Distributed Approaches to Control and Coordination of Distributed Energy Resources

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Computing, Control, and Signal Processing Challenges in Future Power Systems
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Outline

1. Introduction
2. Constrained Fair-Splitting Dispatch Problem
3. Robustified Ratio-Consensus Algorithm
4. Constrained Optimal Dispatch Problem
5. Distributed Generation Control in Small-Footprint Power Systems
6. Concluding Remarks
Motivation

- Distributed Energy Resources (DERs) can potentially be utilized to provide ancillary services
  - Reactive power support for voltage control in distribution networks
  - Active power provision for up and down regulation

- DERs play a key role in small-footprint power systems [microgrids]
  - Highly reliable electricity supply at the household level
  - Disaster-relief installations
  - Military installations
Control and Coordination of DERs

Potential solution: centralized architecture (where each DER is commanded by a single decision maker)
- Requires communication between the central controller and each DER
- Requires up-to-date knowledge of DER availability

Alternative approach: employ a distributed architecture for control and coordination of DERs, with several potential advantages:
- Ability to handle incomplete global knowledge of DER availability
- Potential resiliency to faults and/or unpredictable DER behavior

![Centralized vs. Distributed Architecture Diagram]

Centralized

Distributed
Overview of the Presentation

- We discuss iterative distributed algorithms for DER control that perform, at each iteration, simple computations relying on local information
  - This information is acquired via communication with neighboring DERs
- In general, the goal is for the DERs to collectively provide a certain amount of a resource
  - This resource can be either active or reactive power
- We address two different problem settings:
  - There are constraints on DER upper and lower capacity, but no cost associated to each DER
    - Constrained fair-splitting dispatch problem
  - There are constraints on DER upper and lower capacity, and quadratic cost associated to each DER
    - Constrained optimal dispatch problem
- We illustrate the application of the proposed algorithms to the problem of generation control in a small-footprint power system
Generation Control in Small-Footprint Power Systems

Physical Layer

Generator 1

\[
d\frac{d}{dt} \begin{bmatrix} \delta_1 \\ \omega_1 \\ E_{f1} \\ P_{m1} \\ Q_{g1} \end{bmatrix} = f(\begin{bmatrix} \delta_1, \omega_1, E_{f1}, P_{m1} \\ \delta_2, \omega_2, E_{f2}, P_{m2} \\ \delta_n, \omega_n, E_{fn}, P_{mn} \end{bmatrix}^T, [\theta_1, V_1]^T, E'_1, u'_1 + \Delta u'_1)
\]

\[
[\begin{bmatrix} P_{m1} \\ Q_{g1} \end{bmatrix}] = \frac{p_1(\delta_1, \theta_1, V_1)}{q_1(\delta_1, \theta_1, V_1)}
\]

Generator 2

\[
d\frac{d}{dt} \begin{bmatrix} \delta_2 \\ \omega_2 \\ E_{f2} \\ P_{m2} \\ Q_{g2} \end{bmatrix} = f(\begin{bmatrix} \delta_2, \omega_2, E_{f2}, P_{m2} \\ \delta_2, \omega_2, E_{f2}, P_{m2} \\ \delta_n, \omega_n, E_{fn}, P_{mn} \end{bmatrix}^T, [\theta_2, V_2]^T, E'_2, u'_2 + \Delta u'_2)
\]

\[
[\begin{bmatrix} P_{m2} \\ Q_{g2} \end{bmatrix}] = \frac{p_2(\delta_2, \theta_2, V_2)}{q_2(\delta_2, \theta_2, V_2)}
\]

Generator n

\[
d\frac{d}{dt} \begin{bmatrix} \delta_n \\ \omega_n \\ E_{fn} \\ P_{mn} \\ Q_{gn} \end{bmatrix} = f(\begin{bmatrix} \delta_n, \omega_n, E_{fn}, P_{mn} \\ \delta_n, \omega_n, E_{fn}, P_{mn} \\ \delta_n, \omega_n, E_{fn}, P_{mn} \end{bmatrix}^T, [\theta_n, V_n]^T, E'_n, u'_n + \Delta u'_n)
\]

\[
[\begin{bmatrix} P_{mn} \\ Q_{gn} \end{bmatrix}] = \frac{p_n(\delta_n, \theta_n, V_n)}{q_n(\delta_n, \theta_n, V_n)}
\]

\[
\frac{dx}{dt} = f(x, y, u^r + \Delta u^r), \quad 0 = g(x, y)
\]
Cyber Layer Model

1. Each DER has a processor that can perform simple operations.
2. Exchange of information between DERs can be described by a directed graph $G = \{V, E\}$:
   - $V = \{1, 2, \ldots, n\}$ is the node set (each node corresponds to a DER).
   - $E \subseteq V \times V$ is the set of directed edges, where $(j, i) \in E$ if node $j$ can receive information from node $i$ (asymmetric information structure).
   - We allow self-loops for all nodes in $G$, i.e., $(j, j) \in E$ for all $j \in V$.
3. Nodes that can transmit information to node $j$ are said to be in-neighbors of node $j$ and are represented by the set
   \[ N_j^- = \{i \in V : (j, i) \in E\}; \]
   $D_j^- = |N_j^-|$ denotes the in-degree of $j$.
4. Nodes that can receive information from node $j$ are said to be out-neighbors of node $j$ and are represented by the set
   \[ N_j^+ = \{l \in V : (l, j) \in E\}; \]
   $D_j^+ = |N_j^+|$ denotes the out-degree of $j$. 
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Problem Setup

1. Denote by $X$ the total amount of resource to be provided, and assume it is known to an entity outside $\mathcal{V}$, referred to as the leader.
   - The leader can communicate with $m \geq 1$ nodes.

2. Denote by $x_j$ the amount of resource provided by DER $j$ without specifying whether it is active or reactive power.

3. Let $0 < x_j < \bar{x}_j$, for $j = 1, 2, \ldots, n$, denote the minimum ($x_j$) and maximum ($\bar{x}_j$) capacity limits on $x_j$.

4. Assume that $\sum_{j=1}^{n} x_j \leq X \leq \sum_{j=1}^{n} \bar{x}_j$ (ensures feasibility).

5. The total amount of resource $X$ can be collectively provided by having each node $j$ contribute

   $$x_j = x_j + \frac{X - \sum_{l=1}^{n} x_l}{\sum_{l=1}^{n} (\bar{x}_l - x_l)} (\bar{x}_j - x_j)$$

   (fair splitting solution)

6. The goal is for each node $j$ to compute $x_j$ without global knowledge.
Constrained Fair-Splitting Dispatch Algorithm

1. Each node maintains two internal state variables $y_j$ and $z_j$, which are updated at each $k \geq 0$ as follows:

\[ y_j[k + 1] = \sum_{i \in N_j^-} \frac{1}{D_i^-} y_i[k] \]  \hspace{1cm} (1)

\[ z_j[k + 1] = \sum_{i \in N_j^-} \frac{1}{D_i^-} z_i[k] \]  \hspace{1cm} (2)

2. $y_j[0] = \frac{X}{m} - x_j$ if leader communicates with $j$, and $y_j[0] = -x_j$ otherwise; and $z_j[0] = \bar{x}_j - x_j$.

3. At each iteration step, each node $j$ computes

\[ x_j[k] = x_j + \frac{y_j[k]}{z_j[k]} (\bar{x}_j - x_j) \]

4. Can be shown that [D-G, Hadjicostis, '10, '11]

\[ x_j = \lim_{k \to \infty} x_j[k] = x_j + \frac{X - \sum_{l=1}^{n} x_l}{\sum_{l=1}^{n}(\bar{x}_l - x_l)} (\bar{x}_j - x_j) \]
4-Node Example

- \( X = 1 \)
- Minimum and maximum capacities:

\[
\underline{x} = [0.1, 0.05, 0.12, 0]^T
\]
\[
\overline{x} = [0.35, 0.3, 0.26, 0.24]^T
\]

Collective dynamics of both iterations:

\[
y[k + 1] = P y[k]
\]
\[
z[k + 1] = P z[k]
\]

where

\[
y[k] = [y_1[k], y_2[k], y_3[k], y_4[k]]^T
\]
\[
z[k] = [z_1[k], z_2[k], z_3[k], z_4[k]]^T
\]

\[
P = \begin{bmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\
\frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{2} \\
\frac{1}{3} & 0 & \frac{1}{3} & 0 \\
0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{2}
\end{bmatrix}
\]

- \( P \) is column stochastic and primitive as long as the underlying directed graph is strongly connected
Resource provided by each node given by

\[ x = [x_1, x_2, x_3, x_4]^T = [0.3074, 0.2574, 0.2361, 0.1991]^T \]

Total resource provided equals the demand: \[ \sum_{i=1}^{4} x_i = 1 = X \]
A Generalization: Ratio Consensus

Each node maintains two internal state variables \( y_j \) and \( z_j \), which are updated at each \( k \geq 0 \) as follows:

\[
y_j[k + 1] = \sum_{i \in \mathcal{N}_j^-} \frac{1}{D_i^+} y_i[k] \quad \text{(3)}
\]

\[
z_j[k + 1] = \sum_{i \in \mathcal{N}_j^-} \frac{1}{D_i^+} z_i[k] \quad \text{(4)}
\]

\[
y[k + 1] = P y[k]
\]

\[
z[k + 1] = P z[k]
\]

Lemma: Let \( y_j[k], \forall j \), be the result of iteration (3) for some \( y_j[0], \forall j \), and \( z_j[k], \forall j \), be the result of iteration (4) for some \( z_j[0], \forall j \). Then,

\[
\gamma := \lim_{k \to \infty} \frac{y_j[k]}{z_j[k]} = \frac{\sum_{l=1}^{n} y_l[0]}{\sum_{l=1}^{n} z_l[0]}
\]

Ratio-consensus is a primitive for solving the constrained optimal dispatch problem in a distributed fashion.
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At each time step, there is a nonzero probability that a communication link may drop packets

- Link availability is independent between links and between time steps

Resource provided by each node given by

\[ x = [0.2537, 0.2037, 0.2061, 0.1476]^T \]

Total resource provided does not meet demand:

\[ \sum_{i=1}^{4} x_i = 0.811 \neq X = 1 \]
Robustified Ratio Consensus [D-G, Hadjicostis, Vaidya, ‘12]

- For each of the two (numerator and denominator) iterations, node $j$ calculates several quantities of interest, which we refer to as:
  - Internal state
  - Total broadcasted mass to each out-neighbor $l \in \mathcal{N}_j^+$
  - Total received mass from each in-neighbor $i \in \mathcal{N}_j^-$

- At time instant $k$, the total broadcasted mass of $j$ is the sum up to step $k$ of the weighted value of node $j$ internal state.

- Node $j$ updates the value of the received mass from node $i$ to be either the total broadcasted mass sent by node $i$ if the link from $i$ to $j$ is available, or the most recently received mass value from node $i$.

- Node $j$ updates its internal state to be a linear combination of its own previous internal state value and the sum (over all its in-neighbors) of the differences between the two most recently received mass values.
Numerator Iteration

- As before, let $y_j[k]$ be node $j$’s internal state at time instant $k$
- Let $\mu_{lj}[k]$ denote the mass broadcasted from node $j$ to each of its out-neighbors $l$ (this is the same for each out-neighbor $l$ of node $j$, i.e., for each $l \in \mathcal{N}_j^+$)
- $\nu_{ji}[k]$ denote the mass received at node $j$ from node $i \in \mathcal{N}_j^-$. Then, for all $k \geq 0$,

\[
y_j[k + 1] = \frac{1}{D_j^+} y_j[k] + \sum_{i \in \mathcal{N}_j^-} (\nu_{ji}[k] - \nu_{ji}[k - 1])
\]

\[
\mu_{lj}[k] = \mu_{lj}[k - 1] + \frac{1}{D_j^+} y_j[k] = \sum_{t=0}^{k} \frac{1}{D_j^+} y_j[t]
\]

where

\[
\nu_{ji}[k] = \begin{cases} 
\mu_{ji}[k], & \text{if link } (j, i) \text{ is available at instant } k \geq 0 \\
\nu_{ji}[k - 1], & \text{if link } (j, i) \text{ is not available at instant } k \geq 0 
\end{cases}
\]
Convergence of Robustified Algorithm

- Despite the presence of unreliable communication links, we argue that with the proposed robustified algorithm, nodes can asymptotically obtain the exact solution as

\[
x_j = \lim_{k \to \infty} \left( x_j + \frac{y_j[k]}{z_j[k]}(\bar{x}_j - x_j) \right) = x_j + \frac{X - \sum_{l=1}^{n} x_l}{\sum_{l=1}^{n} (\bar{x}_l - x_l)} (\bar{x}_j - x_j)
\]

calculated whenever \( z_j[k] \) is large enough.

- In particular, we have shown that [D-G, Hadjicostis, Vaidya, ‘12]:
  - \( \lim_{k \to \infty} y_j[k] = \gamma \lim_{k \to \infty} z_j[k] \), for \( \gamma = \frac{X - \sum_{l=1}^{n} x_l}{\sum_{l=1}^{n} (\bar{x}_l - x_l)} \), almost surely
  - \( z_j[k] > 0 \) occurs infinitely often

- Two different proof techniques:
  - Analysis of moment dynamics
  - Analysis of products of irreducible, aperiodic, stochastic matrices
6-Node Example

\[ y[k] \text{ vs } k \]

\[ z[k] \text{ vs } k \]

\[ x[k] \text{ vs } k \]

\[
X = 1
\]

\begin{center}
\begin{tikzpicture}
  \node[draw,circle] (1) at (0,0) {1};
  \node[draw,circle] (2) at (1,0) {2};
  \node[draw,circle] (3) at (2,0) {3};
  \node[draw,circle] (4) at (3,0) {4};
  \node[draw,circle] (5) at (4,0) {5};
  \node[draw,circle] (6) at (5,0) {6};
  \path[->] (1) edge[bend left] (2);
  \path[->] (2) edge[bend left] (1);
  \path[->] (2) edge[bend left] (3);
  \path[->] (3) edge[bend left] (2);
  \path[->] (3) edge[bend left] (4);
  \path[->] (4) edge[bend left] (3);
  \path[->] (4) edge[bend left] (5);
  \path[->] (5) edge[bend left] (4);
  \path[->] (5) edge[bend left] (6);
  \path[->] (6) edge[bend left] (5);
\end{tikzpicture}
\end{center}
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1. Denote by $X$ the total amount of resource to be provided and assume it is known to an entity outside $\mathcal{V}$, referred to as the leader.
2. Denote by $x_j$ the amount of resource provided by DER $j$.
3. Let $\underline{x}_j$ ($\bar{x}_j$) denote the minimum (maximum) capacity limit on $x_j$.
4. Capacity-constrained optimal dispatch [assuming quadratic costs]:

$$x^* = \text{arg min} \sum_{j=1}^{n} \frac{(x_j - \alpha_j)^2}{2\beta_j}$$

subject to

$$\sum_{j=1}^{n} x_j = X$$

$$0 < \underline{x}_j \leq x_j \leq \bar{x}_j, \forall j$$

(5)

where $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$; and $\alpha_j \leq 0$, and $\beta_j > 0$ are reals.

5. The goal is for each node $j$ to distributively compute $x_j^*$ without global knowledge.
The Lagrange Dual Problem

- The DER optimal dispatch problem in (5) is convex and has a separable structure.
- The Lagrange dual problem of (5) is given by

\[
\text{maximize } \lambda \in \mathbb{R}^+ \lambda X + \sum_{j=1}^{n} f_j(\lambda)
\]  

(6)

where \( f_j(\lambda), \ \forall \ j, \) are

\[
f_j(\lambda) = \begin{cases} 
\frac{(x_j - \alpha_j)^2}{2\beta_j} - \lambda x_j, & 0 \leq \lambda < \lambda_{2j-1} \\
-\lambda (\alpha_j + \lambda \frac{\beta_j}{2}), & \lambda_{2j-1} \leq \lambda \leq \lambda_{2j} \\
\frac{(x_j - \alpha_j)^2}{2\beta_j} - \lambda \bar{x}_j, & \lambda_{2j} < \lambda 
\end{cases}
\]  

(7)

with \( \lambda_{2j-1} = \frac{x_j - \alpha_j}{\beta_j} > 0 \) and \( \lambda_{2j} = \frac{\bar{x}_j - \alpha_j}{\beta_j} > 0 \)

- The Lagrange dual problem provides the optimal solution to the primal problem in (5).
A Centralized Algorithm [Madrigal, Quintana, ’00]

- All the $f_j$’s are continuously differentiable; therefore, the cost function in (6) is also continuously differentiable.
- Thus, if (6) is feasible, the optimal solution $\lambda^*$ must satisfy

$$X - \sum_{j=1}^{n} g_j(\lambda^*) = 0 \quad (8)$$

where $g_j(\lambda) = -\frac{df_j(\lambda)}{d\lambda}$, i.e.,

$$g_j(\lambda) = \begin{cases} 
  x_j, & 0 \leq \lambda < \lambda_{2j-1} \\
  \alpha_j + \lambda \beta_j, & \lambda_{2j-1} \leq \lambda \leq \lambda_{2j} \\
  \overline{x}_j, & \lambda_{2j} < \lambda 
\end{cases} \quad (9)$$

- Obtaining the $\lambda^*$ that satisfies (8) is equivalent to finding the point at which the functions $g(\lambda) := \sum_{j=1}^{n} g_j(\lambda)$ and $h(\lambda) = X$ intersect.
Graphical Interpretation for $n = 3$

The values of $\lambda$ for which the slope of $g(\lambda)$ changes correspond to values of $\lambda$ for which the slope of the individual $g_j(\lambda)$'s changes.

Key is to find $\lambda^-$ and $\lambda^+$ and then interpolate to find $\lambda^*$:

$$X = 220$$
$$\lambda^- = 203$$
$$\lambda^+ = 613$$

$$\lambda^* = \lambda^+ - \left[g(\lambda^+) - X\right] \frac{\lambda^+ - \lambda^-}{g(\lambda^+) - g(\lambda^-)} = 425$$  \hspace{1cm} (10)
Developing a Distributed Solution

- While (10) provides the unique global solution, it requires a centralized decision maker with knowledge of all the individual $g_j(\lambda)$’s and $X$
- Rewrite (10) as follows

$$\lambda^* = \lambda^+ - \left[ \frac{g(\lambda^+)}{X} - 1 \right] \frac{\lambda^+ - \lambda^-}{g(\lambda^+) - g(\lambda^-)} \quad (11)$$

- By inspection of (11) we note that, for each node $j$ to be able to obtain $\lambda^*$, it needs to learn the following quantities:
  - $\lambda^-$
  - $\lambda^+$
  - $\frac{g(\lambda^+)}{X} = \sum_{l=1}^{n} g_l(\lambda^+)$
  - $\frac{g(\lambda^-)}{X} = \sum_{l=1}^{n} g_l(\lambda^-)$

- A message-passing protocol together with ratio-consensus provides a distributed solution to the problem [D-G, Cady, Hadjicostis, ’12]
Evolution of Distributed Optimal Dispatch for 3-nodes

\[
g(\lambda^+)/X, \quad g(\lambda^-)/X
\]

Graph of 3-node network

Graph of 3-node network
# Large Power System Generation Control Architecture

<table>
<thead>
<tr>
<th>Level</th>
<th>Control Type</th>
<th>Time</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>Governor Control</td>
<td>≈ 0.1 s</td>
<td>Balance generation and demand</td>
</tr>
<tr>
<td>Secondary</td>
<td>Automatic Generation Control</td>
<td>≈ 2 - 5 s</td>
<td>Regulate Frequency, ACE, Participation Factors</td>
</tr>
<tr>
<td>Tertiary</td>
<td>Optimal Dispatch</td>
<td>≈ 5 min</td>
<td>Minimize overall generation cost</td>
</tr>
</tbody>
</table>

- **AGC and optimal dispatch are implemented over a centralized architecture**
  - Complete knowledge of the devices in the system
  - Communication link from the controller to each generator

Traditionally Centralized

Decentralized
Decentralizing the Top Two Layers

- Control objectives are similar to large power systems
  - Balance generation and demand
  - Regulate frequency
  - Coordinate generators to maximize overall efficiency

- Without a centralized controller, the system is more adaptable
  - New generators can be added or removed easily
  - More resilient to individual failures
  - Requires less planning to begin operation

- Utilize communication links between DERs and computations performed by each local controller

- Distributively determine the output of each DER to regulate frequency and minimize overall cost (optimal dispatch)
\[ \frac{dx}{dt} = f(x, y, u^r + \Delta u^r), \quad 0 = g(x, y) \]
Frequency Regulation and Optimal Dispatch Timeline

- Power system dynamics: \( \frac{dx}{dt} = f(x, y, u^r + \Delta u^r), \ 0 = g(x, y) \)
- While frequency is not at 60 Hz
  - Fair-splitting (FS) algorithm is triggered at instant \( r < s \)
  - After each execution of the FS algorithm is completed, each generator computes \( \Delta u^r_i \) and updates its command to \( u^{r+1}_i = u^r_i + \Delta u^r_i \)
- When frequency returns to 60 Hz (instant \( r = p \)):
  - Optimal dispatch (OD) algorithm is triggered and each generator computes \( \Delta u^s_i \) and updates its command to \( u^{s+1}_i = u^s_i + \Delta u^s_i \)
  - After OD has been executed, the procedure repeats
Ongoing Work: Experimental Testbed

Physical layer hardware

Cyber layer hardware
Two load increases at time $t = 100$ s and $t = 400$ s, and a subsequent generator re-dispatch at time $t = 775$ s.

First load change at bus 3 from $313$ W to $396$ W.

Second load change at bus 2 from $396$ W to $465$ W.
Two load increases at time $t = 100$ s and $t = 300$ s, and a subsequent generator re-dispatch at time $t = 500$ s

First load change at bus 6 from 390 W to 365 W

Second load change at bus 2 from 365 W to 352 W
Addition of Spinning Reserve

Bus 4

Bus 6

Bus 5

Bus 2

Bus 3

Bus 1

G_2

L_1

P_6

G_3

G_4

L_2

P_3

G_1

G_2

G_3

P_6

P_3

L_2

L_1

Bus 1

Bus 2

Bus 3

Bus 4

Bus 5

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Response with Spinning Reserve

- DERs $G_1$, $G_2$, and $G_3$ reach their maximum output at $t \approx 275$ s
- DER $G_4$ obtains a value of $\psi^r > 1$ and determines demand $> \text{capacity}$
- After reserve adjusts maximum output (at $t \approx 360$ s) speed is increased to within bounds
- DERs are optimally re-dispatched at $t \approx 650$ s
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Proposed distributed control algorithms for:

- Constrained fair-splitting dispatch problem
- Constrained optimal dispatch problem

Strategies allow DERs in distribution systems to become active and reactive power support resources, with minimal infrastructure support.

Ongoing work:

- Demonstrate how these algorithms can be used to provide reactive power support for voltage control in distribution networks
- Integrate other generation resources