Flat Versus Metered Rates, Bundling, and "Bandwidth Hogs"

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Abstract

The current push for bandwidth caps, tiered usage pricing, and other measures in both wireless and wireline communications is usually justified by invoking the specter of "bandwidth hogs" consuming an unfair share of the transmission capacity and being subsidized by the bulk of the users. This paper presents a conventional economic model of flat rates as a form of bundling, in which consumption can be extremely unequal, and can follow the ubiquitous Pareto distribution. For a monopoly service provider with negligible marginal costs, flat rates turn out to maximize profits in most cases. The advantage of evening out the varying preferences for different services among users overcomes the disadvantage of the heaviest users consuming more than the average users. The model is tractable enough that it allows for exploration of the effects of non-zero marginal costs (which in general strengthen the case for metered pricing), and of welfare effects.

1. Introduction

The telecommunications industry has often been the scene of pricing controversies, starting with regular postal services [7]. The general pattern has not changed much over the centuries. Usually, industry leaders, in modern times supported by economists, argue for fine-scale metering, while consumers fight for simplicity. The confusing scene in pricing of telecommunication services as well as many types of information goods results from a variety of conflicting factors. It is likely that in the future, just as in the past, there will be a mix of pricing models that varies depending on technologies and local conditions. Many of the factors affecting pricing were observed centuries ago, and now they are being investigated more intensively. Some references are [1], [4], [6], [7], [8], and [11], as well as the papers listed there.

This paper does not consider the various behavioral economics aspects of pricing, such as consumer willingness to pay more for flat rates. We take just the conventional economic point of view, in which consumers and producers have value functions that are well-defined and known precisely only to themselves, which they try to maximize. Even in this setting, flat rates can often be shown to be advantageous to sellers (and often to buyers as well), as they are a form of bundling, selling several goods or services in a single package. Bundling has been a standard business practice for thousands of years. The justification for it in the standard economic model, as a way to take advantage of uneven valuations for different goods among consumers, has been developed over the last half of century, starting with the work of Stigler in 1963 [10]. The literature on bundling is vast, and we mention just a few of the seminal works, as well as some of the most recent publications, e.g. [3], [9], and [12]. Most of this literature is concerned with just a small number of goods (often with a particular good bought in varying quantities), and aims to explicate the degree to which non-zero marginal costs as well as complementarity or substitutability of the goods affect the gains to be obtained from bundling.

In telecommunications, flat rates can be viewed as a form of bundling a very large number of goods, such as access to hundreds of millions of websites or phone calls to potentially billions of people. Such settings were considered in [2], [4], and [5]. This paper can be considered an extension of those works. The papers of Bakos and Brynjolfsson [2] and of Geng, Stinchcombe, and Whinston [5] demonstrate that for wide classes of valuations on information goods (i.e., goods with zero marginal cost), bundling is more profitable than separate sales for a monopoly seller when there are many goods. (Their results are outlined in Section 2.)

There is a limitation to the models of [2] and [5]. In these models, when bundling is shown to be optimal, the seller extracts essentially the full amount that buyers are willing to pay, and almost all buyers have roughly the same budgets. Thus there is no room for "bandwidth hogs," and bundling is optimal not just for the seller, but for almost all buyers. Further, those buyers spend almost all they are able to do, so there is practically no "consumer surplus." Thus this is a rare situation where (almost) everybody wins.

This paper proposes a model that avoids the above limitation. It is similar in principle to those of [2] and [5], but allows for more elaborate forms for the valuations of goods by consumers. We consider J buyers, and I

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goods, such as songs to download, web sites to view, or phone calls to make. J can be small or large, while I is best thought of as very large, and many results will hold only for large I. We assume that buyer j values good i, at $U^j(x_i) = \omega^j \times \nu_i^j$, $1 \leq i \leq I$, $1 \leq j \leq J$, where ω^j and ν_i^j are independent random variables, with different distributions for ω and ν. (See Section 3 for restrictions on the distributions.) We assume that buyers have unit demands, so purchase either one or no units of a particular good. The parameters ν_i^j (and thus $U^j(x_i)$) can be zero most of the time, corresponding to most goods on offer being of no interest to any single user. We also assume the value of a collection of goods to a consumer is the sum of the values of individual goods.

The parameters ω^j's are an indirect way to introduce a budget constraint on consumers. For large I, buyer j will usually have willingness to pay for all the goods on offer close to $\omega^j I E[\nu]$. Hence if the distribution of ω^j is, say, the common Pareto one that is frequently observed in practice, we have a very unbalanced willingness to spend among buyers, with rules like the frequent "top 10% of users account for 90% of consumption."

The main results of this paper show that for zero marginal costs in the model sketched above, bundling is, for large number of goods I, almost always more profitable for the seller than separate sales. (There are some rare technical conditions, described in Section 4, under which separate sales can produce larger profits, but that happens seldom and the gain is marginal and declines with growing I.) However, willingness to spend by consumers varies widely, and so transactions often leave substantial consumer surplus. On the other hand, bundling, while it maximizes seller profit, often results in prices for the bundle that are not affordable for a substantial fraction of users ("digital exclusion"). Thus the model allows for explorations of more interesting phenomena than previous ones, in particular of welfare effects.

Our model does allow for non-zero marginal costs, and some examples are presented. When those costs are high enough, separate sales lead to maximal profits, consistent with general observationsin the literature.

For the sake of simplicity, we assume in the current draft that valuations of goods are uncorrelated. Extensions of our methods to cases where there are dependencies, along the lines of [2] and [5], will require further work.

The "bandwidth hogs" that are being stigmatized by the telecom industry can be modeled in our setting by a combination of a substantial probability that ω is very small but non-zero and that ν is very small but non-zero. Bundling would then lead to high usage that would be suppressed by even a low price per good under separate sales. Our result that profits are maximized through bundling applies to such cases, but in full generality only for zero marginal costs. When those costs are non-zero, it is necessary to work with particular distributions of ω and ν to find out the threshold on marginal costs beyond which separate sales are more profitable. In such situations it is also possible to explore the effects of imposing usage caps.

In a model such as ours, with a heterogenous distribution of budgets, there are several factors that operate. Bundling smooths out distribution of valuations of goods. On the other hand, it allows some to obtain collections of goods for far less than their willingness to spend, while preventing others, with low budgets, from purchasing anything. The main contribution of the paper is to show in a precise quantitative way that the advantages of bundling for the seller overcome the disadvantages.

This paper is organised as follows: In Section 2, we review the models proposed by Bakos and Brynjolfson [2] and Geng at al. [5]. In Section 3, we describe our model. In Section 4, we prove that in this model, bundling is the optimum strategy, or close to it, for the seller. In Section 5, we consider our model for several specific types of distributions of willingness to pay, and consider the effects of non-zero marginal costs, "digital exclusion," and social welfare. Finally, in Section 6, we present the conclusions and outline potential extensions of our model. The Appendix presents some details of the examples discussed in Section 5.

2. Related Works

Our model is closest to that of Bakos and Brynjolfson [2] and Geng at al. [5]. Since there are some technical problems with the proofs in [2], as was pointed out in [5], which has corrected statements and proofs, we concentrate on the latter paper. Simplifying somewhat, [5] considers value functions of the form $U^j(x_i) = \nu_i^j$ (i.e., with ω identically 1), where ν_i^j's are independent, and have the same distributions for each j, but those distributions may depend on i.

Geng at al. [5] show that when the expected value of $U^j(x_1) + U^j(x_2) + ... + U^j(x_I)$ is large compared to its variance, bundling is close to optimal (in that it extracts revenues close to the maximal willingness to pay). That paper also obtains conditions for this mean to be large compared to the variance, basically requiring that the mean value of ν_i^j as a function of i vary smoothly. The required relation between mean and variance in [5] is very similar to ours, the result of both papers relying on the Chebyshev inequality.

For our proofs to be valid, we require the distributions of all ν_i^j's to be identical. We also require, at least in the present version, that ν_i^j's be independent. Both Bakos and Brynjolfson [2] and Geng at al. [5] do consider some dependencies among the information goods. Some of the dependencies assumed there can be accommodated in our model without any additional technical work, since we can vary the distributions of ν_i^j as we let $I \to \infty$, say.

Geng at al. [5] produce a counterexample to some of the claims in [2]. Their construction involves expected values of ν_i^j declining rapidly as $i \to \infty$. In that particular counterexample, separate sales can be about twice as profitable as bundling. However, there is considerable variation in both total budgets of individual buyers, and in valuations of individual goods. In our model, bundling is optimal even when individual budgets vary dramatically. Hence, combined with the results of [5], we are led to the suggestion that optimality of bundling depends far more on similarity in distribution of valuations of goods than in the budgets of individuals.

It is well known that mixed bundling (selling both bundles and separate goods at the same time, but with prices higher than they would be for either pure bundling or pure separate sales) is generally more profitable than either extreme strategy. We show an example of this effect, but we concentrate on comparing the extreme cases of either pure bundling or pure separate sales.

3. Model Description

In this paper, we assume that there is a single seller producing I information goods for a population of J buyers with *unit demand* (i.e., each buyer purchases one or zero units of each good). We denote the collection of all I goods by $\mathcal{X}$.

We assume that there exist a *real-valued, non-negative* function that represents each buyer's willingness to pay (w.t.p.) for a single product, with the w.t.p. function of buyer j for product i given by

$$U^j(x_i) = \omega^j \times \nu_i^j, 1 \leq i \leq I, 1 \leq j \leq J. \qquad (1)$$

We further assume these funcitons are additive, so that

$$U^j(\mathcal{X}) = \sum_{i=1}^{I} U^j(x_i) = \omega^j \times (\nu_1^j + ... + \nu_I^j), 1 \leq j \leq J \qquad (2)$$

represents the *w.t.p. function for the bundle of all I goods* of buyer j.

We assume the ω^j, $1 \leq j \leq J$, are non-negative *i.i.d.* random variables with finite mean $E[\omega]$. The $\nu_i^j, 1 \leq i \leq I, 1 \leq j \leq J$, are non-negative *i.i.d.* random variables with finite mean $E[\nu]$ and finite variance σ^2.

We assume ω^j's are also independent of ν_i^j's, so that the $U^j(x_i), 1 \leq i \leq I, 1 \leq j \leq J$ are non-negative *i.i.d.* random variables with finite mean. Note that we allow ω to have infinite variance, and so $U^j(x_i)$ can have infinite variance.

We denote the C.D.F. of ω by $F_\omega(x) = Prob\{\omega \leq x\}, \forall x \in \mathcal{R}$, and similarly for $F_\nu(x)$ and $F_{\omega\nu}(x)$.

An important observation is that the distribution of $U^j(x_i)$ is the same for every i. Thus the goods in our model are homogenous in this sense, although their valuations do vary widely (with potentially infinite variance).

While many of our results hold only for large numbers I of goods, generally the number J of buyers can be arbitrary, even 1 or 2. The ω^j can even be completely deterministic (with the restriction that the seller might know the exact distribution of the actual ω^j, but would not know individual values of ω^j). However, for simplicity we will assume ω^j's are random variables. Our results are valid for finite values of I, not just asymptotically, and the distributions of ω and ν can vary, and do not have to be held fixed as we increase I, say. For the validity of our main results on optimality of bundling we basically need only that σ^2 be very small compared to $IE[\nu]^2$.

A. Seller's maximization problem

In our basic setting we assume zero marginal cost for the seller, so that the revenue is the profit, and we compare only the two extreme alternatives of either selling all goods as a bundle, or selling each separately.

1) Separate sales: When selling each good separately, symmetry of the distributions implies that the profit-maximizing strategy is to sell all goods at the same price.

Let p be the common price for each good. Then

$$\mathcal{D}^S(p) = J \times \big(1 - F_{\omega\nu}(p)\big) \qquad (3)$$

is the expected number of buyers who are willing to purchase any particular item at price p, and the expected revenue (and thus profit) of the seller is given by

$$\pi^S(p) = pI\ \mathcal{D}^S(p). \qquad (4)$$

We use p^* to denote a price (possibly more than one) which maximizes $\pi^S(p)$.

The maximum possible profit for the seller (with separate sales) is equal to the sum of the average of each user's w.t.p. for each good. To measure how close the seller's profit is to the optimum profit we define

$$\rho^S = \frac{\pi^S(p^*)}{IJE[U(x)]} = \frac{\pi^S(p^*)}{IJE[\omega]E[\nu]}, \qquad (5)$$

which has the property that $0 \leq \rho^S \leq 1$.

2) Bundling: In this paper, we mainly consider the case of pure bundling, so that each buyer either purchases all goods for the single price or buys nothing. By our assumptions, the expected number of buyers who are

If the seller has complete information about each buyer's valuation, is not facing legal restraints, and can prevent resale, she can practice first degree price discrimination and capture the entire consumer surplus. We exclude such practices in our model.

willing to buy the entire bundle is given by

$$\mathcal{D}^B(q) = J \times \big(1 - F_{U(\mathcal{X})}(q)\big), \tag{6}$$

where

$$F_{U(\mathcal{X})}(q) = Prob\{\omega \cdot (\sum_{i=1}^{I} \nu_i) \le q\}.$$

Let q be the price of the bundle. Then the expected profit from bundling is

$$\pi^B(q) = q \cdot \mathcal{D}^B(q). \tag{7}$$

We let q^* denote a price q that maximizes $\pi^B(q)$.

To measure how close the seller's profit with bundling comes to the optimum profit we use

$$\rho^B = \frac{\pi^B(q^*)}{JE(U[\mathcal{X}])} = \frac{\pi^B(q^*)}{IJE[\omega]E[\nu]}, \tag{8}$$

which again has the property that $0 \le \rho^B \le 1$.

B. Buyers' surplus and market exclusion

We consider two standard measures to determine social welfare, *market exclusion* and *buyers' surplus*. Market exclusion measures how many people are not able to purchase anything at the (profit-maximizing) price offered by the seller. Buyers' surplus measures the difference of average willingness to pay from the price offered by the seller for the items that are actually purchased.

With bundling, market exclusion occurs when there is a j with $U^j(\mathcal{X}) < q^*$. With separate sales, market exclusion occurs when there exists at least one buyer, j, with the property that $U^j(x_i) < p^*, 1 \le i \le I$.

The expected consumer surplus for each buyer, averaged over each good, when the bundle price is set by seller at q^*, is

$$\varsigma^B = E[\big(\omega(\sum_{i=1}^{I} \nu_i) - q^*\big)^+]/I. \tag{9}$$

With separate sales, each buyer's expected surplus from purchasing any single good is

$$\varsigma^S = E[\big(U(x) - p^*\big)^+] = E[\big(\omega\nu) - p^*\big)^+]. \tag{10}$$

4. Optimality of Bundling

Define

$$\Psi = \max_{p>0} p \int_0^\infty \big(1 - F_\omega(\frac{p}{x})\big) dF_\nu(x). \tag{11}$$

Then by Eq. (4), the maximum expected profit from selling separately is equal to

$$\pi^S(p^*) = IJ\Psi. \tag{12}$$

Recall that σ is the standard deviation of ν, and define

$$\Phi = \max_{t>0} t \, \big(1 - F_\omega(t)\big). \tag{13}$$

Theorem 1: For any $\alpha > 0$, the maximum expected profit from bundling satisfies

$$\pi^B(q^*) \ge IJ\Phi E[\nu]\big(1 - \frac{\alpha}{IE[\nu]}\big)\big(1 - \frac{I\sigma^2}{\alpha^2}\big).$$

Proof: The intuition behind the proof is that buyer j will usually value the bundle close to $\omega^j \cdot IE[\nu]$. Using the assumption that ν has finite second moment, we find from the Chebyshev inequality that for any $\alpha > 0$,

$$Prob\{|\sum_{i=1}^{I} \nu_i^j - IE[\nu]| \ge \alpha\} \le \frac{I\sigma_\nu^2}{\alpha^2}. \tag{14}$$

We consider only those buyers j who have

$$\sum_{i=1}^{I} \nu_i^j \ge IE[\nu] - \alpha.$$

The expected number of them is, by the Chebyshev inequality, at least

$$J \cdot (1 - \frac{I\sigma_\nu^2}{\alpha^2}).$$

Buyer j in this category will certainly purchase the bundle at price q if

$$\omega^j \times \big(IE[\nu] - \alpha\big) \ge q,$$

and so for any $q \ge 0$,

$$\pi^B(q) \ge qJProb\{\omega\big(IE[\nu] - \alpha\big) \ge q\} \cdot \big(1 - \frac{I\sigma_\nu^2}{\alpha^2}\big).$$

Suppose the maximum that defines Φ is attained at $t = t^*$. Then we set the bundle price

$$q' = t^* \cdot \big(IE[\nu] - \alpha\big),$$

and obtain the lower bound of the theorem. ■

Theorem 2: For random variables ω and ν that are independent and non-negative, with ν having finite mean,

$$\Psi \le \Phi E[\nu]. \tag{15}$$

Proof: By definition of Eq. (11),

$$\Psi = \max_{p>0} \int_0^\infty \frac{p}{x} \cdot \big(1 - F_\omega(\frac{p}{x})\big) x dF_\nu(x) \le$$

$$\Phi \int_0^\infty x \cdot dF(x) = \Phi E[\nu].$$

■

Theorem 3: If $\Psi < \Phi E[\nu]$, then for a sufficiently large number of goods, the expected revenue from bundling will be strictly larger than the expected revenue from separate sales. If $\Psi = \Phi E[\nu]$, the ratio of expected revenue from bundling to the expected revenue from separate sales will be $\geq 1-\delta$ where $\delta \to 0$ as the number of goods grows (i.e., $I \to \infty$).

Proof: We choose an approximately optimal α as

$$\alpha = I^{2/3} E[\nu]^{1/3} \sigma_\nu^{2/3}.$$

Then

$$\big(1 - \frac{\alpha}{IE[\nu]}\big)\big(1 - \frac{I\sigma_\nu^2}{\alpha^2}\big) = \big(1 - \frac{\sigma^{2/3}}{I^{1/3}E[\nu]^{2/3}}\big)^2.$$

As $I \to \infty$, this goes to 1 at the rate of $I^{-1/3}$, and the theorem follows.

■

Note that the proof actually gives us precise estimates for how large I has to be for bundling to be strictly more profitable than separate sales when Theorem 2 holds with strict inequality, and for it to be within some fixed fraction, say 1%, of the profit of separate sales when equality holds in that result.

For most distributions of ω and ν strict inequality holds in Theorem 2.

5. Examples of specific W.T.P. functions

This section applies the basic model for some special distributions of ω and ν. Greater specificity enables us to present results that are easier to understand, and also to explore effects of non-zero marginal costs, digital exclusion, and consumer surplus. Details on the exact computations and some additional graphs are presented in the Appendix.

For simplicity, from now on we will assume that I, the number of information goods, is very large, so that for most buyers j,

$$\sum_{i=1}^{I} \nu_i^j \approx IE\nu.$$

In this asymptotic limit of $I \to \infty$, we can then approximate $\pi^B(q^*)$ by $IJ\Phi E[\nu]$ (for marginal costs zero), and of course we still have the exact relation $\pi^S(p^*) = IJ\Psi$. We can rewrite the demand for bundles as

$$\mathcal{D}^B(q) = J \times \big(1 - F_\omega(\frac{q}{IE[\nu]})\big).$$

The equations and choices of parameters we write down are just the leading term in the asymptotic expansion, and to obtain fully rigorous results we would need to toss in factors that behave like $I^{-1/3}$ as $I \to \infty$.

A. Constant ω

***Example** 5.1:* We assume that $\omega^j = 1$ for all j. This reduces to the problem studied by Bakos and Brynjolfson [2], and so we are basically rederiving their results. For simplicity, let us further specify that $\nu \sim \mathcal{U}(0,1)$ where $\mathcal{U}(0,1)$ is the continuous uniform distribution on the interval from 0 to 1, so that $E[\nu] = 1/2$. Then the w.t.p. function for a bundle of all goods will be equal to

$$U^j(\mathcal{X}) = \sum_{i=1}^{I} \nu_i^j \approx IE[\nu] = I/2, \;\; 1 \leq j \leq J. \qquad (16)$$

Hence bundling will produce revenues of about $IJ/2$, whereas (see the Appendix, in particular Fig. 3) separate sales produce only half as much, $IJ/4$. Bundling captures the maximum possible profit, and leaves no consumer surplus. Separate sales do have consumer surplus of about one half of the revenues in that case. There is practically no "digital exclusion," as almost all buyers do purchase something, but consumption is larger under bundling.

***Example** 5.2:* Let's assume in Example (5.1) that it costs $c_i = c > 0, 1 \leq i \leq I$ to produce or distribute each good. This implies that it costs $\sum_{i=1}^{I} c = Ic$ to produce a bundle of I goods.

Bundling continues to yield a greater profit as long as $c < \sqrt{2}-1$, otherwise separate sales are more profitable. As in the previous example (which has $c = 0$), almost every one buys in either case.

Consumer surplus with bundling will remain zero ($\varsigma^B = 0$). Under separate sales, consumer surplus will be positive but will decline with cost, $\varsigma^S = (1-c)^2/8$.

B. Product of Pareto and uniform distributions

***Example** 5.3:* We now assume ω has a Pareto distribution with parameters $\tau > 0$ and $\alpha > 1$, so that $F_\omega(x) = (\tau/x)^\alpha$ for $x \geq \tau$ and $E[\omega] = \alpha\tau/(\alpha-1)$. The assumption that $\alpha > 1$ guarantees that $E[\omega] < +\infty$. Larger values of α mean smaller fraction of buyers with very high incomes, and for very large α we are close to the first example of this section, in which ω is constant. We also assume that, as in the previous example, $\nu \sim \mathcal{U}[0,1]$.

The direct computation (see the Appendix) shows, as guaranteed by our theorems, that bundling maximizes profits. Selling separately can capture no more than half the maximal profit, and the half can be approached only for large α. See Fig. 1.

There is no significant market exclusion in either case. Buyers' surplus depends on α and is sometimes maximized with bundling and sometimes with separate sales. The profit-maximizing bundle is less expensive than buying them separately, $Ip^* > q^*$.

***Example** 5.4:* We next consider the effect of introducing non-zero marginal costs in the previous example. When c, the cost of each good, is low enough, bundling continues to be more profitable than separate sales, with

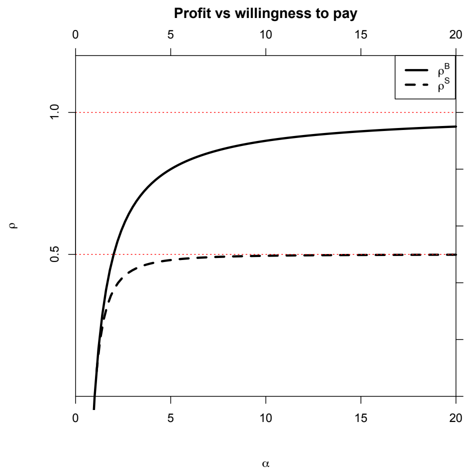

Fig. 1. Example (5.3): By selling bundles, the seller can come close to capturing the maximal possible profit for large α, but separate sales never capture more than half the maximal possible profit.

the cross-over depending on α and τ. See Fig. 6 in the Appendix for an illustration.

C. *Discrete distribution for* ω

Example *5.5:* In this example we assume that consumers fall into two income classes, with those in one class having about twice the income of the others, and the relative populations of the two classes varying. More precisely, take $a > 0$ and define

$$\omega \in \{a, 2a\}, \text{ with } Prob\{\omega = a\} = x \in [0, 1].$$

As before, we assume $\nu \sim \mathcal{U}(0, 1)$. Results are illustrated in Fig. 2.

By Theorem 3 or direct computation, bundling yields greater profit than separate sales. There will be no significant market exclusion for $x \geq 1/2$, but a positive fraction of buyers will be excluded when $x < 1/2$. There is no significant market exclusion with separate sales.

When $x < 1/2$ the price of the optimally-priced bundle is higher than the cost of buying I goods separately at the optimal price for separate sales, $q^* \geq Ip^*$. When $x \geq 1/2$, it costs less to buy the bundle.

With either separate sale or bundling, the seller can capture the maximum possible profit only when there is only one class of income, *i.e.,* $x = 0$ *or* $x = 1$. Separate sales can at most capture half of the possible maximum profit, although they provide a greater surplus to buyers.

Example *5.6:* The distributions of ω and ν are the same as in Example 5.5. However, this time the seller engages in *mixed bundling*, selling both a bundle and separate goods. We assume each buyer with w.t.p. for the bundle that exceeds the price of the bundle will purchase it.

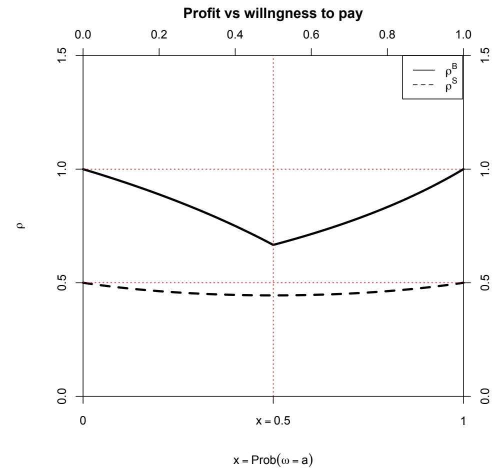

Fig. 2. Example (5.5): The seller's expected profit with bundling can reach the maximal possible profit when either $x = 0$ or $x = 1$, but as $x \uparrow 1/2$, this declines, and at $x = 1/2$ only $2/3$ of the maximal profit can be attained. With separate sales, just as with bundling, the seller's profit is closest to optimum profit when either $x = 0$ or $x = 1$, but it cannot exceed half of the optimum profit.

Mixed bundling in this example generates higher profit than pure bundling when $x < \frac{2}{3}$. See the Appendix for details.

6. Conclusions

This paper presents a new model of demand for information goods. Unlike the most prominent models in the literature, it allows for wide variation in consumers' budgets. It is tractable enough to yield a general result that with zero or very low marginal costs, bundling is almost always more profitable to the monopoly seller than separate sales, even when there are some buyers with disproportionately large usage. On the other hand, bundling often excludes a substantial fraction of potential consumers from the market. Consumer surplus varies, and sometimes is maximized with bundling, sometimes with separate sales.

The model of this paper is flexible enough to allow for non-zero marginal costs. It also offers possibility of extension to goods that are partial complements or substitutes for each other.

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Appendix

A. Example 5.1

With separate sales, the seller chooses $p^* = \frac{1}{2}$ and obtains the maximum expected revenue equal to $\pi^S(p^*) = \frac{1}{4}IJ$.

When bundling, the seller chooses $q^* = IE[\nu] = \frac{1}{2}I$ (more precisely, a price within $I^{-1/3}$ of $1/2$, something that we will not mention from now on) and receives the maximum expected revenue almost equal to $\pi^B(q^*) = \frac{1}{2}IJ$, which is twice the maximal profit realizable with separate sales. Thus in the limit as $I \to \infty$,

$$\rho^B = 1 \quad > \quad \rho^S = \frac{1}{2}.$$

In this example, there is practically no market exclusion with separate sales, since for large I, almost every buyer will value some good at more than the price $p^* = 1/2$. Similarly, almost everybody buys the bundle.

Almost all buyers will have valuations for the approximately $I/2$ goods that they buy at the price of $\frac{1}{2}$ uniformly distributed between $\frac{1}{2}$ and 1, so the consumer surplus per user and per good will be close to $\varsigma^S = 1/8$. With bundling consumer surplus is essentially zero.

B. Example 5.2

If the cost is too high, $c > 1/2$, selling a bundle at a profit would force the price to be higher than willingness to pay of all but a negligible fraction of users. If $c < 1/2$, a seller engaging in bundling chooses $q^* = I/2$ to maximize

$$\pi^B(q) = (q - Ic) \cdot \mathcal{D}^B(q^*),$$

and obtains the maximum expected revenue equal to

$$\pi^B(q^*) = \begin{cases} IJ(\frac{1}{2} - c), & if \quad c < \frac{1}{2} \\ 0, & if \quad c \geq \frac{1}{2} \end{cases}$$

With separate sales, as long as $c < 1$ the seller can choose $p > c$ and obtains expected profit equal to

$$\pi^S(p) = IJ(p - c)(1 - p),$$

which is maximized by choosing $p^* = (1 + c)/2$, and produces maximum expected profit equal to

$$\pi^S(p^*) = \begin{cases} IJ(1 - c)^2/4, & if \quad c < 1 \\ 0, & if \quad c \geq 1 \end{cases}$$

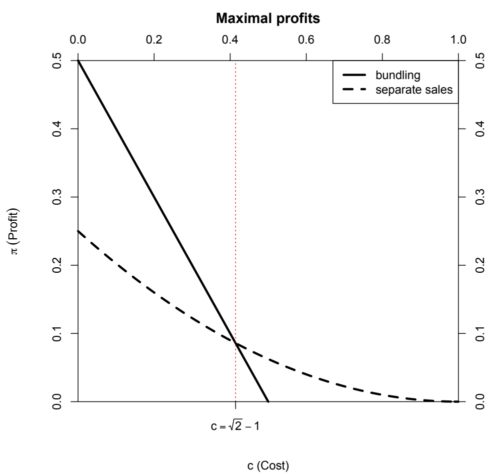

Fig. 3. Example (5.2): Profit from bundling (π^B/IJ) is higher than profit from separate sales (π^S/IJ) when marginal cost is low. The cross-over happens at $c = \sqrt{2} - 1$.

C. Example 5.3

With separate sales, demand from each buyer is equal to

$$\mathcal{D}^S(p)/J = \begin{cases} (\frac{\tau}{p})^\alpha \frac{1}{(\alpha+1)}, & if \quad p \geq \tau \\ 1 - \frac{p}{\tau} \cdot \frac{\alpha}{\alpha+1}, & if \quad 0 \leq p \leq \tau \end{cases} \tag{17}$$

The profit-maximizing price p^* for the seller is

$$0 \le p^* = \frac{\tau(\alpha+1)}{2\alpha} \le \tau \tag{18}$$

and yields the maximum expected profit

$$\pi^S(p^*) = IJ\frac{(\alpha+1)\tau}{4\alpha}. \tag{19}$$

In this setting, selling separately can never capture more than half of the optimum profit, as

$$\rho^S = \frac{\alpha^2-1}{2\alpha^2} < \frac{1}{2}, \tag{20}$$

and it is only for large α that ρ^S can come close to that $1/2$.

If there were any market exclusion with separate sales, there would be a j such that

$$U^j(x_i) = \omega^j \times \nu_i^j < p^* < \tau, 1 \le i \le I. \tag{21}$$

However, since $\nu_i^j \sim \mathcal{U}(0,1)$, for large I the maximal ν_i^j will be close to 1 for most values of j, and so most buyers j will have $U^j(x_i) > p^*$ for at least one good i. Hence there will be no significant market exclusion in this example.

With bundling the probability that a buyer will purchase at price q is (asymptotically as $I \to \infty$) equal to

$$\mathcal{D}^B/J = \begin{cases} (\frac{\tau I}{2q})^\alpha, & if \quad q \ge \tau I/2 \\ 1, & otherwise. \end{cases} \tag{22}$$

To maximize profits, the seller chooses

$$q^* = \frac{1}{2} \cdot \tau \cdot I \tag{23}$$

and obtains the expected profit

$$\pi^B(q^*) = IJ\tau/2. \tag{24}$$

The expected profit from bundling, given in Eq. (24), is always larger than the expected profit from separate sales, given in Eq. (4).

By selling bundles, the seller can come close to capturing the maximal possible profit only for large α, as $\rho^B = 1 - 1/\alpha$.

At the asymptotic price $q^* = I\tau/2$, almost everybody buys the bundle and therefore, market exclusion is essentially zero. With bundling, buyers will have a positive surplus,

$$\varsigma^B = \frac{\tau}{2(\alpha-1)}. \tag{25}$$

Buyers' surplus with separate sales is equal to

$$\varsigma^S = \int_\tau^\infty \frac{\alpha\tau^\alpha}{x^{\alpha+1}} \int_{\min\{p^*/x,1\}}^1 (yx - p^*)dydx =$$

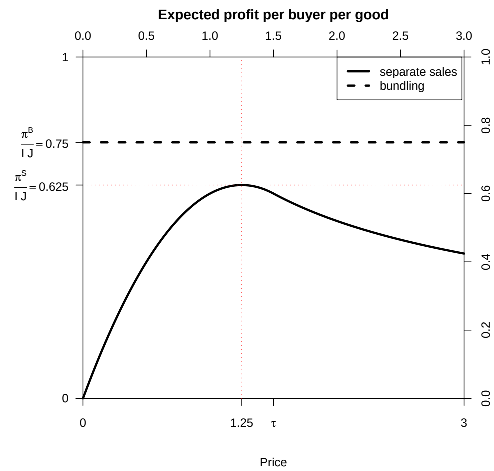

Fig. 4. Example (5.3): Expected profit from bundling ($\pi^B(q^*)/IJ$), dominates the expected profit from separate sell ($\pi^S(p)/IJ$). In this graph we assume $\tau = \alpha = 3/2$.

$$\frac{\tau(\alpha^2+3)}{8\alpha(\alpha-1)}. \tag{26}$$

In spite of the complicated expressions, comparison of buyers' surplus with bundling and with separate sales is simple, and which one is larger turns out to depend only on α:

$$\begin{cases} \varsigma^S < \varsigma^B & \alpha < 3 \\ \varsigma^S = \varsigma^B & \alpha = 3 \\ \varsigma^S > \varsigma^B & \alpha > 3 \end{cases} \tag{27}$$

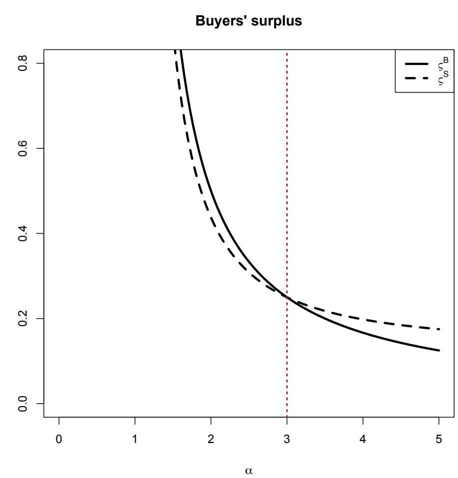

Fig. 5. Example 5.3: Buyer's surplus is greater with bundling (ς^B) when $\alpha < 3$. This show that consumer surplus is not necessary greater with bundle sale, although bundling yields greater profit.

D. Example 5.4

The expected profit for the seller with separate sales and bundling respectively are given by

$$\pi^S(p) = I \cdot (p - c) \cdot \mathcal{D}^S(p)$$

and

$$\pi^B(q) = (q - Ic) \cdot \mathcal{D}^B(q)$$

It turns out that maximum profit from bundling is equal to

$$\frac{\pi^B(q^*)}{IJ} = \begin{cases} \frac{1}{2}(\tau - 2c), & c \le c_1 \\ \frac{1}{2}\left(\frac{\tau}{\alpha}\right)^{\alpha}\left(\frac{\alpha-1}{2c}\right)^{\alpha-1}, & c \ge c_1 \end{cases} \tag{28}$$

where $c_1 = \frac{\tau(\alpha-1)}{2\alpha}$.

Maximum profit from separate sales is equal to

$$\frac{\pi^S(p^*)}{IJ} = \begin{cases} \max\left\{\frac{\tau-c}{\alpha+1}, \frac{(\tau\alpha+\tau-\alpha c)^2}{4\alpha\tau(\alpha+1)}\right\} & c \le 2c_1 \\ \max\left\{\frac{\tau-c}{\alpha+1}, \frac{\tau^{\alpha}(\alpha-1)^{\alpha-1}}{(\alpha+1)\alpha^{\alpha}c^{\alpha-1}}\right\}, & c \ge 2c_1 \end{cases} \tag{29}$$

Profit from bundling is greater than from separate sales as long as c is sufficiently small. The cross over happen precisely when

$$c = \frac{-\alpha\tau - \alpha^2\tau + \sqrt{2(\alpha^3\tau^2 + \alpha^4\tau^2)}}{\alpha^2}$$

See the figure below.

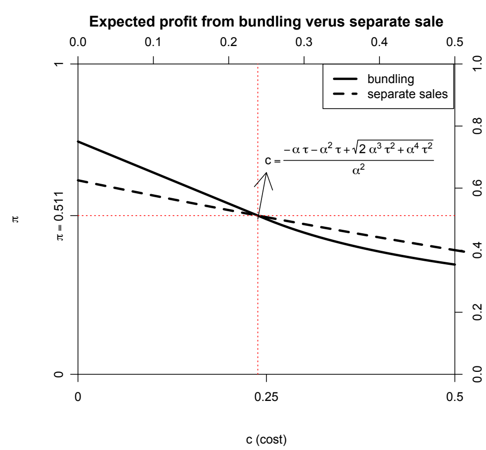

Fig. 6. Example 5.4: If we choose $\alpha = \tau = \frac{3}{2}$, we can see that bundling continues to be more profitable than separate sales when $c < 0.23$.

E. Example 5.5

With bundling the demand is equal to

$$\mathcal{D}^B(q) = \begin{cases} J & q \le \frac{1}{2}aI \\ J(1-x) & \frac{1}{2}aI < q \le aI \\ 0 & q > aI \end{cases} \tag{30}$$

There are two cases to consider, depending on which class of buyers is more numerous.

Case 1: $x < 1/2$. The seller of a bundle in this situation chooses $q^* = aI$ (or, more precisely, a price within $I^{2/3}$ of that, and receives the expected maximum profit

$$\pi^B(q^*; x < 1/2) = aIJ(1-x). \tag{31}$$

Then buyers with $\omega = \{a\}$ will be excluded from the market, so asymptotically

$$\mathcal{M}^B = x.$$

In this case ($x < 1/2$)

$$\rho^B = \frac{2(1-x)}{2-x},$$

so for $x = 0$ bundling yields the maximal possible profit, but as $x \uparrow 1/2$, this declines, and at $x = 1/2$ only $2/3$ of the maximal profit can be attained.

Case 2: $x \ge 1/2$. Here the seller of a bundle chooses $q^* = Ia/2$. The expected profit is equal to

$$\pi^B(q^*; x \ge 1/2) = \frac{a}{2}IJ. \tag{32}$$

Under these condition everybody buys and there is no market exclusion, $\mathcal{M}^B = 0$. We also have in this case

$$\rho^B = \frac{1}{2-x}.$$

The seller's expected profit with bundling can reach the maximal possible profit (of bundling) when either $x = 0$ or $x = 1$. With separate sales, demand is equal to

$$\mathcal{D}^S(p) = \begin{cases} J[1 - \frac{p}{2a}(1+x)], & p < a \\ J[(1 - \frac{p}{2a})(1-x)], & a \le p < 2a \\ 0, & p \ge 2a \end{cases} \tag{33}$$

A short calculation shows the profit maximizing price is $p^* = a/(1+x)$, and yields profit

$$\pi^S(p^*) = \frac{IJa}{2(1+x)}. \tag{34}$$

With separate sales, just as with bundling, the seller's profit is closest to optimum profit when either $x = 0$ or $x = 1$, but it cannot exceed half of the optimum profit, as we have

$$\rho^S = \frac{1}{(1+x)(2-x)}$$

With separate sales there is practically no market exclusion, since $p^* < a$.

With bundling, for values of $x < 1/2$, there is essentially no consumer surplus, as the seller extracts the maximal willingness to pay from the buyers with $\omega = \{2a\}$ and those with $\omega = \{a\}$ are excluded, so $\varsigma^B = 0$.

For $x \geq 1/2$, the optimum price of the bundle will be $q^* = Ia/2$, and buyers with $\omega = \{2a\}$ will be the only ones with positive surplus. So the average surplus per buyer per good will be

$$\varsigma^B = a(1-x)/2.$$

With separate sales, given $p^* = a/(1+x)$, buyers with $\omega = \{a\}$ on average have surplus equal to

$$\frac{ax^2}{2(1+x)^2}$$

and buyers with $\omega = \{2a\}$ on average have surplus equal to

$$\frac{(a + 4ax + 4ax^2)}{4(1+x)^2}.$$

Therefore, the average surplus per buyer will be

$$\varsigma^S = a\frac{1 + 3x - 2x^3}{4(1+x)^2}.$$

F. Example 5.6

In the previous example, if the seller of the bundle chooses $q \leq \frac{1}{2}Ia$ then almost everybody buys the bundle so the entire revenue comes from the bundling. If $q \geq Ia$ then nobody buys the bundle and the seller can only sell separately. If the seller selects $\frac{Ia}{2} < q < Ia$, revenue from bundling will be $q(1-x)$, but Jx buyers with $\omega = \{a\}$ will be excluded from the market. Mixed bundling allows the seller to offer individual goods at a price p.

Profit from mixed bundling is equal to

$$\pi^m(p,q) = q\big(\sum_{1\leq j\leq J} Prob\{\omega^j IE[\nu] \geq q\}\big) + Ip\big(\sum_{j:\omega^j IE[\nu]<q} Prob\{\omega^j \nu \geq p\}\big). \quad (35)$$

The seller chooses

$$(p^*, q^*) \in \arg\max \pi^m(p,q)$$

to maximize his or her expected profit from mixed bundling. In our case, it is easy to see that $p^* = \frac{a}{2}$ and $q^* = Ia$ and the maximum expected profit from mixed bundling is equal to

$$\pi^m(p^*, q^*) = IJa(1 - \frac{3}{4}x). \quad (36)$$