MINIMUM REDUNDANCY FILTERBANK PRECODER FOR BLIND CHANNEL IDENTIFICATION IRRESPECTIVE OF CHANNEL NULLS *

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Abstract - *In this paper we propose a blind deterministic method for channel estimation based on the introduction of minimal redundancy on the transmitted data sequence through linear precoders that map consecutive blocks of information symbols onto higher size blocks. This transmission scheme incorporates, for example, OFDM and CDMA systems. We prove that even allowing the transmitted data block to be longer than the information block by only one sample is sufficient to guarantee the channel identification uniquely with a deterministic algorithm, without any restriction on the channel zero location and under sufficient conditions on the precoder only, which can be easily checked a-priori. Our proposed blind estimation method is able to work with any amount of extra redundancy and this renders the method particularly useful for all applications where the channels have long impulse responses, such as in wired as well as in wireless macrocellular communication systems.*

I. INTRODUCTION

Reliability and efficiency are the most desirable and conflicting requirements in the design of a communication system. In general, adding redundancy is a means to increase reliability. However, extra redundancy comes at the price of extra bandwidth or power, which are both critical resources in mobile systems. Furthermore, channel variability due to Doppler effects or carrier asynchronism puts a limit on the maximum decoding delay and then precoding depth. Therefore, the optimal trade-off between reliability and efficiency pushes the research towards methods capable of providing reliable channel estimates using short data blocks, which do not rely upon much a-priori information about the channel, possibly without sending long training sequences. Although not explicitly acknowledged, *all blind* communication systems introduce redundancy relative to what is dictated by Shannon's limit. Higher-order schemes, such as CMA [2], do so by requiring non-Gaussianity of the input. Fractionally spaced approaches require input redundancy in the form of excess bandwidth, while multiple antennas entail spatial redundancy. On the other

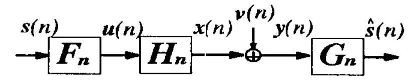

Figure 1. - Block model for filterbank transmission systems

hand, multi-carrier (e.g., OFDM) and spread-spectrum (e.g., CDMA) systems capitalize on code-redundancy in order to mitigate frequency-selective fading. A common framework describing all redundancy-inducing systems was introduced in [3] by using multirate filterbank transceivers. More specifically, the precoding schemes considered in [3], [4] operate a linear mapping of blocks of M information symbols into expanded blocks of P samples, with $P > M$, which are transmitted through the digital link. Because $P > M$ the information rate is lower than the transmission rate, and this loss in efficiency is the price to be paid to obtain several advantages. In [3] it was proved that these schemes offer the possibility to invert *any* FIR channel using FIR filterbank equalizers and the sufficient conditions for the existence of a ZF equalizer for any FIR channel were expressed only in terms of the coding scheme and required only an overestimation of the FIR channel order, which is usually available. However, the blind channel estimation method of [4] considered only precoders with null guard intervals of length at least equal to the channel order.

In this paper we extend the blind deterministic channel estimation method of [4] to the most general class of precoders using *any* amount of redundancy and in particular much less than the channel memory. This makes the new method particularly attractive for communications over very long impulse response channels.

II. SYSTEM MODEL

Our precoder operates as a linear redundant block encoder that maps blocks of M input symbols into blocks of P encoded data, which are subsequently modulated and sent through the channel. The information rate is $(M/P)/T$, where $1/T$ is the transmission rate. Block coding schemes can be compactly described using block,

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or vector, convolutions (see Fig.1). This model can represent a single user system, e.g. an OFDM, or the downlink channel of a multi-user system. In the first case, the $M \times 1$ information symbol vectors $s(n)$ are formed by a serial to parallel conversion of the single user information data stream, i.e. $s(n) := (s(nM), \ldots, s(nM+M-1))^T$ whereas, in the second case, the mth component of $s(n)$ is the symbol pertaining to the mth user information stream $s(n) := (s_1(n), \ldots, s_M(n))^T$. The encoder and decoder operate the following mapping

$$u(n) = \sum_{i=-\infty}^{\infty} F_i s(n-i), \quad \hat{s}(n) = \sum_{j=-\infty}^{\infty} G_j y(n-j), \quad (1)$$

where F_i is a sequence of $P \times M$ encoding matrices, with $P > M$, $u(n)$ represents the $P \times 1$ encoded vector, G_j are the $M \times P$ decoding matrices and, similar to $u(n)$, $y(n) := (y(nP), \ldots, y(nP+P-1))^T$ is the received data block, while $\hat{s}(n)$ is the reconstructed block of symbols that, in the absence of noise, should be equal to $s(n)$. The redundant mapping described in (1) can be implemented via a multirate filterbank [3]. Most existing encoding systems are of order zero, i.e. $F_i = F_0\delta(i)$, although there could be some advantages in using higher order encoders (see also [6]). In a zero-order encoder, the columns of F_0 in the multiuser case are the so called spreading codes, where P is the spreading factor, usually equal to the code length, while in OFDM systems they are the complex exponentials [3], i.e. $\{F_0\}_{n,m} = e^{j\frac{2\pi}{M}mn}$ for $m \in [0, M-1]$, $n \in [0, P-1]$, where $P = M + L$ to include the cyclic prefix. In this paper we will concentrate on zero-order precoders, although most of the proposed techniques can be extended to precoders with block-memory.

With reference to Fig. 1, the channel output can also be represented as a block convolution

$$x(n) = \sum_{l=-\infty}^{\infty} H_l u(n-l), \quad (2)$$

where, denoting by $h(n)$ the channel impulse response, H_l are $P \times P$ Toeplitz matrices with entries

$$\{H_l\}_{k,p} = h(lP + k - p) \quad p, k \in [0, P-1]. \quad (3)$$

It is important to remark that, even if the precoder is block memoryless, the blocks at the output of the channel are in general superimposed. Let us make the following assumptions:

(a0) Channel $h(l)$ is Lth order FIR with $h(0), h(L) \neq 0$;
(a1) (P, M, L) are chosen such that the triplet (P, M, L) satisfies: $K := P - M > 0$ and $P > L$;
(a2) The precoder F_i is block memory less and full column rank and the decoder G_k is Q-th order.

Considering the noise free case ($v(n) = 0$ and $y(n) = x(n)$), in force of (a1), from (3) and (2) it follows that $x(n) = H_0 u(n) + H_1 u(n-1)$. Matrix H_1 has all zero elements except for the first L rows and last L columns, thus in every received block the first L samples of $x(n)$ are affected by interblock interference (IBI). Nonetheless, the introduction of redundancy allows perfect symbol recovery even using a *finite* order decoder. In fact, building the vector $\chi(n)$ by stacking Q consecutive vectors $x(n)$ as $\chi(n) := (x^T(Qn), \ldots, x^T(Qn + Q-1))^T$, we can relate the vector $\chi(n)$ to Q transmitted blocks $\psi(n) := (u^T(Qn), \ldots, u^T(Qn + Q - 1))^T$ through the $(QP - L) \times QP$ fat Sylvester matrix $\mathcal{H}$ with first column $(h(L), 0, \ldots, 0)^T$, and first row $(h(L), \ldots, h(0), 0 \ldots, 0)$, writing

$$(0, I_{(QP-L)\times(QP-L)})\chi(n) = \mathcal{H}\psi(n). \quad (4)$$

Let us define the $QP \times QM$ block diagonal matrix $\mathcal{F}$ and the $M \times QP$ equalizing matrix $\mathcal{G}$ as

$$\mathcal{F} := I_{Q\times Q} \otimes F_0, \quad \mathcal{G} = (G_{Q-1}, \ldots, G_0) \quad (5)$$

where $\otimes$ stands for Kronecker product and, in particular, $G_{Q-1} := (0_{M\times L}\ , \tilde{G}_{Q-1})$ is equipped with L leading zeros to cancel the IBI. Because $\psi(n) \equiv \mathcal{F}(s^T(Qn), \ldots, s^T(Qn+Q-1))^T$ and, defining $\xi(n) := (s^T(Qn), \ldots, s^T(Qn + Q - 1))^T$, we can write

$$(0, I_{(QP-L)\times(QP-L)})\chi(n) = \mathcal{HF}\xi(n). \quad (6)$$

Since the matrix $\mathcal{HF}$ is $(QP - L) \times QM$, by a proper selection of the parameters P and M, for a given L, it is possible to invert the system (6) exactly, using a finite order Q. Moreover, every block is observed Q times, therefore the equalizer $\mathcal{G}$ can be also optimized with respect to the block delay, selecting the set of M consecutive rows of $(\mathcal{HF})^\dagger$ that have minimum norm. Conditions necessary to guarantee the invertibility of $\mathcal{HF}$, for *any* channel are given in [3].

III. Blind Channel Estimation

Building upon the results of [4], this section shows that not only $P = M + 1$ is sufficient to allow perfect ZF equalization with FIR filterbank equalizer for any channel $\mathcal{H}$, but also that, under much wider conditions on the transmit filterbank F_0, *it is possible to identify the channel h blindly, with a deterministic algorithm, from a finite number of blocks $x(n)$, without any restriction on the channel zeros locations.*

Recalling the notation introduced in Section II, collecting $Q + N$ consecutive blocks from the data we can form a $(QP - L) \times N$ block Hankel matrix $\mathcal{X}(n)$ given by:

$$\mathcal{X}(n) = \mathcal{HFS}(n), \quad (7)$$

where $\mathcal{S}(n)$ is the $QM \times N$ block Hankel matrix with first column $(s^T(Qn), \ldots, s^T(Qn+Q-1))^T$ and last M rows $(s(Qn+Q-1), \ldots, s(Qn+N+Q-1))$. Let us make the following extra assumptions:

(a4) the input data are persistently exciting in the sense that for N large enough and $N \geq QM$, the $QM \times QM$ matrix $\mathcal{S}(n)\mathcal{S}^H(n)$, is full rank;

(a5) $P = M + K$, with $K > 0$ and Q is such that $QK \geq L+1$, so that $\mathcal{X}(n)\mathcal{X}^H(n)$ has dimensionality

$$QP - L = QM + (QK - L) > QM. \qquad (8)$$

Invoking (a4) and observing that $\mathcal{HF}$ has more rows than columns, (7) yields:

$$\begin{aligned} \nu(\mathcal{X}(n)\mathcal{X}^H(n)) &= \nu(\mathcal{HF}\mathcal{S}(n)\mathcal{S}^H(n)\mathcal{F}^H\mathcal{H}^H) \qquad (9)\\ &= \nu(\mathcal{HF}\mathcal{F}^H\mathcal{H}^H) \\ &\geq QP - L - QM = (QK - L) \geq 1, \end{aligned}$$

Hence the null space of the matrix $\mathcal{X}(n)\mathcal{X}^H(n)$ does not contain only the null vector. In particular, let us denote by $\tilde{\boldsymbol{u}} \neq \mathbf{0}$ a non null vector lying in the null space $\mathcal{N}(\mathcal{X}(n)\mathcal{X}^H(n))$ and by $\tilde{\mathcal{U}}$ the $(L+1) \times QP$ Hankel matrix with first column $(0, \ldots, u^*(0))$ and last row $(\tilde{\boldsymbol{u}}^H, \mathbf{0}_{L\times 1})$. Assuming, without loss of generality, that the precoder matrix can be decomposed as

$$\boldsymbol{F}_0 = \left(\boldsymbol{I}_{M\times M}, \boldsymbol{\Phi}^T\right)^T \boldsymbol{F} \qquad (10)$$

we introduce the polynomials

$$\begin{aligned} (\Phi_0(z) - z^{-M}, \ldots, \Phi_{P-M-1}(z) - z^{-P})^T &:= \\ (\boldsymbol{\Phi}, -\boldsymbol{I}_{K\times K})(1, z^{-1}, \ldots, z^{-P+1})^T \qquad (11) \end{aligned}$$

and state the following assumption, necessary for the ensuing theorem:

(a6) starting from matrix $\boldsymbol{\Phi}$, defined in (10), there does not exist pairs of Q-th order polynomials $A_k(z)$ and $A'_k(z)$ with at least one distinct root such that the ratio

$$\frac{\sum_{k=0}^{K-1}(\Phi_k(z) - z^{-M-k})A_k(z^P)}{\sum_{k=0}^{K-1}(\Phi_k(z) - z^{-M-k})A'_k(z^P)} \qquad (12)$$

simplifies into the ratio of polynomials of degree $< L$.

The following theorem establishes the identifiability conditions, which guarantee that the channel impulse response can be estimated blindly within a scale ambiguity (see [5] for the proofs of this theorem and of the following lemmas):

Theorem 1. *Assume (a0)-(a6), and define the $(QP - L) \times N$ data matrix $\mathcal{X}(n)$ as in (7). The matrix $\mathcal{X}(n)$ is such that $\nu(\mathcal{X}(n)\mathcal{X}^H(n)) \geq 1$ and considering a vector $\tilde{\boldsymbol{u}} \neq \mathbf{0}$ lying in the null space $\mathcal{N}(\mathcal{X}(n)\mathcal{X}^H(n))$, the $(L+1) \times QM$ matrix $\tilde{\mathcal{U}}\mathcal{F}$ has nullity $\nu(\tilde{\mathcal{U}}\mathcal{F}\mathcal{F}^H\tilde{\mathcal{U}}^H) = 1$ and the channel impulse response spans its null space, i.e.:*

$$\boldsymbol{h}^T\tilde{\mathcal{U}}\mathcal{F}\mathcal{F}^H\tilde{\mathcal{U}}^H = \mathbf{0}. \qquad (13)$$

Therefore the channel can be identified uniquely from the data up to a scalar factor by solving (13).

Assumption (a6) seems rather complicated to verify in general, except for the following interesting cases:

Lemma 1. *(Minimal redundancy) Under assumptions (a0)-(a5), if $P = M + 1$, for any $\boldsymbol{F}_0$ of rank M, the channel $\boldsymbol{h}$ can be identified uniquely up to a scale ambiguity, solving (13).*

Lemma 2. *(Cyclic prefix) If cyclic prefixes are used, i.e. $\boldsymbol{\Phi} = (\boldsymbol{I}_{K\times K}, \mathbf{0})$ in (10), and $K > 1$, the channel cannot be estimated uniquely through (13).*

An interesting aspect of Theorem 1 is that there are conditions for channel identifiability that depend only on the structure of $\mathcal{F}$, which is under our control. The questions that arise naturally are: i) can one apply the blind method to existing precoding techniques, even without satisfying assumption (a6), which seems to be too restrictive? ii) whenever $\nu(\mathcal{X}(n)\mathcal{X}(n)^H) > 1$, can we use the other vectors in the null space to estimate channels which are not identifiable resorting only to one null vector $\tilde{\boldsymbol{u}}$? From (9) it is clear that increasing Q, for given K and L, the dimension of the null-space of $\mathcal{X}(n)\mathcal{X}^H(n)$ increases and we can then extend our algorithm to incorporate all independent vectors of $\mathcal{N}(\mathcal{X}(n)\mathcal{X}^H(n))$ in the channel estimate. In fact, in force of (a4), $\mathcal{N}(\mathcal{X}(n)\mathcal{X}^H(n)) \equiv \mathcal{N}(\mathcal{HF}\mathcal{F}^H\mathcal{H}^H)$ and denoting by $\{\boldsymbol{u}_1, \ldots \boldsymbol{u}_J\}$ a basis of $\mathcal{N}(\mathcal{X}(n)\mathcal{X}^H(n))$, for each $\boldsymbol{u}_j$, $j = 1, \ldots, J$ we can introduce the Hankel matrix $\mathcal{U}_j$, built as $\mathcal{U}$. Then, arguing as in (13) in Theorem 1, the vector $\boldsymbol{h}$ can be found in the intersection of the left null spaces of the matrices $\mathcal{U}_j\mathcal{F}$, i.e.

$$\boldsymbol{h}^T\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^H = \mathbf{0}, \qquad \boldsymbol{\Upsilon} := (\mathcal{U}_1\mathcal{F}, \ldots \mathcal{U}_J\mathcal{F}) \qquad (14)$$

and condition (a6) is only sufficient but not necessary to ensure that $\nu(\boldsymbol{\Upsilon}) = 1$. Thus, if identifiability is lost by using the minimum K such that $\nu(\mathcal{X}(n)\mathcal{X}^H(n)) > 0$, increasing Q one can exploit the extra null vectors to remove the ambiguity. The numerical results shown in Section IV will confirm these statements.

There are two relevant aspects related to Theorem 1: i) the conditions for channel invertibility or identifiability are not equivalent because even if the channel cannot be equalized, it can be still identified because invertibility of $\mathcal{HF}$ is not required in the estimation procedure - a significant example of this statement is given by OFDM with cyclic prefix; ii) Theorem 1 does not require the channel order to be known, as long as an upper-estimate is available.

IV. PERFORMANCE AND NUMERICAL RESULTS

In this section we provide numerical results that substantiate the theory developed in Sections III. Before illustrating the examples, we briefly recall the basic assumptions underlying our experiments. In general, to form a $QP \times N$ block Hankel data matrix $\boldsymbol{\mathcal{X}}(n)$, the number of data-blocks needed is $(Q + N)$, which results in a total number of samples $N_{samples} := (Q + N)P$ corresponding to a number of information symbols $N_{symbols} := (Q + N)M$, which boils down to an information rate $\mathcal{E} := M/P = M/(M + K)$, where K is our ***controlled*** redundancy. We evaluate the performance in terms of mean square error (MSE) and bit error rate (BER) versus the ratio between the average energy per symbol E_s and the noise spectral density N_0. The symbols constellation is QPSK. Finally, the blind channel estimation method, as any blind method, is able to provide a channel estimate, up to a constant scale factor. To avoid any further complication not directly related to the channel estimation method, in our simulations we multiply the solution of (13) (or (14)) by its correlation coefficient with the true channel vector $\boldsymbol{h}$.

Example 1 (Average channel MSE over Rayleigh fading): In this experiment we test the average performance of our estimation method over frequency selective Rayleigh fading channels, modeled as FIR filters of order $L = 3$ with complex, equally distributed, uncorrelated, Gaussian taps, each of variance $1/\sqrt{L}$. Each encoded symbol block is of size $M = 8$. We analyze the performance of OFDM precoders with cyclic prefixes of variable length, i.e. $K = 1, 2$, and 4, to study the effects of introducing different amounts of redundancy. Each estimate is obtained using the minimum value of N, i.e. $N = QP$. $K = 1, 2, 4$, corresponds respectively to transmit at information rates: $\mathcal{E} \approx 0.89, 0.80, 0, 67$. In Fig.2 a) we show the MSE, averaged over 500 independent random channels, obtained by using the minimum value of $Q = (L + 1)/K$ to satisfy (a5), which corresponds also to the minimum number of samples, i.e. $N_{samples} := Q(1 + P)P = 360, 220, 156$, for $K = 1, 2$, and 4, respectively. Since the number Q of blocks in the super block is minimum, for each K, we can rely upon the presence of at most one null vector $\tilde{\boldsymbol{u}}$ to estimate the channel by solving (13). From Fig.2 a) we observe that increasing the amount of redundancy by increasing K, for the same M, does not bring any benefit to channel identifiability and this result is rather counterintuitive. Nevertheless, this behavior is perfectly understandable on the basis of our theory. In fact, according to Lemma 1, the identifiability through (13) is guaranteed for $K = 1$ and this justifies the good behavior of the curve referring to $K = 1$. On the other hand, Lemma 2 proves that, for $K > 1$, using cyclic prefixes some chan-

(a)
(b)

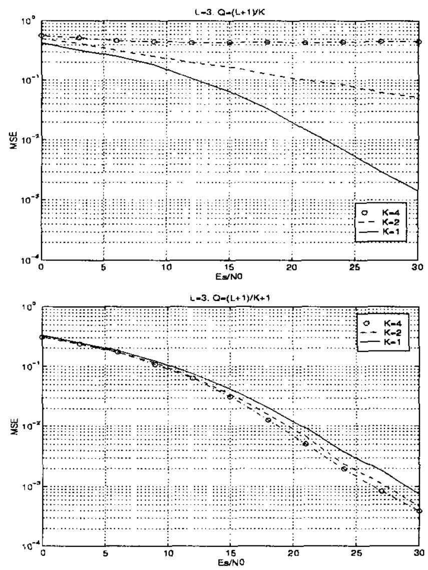
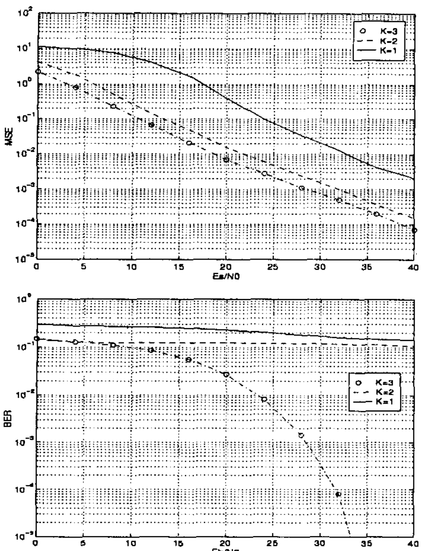


Figure 2. Average channel MSE for OFDM $(M, L) = (8, 3)$ cyclic prefix length $K = 1, 2, 4$, (a) Q=4,2,1; (b) Q=5,3,2.

nels may be estimated ambiguously and this, together with the smaller number of data observed, justifies the performance loss of the cases referring to $K > 1$ with respect to the case $K = 1$. The total lack of identifiability becomes evident by looking at the curve relative to $K = 4$, where $Q = 1$ is sufficient to build a rank deficient matrix. In this case, in fact, the same Lemma 2 shows that for $L = 3$ and $K = 4$, the identifiability is lost for *any channel* of order $L = 3$. However, according to the estimation method based on (14), the cure for this undesired behavior is found by increasing Q, for any given K, and thus expanding the null-space of the data matrix while preserving, at the same time, the information rate. The results obtained using the algorithm based on (14), using $Q = (L + 1)/K + 1$, are shown in Fig. 2 b). The data samples are in this case $N_{samples} = Q(1 + P)P = 450, 330, 312$ and we can see that the methods perform now equally well for all values of K.

Example 2 (Estimation versus ZF-equalization): In this case, we compare the MSE and the BER obtained with different values of K, in cases where the channel can be identified from the data but the symbols cannot be recovered through a linear equalizer. In our simulations we used again the minimum number of data to perform channel estimation which is $N = QP$ in conjunction with the minimum $Q = \lceil (L + 1)/K \rceil$. More specifically, we simulate the downlink of a CDMA system using Hadamard codes of length $M = 12$, with 11, 10 and 9 active users,



(a)

(b)

Figure 3. (a) Channel MSE (b) BER , $L = 6$, $M = 11, 10, 9$ Hadamard codes with $K = 1, 2, 3$ and $Q = 7, 4, 3$.

which corresponds to $K = 1, 2$ and 3, respectively. To test the ability of our estimation method when the necessary conditions for ZF-FIR channel equalization are violated, we consider a 6th order channel with zeros at $[(0.5\exp(j2\pi/P),\ 0.5\exp(j4\pi/P),\ 0.5\exp(j6\pi/P),$ $0.2\exp(j8\pi/P), 2.5\exp(-j4\pi/P), 1.5\exp(-j8pi/P))]$. Three of these zeros lie on a circle of radius 0.5 and, according to condition (1.a) of Theorem 1 in [3], this channel is not invertible through a linear equalizer when $(P - M) = K < 3$. This implies that, for $K = 1, 2$ the ZF equalizer performance will drop down, while for $K = 3$, this does not apply and indeed the BER curves shown in Fig. 3(b) confirm what predicted by the theory. Conversely, the MSE curves shown in Fig. 3(a) decrease asymptotically as $1/SNR$, thus proving that channel identifiability is not affected by the circumstance that the matrix $\mathcal{HF}$ is not invertible. In summary, channel estimation and equalization are distinct issues and do not impose the same restrictions on the precoder. As far as the channel estimation is concerned, the choice $K = 1$ clearly appears to be the most robust and convenient in terms of efficiency, but, unless sufficiently long scrambling codes are used, $K = 1$ is the most vulnerable choice for channel equalization.

Example 3 (Blind decision directed (DD) method): In this example, we take the samples at the output of our ZF receiver, built by using the blind channel estimation, and decide about the transmitted symbols. The decided symbols are then used as training data to update the

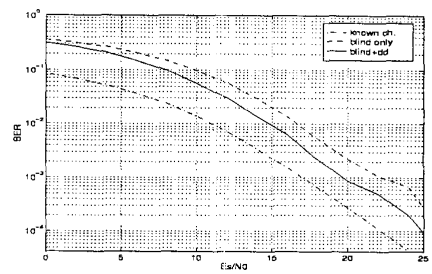


Figure 4. BER for $(M, L, K) = (10, 2, 2)$ an Hadamard precoder with and Q=3.

channel estimate. The performance is again obtained by averaging over 500 independent Rayleigh channels, generated as in Example 1. We simulated a precoder using Hadamard codes of length $P = 12$ with $M = 10$ and thus, $K = 2$ and $\mathcal{E} = 0.83$. We also used $Q = 3$ and $N = 46$ blocks, for a total amount of samples (or chips) of $N_{samples} = (Q + N)P = 588$ used in each iteration of the batch algorithm (corresponding to 49 information symbols fro each user). Fig. 4 shows the BER obtained i) assuming known channel (dash-dotted line); ii) using blind channel estimate (dashed line); iii) using the DD method described above (solid line). We observe that the DD method recovers most of the loss due to the initial blind estimation and its loss with respect to the ideal case, when the channel is perfectly known at the receiver, is moderate, although the method is still fully blind.

References

[1] G. B. Giannakis, "Filterbanks for blind channel identification and equalization," *IEEE Sig. Proc. Letters*, pp. 184-187, June 1997.

[2] D. N. Godard, "Self-recovering equalization and carrier-tracking in two-dimensional data communication system", *IEEE Trans. on Comm.*, Vol.COM-28, pp.1867-1875, 1980.

[3] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant Filterbank Precoders and Equalizers, Part I: Unification and Optimal Designs," *IEEE Trans. on on Sig. Proc.*,Vol. 47, pp.1988–2006, July 1999.

[4] ——, "Redundant Filterbank Precoders and Equalizers, Part II: Blind Channel Estimation, Synchronization and Direct Equalization," *IEEE Trans. on Sig. Proc.*, Vol. 47, pp.2007–2022, July 1999.

[5] ——, "Channel invertibility and identifiability using a controlled redundancy," submitted to *IEEE Trans. on Sig. Proc.*.

[6] G.W. Wornell, "Spread-signature CDMA: Efficient multiuser communications in the presence of fading," *IEEE Trans. on Inf. Th.*, vol.41, pp. 1418-1438, September 1995.

[7] G. Xu, H. Liu, L.Tong, T. Kailath, "A Least-Squares Approach to Blind Channel Identification", *IEEE Trans. on Sig. Proc.*, pp.2982-2993, Dec. 1995.