-noise ratio γ is given in this case by [17, page 255]

$$\epsilon(\gamma) = Q(\sqrt{2\gamma}) := \int_{\sqrt{2\gamma}}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \quad (13)$$

The derivative of $\epsilon(\gamma)$ with respect to γ is then calculated as follows:

$$\frac{d\epsilon(\gamma)}{d\gamma} = -\frac{1}{\sqrt{2\pi}}\frac{e^{-\gamma}}{\sqrt{2\gamma}} := -\phi(\gamma). \quad (14)$$

Substituting (14) into (12), we can express the optimal energy allocation in the following form:

$$x_i^* = \phi^{-1}\left((\nu^* - \lambda_i^*)2^i\frac{N_0}{\mathcal{E}}\right)\frac{N_0}{\mathcal{E}}, \quad i = 1,\dots,N. \quad (15)$$

Noticing that the domain of $\phi(\gamma)$ defined in (14) is $(0,+\infty)$, from the complementary slackness conditions in (9), we deduce that $\lambda_i^* = 0$, for all i, and finally, we obtain

$$x_i^* = \phi^{-1}\left(\nu^* 2^i\frac{N_0}{\mathcal{E}}\right)\frac{N_0}{\mathcal{E}}, \quad i = 1,\dots,N, \quad (16)$$

where ν^* is a constant chosen to enforce the constraint $\sum_{i=1}^N x_i^* = 1$.

Equation (16) is intuitively appealing, because the monotonicity of $\phi(\gamma)$ ensures that each bit is allocated energy according to its significance: the smaller the i is, the more significant bit i is, and the more energy is allocated to bit i.

2.2.2. Binary orthogonal signaling with envelope detection

It is well known that binary orthogonal signals such as binary frequency-shift keying (FSK) or pulse-position modulation (PPM) can be demodulated using noncoherent envelope detection [17, pages 307–310]. In this case, the cross-over probability expressed in terms of the bit energy-to-noise ratio is given by

$$\epsilon(\gamma) = \frac{1}{2}e^{-\gamma/2}. \quad (17)$$

The derivative is then

$$\frac{d\epsilon(\gamma)}{d\gamma} = -\frac{1}{4}e^{-\gamma/2} := -\varphi(\gamma). \quad (18)$$

Substituting (18) into (12), we obtain

$$x_i^* = \varphi^{-1}\left((\nu^* - \lambda_i^*)2^i\frac{N_0}{\mathcal{E}}\right)\frac{N_0}{\mathcal{E}}. \quad (19)$$

Noticing that the function $\varphi(\gamma)$ has domain $[0,+\infty)$ and range $(0,\varphi(0) = 1/4]$, and supposing that $\lambda_i^* = 0$, for all i, we must have $\nu^* 2^N N_0/\mathcal{E} \le 1/4$. Furthermore, the condition $\sum_{i=1}^N x_i^* = 1$ is not guaranteed to be satisfied when ν^* is bounded. Based on (9), we can simplify (19) as follows:

$$\begin{aligned} x_i^* &= \varphi^{-1}\left(\min\left\{\frac{1}{4}, \nu^* 2^i\frac{N_0}{\mathcal{E}}\right\}\right)\frac{N_0}{\mathcal{E}} \\ &= 2\frac{N_0}{\mathcal{E}}\ln\left(\frac{1}{4(\min\{1/4, \nu^* 2^i(N_0/\mathcal{E})\})}\right). \end{aligned} \quad (20)$$

Equation (20) implies that it is possible to have $x_i^* = 0$ for some large i's. In fact, when $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, the problem in (8) can be readily shown to be convex which not only implies that the optimal solution is guaranteed to exist and is unique, but also can be found using a numerically efficient search.

The optimal energy allocation for a special case will be examined in Section 5, where we will confirm that a certain number of less significant bits should not be allocated any energy.

2.3. Circuit energy consumption

Till now, we have neglected the fact that the circuit itself will also consume a certain amount of energy when transmitting the quantization bits b_i. Optimization in (8) implies that if we ignore circuit energy consumption and optimally allocate the transmission energy among bits, then the achieved reconstruction error bound will decrease as we increase the number of quantization bits N. However, this will not be true when the circuit energy consumption is taken into account. The reason is that the energy consumed by the circuit will also increase as the number of bits grows larger. To quantify this tradeoff, we adopt the model in [3], where the power of the circuit electronics (excluding the RF transmission power) is assumed to be P_{on} when the sensor is transmitting each quantization bit. The energy consumption of the circuit electronics during the sleep and transition modes is assumed to be very small and can be neglected. Letting T_0 denote the bit period, when N quantization bits are transmitted, we can express the circuit energy consumption as $\mathcal{E}_c = NT_0P_{\text{on}}$. With the total energy budget per measurement transmission being $\mathcal{E}$, the remaining energy for RF transmission of the N bits will be $\mathcal{E}_r = \mathcal{E} - \mathcal{E}_c = \mathcal{E} - NT_0P_{\text{on}}$. In order to have $\mathcal{E}_r > 0$, we obviously need to make sure that $N < \mathcal{E}/(T_0P_{\text{on}})$. Now, with the circuit energy consumption considered, let us revisit the issue of optimizing the number of quantization bits, which we have examined in the previous subsections.

2.3.1. Optimal number of quantization bits with equal energy allocation

Let us first assume that the residual energy $\mathcal{E}_r$ is equally allocated among the N quantization bits. Similar to (5), we can upper bound the mean-absolute reconstruction error as

$$\begin{aligned} \mathsf{E}|A - \hat{A}| &\le 2^{-N} + (1 - 2^{-N})\epsilon\left(\frac{\mathcal{E}_r}{NN_0}\right) \\ &= 2^{-N} + (1 - 2^{-N})\epsilon\left(\frac{\mathcal{E} - NT_0P_{\text{on}}}{NN_0}\right) := f_c(N) \end{aligned} \quad (21)$$

from which the optimal value of N can be obtained as

$$P_{\text{opt}} = \arg\min_N f_c(N). \quad (22)$$

Comparing the latter with (6), we recognize that similar comments apply regarding the existence, uniqueness and numerical evaluation of the optimal N when circuit energy is accounted for.

2.3.2. Optimal number of quantization bits with optimal energy allocation

When the measurement A is quantized to N bits, the optimal strategy of allocating the residual energy $\mathcal{E}_r = \mathcal{E} - NT_0P_{\text{on}}$ is the solution of the following optimization problem:

$$\begin{aligned} \text{minimize}_{\mathbf{x}}\ & f_0(\mathbf{x}; N) := 2^{-N} + \sum_{i=1}^{N} \epsilon\left(\frac{\mathcal{E}_r x_i}{N_0}\right) 2^{-i} \\ \text{subject to}\ & f_i(\mathbf{x}) := -x_i \le 0, \quad i = 1, \dots, N, \\ & h(\mathbf{x}) := \sum_{i=1}^{N} x_i = 1. \end{aligned} \tag{23}$$

Denoting the optimal solution by $\mathbf{x}_N^*$, we can obtain the optimal number of quantization bits as

$$N_{\text{opt}} = \arg\min_N f_0(\mathbf{x}_N^*; N). \tag{24}$$

In Section 5, we will find N_{opt} for specific system setups in the case of equal energy allocation and optimal energy allocation when energy consumption of the underlying circuitry is taken into account.

3. MULTISENSOR COOPERATION IN ESTIMATING A PARAMETER

Let us now consider the multisensor setup depicted in Figure 2, where each sensor k has available local bounded noisy observation $X_k = A + n_k$, and n_k is zero mean with variance σ_k^2 and independent of n_l for $l \neq k$. After normalization, we have $X_k \in [0, 1]$. Sensor k quantizes its local observation X_k to the N_k most significant bits, that is, with $X_k = \sum_{i=1}^{\infty} b_i^{(k)} 2^{-i}$, we have $(X_k)_Q = \sum_{i=1}^{N_k} b_i^{(k)} 2^{-i}$. Bits $\{b_i^{(k)}\}_{i=1}^{N_k}$ are then transmitted through the wireless channel, which is again modeled as a BSC with cross-over probability ϵ_k. The fusion center reconstructs X_k with the demodulated bits $\{\hat{b}_i^{(k)}\}_{i=1}^{N_k}$ to obtain

$$\hat{X}_k = \sum_{i=1}^{N_k} \hat{b}_i^{(k)} 2^{-i}. \tag{25}$$

When we have available unquantized real valued observations $X_k = A + n_k$, $k = 1, 2, \dots, K$, the best linear unbiased estimator (BLUE) of A is known to be [18]

$$\hat{A}_{\text{BLUE}} = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{X_k}{\sigma_k^2}. \tag{26}$$

This motivates us to form the following estimator for the parameter A when the noise variances are known at the fusion center, where we have only available $\hat{X}_k$, $k = 1, 2, \dots, K$:

$$\hat{A} = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k}{\sigma_k^2}. \tag{27}$$

The problem we are interested in can be formulated as follows.

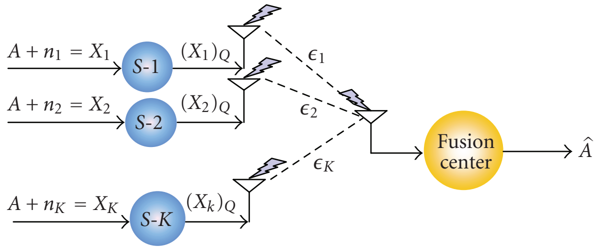


FIGURE 2: Multisensor cooperation in estimating a scalar parameter with quantized observations.

For a fixed number of quantization bits (N_k) per sensor, what is the optimal scheme for allocating the total energy E_T prescribed to all sensors so that the mean-square estimation error $\mathsf{E}|\hat{A} - A|^2$ is minimized? Furthermore, what is the optimal number of quantization bits per sensor so that this energy allocation scheme achieves the minimum possible estimation error?

In this section, we will neglect the circuit energy consumption. Generalization to the case where the energy consumption by the circuit electronics is nonnegligible is rather straightforward using the model described in Section 2.3. Furthermore, we assume that the energy allocated per sensor will be equally distributed among the quantization bits. Now, let us take a look at the estimation error

$$\begin{aligned} \hat{A} - A &= \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k - A}{\sigma_k^2} \\ &= \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{\hat{X}_k - X_k + n_k}{\sigma_k^2}. \end{aligned} \tag{28}$$

Upon defining the reconstruction error $\tilde{X}_k := \hat{X}_k - X_k$, we have

$$\begin{aligned} \mathsf{E}|\hat{A} - A|^2 &= \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-2} \mathsf{E}\left|\sum_{k=1}^{K} \frac{\tilde{X}_k + n_k}{\sigma_k^2}\right|^2 \\ &= \frac{\mathsf{E}\left|\sum_{k=1}^{K} (\tilde{X}_k/\sigma_k^2)\right|^2}{\left(\sum_{k=1}^{K} (1/\sigma_k^2)\right)^2} \\ &\quad + \frac{\mathsf{E}\left[\left(\sum_{k=1}^{K} (\tilde{X}_k/\sigma_k^2)\right)\left(\sum_{k=1}^{K} (n_k/\sigma_k^2)\right)\right] + \sum_{k=1}^{K} (1/\sigma_k^2)}{\left(\sum_{k=1}^{K} (1/\sigma_k^2)\right)^2}. \end{aligned} \tag{29}$$

Since $\hat{X}_k - (X_k)_Q = \sum_{i=1}^{N_k} (\hat{b}_i^{(k)} - b_i^{(k)}) 2^{-i}$, it follows that $\hat{X}_k - (X_k)_Q$ and n_k are uncorrelated. Furthermore, as shown in [19], when the characteristic function of n_k is bandlimited to $2\pi/\Delta$, where $\Delta = 2^{-N_k}$ is the quantization step size, the quantization error $(X_k)_Q - X_k$ is uncorrelated with the input $X_k = A + n_k$. (In a uniform quantizer with step size Δ, the correlation between input $\mathcal{X}$ and the quantization error ϵ is given by [19] $\mathsf{E}[\mathcal{X}\epsilon]/\sqrt{\mathsf{E}[\mathcal{X}^2]\mathsf{E}[\epsilon^2]} = [\sqrt{3}/(\pi\sqrt{\mathsf{E}[\mathcal{X}^2]})]\sum_{k\neq 0}[(-1)^k/k]\dot{\phi}(2\pi k/\Delta)$, where $\phi(\omega) := \mathsf{E}[e^{j\omega\mathcal{X}}]$ and $\dot{\phi}(\omega) := d\phi(\omega)/d\omega$. Therefore, as long as the $\phi(\omega)$ energy is concentrated in the interval $[-2\pi/\Delta, 2\pi/\Delta]$, one can safely consider $\mathcal{X}$ and ϵ as uncorrelated.) Hence practically, as long as the quantization step $\Delta = 2^{-N_k}$ is sufficiently small relative to σ_k, one can safely assume the reconstruction error $\widetilde{X}_k = \widehat{X}_k - X_k = \widehat{X}_k - (X_k)_Q + (X_k)_Q - X_k$ is statistically uncorrelated with the observation noise n_k. Thus, the second summand in the numerator disappears. Hence, minimizing $\mathsf{E}|A - \widehat{A}|^2$ reduces to minimizing $\mathsf{E}|\sum_{k=1}^{K}(\widetilde{X}_k/\sigma_k^2)|^2$. Because for any bounded random variable $Z \in [-U, U]$ with pdf $p(z)$, we have $\mathsf{E}|Z|^2 = \int_{-U}^{U}|z|^2 p(z)dz \leq \int_{-U}^{U} U|z|p(z)dz = U\mathsf{E}|Z|$, noticing that $\sum_{k=1}^{K}(\widetilde{X}_k/\sigma_k^2)$ is bounded, we can instead minimize $\mathsf{E}|\sum_{k=1}^{K}(\widetilde{X}_k/\sigma_k^2)|$, which we upper bound as

$$\mathsf{E}\left|\sum_{k=1}^{K}\frac{\widetilde{X}_k}{\sigma_k^2}\right| \leq \sum_{k=1}^{K}\frac{\mathsf{E}|\widetilde{X}_k|}{\sigma_k^2} \leq \sum_{k=1}^{K}\frac{2^{-N_k} + (1-2^{-N_k})\epsilon_k}{\sigma_k^2}, \tag{30}$$

where ϵ_k is the cross-over probability of the BSC between sensor k and the fusion center.

3.1. Identical number of bits per sensor

For clarity in exposition, we first consider here a simple situation where each sensor transmits the same fixed number of bits N (i.e., $N_k = N$, for all k). With x_k denoting the fraction of the total energy E_T allocated to sensor k, we can express ϵ_k as $\epsilon_k(E_T x_k/(NN_0))$, where N_0 is the noise level at the receiver of the fusion center which is assumed common to all channels. The optimal energy allocation scheme will be the solution of the following optimization problem ($\mathbf{x} := [x_1, \ldots, x_K]^T$):

$$\begin{aligned}
&\text{minimize}_{\mathbf{x}}\ f_0(\mathbf{x}; N) := \sum_{k=1}^{K}\frac{1}{\sigma_k^2}\left[\frac{1}{2^N} + \left(1 - \frac{1}{2^N}\right)\epsilon_k\left(\frac{E_T x_k}{NN_0}\right)\right]\\
&\text{subject to}\ f_k(\mathbf{x}) := -x_k \leq 0, \quad k = 1, \ldots, K,\\
&\qquad h(\mathbf{x}) := \sum_{k=1}^{K} x_k = 1.
\end{aligned} \tag{31}$$

As in Section 2, we can write down the KKT conditions for the optimal solution $\mathbf{x}^* := [x_1^*, \ldots, x_K^*]^T$ as follows:

$$x_k^* \geq 0, \quad \lambda_k^* \geq 0, \quad \lambda_k^* x_k^* = 0, \quad k = 1, 2, \ldots, K, \tag{32}$$

$$\sum_{k=1}^{K} x_k^* = 1, \tag{33}$$

$$\nabla f_0(\mathbf{x}^*; N) + \sum_{k=1}^{K}\lambda_k^*\nabla f_k(\mathbf{x}^*) + \nu^*\nabla h(\mathbf{x}^*) = 0. \tag{34}$$

From (34), we have

$$\frac{1}{\sigma_k^2}\frac{E_T}{NN_0}\frac{d\epsilon_k(\gamma)}{d\gamma}\bigg|_{\gamma=(E_T/NN_0)x_k} - \lambda_k^* + \nu^* = 0, \quad k = 1, \ldots, K. \tag{35}$$

To delve further into (35), we consider a particular system setup. Letting κ denote the path loss exponent [20] of the wireless channel (d_k is the distance between sensor k and the fusion center), and supposing BPSK modulation, we can express the cross-over probability in the presence of AWGN as $\epsilon_k(\gamma) = Q(\sqrt{2\gamma\mathcal{C}/d_k^\kappa})$ with $\mathcal{C}$ being a constant. Under these operating conditions, (35) and (32) yield

$$\begin{aligned}
x_k^* &= \phi_k^{-1}\left(\nu^*\sigma_k^2\frac{NN_0}{E_T}\right)\frac{NN_0}{E_T},\\
\phi_k(\gamma) &:= \frac{1}{\sqrt{2\pi}}\frac{e^{-\gamma(\mathcal{C}/d_k^\kappa)}}{\sqrt{2\gamma}}\sqrt{\frac{\mathcal{C}}{d_k^\kappa}},
\end{aligned} \tag{36}$$

where ν^* is chosen such that $\sum_{k=1}^{K} x_k^* = 1$. In Section 5, we will examine a specific system and find the corresponding optimal energy allocation to gain further insight into these closed-form expressions.

In fact, when $\epsilon_k(\gamma)$, for all k, is convex in γ, the problem in (31) turns out to be convex, which implies that the global optimum exists and can be easily found numerically. In most cases, convexity is guaranteed, for example, when $\epsilon_k(\gamma)$ is expressible in terms of $Q(\sqrt{2\gamma})$ or $(1/2)e^{-\gamma/2}$.

Subsequently, the optimal number of quantization bits N_{opt} can be easily found using one-dimensional numerical search to solve the optimization problem

$$N_{\text{opt}} = \arg\min_N f_0(\mathbf{x}^*; N), \tag{37}$$

where $f_0(\mathbf{x}^*; N)$ is the optimal value of the objective function in (31) when the number of quantization bits per sensor is N. In Section 5, we will show an example of the functional relationship between $f_0(\mathbf{x}^*; N)$ and N, from which N_{opt} can be readily determined.

3.2. Different number of bits per sensor

Now, let us consider the case where sensor k transmits N_k quantization bits, $k = 1, \ldots, K$. From (30), we can see that the optimal energy allocation scheme which minimizes the estimation error is the solution of the following optimization problem:

$$\begin{aligned}
&\text{minimize}_{\mathbf{x}}\ f_0(\mathbf{x}; N_k,\ k = 1, \ldots, K)\\
&\qquad := \sum_{k=1}^{K}\frac{1}{\sigma_k^2}\left[\frac{1}{2^{N_k}} + \left(1 - \frac{1}{2^{N_k}}\right)\epsilon_k\left(\frac{E_T x_k}{N_k N_0}\right)\right]\\
&\text{subject to}\ f_k(\mathbf{x}) := -x_k \leq 0, \quad k = 1, \ldots, K,\\
&\qquad h(\mathbf{x}) := \sum_{k=1}^{K} x_k = 1.
\end{aligned} \tag{38}$$

(i) At lth step, with $N_k = N_k^{(l)},\ k = 1,\ldots,K$, find $\mathbf{x}^{(l)} = [x_1^{(l)},\ldots,x_K^{(l)}]^T$ as the optimal solution of (38).
(ii) Update $N_k^{(l)}$ to $N_k^{(l+1)}$ based on the iteration

$$N_k^{(l+1)} = \arg\min_{N_k}\left[\frac{1}{2^{N_k}} + \left(1 - \frac{1}{2^{N_k}}\right)\epsilon_k\left(\frac{E_T x_k^{(l)}}{N_k N_0}\right)\right]. \tag{39}$$

(iii) Go to $(l+1)^{\text{st}}$ step.

Algorithm 1

Given the set of N_k, $k = 1,\ldots,K$, the solution to (38) is similar to (35). The problem we are interested in is the minimization of $f_0(\mathbf{x}; N_k,\ k = 1,\ldots,K)$ with respect to $N_k,\ k = 1,\ldots,K$ and $x_k,\ k = 1,\ldots,K$. In order to find optimal $N_k,\ k = 1,\ldots,K$ and $x_k,\ k = 1,\ldots,K$ jointly, we advocate Algorithm 1.

When $\epsilon_k(\gamma)$ is convex and $\{N_k\}_{k=1}^K$ are fixed, the problem in (38) is clearly convex, which implies that the optimal energy allocation vector $\mathbf{x}^*$ can be found using standard numerically efficient search schemes [16]. Hence, step (i) of Algorithm 1 is easily carried out. It is also easy to prove that the objective function is always decreasing from one iteration to another. The argument is as follows:

$$\begin{aligned} f_0(\mathbf{x}^{(l+1)}; N_k^{(l+1)},\ k = 1,\ldots,K) &\\ \le f_0(\mathbf{x}^{(l)}; N_k^{(l+1)},\ k = 1,\ldots,K) &\\ \le f_0(\mathbf{x}^{(l)}; N_k^{(l)},\ k = 1,\ldots,K). & \end{aligned} \tag{40}$$

Our experience with simulations is that Algorithm 1 typically converges after 3-4 iterations. In Section 5, we will utilize this approach to jointly optimize N_k and x_k, $k = 1,\ldots,K$, for a specific wireless sensor network.

Remark 1. In this section, we have dealt with energy and quantization optimization for multiple sensors that are cooperating in the estimation of a common parameter. The optimum scheme will be first derived in a centralized manner and then released to each individual senor, which may create a lot of scheduling overhead. However, in practice, we will not need to update the optimum scheme frequently unless there is a major change in the configuration of the sensor network.

4. EFFECTS OF CHANNEL CODING

In the preceding sections, we have limited our consideration to uncoded transmissions. In this section, we will examine the performance limit of our reconstruction problem when error control codes are adopted before the quantized bits enter the BSC. The total energy budget for RF transmission of $A \in [0,1]$ is again constrained to be $\mathcal{E}$. The energy consumption of the circuit electronics will be neglected here for clarity in exposition.

4.1. Single measurement transmission

Suppose now that A is quantized to N_Q bits as in (1), $A_Q = \sum_{i=1}^{N_Q} b_i 2^{-i}$, and that a $(2^{N_Q}, N)$ channel code is constructed to transmit the N_Q bits over the BSC by using the channel N times. Letting $P_e^{(N)}$ denote the average error probability of maximum likelihood (ML) decoding and $\widehat{A} := \sum_{i=1}^{N_Q} \widehat{b}_i 2^{-i}$ denote the reconstruction of A at the fusion center, we have the following upper bound for the reconstruction error $|A - \widehat{A}|$ at the fusion center:

$$\begin{aligned} \mathsf{E}[|A-\widehat{A}|] &= (1-P_e^{(N)})\mathsf{E}[|A-\widehat{A}| \mid \text{correct decoding}] \\ &\quad + P_e^{(N)}\mathsf{E}[|A-\widehat{A}| \mid \text{decoding error}] \\ &\le (1-P_e^{(N)})\mathsf{E}[\,|A-A_Q|\,] + P_e^{(N)}\cdot 1 \\ &\le 2^{-N_Q} + P_e^{(N)}, \end{aligned} \tag{41}$$

where in deriving (41) we have used the fact that A and $\widehat{A}$ both lie in $[0,1]$.

In order to proceed, we need the following result from [21, Chapter 5].

Theorem 1 (Random coding theorem). *For a discrete memoryless channel $(\mathcal{X}, p(x \mid y), \mathcal{Y})$, there exists a (e^{NR}, N) block channel code with average error probability of ML decoding satisfying*

$$P_e^{(N)} \le e^{-NE_r(R)}, \tag{42}$$

where $E_r(R)$ is the random coding exponent which is defined as

$$E_r(R) = \max_{\rho\in[0,1]}\max_{p(x)}\left[-\ln\sum_{y\in\mathcal{Y}}\left[\sum_{x\in\mathcal{X}} p(x)p(y\mid x)^{1/(1+\rho)}\right]^{1+\rho} - \rho R\right]. \tag{43}$$

The random coding exponent for a BSC with cross-over probability $\epsilon < 1/2$ is [21]

$$E_r(R,\epsilon) = \begin{cases} T_\epsilon(\delta) - H(\delta), & R = \ln 2 - H(\delta),\ \rho \in [0,1], \\ \ln 2 - 2\ln\left(\sqrt{\epsilon} + \sqrt{1-\epsilon}\right) - R, & R < \ln 2 - H\left(\dfrac{\sqrt{\epsilon}}{\sqrt{\epsilon} + \sqrt{1-\epsilon}}\right), \end{cases} \tag{44}$$

where $\delta := \epsilon^{1/(1+\rho)}/[\epsilon^{1/(1+\rho)} + (1-\epsilon)^{1/(1+\rho)}]$, $H(\delta) := -\delta\ln\delta - (1-\delta)\ln(1-\delta)$, and $T_\epsilon(\delta) := -\delta\ln\epsilon - (1-\delta)\ln(1-\epsilon)$.

In our case, since we use the channel N times, the bit energy per transmission will be effectively reduced to $\mathcal{E}/N$, and thus, the equivalent BSC's cross-over probability will become $\epsilon(\mathcal{E}/(NN_0))$. For our $(2^{N_Q}, N)$ channel code, the rate in *nats/channel use* will be $(N_Q/N)\ln 2$. Thus, applying Theorem 1 with an appropriate channel code, we can use (42) to bound (41) as

$$\begin{aligned} \mathsf{E}[|A-\widehat{A}|] &\le 2^{-N_Q} + e^{-NE_r((N_Q/N)\ln 2,\,\epsilon(\mathcal{E}/NN_0))} \\ &:= f_{\text{code}}(N_Q, N). \end{aligned} \tag{45}$$

Clearly, N_Q and N can be optimally selected to minimize $f_{\text{code}}(N_Q, N)$ and thus the reconstruction error. In Section 5, we will compare this upper bound with the bound achieved with the uncoded transmission schemes we developed in Section 2.

4.2. Multiple simultaneously transmitted measurements

The exponentially decreasing behavior of the decoding error probability described by the random coding theorem favors large block sizes. However, in the single measurement transmission case, when the block size N becomes large, the capacity of the underlying BSC goes to zero. To resolve this tradeoff, we can transmit multiple measurements together. In practice, for some application scenarios, it may not be necessary for the remote sensor to transmit its measurement back to the fusion center immediately, that is, one can wait until $L > 1$ measurements $\{A_1, A_2, \dots, A_L\} \in [0,1]^L$ are acquired, and then transmit them jointly to the destination. The critical difference here is that the energy budget increases to $L\mathcal{E}$. We assume no probabilistic model for the source and, again, employ the universal uniform quantizer to quantize each measurement to N_Q bits. As a result, the total number of bits to be transmitted is LN_Q. Adopting an appropriate $(2^{LN_Q}, LN)$ channel code with error probability $P_e^{(LN)}$, as in (41) and (45), we thus obtain

$$\begin{aligned}\frac{1}{L}\sum_{l=1}^{L} \mathsf{E}\left[\,|A_l - \hat{A}_l|\,\right] &\le 2^{-N_Q} + P_e^{(LN)} \\ &\le 2^{-N_Q} + e^{-LNE_r((N_Q/N)\ln 2, \epsilon(L\mathcal{E}/(LNN_0)))} \\ &:= \tilde{f}_{\text{code}}(N_Q, N).\end{aligned} \tag{46}$$

Comparing the bound for a single measurement transmission, $f_{\text{code}}(N_Q, N)$ in (45), with the bound for multiple simultaneously transmitted measurements, $\tilde{f}_{\text{code}}(N_Q, N)$ in (46), we can see

$$\begin{aligned}&f_{\text{code}}(N_Q, N) > \tilde{f}_{\text{code}}(N_Q, N), \\ &\qquad \text{when } E_r\left(\frac{N_Q}{N}\ln 2, \epsilon\left(\frac{\mathcal{E}}{NN_0}\right)\right) > 0, \quad \forall L > 1.\end{aligned} \tag{47}$$

Equation (47) shows clearly that it is preferable to transmit multiple measurements simultaneously in energy-limited communication settings. However, when we directly transmit uncoded quantization bits, there is no preference between transmitting a single or multiple measurements.

Certainly, judicious selection of N_Q or/and N should minimize the $\tilde{f}_{\text{code}}(N_Q, N)$ bound to ensure reliable performance in reconstruction. To this end, let us explore further the characteristics of $\tilde{f}_{\text{code}}(N_Q, N)$ with large L. As long as $R < \ln 2 - H(\epsilon)$, we know that $E_r(R, \epsilon) > 0$, see (44). Thus, as $L \to \infty$, we have

$$\begin{aligned}\lim_{L\to\infty}&\left\{\arg\min_{N_Q} 2^{-N_Q} + e^{-LNE_r((N_Q/N)\ln 2, \epsilon(L\mathcal{E}/(LNN_0)))}\right\} \\ &= N\left(1 - \frac{H(\epsilon(\mathcal{E}/NN_0))}{\ln 2}\right) := N_Q^*(N);\end{aligned} \tag{48}$$

$$\lim_{L\to\infty}\left\{\min_{N_Q} 2^{-N_Q} + e^{-LNE_r((N_Q/N)\ln 2, \epsilon(L\mathcal{E}/(LNN_0)))}\right\} = 2^{-N_Q^*(N)}. \tag{49}$$

Equation (49) implies that the number of channel uses to transmit one measurement can be optimally chosen to be

$$N^* = \arg\max_N N\left(1 - \frac{H(\epsilon(\mathcal{E}/(NN_0)))}{\ln 2}\right). \tag{50}$$

Accordingly, the optimal number of quantization bits is $N_Q^*(N^*)$ given by (48).

In Section 5, we will study how L, the number of simultaneously transmitted measurements, affects the achievable distortion in source reconstruction with numerical examples.

5. NUMERICAL EXAMPLES

In this section, we provide numerical examples to corroborate the analytical results we derived in the previous sections.

5.1. Optimal number of quantization bits

As discussed in Sections 2.1 and 2.3.1, when the total energy budget is uniformly allocated among quantization bits, there is an optimal value of N which minimizes the mean-absolute reconstruction error upper bound both when the circuit energy is neglected and also when circuit energy consumption is accounted for. Here, we first consider the channel to be AWGN and use BPSK modulation. The BSC crossover probability as a function of the bit energy-to-noise ratio is $\epsilon(\gamma) = Q(\sqrt{2\gamma})$. Figure 3 depicts the bound $f(N)$ in (5) together with the simulated actual mean-absolute reconstruction error $\mathsf{E}|A - \hat{A}|$, and the bound $f_c(N)$ in (21) with $\mathcal{E}/N_0 = 20$ and $T_0 P_{\text{on}}/N_0 = 1$. It can be seen that the bound $f(N)$ is pretty tight and numerical minimization yields $N_{\text{opt}} = 7$ in the first case and $N_{\text{opt}} = 6$ in the second case. In Figure 3, we also plot $f(N)$ and $f_c(N)$ when $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, which is the BER when binary orthogonal modulation is used along with envelope detection; N_{opt} here turns out to be 5 and 4, respectively.

5.2. Optimal bit energy allocation

In Section 2.2, we derived an optimal energy allocation scheme per bit to minimize the reconstruction error. Considering envelope detection of binary orthogonal signals as in Section 2.2.2, with $\mathcal{E}/N_0 = 20$ and $N = 10$, we can find the optimal energy allocation by solving the convex optimization problem in (8) using the interior-point method outlined in [16, Chapter 11]; Figure 4 depicts the result.

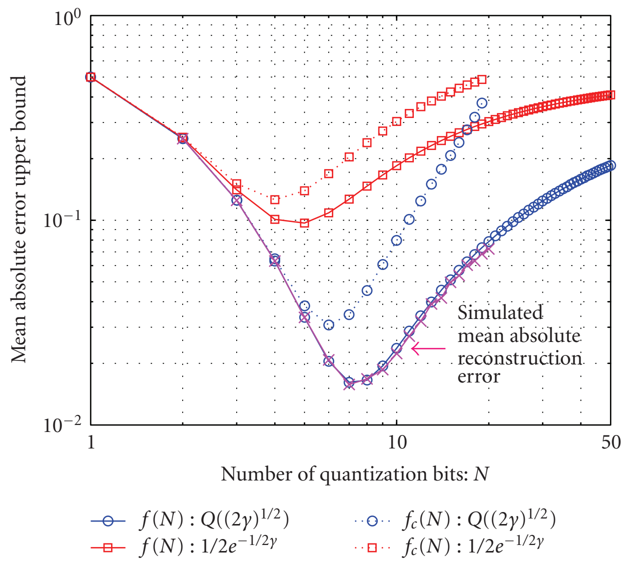


Figure 3: The bound of $\mathsf{E}|A-\hat{A}|$ whose minimum yields the optimum number of quantization bits.

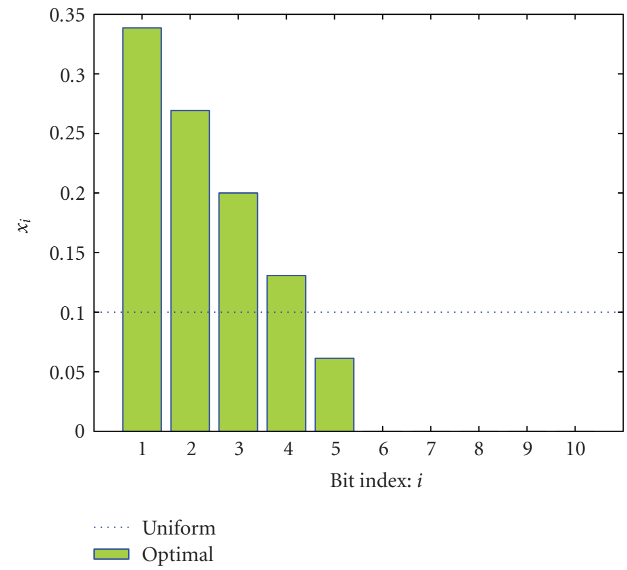


Figure 4: Optimal energy allocation over a fixed number of quantization bits ($N = 10$).

For the same $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, Figure 5 compares the reconstruction error between the optimal energy allocation scheme in (8) and the equal energy distribution scheme in (5) with a different number of quantization bits N. We observe that the reconstruction error decreases to a floor as N increases with optimal energy allocation, which is different from the equal energy allocation scheme. The intuitive explanation for this behavior is that as N increases, equal energy allocation increases the cross-over probability for all transmitted bits; on the other hand, optimal energy allocation does not experience this problem. As already noticed in Figure 4, when N is large enough, the optimal scheme just assigns no (or very little) energy to less significant bits.

In Figure 5, we also plot the reconstruction error as a function of N when circuit energy consumption is taken into account with optimal allocation of the residual energy to the quantization bits as in (23); here we take $T_0P_{\text{on}}/N_0 = 1$. The optimal number of quantization bits in (24) is easily seen to be $N_{\text{opt}} = 6$.

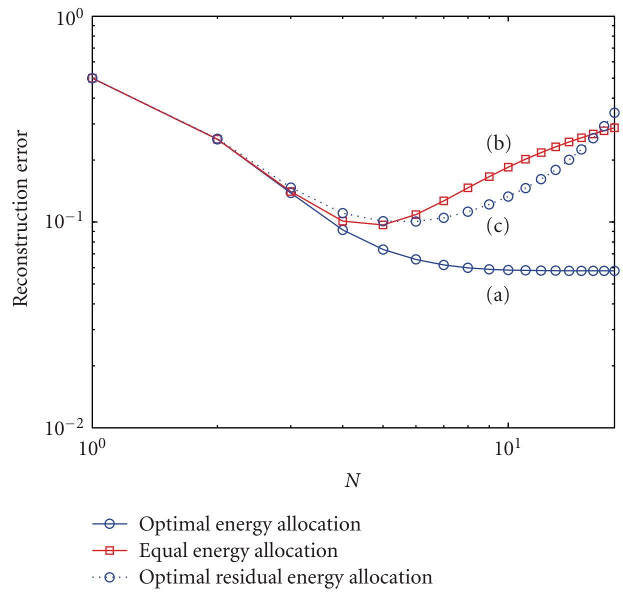


Figure 5: (a) Reconstruction error with optimal energy allocation among bits as in (8). (b) Reconstruction error with equal energy allocation as in (5). (c) Reconstruction error in (23) taking into account the energy consumption of circuit electronics.

5.3. *Optimal energy allocation among sensors*

Suppose that $K = 10$ sensors are deployed with local observation noise variances denoted by $\sigma_1^2, \sigma_2^2, \ldots, \sigma_{10}^2$, the path loss exponent of the wireless channel is $\kappa = 2$ (free space), and accordingly, the cross-over probability is given by $\epsilon_k(\gamma) = Q(\sqrt{2\gamma\mathcal{C}/d_k^2})$, where d_k is the distance between sensor k and the fusion center. Parameter $\mathcal{C}$ is set to be 1 here. In the following, we set the total energy budget to be $E_T/N_0 = 200$.

5.3.1. *Identical* $N_k = N,\ k = 1, \ldots, K$

Using the aforementioned parameters, Figure 6 compares the normalized value of the objective function in (31) between the equal energy allocation and the optimal energy allocation scheme for a variable number of bits N while choosing a specific set of values for $\{d_k\}_{k=1}^{10}$ and $\{\sigma_k^2\}_{k=1}^{10}$. For this particular setup, the optimal value of N in (37) turns out to be $N_{\text{opt}} = 6$. With different sets of values for $\{d_k\}_{k=1}^{10}$ and $\{\sigma_k^2\}_{k=1}^{10}$, when N is accordingly chosen to be optimal, the corresponding optimal energy allocation schemes, that is, the numerical solutions of the convex problem in (31), are depicted in Figure 8.

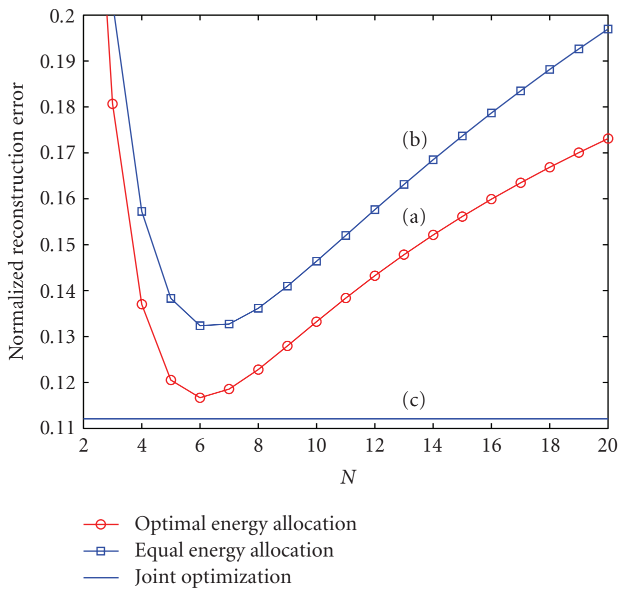


FIGURE 6: With $\sigma_k^2 = 0.01 \times k$, $k = 1, 2, \dots, 10$, and $\{d_1, d_2, \dots, d_{10}\} = \{1, 5, 1, 5, 1, 1, 5, 5, 1, 5\}$. (a) Normalized value of the objective function in (31) with optimal energy allocation among sensors. (b) Normalized value of the objective function in (31) with equal energy allocation among sensors. (c) Normalized value of the objective function in (38) with joint optimization.

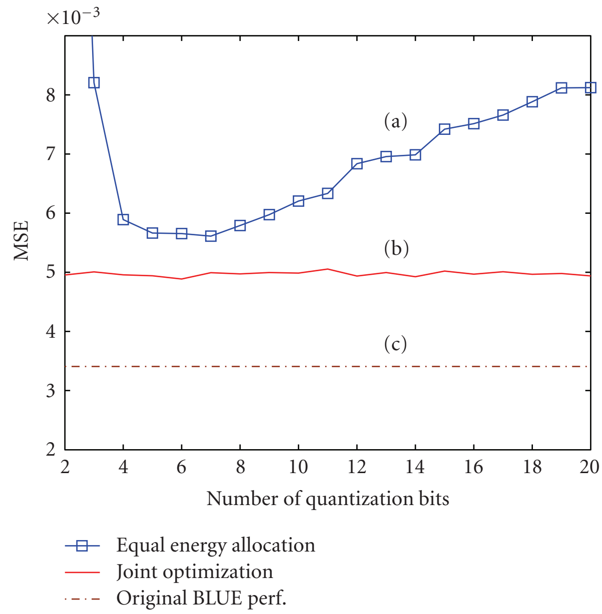


FIGURE 7: With $\sigma_k^2 = 0.01 \times k$, $k = 1, 2, \dots, 10$, and $\{d_1, d_2, \dots, d_{10}\} = \{1, 5, 1, 5, 1, 1, 5, 5, 1, 5\}$. (a) Simulated mean-square estimation error with equal energy allocation among sensors. (b) Simulated mean square estimation error with joint optimization of energy allocation and a number of quantization bits per sensor. (c) Simulated MSE of the unquantized BLUE.

5.3.2. *Jointly optimized* $\{N_k, x_k\}_{k=1}^K$

As explained in Section 3.2, we can find the optimal N_k and x_k, $k = 1, \dots, K$, jointly by utilizing Algorithm 1. The resulting optimal energy allocation scheme and the optimal number of quantization bits per sensor are depicted in Figures 8 and 9, respectively. Through joint optimization, the normalized minimum value of the objective function in (38) is plotted in Figure 6. The gain over the case where each sensor transmits the same number of quantization bits is clear. Furthermore, we have also plotted the simulated mean-square estimation error of different schemes in Figure 7, which again demonstrates the benefits of energy and quantization optimization.

5.4. *Effects of channel coding*

5.4.1. *Single measurement transmission*

With envelope detection of binary orthogonal signals, the underlying BSC's cross-over probability is $\epsilon(\gamma) = (1/2)e^{-\gamma/2}$, where γ denotes the bit energy-to-noise ratio. Assuming a total energy budget $\mathcal{E}/N_0 = 100$, the reconstruction error upper bound in (41) is depicted in Figure 10, where we also plot the optimal bounds achieved with uncoded transmission schemes (cf. (5) and (8)). From these plots, it is evident that the bound $f_{\text{code}}(N_Q, N)$ derived for randomly coded transmission is not tight and is easily achieved with uncoded transmissions.

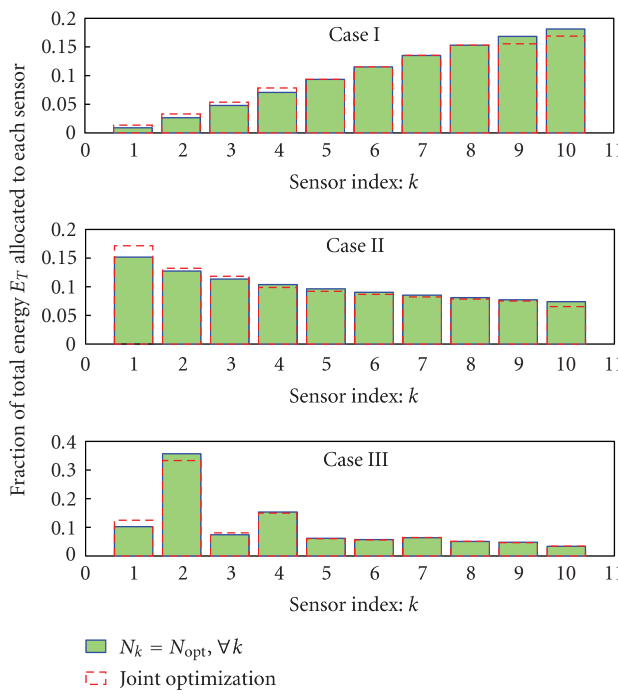


FIGURE 8: Optimal energy allocation scheme for $N_k = N_{\text{opt}}$, for all k, and jointly optimized N_k, $k = 1, \dots, K$. Case I: $d_k = k/4$, $\sigma_k^2 = 0.01$, for all k, $N_{\text{opt}} = 6$. Case II: $d_k = 1$, $\sigma_k^2 = 0.01 \times k$, for all k, $N_{\text{opt}} = 8$. Case III: $\{d_1, d_2, \dots, d_{10}\} = \{1, 5, 1, 5, 1, 1, 5, 5, 1, 5\}$ and, $\sigma_k^2 = 0.01 \times k$, for all k, $N_{\text{opt}} = 6$.

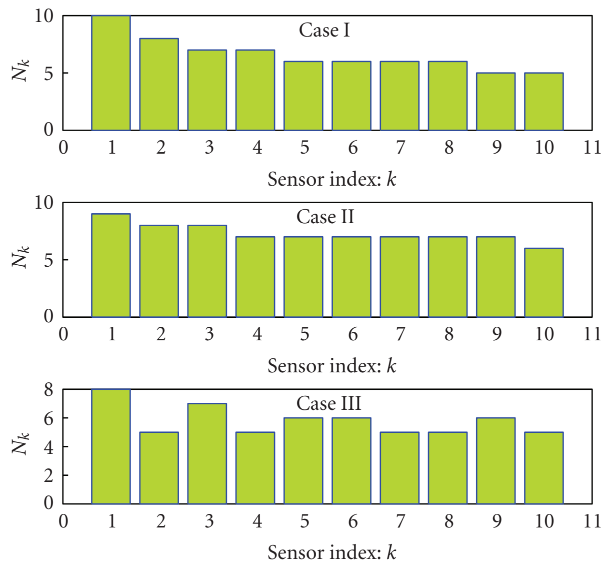
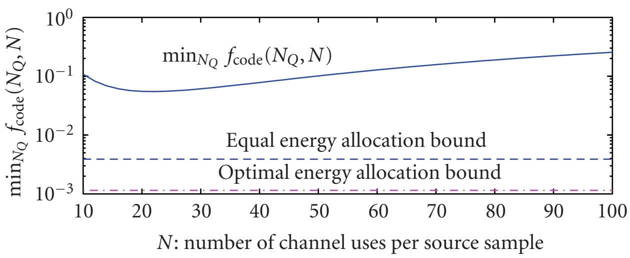
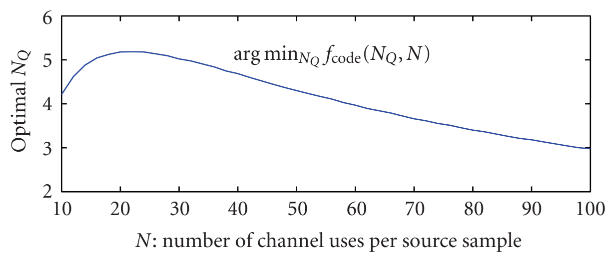


FIGURE 9: Jointly optimized number of quantization bits per sensor: N_k, $k = 1, \ldots, K$. Case I: $d_k = k/4$, $\sigma_k^2 = 0.01$, for all k. Case II: $d_k = 1$, $\sigma_k^2 = 0.01 \times k$, for all k. Case III: $\{d_1, d_2, \ldots, d_{10}\} = \{1, 5, 1, 5, 1, 1, 5, 5, 1, 5\}$ and $\sigma_k^2 = 0.01 \times k$, for all k.



(a)



(b)

FIGURE 10: Single measurement transmission.

5.4.2. *Multiple measurements transmitted together*

Keeping the same set of parameters, the bound provided in (46) is plotted in Figure 11 for different values of L. We observe that with optimal choices of N_Q and N, the uncoded performance bound is easily achieved by transmitting multiple source samples together with appropriate channel coding. The performance gap between uncoded and coded cases can be large. In this particular example, the reconstruction error with coding can be less than 1% of the one without coding.

6. CONCLUSIONS

Motivated by stringent energy requirements that are prevalent in wireless sensor networks, we have pursued optimal quantization at sensor nodes to effect optimal reconstruction at the fusion center. (De)modulation and propagation were captured through the use of the classical binary symmetric channel model. In particular, we have found the number of quantization bits for minimizing the mean-absolute reconstruction error bound in a point-to-point link when each bit is allocated the same energy. We also derived an optimal scheme for energy allocation across quantization bits. Both transmission energy as well as circuit energy were considered. When multiple sensors collaborate to estimate a parameter in noise, we also obtained the optimal energy allocation scheme to distribute the limited total energy among different sensors when each sensor assigns the same energy to all its quantization bits. It turned out that this allocation scheme depends on the prescribed number of quantization bits $\{N_k\}_{k=1}^K$, and thus the optimal $\{N_k\}_{k=1}^K$ can also be found with the help of our convex optimization formulation. We also studied the effects of channel coding on energy constrained quantization and optimized the number of quantization levels and the number of channel uses per source sample.

APPENDIX

A. PROOF OF $f_0(\mathbf{x}_n^*; n) < f_0(\mathbf{x}_{\tilde{n}}^*; \tilde{n})$, FOR ALL $N > \tilde{N}$

Because $N > \tilde{N}$, we can construct the following N-dimensional vector

$$\mathbf{x}_N := \left[x_1, \ldots, x_N\right]^T = \left[\mathbf{x}_{\tilde{N}}^{*T}, 0, \ldots, 0\right]^T, \tag{A.1}$$

where $\mathbf{x}_{\tilde{N}}^* := [\tilde{x}_1^*, \ldots, \tilde{x}_{\tilde{N}}^*]^T$ is the optimal solution of (8) when transmitting $\tilde{N}$ quantization bits. By construction, $\mathbf{x}_N$ is a feasible point for the optimization problem in (8) when transmitting N bits because

$$x_i \geq 0, \quad i = 1, \ldots, N, \quad \sum_{i=1}^{N} x_i = \sum_{i=1}^{\tilde{N}} \tilde{x}_i^* = 1. \tag{A.2}$$

As $\mathbf{x}_N^*$ is the optimal solution when transmitting N bits, we have

$$f_0(\mathbf{x}_N; N) \geq f_0(\mathbf{x}_N^*; N). \tag{A.3}$$

We can write down $f_0(\mathbf{x}_N; N)$ explicitly as

$$\begin{aligned} f_0(\mathbf{x}_N; N) &= 2^{-N} + \sum_{i=1}^{N} \epsilon\left(\frac{\mathcal{E} x_i}{N_0}\right) 2^{-i} \\ &= 2^{-N} + \sum_{i=1}^{\tilde{N}} \epsilon\left(\frac{\mathcal{E} \tilde{x}_i^*}{N_0}\right) 2^{-i} + \sum_{i=\tilde{N}+1}^{N} \epsilon(0) 2^{-i} \\ &= 2^{-N} - 2^{-\tilde{N}} + f_0\left(\mathbf{x}_{\tilde{N}}^*; \tilde{N}\right) + \frac{1}{2} \sum_{i=\tilde{N}+1}^{N} 2^{-i} \\ &= 2^{-N-1} - 2^{-\tilde{N}-1} + f_0\left(\mathbf{x}_{\tilde{N}}^*; \tilde{N}\right) < f_0\left(\mathbf{x}_{\tilde{N}}^*; \tilde{N}\right), \end{aligned} \tag{A.4}$$

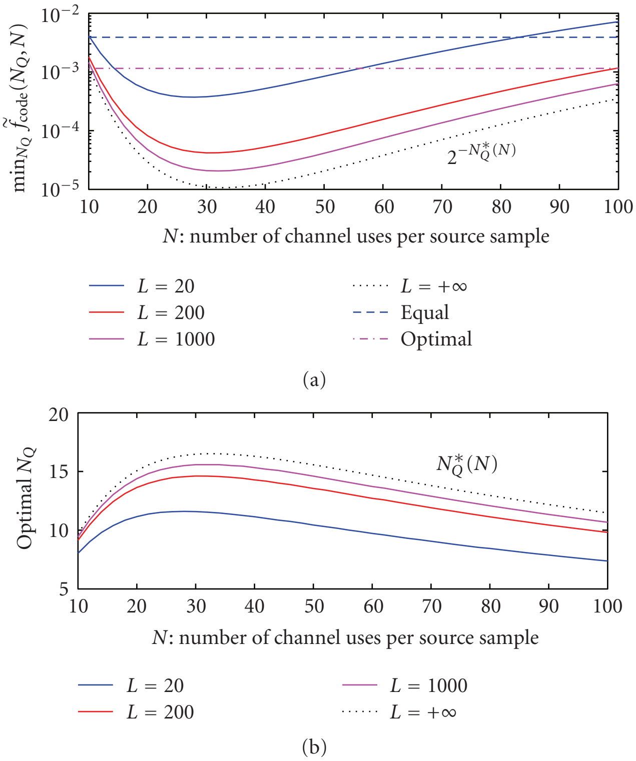


FIGURE 11: Multiple measurements transmitted together.

where we have used the property that $\epsilon(0) = 1/2$. From (A.3) and (A.4), it follows readily that $f_0(\mathbf{x}_N^*; N) < f_0(\mathbf{x}_{\tilde{N}}^*; \tilde{N})$.

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