Earth Simulator Computer

Magnetohydrodynamic Simulations

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MINNEAPOLIS, MINNESOTA
Digital Technology Center
Contents

• Earth Simulator (ES) and GeoFEM Project
  - Introduction of Earth Simulator project
  - Architecture of ES
  - GeoFEM’s Approach for ES
  - Geodynamo simulation using GeoFEM

• Results

• Methods

• Introduction

• Conclusions and future study
Objectives

- Forecast earth dynamics by "Virtual Earth"
- Enhance information science & technology
- Development of parallel computer "Earth Simulator"
- Advanced software

MEXT (STA), 1997-2001 F/Y
Earth Simulator Project
Inside of the Earth Simulator Building

- max 16 Gbytes/sec x 2
- single-stage crossbar network
- 640 nodes (8 vector processors / node)

(http://www.es.jamstec.go.jp)
Earth Simulator (http://www.es.jamstec.go.jp)

Linpack 3.5 Tflips (March, 2002)

Peak speed : 40 Tflips
Total main memory : 10 Tbytes
<table>
<thead>
<tr>
<th>Year</th>
<th>Nmax</th>
<th>Processors</th>
<th>Nhalf</th>
<th>Nmax</th>
<th>Nhalf</th>
<th>Rpeak (GFlops)</th>
<th>Rmax (GFlops)</th>
<th>Year</th>
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<td>2001</td>
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<td>179000</td>
<td>518096</td>
<td>12288</td>
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<td>2000</td>
<td>2000</td>
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<tr>
<td>366240</td>
<td>1075200</td>
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<td>1075200</td>
<td>366240</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Earth-Simulator

ASC1 Red

ES45/1 GHz

AlphaServer SC

MZH 16 way

Power3 375

SP

ASC1 Red

ES45/1 GHz

AlphaServer SC

MZH 375

SP

ASC1 White, SP

ASC1 Red

ES45/1 GHz

AlphaServer SC

Earh-Simulator

Rmax (GFlops)
Objectives

- Forecast earth dynamics by "Virtual Earth"
- Enhance information science & technology

Development of

- Parallel computer "Earth Simulator"
- Advanced software

Target of GeoFEM

Solid earth : crust-mantle-core dynamics
Fluid earth : ocean-atmosphere dynamics

MEXT (STA), 1997-2001 F/Y
Global scale
\(10^4\) km, \(10^8\) s
- Mantle/Core Dynamics and Interaction
- Interior Earth Structure
- Global scale (10 km, 10^-2 s)

Regional scale (10^3 km, 10^4-10^8 y)
- Quasi-Static Earthquake Generation Cycle
- Dynamic Rupture
- GPS Tectonic Data Assimilation

Local scale (10^-1, 10^2 s)
- Earthquake Generation
- Seismic Wave Propagation
- 3 targets for the simulations - What is “Solid Earth”?

Fluid dynamics is required for understanding
3 targets for the simulations

- Mantle-Core Dynamics & Plumes
  Final Target: $10^9$ nodes (10km mesh)

- Crustal movements & tectonic deformation
  Final Target: $10^9$ nodes (1km mesh)

- Seismic wave generation & propagation
  Final Target: $10^{11}$ nodes (20m mesh)
Parallel Finite Element Simulator for Solid Earth

GeoFEM

* Research Organization for Information Science and Technology

sponsored by Ministry of Education, Culture, Sports, Science and Technology

Univ. of Tokyo, Ochanomizu Univ. and RIST
Parallel Efficiency (Thermal Fluid)

- Estimated time from 4000 steps by Alpha cluster in HPCEC, Compaq Corp. (USA)

Graphs showing iteration counts, elapsed time for solving the Poisson equation, and elapsed time for time evolution. Colors represent different meshes: Red (Mesh 1a), Yellow (Mesh 1b), Green (Mesh 1c), Purple (Mesh 1d).
Re-Ordering Technique for Vector/Parallel Architectures (K. Nakajima)

- Cyclic DDS for SMP unit
- DDS re-ordering: different ordering for L/U computation
- CMC (Cyclic Multicolor) re-ordering
- RCM (Reverse Cutihl-Mckee) re-ordering
  and Maruyama (NEC)

• Cyclic DDS (RCM+CMC) Re-Ordering (Doi, Washio, Osoda)

- Traditional remedy -> RCM: not efficient for SMP parallel
- Localized operation for inter node/partitioning operation through MPI
- Difficult for vectorization/parallelization
  for forward/backward substitution process
- ILU/IC type preconditioning requires global data dependency
 
ILU/IC type preconditioning requires global data dependency.
Programming Models for Earth Simulator

Hybrid Parallel Programming Models for Earth Simulator
3 levels of Optimization
- Intra Processor: Vectorization
- Intra Node: OpenMP Parallelization for each domain
- Inter Node: MPI with domain decomposition

Toward Earth Simulator
Parallel/Vector Performance (NO SMP) (K. Nakajima)

- NEC SX-4 (JAERI/CCSE): 1.024 PE's, 300 GFlops, 12 PEs, 970 Mflops (~50% of peak speed)
- Hitachi SR2201 (U.Tokyo): 1.024 PE's, 300 GFlops, 1024 PEs, 300 GFlops, peak

Pseudo vector: distributed memory architecture
We have only this results on ES...

Test on 1 node of ES (Peak: 8x8=64 GFLOPS)

(K. Nakajima)
<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Solver</th>
<th>MFLOPS/PE</th>
<th>Elapsed time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAL-SK-2-F-F-SMP</td>
<td>SOLVER - VCG33</td>
<td>198.42</td>
<td>301.65</td>
</tr>
<tr>
<td></td>
<td>FEM-SK-VELICO</td>
<td>45.86</td>
<td>6.17</td>
</tr>
<tr>
<td></td>
<td>FEM-SK-VECT-P</td>
<td>15.44</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>FEM-SK-LORENTZ</td>
<td>7.36</td>
<td>27.42</td>
</tr>
<tr>
<td></td>
<td>FEM-SK-DIFFUSION</td>
<td>10.96</td>
<td>10.96</td>
</tr>
<tr>
<td></td>
<td>FEM-DIV-VELO</td>
<td>7.65</td>
<td>34.06</td>
</tr>
<tr>
<td></td>
<td>FEM-SK-INERTIA</td>
<td>14.16</td>
<td>14.16</td>
</tr>
<tr>
<td></td>
<td>FEM-SK-MULTI-PASS</td>
<td>301.65</td>
<td>301.65</td>
</tr>
<tr>
<td></td>
<td>SR8000</td>
<td>323.51</td>
<td>257.63</td>
</tr>
<tr>
<td></td>
<td>SX-4</td>
<td>35.31</td>
<td>35.31</td>
</tr>
<tr>
<td></td>
<td>SR8000</td>
<td>946.4</td>
<td>379.5</td>
</tr>
<tr>
<td></td>
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<td>1310.9</td>
<td>1242.8</td>
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<td>SX-4</td>
<td>1268.8</td>
<td>1238.6</td>
</tr>
<tr>
<td></td>
<td>SX-4</td>
<td>1333.2</td>
<td>1313.2</td>
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<tr>
<td></td>
<td>SX-4</td>
<td>1396.7</td>
<td>1313.2</td>
</tr>
</tbody>
</table>

**Performance on SX-4 and SR8000**

- 10 steps of Dynamo Benchmark case 1 (MHD)
- 130k elements
- SX-4 x 1PE (Max. 2GFlops)
- SR8000 x 1node (Max. 1GFlops x 8)
The present simulation will be performed on Earth Simulator (ES). On ES, $E = 10^{-7}$ ($T_a = 10^{14}$) is considered to be a target of the present MHD simulation. A simulation with $1 \times 10^8$ elements can be performed if 600 nodes of ES can be used. The size of mesh data for a snapshot of 330k element mesh is approximately 60MB, whereas for 1E8 mesh elements, it is around 8.9x10^7 (5.3x10^7) to 9.7x10^7 (5.5x10^7) bytes.

### Challenge for the Earth Simulator

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Num. of Element</th>
<th>Estimated Truncation Level</th>
<th>Nele on a Sphere</th>
<th>Nele for Radial</th>
<th>Nele for Element</th>
<th>Num. of Nele (Outer Core)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>240</td>
<td>221184</td>
<td>221 (240)</td>
<td>8.9x10^7</td>
<td>5.3x10^7</td>
<td>5.3x10^7</td>
</tr>
<tr>
<td>3</td>
<td>360</td>
<td>345600</td>
<td>345 (160)</td>
<td>160</td>
<td>9.7x10^7</td>
<td>5.5x10^7</td>
</tr>
<tr>
<td>4</td>
<td>340</td>
<td>345600</td>
<td>345 (160)</td>
<td>160</td>
<td>9.7x10^7</td>
<td>5.5x10^7</td>
</tr>
</tbody>
</table>

A simulation with $1 \times 10^8$ elements can be performed on ES. MHD simulation on ES is considered to be a limit of the present simulation.
PMR (Parallel Mesh Relocator) (S. Ezure)

The present simulation will be performed on Earth Simulator (ES).

- On ES, E=10^{-7} (Ta=10^{14}) is considered to be a target of the present MHD simulation.
- A simulation with 1x10^8 elements can be performed if 600 nodes of ES can be used.

Size of mesh data: 60MB/byte for 330k element mesh.

<table>
<thead>
<tr>
<th>Mesh 1</th>
<th>Mesh 2</th>
</tr>
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<tbody>
<tr>
<td>221184</td>
<td>8.9x10^7</td>
</tr>
<tr>
<td>240</td>
<td>(5.3x10^7)</td>
</tr>
<tr>
<td>360</td>
<td>(5.5x10^7)</td>
</tr>
<tr>
<td>(160)</td>
<td>(160)</td>
</tr>
</tbody>
</table>

How to obtain such huge mesh data??

8.9x10^7 (5.3x10^7)

9.7x10^7 (5.5x10^7)

240221184360 (240) Mesh
2360345600240 (160) Mesh

Estimated truncation level Nele on a Sphere

240

18 GB/Snap Shot for 1E8 mesh??

Size of mesh data: 60MB/byte for 330k element mesh.
How do we create these mesh data?

Run simulation on "Earth Simulator"

Mesh generation on "Single PE"

Partitioning on "Single PE"

At most: $10^9 \sim 10^8$ nodes

At most: $10^8 \sim 10^7$ nodes

Impossible!!

At most: $10^7 \sim 10^6$ nodes

(S. Ezure)
Run simulation on "Earth Simulator".

Mesh refinement on "Earth Simulator".

Mesh generation on "Single PE".

Partitioning on "Single PE".

PMR - Parallel Mesh Relocation.
Example of PMR

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>PE#0</th>
<th>PE#1</th>
<th>PE#2</th>
<th>PE#3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Mesh</td>
<td>795</td>
<td>795</td>
<td>795</td>
<td>795</td>
<td>795</td>
</tr>
<tr>
<td>Partitioned Mesh</td>
<td>467</td>
<td>438</td>
<td>422</td>
<td>414</td>
<td>438</td>
</tr>
<tr>
<td>Partitioned and Refined Mesh</td>
<td>2,702</td>
<td>2,397</td>
<td>2,481</td>
<td>2,316</td>
<td>2,767</td>
</tr>
</tbody>
</table>

**Model:** carbon block

**Example of PMR**
Visualization in GeoFEM

The present simulation will be performed on Earth Simulator (ES).

- On ES, \( E = 10^{-7} \) (\( T_a = 10^{14} \)) is considered to be a target of the present MHD simulation.
- A simulation with \( 1 \times 10^8 \) elements can be performed if 600 nodes of ES can be used.

Data size: 178MB per snapshot for 330k element mesh.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Num. of elements</th>
<th>Estimated truncation level</th>
</tr>
</thead>
<tbody>
<tr>
<td>221,184</td>
<td>360 ( (240) )</td>
<td>8.9 \times 10^7 ( (5.3 \times 10^7) )</td>
</tr>
<tr>
<td>240</td>
<td>160 ( (160) )</td>
<td>5.5 \times 10^7 ( (5.5 \times 10^7) )</td>
</tr>
</tbody>
</table>

How to visualize such huge results?
Framework of Parallel Visualization (L. Chen et al.)
Parallel Visualization Techniques in GeoFEM (L. Chen et al.)

Scalar Field
Vector Field
Tensor Field

Cross-sectioning
Isosurface-fitting
Volume Rendering
Streamlines
Particle Tracking
Topological Map
LIC
Interval Volume-fitting
Volume Rendering

Tensor Field
Vector Field
Scalar Field

Requirement of MHD

Not developed
Not SMP/Vectorized
SMP/Vectorized

Requirement of MHD

Module
Module
Module

Hyperstreamlines
Simulation Processes on ES for Dynamo Simulation

- Test Version
- Complete version
- Earth Simulator

- Mesh Data
- PVR results
- Field line results
- Whole result
- Surface results
- Averaged values
- Single domain data
- Multi-domain data
- PVR
- PSR
- PMR

- Original mesh
- Mesh Data
- Averaged values
- PSR
- PVR

MeshData

Single domain data

Multi-domain data

Averaged values

Field line results

PVR results

Surface results
Visualization Process (for MHD dynamo)

Cross sections
Time dependent data
PGPLOT
GIF Files
Movie for time evolution
BMP Files
AVS
PNG Files
PGPLOT
Snapshots
TIFF Files
GPPView
BMP Files
Snapshots
TIFF Files
GPPView
PNG Files
AVS
PGPLOT
GPPView
AVS
Super

For MHD dynamo
This is my main study...
Simple Model for the Earth's Outer Core
\[ \mathbf{B} = \nabla \times \mathbf{A} \]

For insulator:
\[ 0 = \nabla \cdot \mathbf{A} \]
\[ \nabla \cdot \mathbf{A} = 0 \]

For conductor:
\[ 0 = \nabla \cdot \mathbf{A} \]
\[ (\mathbf{B} \cdot \mathbf{n}) + \nabla \times \left( \frac{\mu}{\nabla \cdot \mathbf{A}} \frac{d}{dt} \right) + \nabla \times \mathbf{A} = \frac{\mu \varepsilon}{\nabla \cdot \mathbf{A}} \]

For conductive fluid:
\[ \mathcal{L} \cdot \mathbf{A} = \left( \mathcal{L} \cdot \mathbf{n} \right) + \frac{\mu \varepsilon}{\mathcal{L} \cdot \mathbf{A}} \]
\[ 0 = \nabla \cdot \mathbf{A} \]
\[ \mathbf{B} \cdot \left( \mathbf{B} \cdot \mathbf{n} \right) d + \mathcal{L} \left( \mathcal{L} \cdot \mathbf{n} \right) d \]
\[ \left( \mathbf{n} \cdot \nabla \right) \left( \mathbf{B} \cdot \mathbf{n} \right) d + \mathcal{L} \left( \mathcal{L} \cdot \mathbf{n} \right) d = \left( \mathbf{n} \cdot \mathbf{n} \right) + \frac{\mu \varepsilon}{\mathcal{L} \cdot \mathbf{A}} \]

Introduction - Basic Equations for MHD dynamo - Lorentz, Coriolis, Conductivity term
Estimated values for the Outer core:

- Rayleigh number: \( R = 10^{30} \)
- Prandtl number: \( P = 0.1 \)
- Magnetic Prandtl number: \( P_m = 10^{-6} \)
- Taylor number: \( T = 2 \times 10^7 \)

Dimensionless Numbers:

- Magnetic diffusion
- Viscosity
- Thermal diffusion
- Viscosity
- Coriolis force
- Viscosity
- Thermal diffusion
- Buiyancy

For the Outer core.
High spatial resolution is required! To approach such large parameters, 

\[ \rho_l \]

Estimated values for the outer core

- Dimensionless Numbers -

Introduction
Introduction - Why FEM is chosen?

<table>
<thead>
<tr>
<th>Few</th>
<th>Many</th>
<th>Simulation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficult</td>
<td>Easy to apply</td>
<td>Boundary Condition for B</td>
</tr>
<tr>
<td>Easy</td>
<td>Complex and difficult and parallelization</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>High</td>
<td>Accuracy</td>
</tr>
</tbody>
</table>

| FEM | Spectral | |

- Why FEM is chosen?
Purposes

• Develop a MHD simulation code for a fluid in a Rotating Spherical Shell by parallel FEM
• Construct a scheme for treatment of the magnetic field in this simulation code
• Develop a MHD simulation code for
What is difficult for dynamo simulation?

<table>
<thead>
<tr>
<th>Few</th>
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<tr>
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<tr>
<td>Easy</td>
<td>Complex and difficult</td>
<td>Parallelization</td>
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<tr>
<td>Low</td>
<td>High</td>
<td>Accuracy</td>
</tr>
<tr>
<td>FEM</td>
<td>Spectral</td>
<td></td>
</tr>
</tbody>
</table>
Boundary conditions on CMB

**Dipole field**

\[ B = r Y_{10}^\dagger \] on CMB

**Octopole field**

\[ \partial r B_{30} + r B_{30} = 0 \] on CMB

Composition of dipole and octopole

Boundary conditions on CMB on CMB

Octopole field

\[ 0 = \frac{\partial}{\partial \varphi} \left( \frac{1}{r} \right) B_{30}^\dagger + \frac{\partial}{\partial \varphi} B_{30} \]

Dipole field

\[ 0 = \frac{\partial}{\partial \varphi} \left( \frac{1}{r} \right) B_{10}^\dagger + \frac{\partial}{\partial \varphi} B_{10} \]
Treatment of the Magnetic Field

- Finite Element Mesh is considered for the outside of the fluid shell.
- Vector potential in the fluid and insulator is solved simultaneously.

\[
\nA = B, \quad \nabla \cdot A = 0
\]

- Consider the vector potential defined as
Treatment of the Magnetic Field - Finite Element Mesh -

- Element type
  - Tri-linear hexahedral element

- Mesh for the fluid shell

- Grid pattern for the center

Requirement
- Considered to the outside of the core

Based on cubic pattern
- Entire mesh

\[ r_m = 14.8(t_0 - t_i) = 5.09R \]

Filled to the Center

Filled to the Center
Treatment of the Magnetic Field

- Finite Element Mesh is considered for the outside of the fluid shell.

- Consider the vector potential defined as:
  \( \mathbf{A} = \mathbf{B}, \ \mathbf{A} \cdot \mathbf{A} = 0 \)

- The vector potential in the fluid and insulator is solved simultaneously.
\[ 0 = \mathbf{B} \cdot \mathbf{n} \]
\[ \left( \mathbf{B} \cdot \mathbf{n} \right) \nabla \cdot \nabla + \mathbf{B} \cdot \nabla \frac{d}{dt} = \frac{\rho}{\mathbf{B} \cdot \mathbf{n}} \]
\[ \mathcal{L} \cdot n = \left( \mathcal{L} \cdot n \right) + \frac{\rho}{\mathcal{L} \cdot n} \]
\[ 0 = \mathcal{Q} \cdot n = n \cdot n \]

\[ \left\{ \mathbf{B} \cdot \left( \mathcal{B} \cdot \mathcal{J} \right) \right\} \nabla \cdot \mathbf{d} \mathcal{J} + \left\{ \mathcal{J} \left( \mathcal{B} \cdot \mathcal{B} \right) \right\} \nabla \cdot \mathbf{d} \mathcal{B} \]
\[ \left( n \cdot \mathcal{J} \right) \nabla \cdot \left( \mathcal{L} \cdot \mathcal{J} \right) \mathcal{L} \nabla \cdot \mathbf{d} \mathcal{J} = \left( n \cdot \mathcal{B} \right) \nabla \cdot \mathbf{d} \mathcal{B} + \frac{\rho}{\mathcal{Q} \cdot \mathcal{B}} \]

**Treatment of the Magnetic Field - Basic Equations for Spectral Method**

Basic Equations for Spectral Method - Treatment of the Magnetic Field

**Basic Equations for Spectral Method - Treatment of the Magnetic Field**
\[ \mathbf{B} = \nabla \times \mathbf{A} \]

For insulator:
\[ 0 = \nabla \cdot \mathbf{A} \]
\[ \nabla \cdot \mathbf{A} = 0 \]

For conductor:
\[ 0 = \nabla \cdot \mathbf{A} \]
\[ (\mathbf{B} \cdot \mathbf{n}) + \nabla \cdot \frac{\mathbf{u}}{d} + \nabla \cdot \mathbf{A} = \frac{\epsilon}{\mathbf{V}\rho} \]

- Basic Equations for GeoFEM/MHD

Treatment of the Magnetic Field - Lorentz term and Coriolis term.
Process for Time Integration

3x3 Solver for conductor and insulator

Poisson Solver for fluid

Poisson solver for fluid

3x3 Solver for conductor and insulator

Poisson solver for fluid

3x3 solver for fluid
Model of the Present Simulation - Current Model and Parameters -

$$Ra = \frac{a g_0 D T L}{\nu^3} = 1.2 \times 10^4$$

$$Pr = \frac{\nu}{\nu} = 1.0$$

$$Pm = \frac{\nu}{\nu} = 1.0$$

Properties for the simulation box

- Conductive fluid
- Insulator

Dimensionless numbers

Simulation box
Model of the Present Simulation

- Boundary Conditions
- Vector potential: Symmetric
- Temperature: Symmetric
- Velocity: Symmetric

Symmetry with respect to the equatorial plane

\[ \begin{align*}
\mathbf{u} & = \mathbf{0} \\
\nabla \mathbf{A} & = \mathbf{0} \\
0 & = \nabla \mathbf{T} \\
0 & = \mathbf{T}
\end{align*} \]

Finite element mesh for the present simulation

- 77760 elements
- 81303 nodes

For the northern hemisphere

Boundary Conditions
- Non-Slip

- Geometry & Boundary Conditions

Finite element mesh for the present simulation
Comparison with Spectral Method

Radial magnetic field for t = 20.0

Time evolution of the averaged kinetic and magnetic energies in the shell

Comparison with spectral method

![Graph showing energy evolution over time with different methods]
Comparison with Spectral Method Cross Sections at $z = 0.35$

Spectral method
GeoFEM

Magnetic field
Vorticity

Cross Sections with Spectral Method
**Dynamo Benchmark Test - Suggested Solution**

<table>
<thead>
<tr>
<th>Case</th>
<th>Required time</th>
<th>Local magnetic field</th>
<th>Local zonal velocity</th>
<th>Local Temperature</th>
<th>Frequency</th>
<th>Drift angular</th>
<th>Averaged magnetic energy</th>
<th>Averaged Kinetic energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4.9280±0.0060</td>
<td>-7.6250±0.0060</td>
<td>-10.1571±0.0020</td>
<td>0.37338±0.00040</td>
<td>58.348±0.050</td>
<td>&gt;12.0</td>
<td>0.37338±0.00040</td>
<td>30.733±0.020</td>
</tr>
<tr>
<td>1</td>
<td>&gt;12.0</td>
<td>0.4281±0.00012</td>
<td>&gt;1.0</td>
<td>0.1824±0.0050</td>
<td>&gt;1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: Values are approximate and may vary based on specific conditions.*
<table>
<thead>
<tr>
<th>Mesh(b)</th>
<th>Mesh(a)</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.75Deg.</td>
<td>77760</td>
<td>139968</td>
</tr>
</tbody>
</table>

### Results for Case 1

- **Radial magnetic field at the outer boundary of the fluid shell**
- **Radial velocity at the mid-depth of the fluid shell**

<table>
<thead>
<tr>
<th>$E_{\text{mag}}$</th>
<th>$U$</th>
<th>$T$</th>
<th>$E_{\text{kin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.2211</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3.1784</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3.1017</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B$</th>
<th>$T$</th>
<th>$U$</th>
<th>$E_{\text{kin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37262</td>
<td>0.36167</td>
<td>0.37338</td>
<td></td>
</tr>
<tr>
<td>0.37262</td>
<td>0.36167</td>
<td>0.37338</td>
<td></td>
</tr>
<tr>
<td>0.37262</td>
<td>0.36167</td>
<td>0.37338</td>
<td></td>
</tr>
</tbody>
</table>

---

**Christensen, 2001**

**GeoFEM for $t=8.0$**
Conclusions

- We have developed a simulation code for MHD dynamo in a rapidly rotating spherical shell using GeoFEM platform.
- This simulation code is designed to perform on the Earth.
- GeoFEM serves as a parallel FEM platform to perform the dynamo simulation.
- We have developed a simulation code for MHD dynamo in a rapidly rotating spherical shell using GeoFEM platform.
- To obtain more accurate solution, the radial resolution should be increased.
- These differences become smaller by using fine FEM mesh.
- The difference in drift angular frequency is 8% from Christensen's solution.
- Results show that our solution has within 8% difference from Christensen et al. (2001) benchmark test.
- GeoFEM is a parallel FEM platform to perform the dynamo verification.
- The simulation code is designed to perform on the Earth.
- To verify the present simulation code, we carried out the Dynamo benchmark test by Christensen et al. (2001).
Near Future Challenge on ES

The Present Simulation will be performed on Earth Simulator (ES).

- On ES, $E=10^{-7}$ ($T_a=10^{14}$) is considered to be a target of the present MHD simulation.
- A simulation with $10^8$ elements can be performed if 600 nodes of ES can be used.
- We consider that the present code can obtain 25% of the peak performance of ES.

<table>
<thead>
<tr>
<th>Elapsed for 1 step</th>
<th>Num. of element</th>
<th>element direction</th>
<th>Num. of Outer Core</th>
<th>E (240)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Sec</td>
<td>0</td>
<td>Sphere</td>
<td>240</td>
<td>9.3x10^7</td>
</tr>
<tr>
<td></td>
<td>240000</td>
<td></td>
<td></td>
<td>8.9x10^7</td>
</tr>
<tr>
<td></td>
<td>345600</td>
<td></td>
<td></td>
<td>9.7x10^7</td>
</tr>
<tr>
<td></td>
<td>360</td>
<td></td>
<td></td>
<td>5.7x10^7</td>
</tr>
<tr>
<td></td>
<td>360</td>
<td></td>
<td></td>
<td>5.3x10^7</td>
</tr>
</tbody>
</table>

- These targets depend on available computation time and performance of ES.
- We consider that the present code can obtain 25% of the peak performance of ES.
- A simulation with $10^8$ elements can be performed if 600 nodes of ES can be used.
- The present simulation will be performed on Earth Simulator (ES).