On Linear Equalization of Multiple FIR Volterra Channels

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Abstract

A deterministic approach for blind equalization of single-input multiple-output (SIMO) Volterra FIR channels is addressed. It is shown that SIMO Volterra channels can be blindly equalized using only linear FIR filters, provided that a generalized Sylvester resultant constructed from Volterra kernels has maximum column rank and that a minimal persistence-of-excitation order is satisfied by the input. Necessary and sufficient conditions for the existence of linear equalizers are established. Simulations comparing the blind with non-blind equalization are provided.

1. Introduction to Blind Equalization

Satellite communication channels exhibit nonlinear characteristics due to the high power amplifiers which operate close to the saturation region in order to maximize output power. Blind equalization of such nonlinear channels is potentially useful especially when mobile users are present. Identification of nonlinear dynamics is also a subject of interest in biomedical and magnetic recording research.

In the first part of this paper, we describe a general approach for blind deconvolution (equalization) and identification of nonlinear SIMO FIR Volterra systems. Although impossible with a single output, multiple outputs make it possible to deconvolve blindly multiple FIR Volterra channels. The approach requires only that a generalized Sylvester resultant, constructed from the channel coefficients, has maximum column rank and that the input signal possesses a certain persistence-of-excitation order - a requirement also encountered with I/O based methods. The input is allowed to be deterministic or random with unknown color or distribution. The estimation approach is not based on higher order statistics of the input/output signals, and the channel

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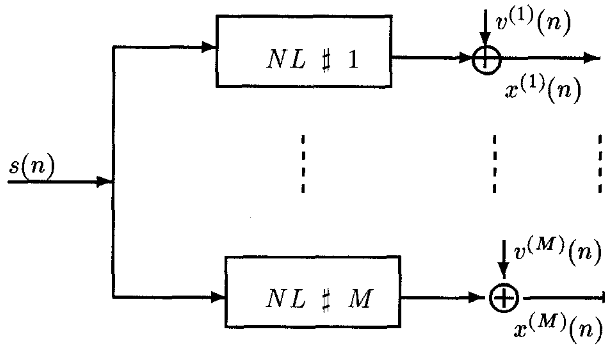

Figure 1. SIMO nonlinear Volterra channel

can be any FIR Volterra channel, which satisfies the above-mentioned rank condition.

Consider the M-channel output $\mathbf{x}'(n) := [x^{(1)}(n) \ldots x^{(M)}(n)]$, for $n = 0, \ldots, N-1$, given by

$$\mathbf{x}(n) = \mathbf{v}(n) + \sum_{p=1}^{P} \sum_{l_1,\ldots,l_p=0}^{L_p} \mathbf{h}_p(l_1,\ldots,l_p)s(n-l_1)\ldots s(n-l_p), \quad (1)$$

where: (i) lower (upper) bold is used for vectors (matrices); (ii) $M \times 1$ vector $\mathbf{h}_p$ corresponding to the pth-order kernel is defined similar to $\mathbf{x}$

$$\mathbf{h}'_p(l_1,\ldots,l_p) := \left[h_p^{(1)}(l_1,\ldots,l_p) \; .. \; h_p^{(M)}(l_1,\ldots,l_p)\right],$$

with $h_p^{(m)}(l_1,\ldots,l_p)$ denoting the pth-order kernel of the mth channel; (iii) the inaccessible scalar input $s(n)$ is allowed to be either deterministic or random; (iv) the range of $(l_1,\ldots,l_p)$ is chosen such that $h_p^{(m)}(l_1,\ldots,l_p)$ is defined over its non-redundant region: $0 \le l_1 \le \ldots \le l_p \le L_p$. (v) $\mathbf{v}(n)$ is additive white Gaussian noise (AWGN) $\mathbf{v}'(n) := [v^{(1)}(n) \ldots v^{(M)}(n)]$. The structure of a SIMO nonlinear channel is depicted in Fig. 1. We view the p-dimensional kernel $\mathbf{h}'_p(l_1,\ldots,l_p)$ as a collection of linear (one-dimensional) kernels defined as:

$\mathbf{h}'_{p,i_1:i_{p-1}}(l) :=$

$$\left[h_p^{(1)}(l, l+i_1, \dots, l+i_{p-1}) \dots h_p^{(M)}(l, \dots, l+i_{p-1})\right] \quad (2)$$

where: $0 \le i_1 \le \dots \le i_{p-1} \le L_p$, and $l = 0, 1, \dots, L_p - i_{p-1}$. In order to compactify notation we, henceforth, use $i_1 : i_{p-1}$ to denote the set $(i_1, \dots, i_{p-1})$ (for $p = 2$, we consider $i_1 : i_1 = i_1$). Similarly, we define the signals

$$s_{p,i_1:i_{p-1}}(l) := s(l)s(l-i_1)\dots s(l-i_{p-1}), \quad (3)$$

with $0 = i_0 \le i_1 \le \dots \le i_{p-1} \le L_p$, and denote $\mathbf{h}_1 := \mathbf{h}_{1,i_0:i_0}$, and $s_1 := s_{1,i_0:i_0}$. Using the change of variables $l_p = l + i_{p-1}$, for $p = 1, \dots, P$, $i_0 = 0$, we rewrite (1) as

$$\mathbf{x}'(n) = \mathbf{v}'(n) + \sum_{p=1}^{P} \sum_{0\le i_1\le\dots\le i_{p-1}\le L_p} \sum_{l=0}^{L_p - i_{p-1}} \mathbf{h}'_{p,i_1:i_{p-1}}(l) s_{p,i_1:i_{p-1}}(n-l). \quad (4)$$

Equation (4) allows us to view a nonlinear SIMO channel as a linear MIMO channel whose inputs are related (c.f. (3)). Given the M-channel system output $\{\mathbf{x}(n)\}_{n=0}^{N-1}$ satisfying (4), we want to blindly deconvolve the system; i.e., we wish to recover both the input sequence $s(n)$ as well as the channel kernels $h_p^{(m)}(l_1, \dots, l_p)$, $p = 1, \dots, P$, $m = 1, \dots, M$, from knowledge of the received data $\mathbf{x}(n)$ only. Specifically, we seek the linear FIR equalizers $\{\mathbf{g}_{p,i_1:i_{p-1}}^{(d)}(k)\}_{k=0}^{K}$ of order K and delay d, which deconvolves the pth-order kernel $\mathbf{h}_{p,i_1:i_{p-1}}(l)$ via

$$\sum_{k=0}^{K} \mathbf{x}'(n-k)\mathbf{g}_{p,i_1:i_{p-1}}^{(d)}(k) = s(n-d)s(n-d-i_1)\dots s(n-d-i_{p-1}), \quad (5)$$

where the $(p-1)$-tuple $(i_1, \dots, i_{p-1})$ and delay d satisfy $0 \le i_1 \le \dots \le i_{p-1} \le L_p$ and $0 \le d \le L_p + K - i_{p-1}$.

2. Direct Blind Equalizers

In this section we present briefly the main results concerning the blind estimation of the linear equalizers (for further details see [2]). We start by introducing some definitions and assumptions. Define the $(L_p + K + 1 - i_{p-1}) \times M(K+1)$ block Toeplitz matrix

$$\mathbf{H}_{p,i_1:i_{p-1}} := \begin{bmatrix} \mathbf{h}'_{p,i_1:i_{p-1}}(0) & \dots & \mathbf{0}' \\ \vdots & \ddots & \vdots \\ \mathbf{h}'_{p,i_1:i_{p-1}}(L_p - i_{p-1}) & \dots & \mathbf{h}'_{p,i_1:i_{p-1}}(L_p - i_{p-1} - K) \\ \vdots & \ddots & \vdots \\ \mathbf{0}' & \dots & \mathbf{h}'_{p,i_1:i_{p-1}}(L_p - i_{p-1}) \end{bmatrix}.$$

Define also $(L_1 + K + 1) \times 1$ and $(L_p + K + 1 - i_{p-1}) \times 1$ vectors $\mathrm{s}_1(n)$ and $\mathrm{s}_{p,i_1:i_{p-1}}(n)$ through the relations

$$\mathrm{s}'_1(\mathrm{n}) := [\ \mathrm{s}(\mathrm{n}) \ \dots \ \mathrm{s}(\mathrm{n} - \mathrm{L}_1 - \mathrm{K})\ ],$$
$$\mathrm{s}'_{\mathrm{p},\mathrm{i}_1:\mathrm{i}_{\mathrm{p}-1}}(\mathrm{n}) := [\mathrm{s}_{\mathrm{p},\mathrm{i}_1:\mathrm{i}_{\mathrm{p}-1}}(\mathrm{n}) \dots \mathrm{s}_{\mathrm{p},\mathrm{i}_1:\mathrm{i}_{\mathrm{p}-1}}(\mathrm{n} - \mathrm{L}_\mathrm{p} - \mathrm{K} + \mathrm{i}_{\mathrm{p}-1})]$$

where $0 \le i_1 \le \dots \le i_{p-1} \le L_p$, and $s_{p,i_1:i_{p-1}}(n)$ as in (3). The noise-free input-output relation (4) can be rewritten in the matrix form

$$\mathbf{X} = \mathbf{S}\ \mathbf{H}, \quad (6)$$

where the $(N-K) \times M(K+1)$ block Hankel matrix $\mathbf{X}$ and $D(L_1, \dots, L_P, K) \times M(K+1)$ block Toeplitz channel matrix $\mathbf{H}$ are given respectively by

$$\mathbf{X} := \begin{bmatrix} \mathbf{x}'(N-1) & \dots & \mathbf{x}'(N-1-K) \\ \vdots & \vdots & \vdots \\ \mathbf{x}'(K) & \dots & \mathbf{x}'(0) \end{bmatrix}, \mathbf{H} := \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_{2,0:0} \\ \vdots \\ \mathbf{H}_{P,L_P:L_P} \end{bmatrix}$$

where $D(L_1, \dots, L_P, K) = L_1 + K + 1$

$$+ \sum_{p=2}^{P} \sum_{i=0}^{L_p} \binom{i+p-2}{p-2} (L_p + K + 1 - i).$$

We adopt the following assumptions:
(a0.1) $\mathbf{v}(n) = 0$, i.e., we consider noise-free data.
(a0.2) $N \ge \max\{K + d + 2M(K+1), K + P^P\}$. This requirement is introduced to assure more equations (data) than unknowns in subsequent equations.
(a1.1) rank$(\mathbf{H}) = D(L_1, \dots, L_P, K)$, condition equivalent to the full column rank of a generalized Sylvester, obtained from $\mathbf{H}'$ by suitably permuting its rows and columns [2]. This implies that $\mathbf{H}$ is a wide matrix i.e.,

$$M(K+1) \ge D(L_1, \dots, L_P, K). \quad (7)$$

(a1.2) $\mathbf{H}$ is square; i.e., (7) is satisfied with equality.
(a2.1) with $L := \max\{L_1, \dots, L_P\}$, input $s(n)$ is such that the matrix $\mathcal{S}^{(0,L+K)}$ defined through[1] $\mathcal{S}^{(0,L+K)} :=$

$$[\mathrm{S}(L+K+1 : N-K, :)\ \ \mathrm{S}(1 : N-2K-L, :)], \quad (8)$$

has maximum column rank; i.e.,

$$\text{rank}(\mathcal{S}^{(0,L+K)}) = 2D(L_1, \dots, L_P, K) - C,$$

where C equals the number of pairs of identical columns of $\mathcal{S}^{(0,L+K)}$.
(a2.2) matrix $\bar{\mathrm{S}}$ in (9) is full column rank

$$\bar{\mathrm{S}} := \begin{bmatrix} s(N-1) & s^2(N-1) & \dots & s^{P^2}(N-1) \\ s(N-2) & s^2(N-2) & \dots & s^{P^2}(N-2) \\ \vdots & \vdots & \ddots & \vdots \\ s(K) & s^2(K) & \dots & s^{P^2}(K) \end{bmatrix}. \quad (9)$$

We review first the case of Volterra systems for which there is only one kernel with maximum memory $L := \max\{L_1, \dots, L_P\}$, and then we focus on models having more than one kernel with maximum memory.

[1]In writing (8), we adopted Matlab's notation $\mathbf{X}(i_1 : i_2, j_1 : j_2)$ to denote the submatrix of $\mathbf{X}$ formed by the i_1 through i_2 rows and the j_1 through j_2 columns of $\mathbf{X}$.

2.1 One Kernel with Maximum Memory

Upon defining $\mathbf{g}_p^{(d)} := [\mathbf{g}_p^{(d)}(0)' \; \dots \; \mathbf{g}_p^{(d)}(K)']'$, and

$\mathcal{X}^{(0,d)} := [\mathbf{X}(d+1 : N-K, :) \;\; -\mathbf{X}(1 : N-K-d, :)]$

for $d = 0, \dots, L_1 + K,\; p = 1, \dots, P$, we have [2]

$$\mathcal{X}^{(0,d)} \boldsymbol{g}_p^{(0,d)} = \mathbf{0} \; , \tag{10}$$

where $\boldsymbol{g}_p^{(0,d)} := [\mathbf{g}_p^{(0)'} \; \mathbf{g}_p^{(d)'}]'$. We have the result [2]

Theorem 1. *Suppose that* **(a0.1)** – **(a2.1)** *are satisfied. If* $L = \max\{L_1, \dots, L_P\}$ *is attained by a single* L_p, *where* $p \in \{1, \dots, P\}$, *then* $\mathbf{g}_{p,0:0}^{(0)}$ *and* $\mathbf{g}_{p,0:0}^{(L+K)}$ *can be uniquely identified (within a scale) from (10) as the null eigenvector of* $\mathcal{X}^{(0,L+K)}$. □

2.2 Many Kernels with Maximum Memory

Now we consider the case when $\dim[\mathcal{N}(\mathcal{X}^{(0,L+K)})] > 1$. Henceforth, w.l.o.g. we suppose that: $L_1 = L_2 = \dots = L_P = L$. Using **(a2.1)** and following the same steps used to derive (10), we find that the pairs of equalizers $\boldsymbol{g}_p^{(0,L+K)}$, $p = 1, \dots, P$, also, satisfy (10). Thus, $\dim[\mathcal{N}(\mathcal{X}^{(0,L+K)})] = P$. Suppose that the null space $\mathcal{N}(\mathcal{X}^{(0,L+K)})$ is spanned by the columns of the $2M(K+1) \times P$ matrix $\mathbf{U}$, which can be easily obtained by performing an SVD on $\mathcal{X}^{(0,L+K)}$. Consider that the pairs of equalizers $\boldsymbol{g}_p^{(0,L+K)}$, $p = 1, \dots, P$, are given by the columns of the matrix

$$\mathbf{G} = \left[\boldsymbol{g}_1^{(0,L+K)} \, \boldsymbol{g}_2^{(0,L+K)} \, \dots \, \boldsymbol{g}_P^{(0,L+K)}\right] .$$

Based on **(a2.1)**, it follows from (10) that $\mathcal{R}(\mathbf{G}) = \mathcal{R}(\mathbf{U}) = \mathcal{N}(\mathcal{X}^{(0,L+K)})$, where $\mathcal{R}$ denotes the range space of a matrix. Since $\mathbf{G}$ and $\mathbf{U}$ are full column rank and both span the same space, there exists a nonsingular $P \times P$ matrix $\mathbf{\Lambda}$ such that

$$\mathbf{G} = \mathbf{U} \, \mathbf{\Lambda} \, . \tag{11}$$

Considering only the first $M(K+1)$ rows of (11), we obtain

$$\mathbf{G}^{(0)} = \mathbf{U}(1 : M(K+1), :) \, \mathbf{\Lambda} \; , \tag{12}$$

where

$$\mathbf{G}^{(0)} = \left[\mathbf{g}_1^{(0)} \, \mathbf{g}_{2,0:0}^{(0)} \, \dots \, \mathbf{g}_{P,0:0}^{(0)}\right] .$$

Since $\mathbf{U}$ is available from the data matrix $\mathcal{X}^{(0,L+K)}$, our goal is to identify $\mathbf{\Lambda}$. Identification of the pth column $\mathbf{\Lambda}(:,p)$ yields the pth column of $\mathbf{G}^{(0)}$; i.e., the pth order equalizer $\mathbf{g}_{p,0:0}^{(0)}$. In order to find $\mathbf{\Lambda}$, we take into account the dependence between the outputs of the equalizers corresponding to the first and pth-order kernels. Equality $[s(n)]^p = s^p(n)$, for $n = N-1, \dots, K$, can be written in matrix-form as:

$$\mathbf{A}_p \, \mathbf{\Lambda}^{[p]}(:,1) = \mathbf{B} \, \mathbf{\Lambda}(:,p) \; , \tag{13}$$

where $\mathbf{A}_p$ and $\mathbf{B}$ are given, respectively, by [2]

$$\mathbf{A}_p = \begin{bmatrix} \mathbf{X}^{[p]}(1,:) \\ \mathbf{X}^{[p]}(2,:) \\ \vdots \\ \mathbf{X}^{[p]}(N-K,:) \end{bmatrix} \mathbf{U}(1 : M(K+1), :)^{[p]} \tag{14}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{X}(1,:) \\ \mathbf{X}(2,:) \\ \vdots \\ \mathbf{X}(N-K,:) \end{bmatrix} \mathbf{U}(1 : M(K+1), :) \; , \tag{15}$$

and $\mathbf{\Lambda}^{[p]}$ denotes the pth-fold Kronecker product of $\mathbf{\Lambda}$. Considering $\tilde{\mathbf{s}}_p := [s^p(N-1) \; \dots \; s^p(K)]'$, $p = 1, \dots, P$, the following result holds [2]

Proposition 1. *Under* **(a0.1)-(a2.2)**, *we have*

$$\begin{aligned} \dim\left[\mathcal{R}(\mathbf{A}_p) \cap \mathcal{R}(\mathbf{B})\right] &= P - p + 1 \; , \\ \mathcal{R}(\mathbf{A}_p) \cap \mathcal{R}(\mathbf{B}) &= \mathrm{span}[\tilde{\mathbf{s}}_p, \dots, \tilde{\mathbf{s}}_P] \; . \quad \square \end{aligned}$$

Making $p = P$ in Proposition 1, we can determine $\tilde{\mathbf{s}}_P$, and then the equalizer $\mathbf{g}_{P,0:0}^{(0)}$. Note that $\mathbf{g}_1^{(0)}$ can be determined provided that there is no ambiguity in taking the Pth-root of $\tilde{\mathbf{s}}_P$; i.e., no ambiguity exists in recovering uniquely $s(n)$ from $s^P(n)$. Such unambiguous recovery is guaranteed, e.g., in the case of a PSK input signal with Q constellation points such that (Q,P) are coprime; $s(n)$ can be uniquely recovered from $s^P(n)$ since a rotation of $s(n)$ by a factor P, brings the constellation points of $s^P(n)$ at distinct locations. Assuming that we can find uniquely $s(n)$ from $s^P(n)$, the determination of all other equalizers is possible since the problem reduces to an I/O identification problem. However, there are cases when $s(n)$ cannot be recovered uniquely from $s^P(n)$; e.g., a PSK input signal with (Q,P) not coprime.

In this case, we proceed to recover uniquely (up to a constant) the equalizer $\mathbf{g}_{P-1,0:0}^{(0)}$. Knowledge of equalizers corresponding to the $(P-1)$st and Pth-order kernels implies access to their outputs $s^{P-1}(n)$, and respectively, $s^P(n)$. Selecting samples $s(n) \neq 0$ and taking the ratio of these two outputs, we obtain the input sequence $s(n) = s^P(n)/s^{P-1}(n)$. Hence, the problem is solved if we can find $\mathbf{g}_{P-1,0:0}^{(0)}$. Towards this goal, we apply again Proposition 1 for $p = P-1$, to infer that:

$$\mathcal{R}(\mathbf{A}_{P-1}) \cap \mathcal{R}(\mathbf{B}) = \mathrm{span}\left[\tilde{\mathbf{s}}_{P-1}, \; \tilde{\mathbf{s}}_P\right] .$$

Consider $[\mathbf{r}_{A_{P-1},B}^{(1)}, \; \mathbf{r}_{A_{P-1},B}^{(2)}]$ to be a basis for $\mathcal{R}(\mathbf{A}_{P-1}) \cap \mathcal{R}(\mathbf{B})$. It follows that there exists a 2×2 nonsingular matrix $\mathbf{F} = [f_{ij}]$ such that:

$$[\mathbf{r}_{A_{P-1},B}^{(1)}, \; \mathbf{r}_{A_{P-1},B}^{(2)}] = [\tilde{\mathbf{s}}_{P-1}, \; \tilde{\mathbf{s}}_P] \; \mathbf{F} \, . \tag{16}$$

Define the vectors $\hat{\mathbf{g}}_{P-1,0:0}^{(0)}$ and $\hat{\hat{\mathbf{g}}}_{P-1,0:0}^{(0)}$ through

$$\mathbf{X} \, \hat{\mathbf{g}}_{P-1,0:0}^{(0)} = \mathbf{r}_{A_{P-1},B}^{(1)} \; , \tag{17}$$

$$\mathbf{X}\,\hat{\hat{\mathbf{g}}}^{(0)}_{P-1,0:0} \;=\; \mathbf{r}^{(2)}_{A_{P-1},B} \;. \qquad (18)$$

Using the relation $[s^{P-1}(n)]^P = [s^P(n)]^{P-1}$ between the outputs of equalizers $\mathbf{g}^{(0)}_{P-1,0:0}$ and $\mathbf{g}^{(0)}_{P,0:0}$, it is shown in [2] that the equalizer $\mathbf{g}^{(0)}_{P-1,0:0}$ can be found as a linear combination of $\hat{\mathbf{g}}^{(0)}_{P-1,0:0}$ and $\hat{\hat{\mathbf{g}}}^{(0)}_{P-1,0:0}$. Thus, we have

Theorem 2. *If* **(a0.1)-(a2.2)** *hold true, then the equalizers corresponding to all kernels and all possible delays can be uniquely identified (within a scale factor).* □

3. Perfect Linear Equalization

In this section we consider the problem of finding the linear equalizers of a SIMO Volterra channel in the non-blind case, i.e., given knowledge of all Volterra channels. This problem can be stated equivalently as: given a Volterra analysis filter bank, find the linear synthesis filter bank which satisfies the perfect reconstruction condition. We may refer to this as a Bezout identity for FIR Volterra filter banks. Specifically, we consider a number of M known FIR Volterra channels, excited by the same common input $s(n)$, and we want to find under what conditions there exist linear FIR equalizers $G_d^{(m)}(z)$, $m = 1,\ldots,M, d = 0,\ldots,L+K$, with perfect reconstruction property. Fig. 2 presents a detailed view of the analysis and synthesis sections. We have seen that each Volterra channel can be represented as a sum of linear FIR filters whose inputs are correlated (4). In Fig. 2, we considered that each Volterra channel is made up of R such linear subchannels $H_r^{(m)}(z)$, $r = 1,\ldots,R$, each one being of order L, and that all the linear equalizers $G_d^{(m)}(z)$ have order K.

The necessary and sufficient condition for perfect reconstruction is

$$\sum_{m=1}^{M} G_d^{(m)}(z)\, H_r^{(m)}(z) = z^{-d}\,\delta(r-1)\;, \qquad (19)$$

for any $m = 1,\ldots,M, r = 1,\ldots,R$ and $d = 0,\ldots,L+K$. Considering the notations $H_r^{(m)}(z) := \sum_{l=0}^{L} h_r^{(m)}(l)z^{-l}$, $G_d^{(m)}(z) := \sum_{k=0}^{K} g_d^{(m)}(k)z^{-k}$, for $m = 1,\ldots,M$ and $r = 1,\ldots,R$, and the $(K+1)\times(L+K+1)$ Toeplitz matrix

$$\mathbf{H}_r^{(m)}(K) = \begin{bmatrix} h_r^{(m)}(0) & \ldots & h_r^{(m)}(L) & \ldots & 0 \\ 0 & \ldots & h_r^{(m)}(L-1) & \ldots & 0 \\ \vdots & \ldots & \ldots & \ldots & \vdots \\ 0 & \ldots & h_r^{(m)}(0) & \ldots & h_r^{(m)}(L) \end{bmatrix}$$

relation (19) can be rewritten in the time-domain as

$$\mathbf{g}_d(K)'\,\mathbf{H}(K) = \mathbf{e}'_d\;, \qquad (20)$$

where

$$\mathbf{g}_d(K)' := \left[g_d^{(1)}(0)\ldots g_d^{(1)}(K)\ldots g_d^{(M)}(0)\ldots g_d^{(M)}(K)\right],$$

$$\mathbf{H}(K) := \begin{bmatrix} \mathbf{H}_1^{(1)}(K) & \mathbf{H}_2^{(1)}(K) & \ldots & \mathbf{H}_R^{(1)}(K) \\ \mathbf{H}_1^{(2)}(K) & \mathbf{H}_2^{(2)}(K) & \ldots & \mathbf{H}_R^{(2)}(K) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_1^{(M)}(K) & \mathbf{H}_2^{(M)}(K) & \ldots & \mathbf{H}_R^{(M)}(K) \end{bmatrix},$$

$$\mathbf{e}'_d := [0\ \ldots\ 0\,1\,0\ \ldots\ 0]\;.$$

The $1\times M(K+1)$ vector $\mathbf{e}'_d$ has the unity in the dth-position. We have the following result:

Proposition 2. *Given M known FIR Volterra channels $H^{(m)}(z)$, $m = 1,\ldots,M$, the necessary and sufficient condition for the existence of the linear equalizers $\mathbf{G}_d^{(m)}(z)$, for $m = 1,\ldots,M$ and $d = 0,\ldots,L+K$, with the perfect reconstruction property (19) is:*

$$\text{rank}(\mathbf{H}_K) - \text{rank}(\mathbf{H}_{K,nl}) = L+K+1\;, \qquad (21)$$

where $\mathbf{H}_{K,nl} := \mathbf{H}_K(:, L+K+2 : R(L+K+1))$. □

If $K \geq L-1$, then condition (21) comes up to the coprimeness condition of the linear parts $H_1^{(m)}(z)$, $m = 1,\ldots,M$, and to the disjointness of the range spaces of $\mathbf{H}_{K,l} := \mathbf{H}_K(:,1 : L+K+1)$ and $\mathbf{H}_{K,nl}$. Further analysis shows that matrices $\mathbf{H}_K$ and $\mathbf{H}_{K,nl}$ have the structure of a generalized Sylvester resultant. It can be shown that if relation (21) holds for K, then it also holds for $K+1$. By expressing the rank of a generalized Sylvester matrix in terms of the dual dynamic indices, [1], [3], of a polynomial matrix, it turns out that condition (21) is equivalent to the difference between two Smith-McMillan degrees of two polynomial matrices be equal to $L+1$ (the length of the linear part), for $K \geq \nu_{max}$ (ν_{max} being the maximum dual dynamic index of a polynomial matrix $H(s)$ constructed from the kernels of all Volterra channels).

We have seen, [2], that in the blind case assumption **(a1.1)** is equivalent to the condition that matrix $\mathbf{H}_K$ is full column rank. If this assumption holds, then also condition (21) is satisfied. It can be shown that the minimal value of K used to test **(a1.1)** and (21) is the same ($K_{min} = \nu_{max}$). Since condition (21) requires besides the coprimeness condition of the linear parts, a certain "coprimeness" between linear and the nonlinear parts, we conclude that condition (21) is more restrictive than the condition encountered in the case of linear FIR filter banks, but it is less restrictive than assumption **(a1.1)**. Finally, we note that the necessary and sufficient conditions for the existence of linear equalizers $G_d^{(m)}(z)$, $m = 1,\ldots,M$, and for a fixed delay $d = d_0$, are slightly different than (21). Due to the limited space, we will provide further details and proofs of these results in a future communication.

4 Simulations

First we considered the blind equalization of a real channel with $\dim[\mathcal{N}(\mathcal{X}^{(0,L+K)})] = 3$. We chose $M = 5$ channels, with $L_1 = L_{2,0} = L_{3,0:0} = L = 1$, and the mth-channel described by

$$x^{(m)}(n) = \sum_{l=0}^{1} h_1^{(m)}(l)s(n-l) + \sum_{l=0}^{1} h_{2,0}^{(m)}(l)s^2(n-l)$$

$$+ h_{2,0}^{(m)}(0)s(n)s(n-1) + \sum_{l=0}^{1} h_{3,0:0}^{(m)}(l)s^3(n-l) + v^{(m)}(n)$$

$\mathbf{h}_1(0) = [1, .5, 2, -0.3, 1]'$, $\mathbf{h}_1(1) = [-2.5, 3, 0, 0.9, 1.5]'$, $\mathbf{h}_{2,0}(0) = [.01, .5, .2, 1, 1.5]'$, $\mathbf{h}_{2,0}(1) = [.2, .3, -.7, .02, 2]'$, $\mathbf{h}_{2,1}(0) = [1.3, -.65, .89, 1.5, .99]'$, $\mathbf{h}_{3,0:0}(0) = [-.6, .2, .9, .76, 3]'$, $\mathbf{h}_{3,0:0}(1) = [-0.9, -0.7, .1, .02, .5]'$. The input sequence was a pseudorandom sequence with normal distribution $\mathcal{N}(0, 1.4)$. Only $N = 500$ samples were used to estimate a $K = 2$ order equalizer. A comparison between the true input $s(n)$ and the equalizer output waveforms for SNR= 30 dB, and respectively, SNR= 40 dB is shown in Figure 3. Then, we repeated the simulation in a non-blind framework, first by preserving the same channels, and then by considering for channels 2 through 5 the same quadratic and cubic parts as those of channel 1. We used the same input as in the blind case. The results are displayed in Fig. 4. In the latter case, note that assumption **(a1.1)** does not hold, thus we can not blindly equalize the modified channel. However, condition (21) is satisfied, which allows determination of the linear equalizers assuming all Volterra channels are known.

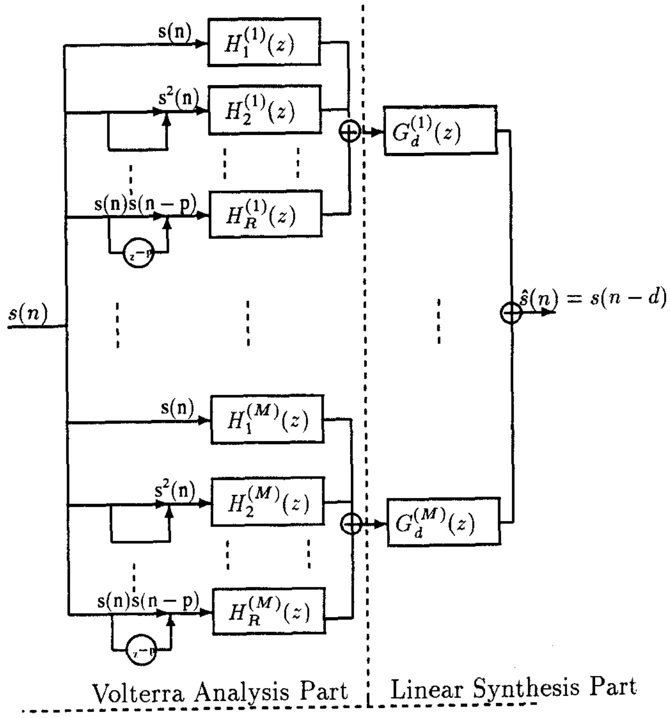

Figure 2. Analysis-synthesis filter banks

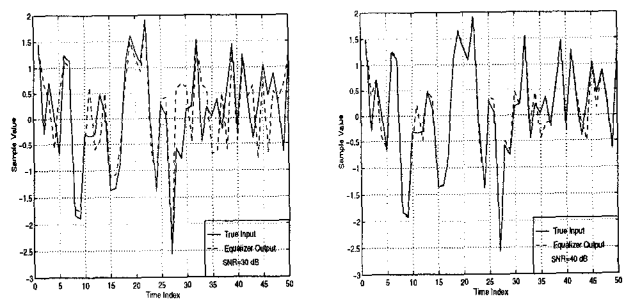

Figure 3. True and blindly equalized inputs

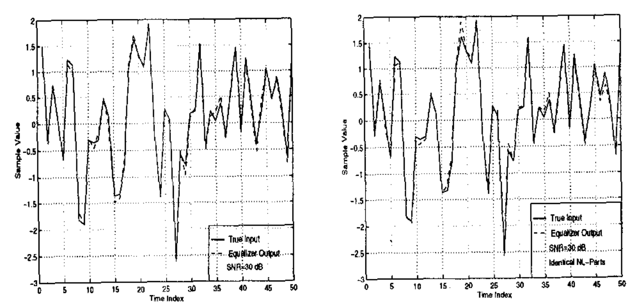

Figure 4. True and non-blind estimate

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