BLIND ESTIMATION AND EQUALIZATION OF TIME- AND FREQUENCY-SELECTIVE CHANNELS USING FILTERBANK PRECODERS

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ABSTRACT

Novel channel estimation and symbol recovery algorithms are proposed for the identification and equalization of time- and frequency- selective channels where the time variation is modeled by a basis expansion. The resulting algorithm enables the usage of a single antenna, dispenses with channel disparity conditions of existing approaches, and allows channel order overestimation. Simulations illustrate the approach.

1. INTRODUCTION

Blind equalization of time-varying channels has gained importance in mobile communications and related fields over the past few decades. Basis expansion models offer finite parametrization of the time variation enabling input/output identification, and with sufficient diversity, blind estimation of these parameters. This prompted the work in [8] which, under rather assumptions on the complex exponential bases, exploited the diversity in the variation in order to identify a single input single output (SISO) time-varying channel. It was shown in [2] that usage of multiple antennas yields a multi input multi output (MIMO) model where the number of antennas needed for identifiability is on the order of the number of basis functions. Here we propose a method that imposes no restrictions on the channel zero locations and is insensitive to channel order overestimation, but introduces a redundancy in the input sequence which can be made arbitrarily small with increasing block lengths.

In Section 2, the framework for this input diversity method is introduced; Section 3 examines the sensitivity to channel order over estimation; Section 4 discusses input recovery, and finally Section 5 exhibits some simulations.

2. INPUT DIVERSITY

Let the input sequence $s(n)$ be passed through a maximally decimated filterbank, as shown in the precoder part of Figure 2. The relationship between $s(n)$ and the channel input $y(n)$ can be expressed as

$$y(n) = \sum_{i=\infty}^{\infty} \sum_{m=0}^{M-1} s(iM+m) f_m(n-iP) \quad . \tag{1}$$

It is well known, [9], that by blocking $s(n)$ in blocks of size M (the downsampling factor) to get $\mathbf{s}(n) := [s(nM) \ldots s(nM+M-1)]^T$ and similarly, $y(n)$ in blocks of size P (the upsampling factor), one obtains an input/output (I/O) relationship for a time-invariant, linear MIMO system: $\mathbf{y}(n) := [y(nP) \ldots y(nP+P-1)]^T = \sum_i \mathbf{F}_i \mathbf{s}(n-i)$. It was shown in [5] that, after the transformed input $y(n)$ goes through an LTI channel, $P \times 1$ blocks of the channel output are given by $\mathbf{x}(n) = \sum_{i,j} \mathbf{C}_j \mathbf{F}_i \mathbf{s}(n-i-j)$, where $\mathbf{C}_j$ are convolution matrices containing the channel coefficients and $\{\mathbf{F}_i\}_{k,m}$. If, in addition, the channel is truncated to be FIR of length L, and P and M are chosen to satisfy $M > L$ and $P = M + L$, then only the terms pertaining to $\mathbf{F}_0$, $\mathbf{C}_0$, and $\mathbf{C}_1$ survive. If the last L rows of $\mathbf{F}_0$ are set to zero (the trailing zeros (TZ) approach of [5]), then the MIMO system has no memory: $\mathbf{x}(n) = \mathbf{HFs}(n)$, where $\mathbf{H} := \mathbf{C}_0$, and $\mathbf{F}$ is the first M rows of $\mathbf{F}_0$. If the channel coefficients are rapidly varying, then the I/O relationship in the absence of noise becomes:

$$\underbrace{\mathbf{x}(n)}_{P\times 1} = \underbrace{\mathbf{H}(n)}_{P\times M} \underbrace{\mathbf{F}}_{M\times M} \mathbf{s}(n) \quad . \tag{2}$$

If, in addition, the scalar I/O relationship of the TV channel is given by the basis expansion model (see also Figure 2),

$$x(n) = \sum_{l=0}^{L_q} \underbrace{\left[\sum_{q=1}^{Q} h_q(l) e^{j\overline{\omega}_q n} \right]}_{h(n;l)} y(n-l) \quad , \tag{3}$$

then (2) becomes

$$\mathbf{x}(n) = \sum_{q=1}^{Q} \mathbf{H}_q \Delta_q \mathbf{Fs}(n) e^{j\omega_q n}, \tag{4}$$

where $\omega_q := P\overline{\omega}_q$, $\mathbf{H}_q$ is a $P \times M$ Toeplitz matrix with $[h_q(0)\ldots h_q(L_q)0\ldots 0]^T$ as its first column and $[h_q(0)0\ldots 0]$ as its first row, and $\mathbf{\Delta}_q := \text{diag}(1, e^{j\overline{\omega}_q}, \ldots, e^{j\overline{\omega}_q(M-1)})$. Notice that as long as $h_q(0) \neq 0$, $\mathbf{H}_q$ will be full column rank. Estimation of the frequencies $\{\omega_q\}_{q=1}^Q$ from the scalar output $x(n)$ and the validity of the basis expansion model is addressed in [7] and [2].

2.1. Channel Estimation

Given a finite record of the vector output $\mathbf{x}(n)$, and the estimates of frequencies $\{\omega_q\}_{q=1}^Q$, we would like to estimate the matrices $\{\mathbf{H}_q\}_{q=1}^Q$ in (4). Let

$$\mathbf{C}_{xx}(\alpha) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1} E[\mathbf{x}(n)\mathbf{x}^H(n)]e^{-j\alpha n}.$$

For identifiability we need to assume:

(a1) $\mathbf{R}_{ss} := E[\mathbf{s}(n)\mathbf{s}^H(n)]$ is full rank. Assumption (a1) means that the input sequence $s(n)$ is persistently exciting (p.e.) of order at least M, which is required even in I/O identification schemes and satisfied by most input streams in digital communications. To facilitate clear exposition of the basic idea we make the following auxiliary assumption:

(a2) The spacing of the frequencies $\omega_1, \ldots \omega_Q$ are such that for $\omega_{q2} \neq \omega_{q_1}$, $\omega_{q_2} - \omega_{q_1} = \omega_{q_4} - \omega_{q_3} \Rightarrow \omega_{q_4} = \omega_{q2}$; i.e., all possible differences of the frequencies are distinct. In the Appendix we discuss how to dispense with this assumption and yet restore identifiability.

Using (4), and (a2), assuming the noise $\mathbf{v}(n)$ is stationary and $\omega_{q_1} \neq \omega_{q_2}$, we obtain

$$\mathbf{C}_{xx}(\omega_{q_1} - \omega_{q_2}) = \mathbf{H}_{q_1}\mathbf{\Delta}_{q_1}\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_{q_2}^H\mathbf{H}_{q_2}^H$$

. Because $\mathbf{R}_{ss}$ is full rank by assumption, $\mathbf{F}$ is full rank by design, and $\mathbf{\Delta}_q$ is always full rank due to its structure, we infer that $\mathcal{R}(\mathbf{C}_{xx}(\omega_{q_1} - \omega_{q_2})) = \mathcal{R}(\mathbf{H}_{q_1})$, where $\mathcal{R}(.)$ stands for range space. Let $\tilde{\mathbf{U}}$ denote the orthogonal complement of the singular vectors that span $\mathcal{R}(\mathbf{H}_{q_1})$. Then, subspace orthogonality implies that $\tilde{\mathbf{U}}^H\mathbf{H}_{q_1} = \mathbf{0}$. Since $\mathbf{H}_{q_1}$ is a convolution matrix, for every $l = 0, \ldots L$, the l^{th} column of $\tilde{\mathbf{U}}$, $\tilde{\mathbf{u}}_l$, can be used to generate the $(L+1) \times M$ Hankel matrix $\mathbf{U}_l$ whose first column is $[\tilde{u}_l(0)\ldots\tilde{u}_l(L)]^T$ and whose last row is $[\tilde{u}_l(L)\ldots\tilde{u}_l(P-1)]$. Let $\mathbf{h}_q := [h_q(0)\ldots h_q(L)]^T$. Since equation

$$\mathbf{h}_{q_1}^H[\mathbf{U}_1\ldots\mathbf{U}_L] = \mathbf{0}\,, \tag{5}$$

and $\tilde{\mathbf{U}}^H\mathbf{H}_{q_1} = \mathbf{0}$ yield identical equations for the unknown coefficients in $\mathbf{h}_{q_1}$, we conclude that $[\mathbf{U}_1\ldots\mathbf{U}_L]$ contains $\mathbf{h}_{q_1}^H$ in its left null space. So the steps involved in estimating $\mathbf{H}_{q_1}$ above are as follows:

Step 1: Estimate $\mathbf{C}_{xx}(\alpha)$ for $\alpha = \omega_{q_1} - \omega_{q_2}$ by $\hat{\mathbf{C}}_{xx}(\alpha) = (1/N)\sum_{n=0}^{N-1}\mathbf{x}(n)\mathbf{x}^H(n)e^{-j\alpha n}$.

Step 2: From the singular value decomposition (SVD) of $\hat{\mathbf{C}}_{xx}(\alpha)$ compute $\hat{\tilde{\mathbf{U}}}$.

Step 3: Calculate $\hat{\mathbf{h}}_{q1}$ as the singular vector corresponding to the minimum singular value of $[\hat{\mathbf{U}}_1\ldots\hat{\mathbf{U}}_L]^H$ mentioned in (5)

The proof of unique (within a scale) recovery of $\mathbf{h}_q$ from the range information of $\mathbf{H}_q$ is addressed in [5], and will not be repeated here.

This method requires only a single antenna for identifiability. The price paid is the reduction in the throughput, since $P = M + L$ symbols are sent for every M information symbols. However, what matters is the ratio M/P which can be made arbitrarily close to 1 for sufficiently large block lengths.

Notice that $h_q(0), h_q(L) \neq 0$ is enough to conclude $\mathbf{H}_q$ is full rank, which implies that the present input diversity scheme like its TI counterpart in [5], but unlike many second-order based output diversity methods (e.g., [4]), does not impose any zero restrictions on the channel coefficients for identifiability.

Another important feature of the input diversity scheme is its robustness to channel order over estimation. In Section 3, we show how using an $\overline{L} > L$, in the input diversity scheme above still yields an accurate estimate of the channel coefficients.

3. ORDER OVERESTIMATION

So far, we have assumed that the orders of $h_q(l)$, L_q have been the same, equal to L and that we know (or have accurately estimated) L. In this section we will show that the proposed input diversity scheme can be shown to work with an upper bound $\overline{L} > L$ (or for different L_q's with an upper bound on $\max_q L_q$). The analysis here pertains to the noise free case, and to simplify the arguments, (a2) will be assumed. In the construction or estimation of a given matrix, to clarify if the true channel order L, or its upper bound $\overline{L}$ is used, (L) or $(\overline{L})$ will be associated with that matrix for clarity.

Suppose we know an upper bound $\overline{L} > L_q, \forall q$. Then we will design our transmitter precoder matrix $\mathbf{F}_0(\overline{L})$ to have $\overline{L}$ trailing zeros and choose $P = M + \overline{L}$. This implies that the last $(\overline{L} - L_q)$ rows of $\mathbf{C}_{xx}(\omega_q - \omega_{q_1})$ will be zero. In estimating $\mathbf{H}_q(\overline{L})$ one relies on the equation $\tilde{\mathbf{U}}^H\mathbf{H}_q(\overline{L}) = \mathbf{0}$, or equivalently,

$$\mathcal{R}(\mathbf{H}_q(\overline{L})) = \mathcal{R}\left(\begin{bmatrix}\mathbf{C}_{xx}^{(L)}(\omega_q - \omega_{q_1})\\ \mathbf{0}_{(\overline{L}-L)\times P}\end{bmatrix}\right). \tag{6}$$

Any solution that we get from (6) will be of the form

$$\hat{\mathbf{H}}_q(\overline{L}) = \begin{bmatrix} \hat{\mathbf{H}}_q(L) \\ 0_{(\overline{L}-L_q)\times M} \end{bmatrix}, \quad (7)$$

where $\hat{\mathbf{H}}_q(L)$ is estimated using $\mathcal{R}(\hat{\mathbf{H}}_q(L)) = \mathcal{R}(\mathbf{C}_{xx}^{(L)}(\omega_q - \omega_{q_1}))$. But this would be the precise equation that would be used to estimate $\hat{\mathbf{H}}_q(L)$ if we had the true order L_q. Note the following remarks:

Remark 1: Since this is a generalization of the TZ method proposed in [5], it is not difficult to see that the filterbank precoding scheme in [5] that assumes a TI channel is also immune to channel order overestimation. This fact was observed in [5] in the simulations, but was not shown analytically.

Remark 2: Even though we have shown here that usage of any $\overline{L}$ that satisfies $\overline{L} > \max_q L_q$ guarantees identifiability, since the amount of redundancy introduced is $\overline{L}$, it is important to choose $\overline{L}$ as close to $\max_q L_q$ as possible in order not to reduce the information rate more than necessary.

4. SYMBOL RECOVERY

In this section we summarize several symbol recovery techniques that could be applied after the channel has been identified. After reviewing a method that relies on parametrizing the null space of a channel matrix, we will go over the zero-forcing and MMSE alternatives.

Having estimated $\{\mathbf{H}_q\}_{q=1}^{Q}$, and having designed $\mathbf{F}$, we let $\mathcal{H}_F := [\mathbf{H}_1\Delta_1\mathbf{F}\ldots\mathbf{H}_Q\Delta_Q\mathbf{F}]$, $\mathbf{s}(n)$ be an $M \times 1$ block of the input, and $\mathbf{s}_b(n) := [e^{j\omega_1 n}\mathbf{s}(n)^T \ldots e^{j\omega_Q n}\mathbf{s}(n)^T]^T$. Then, (4) can be written as follows:

$$\mathbf{x}(n) = \mathcal{H}_F \mathbf{s}_b(n) \quad . \quad (8)$$

We would like to solve (8) for $\mathbf{s}_b(n)$, having knowledge of the output $\mathbf{x}(n)$ and $\mathcal{H}_F$. Since $\mathcal{H}_F$ is a fat matrix, the solution is not unique unless we use the structure of $\mathbf{s}_b(n)$. The set of solutions of (8) can be parametrized by the product $\mathbf{V}\boldsymbol{\theta}(n)$ where the first column of the $QM \times QM - P + 1$ matrix $\mathbf{V}$ is the particular solution $\mathcal{H}_F{}^{\dagger}\mathbf{x}(n)$, and the other columns span the null space of $\mathcal{H}_F$ (hereafter the dependence of $\boldsymbol{\theta}$ on n will be dropped for brevity). The task here is to identify the parameter $\boldsymbol{\theta}$ so that $\mathbf{V}\boldsymbol{\theta} = [e^{j\omega_1 n} \ldots e^{j\omega_Q n}]^T \otimes \hat{\mathbf{s}}(n)$, where $\otimes$ denotes the Kronecker product. Let $\mathbf{V}_i$ be $M \times QM - P + 1$ matrices that satisfy $\mathbf{V} = [\mathbf{V}_1^T \ldots \mathbf{V}_Q^T]^T$. Then, solving for $\boldsymbol{\theta}$ under the structural constraint on $\mathbf{s}_b(n)$ is equivalent to solving the system $\mathbf{V}_i e^{-j\omega_i n}\boldsymbol{\theta} = \mathbf{V}_{i+1}e^{-j\omega_{i+1} n}\boldsymbol{\theta}$, $i = 1, \ldots, Q-1$. After solving for $\boldsymbol{\theta}$, we can compute $\mathbf{V}\boldsymbol{\theta}$ to obtain $\mathbf{s}_b(n)$.

A Viterbi-like approach would be to use the finite alphabet property of the input sequence and for every n construct all possible $\mathbf{s}_b(n)$'s, and choose the one that comes closest to satisfying (8) in the least square sense. The computational complexity of this procedure is $\mathcal{O}(|\mathcal{C}|^M)$, where $|\mathcal{C}|$ is the size of the signal set.

Input recovery can also be established through zero-forcing or MMSE methods. For this purpose, consider the noisy version of (2):

$$\mathbf{x}(n) = \mathbf{H}(n)\mathbf{F}\mathbf{s}(n) + \mathbf{v}(n) .$$

The zero-forcing solution for this scheme is given by $\hat{\mathbf{s}}(n) = (\mathbf{H}(n)\mathbf{F})^{\dagger}\mathbf{x}(n)$. It is well known that zero-forcing schemes do not take the noise into account and suffer at moderate or low SNR's from the effects of noise. This motivates us to consider an equalizer matrix $\mathbf{G}(n)$ that minimizes the following:

$$\hat{\mathbf{G}}(n) = \arg\min_{\mathbf{G}(n)} E|\mathbf{G}(n)\mathbf{x}(n) - \mathbf{s}(n)|^2. \quad (9)$$

Using the orthogonality relationship, it can be shown that the solution to this problem is given by

$$\hat{\mathbf{G}}(n) = \mathbf{R}_{ss}\mathbf{F}^H\mathbf{H}^H(n)(\mathbf{R}_{vv} + \mathbf{H}(n)\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{H}^H(n))^{-1}.$$

The common problem in all input-recovery schemes presented here is that some sort of inversion has to take place for each n which is computationally expensive. Alleviation of this problem by exploiting the known structure of the variation of $\mathbf{H}(n)$ with n is part of our current research.

5. SIMULATIONS

In this section we illustrate the ideas discussed in this paper with computer simulations. In all plots the frequencies were $\omega_1 = 0, \omega_2 = 2\pi/5$ and $\omega_3 = 2\pi/13$, and the precoder matrix was $\mathbf{F} = \mathbf{I}$.

Figure 2 illustrates channel estimation algorithm coupled with the MMSE equalization algorithm with an eye diagram where $N = 1,000$ data were used with $SNR = 15$ dB for randomly generated channels with $(Q, M, L, P) = (3, 5, 3, 8)$. In Figure 3 we illustrate how channel order overestimation does not change, and actually slightly reduces the symbol mean square error (MSE) with usage of bigger block lengths $P = M + \overline{L} = M + L + i, i = 0, \ldots, 3$ at low SNR's. In Figure 3 $N = 5,000$ data were used. In Figure 4, the three input recovery schemes are compared in terms of input MSE. It is observed that the MMSE is the best scheme for all SNR's, and that the ZF scheme as well as the

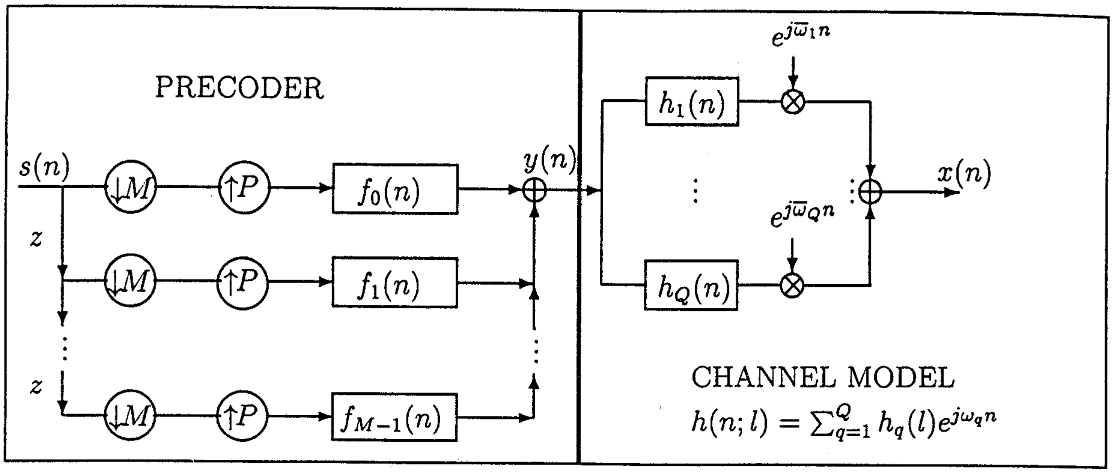

Figure 1. Precoder and TV Channel Model

method that parametrizes the nullspace (parnull) approximates the MMSE closely at high SNR's. In Figure 4, $N = 5,000$ data were used, which are not many considering the implicit modeling of the time variation. Figure 5 illustrates how at SNR=30 dB increasing the data length yields reduced channel root mean square error, which shows the superiority of this averaging-based method to its deterministic counterparts. Finally, in Figure 6, we used the MMSE equalizer with $N = 1000$ data points to obtain a symbol error rate plot. The plot was obtained by counting errors across 500,000 equalized symbols for each value of SNR.

APPENDIX

In this appendix, we will show identifiability without assuming (a2) lifting any restrictive assumptions on the frequencies $\{\omega_q\}$. In general, without assuming (a2), the cyclic correlation matrix is given by

$$\mathbf{C}_{xx}(\omega_{q_2} - \omega_{q_1}) = \sum \mathbf{H}_{q_4}\mathbf{\Delta}_{q_4}\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_{q_3}^H\mathbf{H}_{q_3}^H, \quad (10)$$

where the sum is over q_4, q_3 that satisfy $\omega_{q_4} - \omega_{q_3} = \omega_{q_2} - \omega_{q_1}$.

It is not difficult to see that we can obtain estimates of $\mathbf{H}_Q$ and $\mathbf{H}_1$ using the fact that $\mathbf{C}_{xx}(\omega_Q - \omega_1) = \mathbf{H}_Q\mathbf{\Delta}_Q\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_1^H\mathbf{H}_1^H$, which will enable estimation of $\mathbf{H}_Q$ and $\mathbf{H}_1$ using the output diversity method of Section 2. Let us now consider $\mathbf{C}_{xx}(\omega_Q - \omega_2)$. If ω_Q and ω_2 are the only pair that give rise to $\omega_Q - \omega_2$, then, we can still use the output diversity method to get $\mathbf{H}_2$. If not, the only other pair that can give rise to this difference is ω_{Q-1} and ω_1. In this case we have,

$$\begin{aligned}\mathbf{C}_{xx}(\omega_Q - \omega_2) = \mathbf{H}_Q\mathbf{\Delta}_Q\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_2\mathbf{H}_2^H \\ +\mathbf{H}_{Q-1}\mathbf{\Delta}_{Q-1}\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_1\mathbf{H}_1^H.\end{aligned} \quad (11)$$

Suppose, given $\mathbf{C}_{xx}(\omega_Q - \omega_2)$, $\mathbf{H}_Q$ and $\mathbf{H}_1$, we know how to estimate $\mathbf{H}_{Q-1}$ and $\mathbf{H}_2$. Then, according to (10), $\mathbf{C}_{xx}(\omega_Q - \omega_3)$ can contain at most 3 terms, one of which will be $\mathbf{H}_{Q-1}\mathbf{\Delta}_{Q-1}\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_2\mathbf{H}_2^H$, which has been estimated, and can be subtracted from $\mathbf{C}_{xx}(\omega_Q - \omega_3)$ which will make the problem identical to solving (11) for $\mathbf{H}_{Q-1}$ and $\mathbf{H}_2$ given $\mathbf{C}_{xx}(\omega_Q - \omega_2)$, $\mathbf{H}_Q$ and $\mathbf{H}_1$. The argument is similar for a general $\mathbf{C}_{xx}(\omega_Q - \omega_q)$ for $q > 3$. So, without loss of generality, to establish identifiability, given $\mathbf{C}_{xx}(\omega_Q - \omega_2)$, we need to be able to solve for $\mathbf{H}_{Q-1}$ and $\mathbf{H}_2$ in (11). Since we know $\mathbf{H}_1$, we also know the L Vandermonde vectors $\mathbf{u}_1, \ldots, \mathbf{u}_L$ in its nullspace. Multiplying (11) by $\mathbf{u}_l$ and using the structure of $\mathbf{H}_2^H$, we obtain

$$\mathbf{C}_{xx}(\omega_Q - \omega_2)\mathbf{u}_l = \mathbf{H}_Q\mathbf{\Delta}_Q\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_2\mathbf{U}_l\mathbf{h}_2, \quad (12)$$

$l = 1, \ldots, L$, where, because $\mathbf{u}_l$ is a Vandermonde vector, $\mathbf{U}_l$ is an $M \times (L+1)$ rank 1 matrix obtained by $\mathbf{U}_l = \mathbf{u}(1:M)\mathbf{u}_l^T(1:L+1)$ [1]. So, in order to solve for $\mathbf{h}_2$, we need to use all L equations in (12). Let $\mathcal{U} := [(\mathbf{H}_Q\mathbf{\Delta}_Q\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_2^H\mathbf{U}_1)^T \ldots (\mathbf{H}_Q\mathbf{\Delta}_Q\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^H\mathbf{\Delta}_2^H\mathbf{U}_L)^T]^T$, and $\mathbf{u} := [(\mathbf{C}_{xx}(\omega_Q - \omega_2)\mathbf{u}_1)^T \ldots (\mathbf{C}_{xx}(\omega_Q - \omega_2)\mathbf{u}_L)^T]^T$. Then, (12) becomes

$$\mathcal{U}\ \mathbf{h}_2 = \mathbf{u}\,. \quad (13)$$

It is not difficult to show that the $PL \times (L+1)$ matrix $\mathcal{U}$ has rank L. This means (13) can be solved for $\mathbf{h}_2$ up to a scale ambiguity.

To obtain $\mathbf{H}_{Q-1}$, one can work with the transpose of $\mathbf{C}_{xx}(\omega_Q - \omega_2)$ and apply the same procedure.

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[1]Here, we use the MATLAB notation $\mathbf{u}(i:j)$ to denote the vector formed by the i through j elements of $\mathbf{u}$

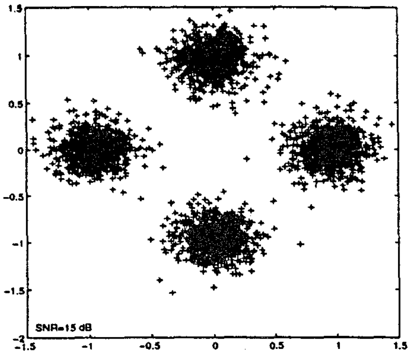

Figure 2. Eye Diagram

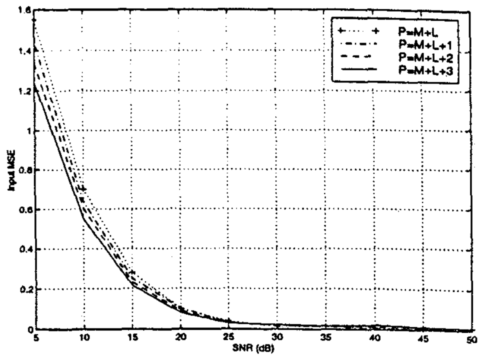

Figure 3. Order overestimation

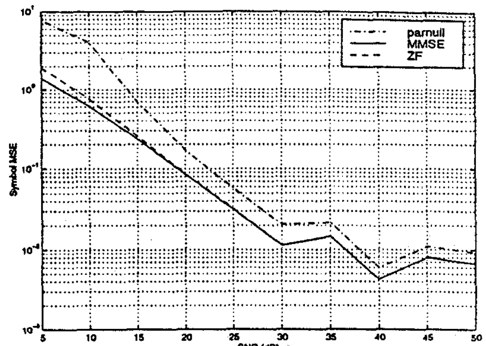

Figure 4. Different Equalization Schemes

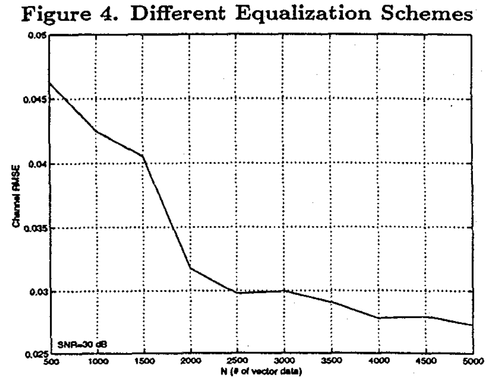

Figure 5. Improvement with data length

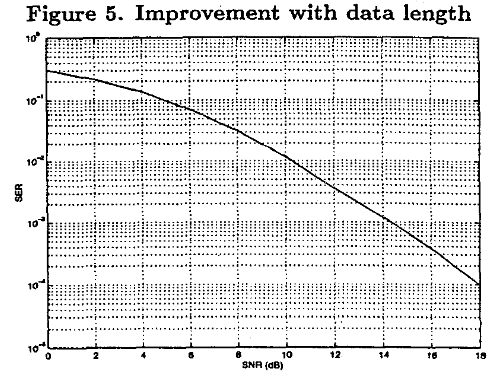

Figure 6. SER for the MMSE equalizer

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