

# Generalized Frequency Hopping OFDMA through Unknown Frequency-Selective Multipath Channels<sup>†</sup>

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## ABSTRACT

The Generalized Frequency Hopping (GFH) OFDMA system developed in this paper permeates FH benefits to multi-carrier CDMA transmissions through unknown frequency-selective multipath channels. No single user suffers from consistent fading effects and multiuser interference is eliminated by design. Constellation-irrespective channel identifiability is guaranteed, and a host of blind channel estimation algorithms are developed trading off complexity with performance. An important variant corresponding to fast-hopping is also addressed. Performance analysis and simulation results illustrate the merits of GFH-OFDMA relative to conventional OFDMA and long code CDMA with RAKE reception.

## I. INTRODUCTION

Introduction of the direct-sequence (DS) CDMA technology to wireless communications has added many desirable features such as robustness to fading, narrow band interference suppression and dynamic spectrum sharing. A practical feature of CDMA is that of aperiodic spreading that distributes users' spectrum over the available bandwidth uniformly and enables differentiation of neighboring base stations in a cellular setting. Although long spreading codes adopted by the IS-95 standard are beneficial to improving capacity of CDMA systems, they do not facilitate usage of existing (and possibly blind) channel estimation and equalization schemes. Blind channel estimation algorithms have been proposed for long-code CDMA, see e.g., [3], [7], [8]. But even when Channel State Information (CSI) is utilized at the receiver, decoding requires high-complexity time-varying equalization. Furthermore, multiuser interference (MUI) is only suppressed statistically.

A low-complexity CDMA scheme, termed as Lagrange-Vandermonde (LV) CDMA, was proposed recently by generalizing orthogonal frequency-division multiple access (OFDMA) transceivers [6]. LV-CDMA eliminates MUI deterministically in the presence of possibly unknown and even rapidly varying multipath, but suffers (similar to OFDMA) from channel fading and may require extra diversity to ameliorate the effect of channel nulls. Among other choices detailed in [6], frequency-hopping (FH) offers such a diversity. For multi-carrier CDMA, adaptive FH was also advocated by [1], assuming that CSI is available at the transmitter through a reliable feedback control channel (see also [4] where FH-OFDMA was proposed for CATV transmissions).

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In this paper, we develop a Generalized FH-OFDMA system which achieves MUI elimination by design and brings FH benefits to long code CDMA over unknown multipath fading. By adopting time-varying Vandermonde-Lagrange codes, GFH-OFDMA is developed as a structured long code CDMA scheme. Exploiting the finite alphabet property of our source, novel blind channel estimators are derived with guaranteed channel identifiability. Symbol recovery can also be assured through fast hopping. Simulation results are presented to support our conclusions.

## II. SYSTEM MODELING AND MOTIVATION

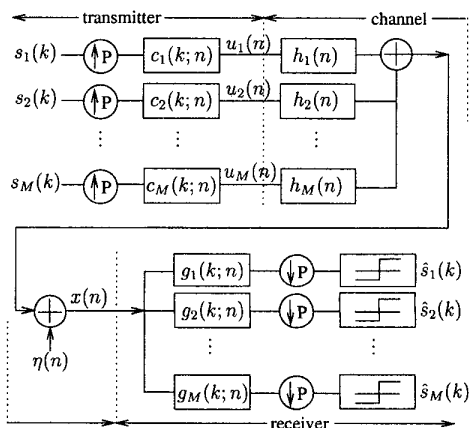


Fig. 1. Discrete-time baseband long code CDMA system

The block diagram in Fig. 1 describes a discrete-time baseband equivalent model of a CDMA system in the uplink scenario, where signals, codes and channels are represented by samples of their complex envelopes taken at the chip rate. Related filterbank models were adopted in [6] and [5] for symbol-periodic coded systems. But similar to [8] and [3], our interest is on CDMA systems with long (or aperiodic) codes. Each of the  $M$  users (say the  $m$ th user) spreads the information symbols  $s_m(k)$  with the upsampler and encodes it using the  $P$ -long time varying code  $c_m(k;n)$ , where  $k$  signifies symbol-dependence of the chips that are indexed by  $n \in [0, P - 1]$ . The  $m$ th user's coded chip sequence  $u_m(n) = \sum_{k=-\infty}^{\infty} s_m(k)c_m(k;n - kP)$  is then pulse shaped and transmitted through a (possibly unknown) dispersive channel. After being filtered by the receive filter (with Nyquist characteristics) the  $m$ th user's received signal at the chip rate can be written as:  $x_m(n) =$

$\sum_{k=-\infty}^{\infty} s_m(k) \sum_{l=-\infty}^{\infty} h_m(l) c_m(k; n-l-kP)$ , where  $h_m(l)$  (with order  $L_m$ ) denotes the equivalent discrete time channel impulse response that includes the  $m$ th user's asynchronism in the form of delay factors as well as transmit-receive filters and multipath effects.

With  $\eta$  denoting filtered AGN and  $L \geq \max(L_1, \dots, L_M)$  being an upper bound on all FIR channel orders, the received signal from all users sampled at chip rate is:  $x(n) := \sum_{m=1}^M x_m(n) + \eta(nT_c)$ , where

$$x_m(n) = \sum_{k=-\infty}^{\infty} s_m(k) \sum_{l=0}^L h_m(l) c_m(k; n-l-kP). \quad (1)$$

The sequence  $x(n)$  is then correlated with the  $N_g$ -long receive-filter  $g_m(k; n)$ , which is equivalent to convolving  $x(n)$  with its conjugated and time-reversed version  $g_m^*(k; N_g - n)$ . The resulting output is downsampled by  $P$ , and the decision is made through the estimate

$$\hat{s}_m(k) = \sum_{n=0}^{N_g-1} g_m(k; n) x(kP + n). \quad (2)$$

To cast (1) and (2) in vector-matrix form, we define the  $P \times 1$  polyphase vectors:  $\mathbf{x}(k) := [x(kP), x(1+kP), \dots, x(P-1+kP)]^T$ ,  $\boldsymbol{\eta}(k) := [\eta(kP), \eta(1+kP), \dots, \eta(P-1+kP)]^T$ ,  $\mathbf{g}_m(k; q) := [g_m(k; qP), g_m(k; 1+qP), \dots, g_m(k; P-1+qP)]^T$ , the  $(L+1) \times 1$  channel vector:  $\mathbf{h}_m := [h_m(0), h_m(1), \dots, h_m(L)]^T$ , and the  $P \times (L+1)$  Toeplitz code matrix

$$\mathbf{C}_m(k; q) = \begin{pmatrix} c_m(k; qP) & \dots & c_m(k; qP-L) \\ c_m(k; qP+1) & \dots & \vdots \\ \vdots & \ddots & \vdots \\ c_m(k; qP+P-1) & \dots & c_m(k; qP+P-L-1) \end{pmatrix}. \quad (3)$$

We then obtain from (1)

$$\mathbf{x}(k) = \sum_{m=1}^M \sum_{k'=-\infty}^{\infty} s_m(k') \mathbf{C}_m(k'; k-k') \mathbf{h}_m + \boldsymbol{\eta}(k). \quad (4)$$

Because the  $P$ -long codes convolved with the FIR channels of maximum order  $L$  yield a sequence of length  $P+L$ , we have from (3) that  $\forall k$ ,  $\mathbf{C}_m(k; q) \equiv \mathbf{0}$  for  $q < 0$  and  $q > \lfloor (P+L)/P \rfloor$ . Hence, (4) has only  $Q_c := \lfloor (P+L)/P \rfloor$  nonzero terms in the summation over  $k'$  and can be re-written as:

$$\mathbf{x}(k) = \sum_{m=1}^M \sum_{q=0}^{Q_c} \mathbf{C}_m(k-q; q) \mathbf{h}_m s_m(k-q) + \boldsymbol{\eta}(k). \quad (5)$$

Recall that  $g_m(k; n)$  in (2) has length  $N_g$ , define  $Q_g := \lfloor N_g/P \rfloor$ , and express (2) as  $\hat{s}_m(k) = \sum_{q=0}^{Q_g} \sum_{n=0}^{P-1} g_m(k; qP+n) x(kP+qP+n)$ , which turns out to be:

$$\hat{s}_m(k) = \sum_{q=0}^{Q_g} \mathbf{g}_m^T(k; q) \mathbf{x}(k+q). \quad (6)$$

Equations (5) and (6) provide a general vector model for time varying CDMA systems that employ long codes as those studied in [3] and [8].

Unlike [8] that models long codes as asymptotically uncorrelated sequences, we in this paper deal with *structured* long codes that (as we shall see next) offer: i) deterministic MUI cancellation with a simple linear receiver; ii) blind channel estimation with simple equalization capabilities.

### III. GENERALIZED FH-OFDMA WITH LONG CODES

In this section we will design our MUI eliminating transceivers and derive several blind channel estimators when Channel State Information (CSI) is not available.

#### A. CSI available - MUI eliminating transceivers

We seek here user codes  $\{c_m(k; n)\}_{m=1}^M$  and receivers  $\{g_m(k; n)\}_{m=1}^M$  that achieve MUI elimination by design. The maximum channel order  $L$  may be large in the completely asynchronous case. In quasi-synchronous (QS) CDMA however, mobile users attempt to synchronize with the base-station's pilot waveform [2]; hence,  $L$  is satisfied with a small value. So with  $P > L$ , we have  $\mathbf{C}_m(k; q) = \mathbf{0}$ , for  $q \neq 0, 1$ , and we can thus rewrite (5) as:

$$\mathbf{x}(k) = \sum_{m=1}^M \mathbf{C}_m(k; 0) \mathbf{h}_m s_m(k) + \sum_{m=1}^M \mathbf{C}_m(k-1; 1) \mathbf{h}_m s_m(k-1) + \boldsymbol{\eta}(k),$$

with the second sum denoting interblock interference (IBI).

Targeting a simple receiver structure, we want to concentrate only on a single block containing the current symbol  $s_m(k)$ . Such an IBI-free reception for the user of interest  $\mu$  is possible, if the receiver  $\mathbf{g}_\mu(k)$  satisfies:  $\mathbf{g}_\mu^T(k) \mathbf{C}_m(k-1; 1) = \mathbf{0}^T, \forall m \in [1, M], m \neq \mu$ , which implies that  $\mathbf{g}_\mu(k)$  must lie in the intersection of left null spaces  $\cap_{m \neq \mu} \mathcal{N}(\mathbf{C}_m(k-1; 1))$ . To characterize the left null space of  $\mathbf{C}_m(k-1; 1)$ , we note that for code lengths  $P > L$ , the only nonzero entries of  $\mathbf{C}_m(k-1; 1)$  in (3) appear in the first  $L$  rows that form an  $L \times L$  full rank (upper triangular) submatrix. Furthermore,  $\mathcal{N}(\mathbf{C}_m(k-1; 1))$  is spanned by the canonical vectors (one unity and all other entries zero) that select the  $P-L$  null rows of  $\mathbf{C}_m(k-1; 1)$ . Hence, the only possibility for IBI-free reception is for  $\mathbf{g}_\mu^T(k)$  to have its first  $L$  entries equal to zero which amounts to discarding the first  $L$  chips of the received block.

Therefore, we have only  $P-L$  chips to convey information from  $M$  users. A necessary condition to guarantee symbol recovery is  $P-L \geq M$ . To avoid overexpansion of bandwidth, we adopt the minimum possible  $P$  in our system design by selecting  $P = M + L$ . To achieve MUI elimination, we design our transceivers as follows: for the  $k$ th symbol of user  $m$ , we assign a complex number  $\rho_{m,k}$  to construct the spreading code as (see also [6]):

$$\mathbf{c}_m^T(k) := [c_m(k; 0) \dots c_m(k; P-1)] = A_m [\rho_{m,k}^{-P+1} \dots \rho_{m,k}^{-1}, 1] \quad (7)$$

where  $A_m$  is the  $m$ th user's amplitude. We term  $\rho_{m,k}$  as user  $m$ 's *signature point* (when  $\rho_{m,k}$  is on the unit circle, it can be thought of as user  $m$ 's subcarrier). Different from [6], we will allow here  $\rho_{m,k}$  to change periodically from symbol to symbol with period  $R$  ( $\rho_{m,k+iR} = \rho_{m,k}, \forall i, \forall m$ ), which leads to long spreading codes with period  $RP$ .

As we argued earlier, prefixing the  $\mu$ th user's  $1 \times P$  receive vector  $\mathbf{g}_\mu^T(k) := [0 \dots 0, \bar{g}_\mu(k; M-1) \dots \bar{g}_\mu(k; 0)]$  by  $L$  leading zeros avoids IBI. The correlation of this vector with the  $l$ th delayed version of  $m$ th user's code is therefore:  $\sum_{n=0}^{M-1} \bar{g}_\mu(k; n) \rho_{m,k}^{-(n+l)} = \rho_{m,k}^{-l} \mathcal{G}_\mu(k; \rho_{m,k})$ , where  $\mathcal{G}_\mu(k; z) := \sum_{n=0}^{M-1} \bar{g}_\mu(k; n) z^{-n}$ . To ensure MUI elimination,  $\mathcal{G}_\mu(k; z)$  must satisfy  $\mathcal{G}_\mu(k; \rho_{m,k}) = \theta_{\mu,k} \delta(m-\mu), \forall m \neq \mu$ , where  $\theta_{\mu,k}$  is a constant. To specify the  $\mu$ th user's receiver transfer function that eliminates MUI, we simply construct  $\mathcal{G}_\mu(k; z)$  as the Lagrange interpolating polynomial through the points  $\rho_{m,k}, m \neq \mu$ :

$$\mathcal{G}_\mu(k; z) = \sum_{n=0}^{M-1} \bar{g}_\mu(k; n) z^{-n} = \frac{1}{A_\mu} \frac{\prod_{m=1, m \neq \mu}^M (1 - \rho_{m,k} z^{-1})}{\prod_{m=1, m \neq \mu}^M (1 - \rho_{m,k} \rho_{\mu,k}^{-1})} \quad (8)$$

Direct substitution from (3), (7), and (8) verifies that  $\mathbf{g}_\mu^T(k) \mathbf{C}_m(k) = \mathbf{0}^T, \forall m \in [1, M], m \neq \mu$  and  $\mathbf{g}_\mu^T(k) \mathbf{C}_\mu(k) = [1, \rho_{\mu,k}^{-1}, \dots, \rho_{\mu,k}^{-L}]$ . Therefore, we have  $\mathbf{g}_\mu^T(k) \mathbf{C}_\mu(k) \mathbf{h}_\mu = \sum_{l=0}^L h_\mu(l) \rho_{\mu,k}^{-l} := H_\mu(\rho_{\mu,k})$ , and the decoder output of user  $\mu$  is obtained as:

$$y_\mu(k) = \mathbf{g}_\mu^T(k) \mathbf{x}(k) = H_\mu(\rho_{\mu,k}) s_\mu(k) + \eta_\mu(k), \quad (9)$$

where  $\eta_\mu(k) := \mathbf{g}_\mu^T(k) \boldsymbol{\eta}(k)$  is additive Gaussian noise. We infer from (9) that not only MUI is eliminated deterministically, but also a simple equalization scheme can be applied:

$$\hat{s}_\mu(k) = y_\mu(k) / H_\mu(\rho_{\mu,k}), \quad (10)$$

to recover the  $\mu$ th user's symbols, provided that  $H_\mu(\rho_{\mu,k}) \neq 0$ .

Although [6] also relied on Vandermonde/Lagrange transceivers like those in (7) and (8), our time-varying design offers additional flexibility. To appreciate it, let us express  $\rho_{m,k}$  as the product of two complex numbers:  $\rho_{m,k} = \rho_m \xi_k$ . Our codes in (7) will then have entries

$$c_m(k; n) = \tilde{c}_m(n) \bar{c}(k; n) := \rho_m^{-P+1+n} \xi_k^{-P+1+n}, \quad (11)$$

where  $\tilde{c}_m(n) = \rho_m^{-P+1+n}$  is a user-specific symbol periodic code, while  $\bar{c}(k; n) = \xi_k^{-P+1+n}$  changes from symbol to symbol, but is common to all users. Recall that in IS-95, the overall spreading code is the product of a short Walsh-Hadamard code with a long PN-sequence with period  $2^{15}$ . By explicitly utilizing the IS-95 like code structure, we can ease our code assignment procedure by: i) constructing each user's symbol periodic code  $\tilde{c}_m(n)$  from sufficiently separated (e.g., equispaced around the unit circle) signature points; ii) changing only  $\bar{c}(k; n)$  from symbol to symbol, with a pattern that is predetermined by the base station and is common to all users. The spreading procedure is illustrated in Fig. 2. With great

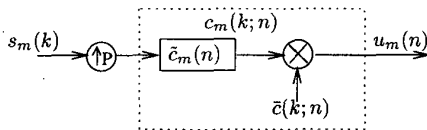


Fig. 2. The  $m$ th user's structured long code model

flexibility in selecting signature points, we highlight next several computationally simple designs.

**Special Case 1:** Here we assign  $\rho_m = \exp(j2\pi m/M)$  and  $\xi_k = \exp(j\theta_k)$ . The overall spreading code vector in (7) will then have entries:

$$c_m(k; n) = e^{-j(\frac{2\pi}{M}m + \theta_k)(P-1-n)}. \quad (12)$$

Substituting  $\rho_{m,k}$  in (8) with the  $c_m(k; n)$  of (12), we find the receiver filter coefficients  $g_m(k; n) = \exp\{j(2\pi m/M + \theta_k)(P-1-n)\}, \forall n \in [L, P-1]$ , which are matched to the transmitted spreading code (12). By setting  $\theta_k = 0$ , (12) corresponds to a conventional OFDMA transmission. On the other hand, with time-varying  $\theta_k$  we obtain an FH-OFDMA system, which explains why we called our system generalized FH-OFDMA. When  $\theta_k$  is a multiple of  $2\pi/M$ , not only the precoding but also the decoding operation can be implemented by an  $M$ -point FFT, as with conventional OFDMA, followed by a constant permutation matrix, which is determined by  $\theta_k$ . For example, setting  $\theta_k = (k \bmod M)2\pi/M = \bar{k}2\pi/M, \bar{k} \in [0, M-1]$ , each user is assigned a frequency that is shifted cyclically over the entire bandwidth. Performing an  $M$ -point FFT of the received block yields the vector  $\bar{\mathbf{x}}(k)$  corresponding to the pre-equalized symbols transmitted over frequencies  $\{0, 2\pi/M, \dots, 2\pi(M-1)/M\}$ . Let  $\mathbf{e}_n$  denote the  $n$ th canonical  $M \times 1$  Euclidean basis vector, and the  $M \times M$  permutation matrix be defined as:  $\prod_k = [\mathbf{e}_{\bar{k}+1}, \dots, \mathbf{e}_M, \mathbf{e}_1, \dots, \mathbf{e}_{\bar{k}}]^T$ . Because the  $k$ th symbol of user  $m$  is transmitted over the subcarrier  $\exp(j2\pi(m + \bar{k})/M)$ , multiplying vector  $\bar{\mathbf{x}}(k)$  by  $\prod_k$  will deliver the corresponding pre-equalized data (9) to each user. Such an FFT based decoder followed by the simple equalizer in (10) constitutes our low complexity receiver. An important extension to the one-step FH could be the  $\Lambda$ -step FH (with  $\Lambda$  an integer  $> M/L$ ) which increases the FH size from  $2\pi/M$  to  $2\pi\Lambda/M$  (see also [4], where  $\Lambda = 4$ ). Successive symbols in such an  $\Lambda$ -step FH use frequencies that are sufficiently separated and not affected by the same channel null. This leads to independently faded symbols and playing a role similar to interleaving, it enhances the effect of error correcting codes.

**Special Case 2:** Here we assign  $\rho_m = r_m \exp(j2\pi m/M)$  that corresponds to equispaced signature points around co-centric circles of possibly unequal radius. We also choose  $\xi_k = r_k \exp(j\theta_k)$ , which offers freedom to tune both the code amplitude  $r_m r_k$  as well as the phase  $2\pi m/M + \theta_k$ . With flexibility in assigning signature points, GFH-OFDMA gains resilience to Doppler effects or carrier offsets over plain OFDMA as argued in [5], where  $\rho_m = [1 + 0.1\cos(\pi m/2)] \exp(j2\pi m/M)$  and  $\xi_k = 1$  was proposed without hopping.

Our derivation in this subsection has shown how FH benefits permeate to general multi-carrier CDMA (and particularly OFDMA) systems through the time varying signature points  $\rho_{m,k}$ . The following proposition summarizes the basic results of this subsection:

**Proposition 1:** *The Generalized FH-OFDMA transceivers of (7) and (8) constitute a structured long code CDMA system, which combines CDMA with frequency-hopping benefits. Proper design of (7) and (8) allows for IBI removal and deterministic MUI elimination in the presence of frequency-*

*selective multipath.*

We will see next how hopping user's signature points facilitates also blind channel estimation.

### B. CSI not available - Blind Receivers

Our channel estimates in this section will require averaging across periods of our long code  $c_m(k; n)$ . Recalling that each  $\rho_{m,k}$  is periodic in  $k$  with period  $R$ , we can write for  $r \in [0, R - 1]$ , our symbol index as  $k = iR + r$ , where  $i$  will henceforth index blocks of symbols. Assigning a double argument  $(i; r)$  to quantities with argument  $k$ , we can e.g., express our received data in (9) as:

$$y_m(i; r) = H_m(\rho_{m,r})s_m(i; r) + \eta_m(i; r), \quad (13)$$

where  $\rho_{m,r}$  depends only on  $r$  (and not  $i$ ) because of the assumed code periodicity.

Unlike existing channel estimation methods that are based on the received chip sequences [8,3], our methods will rely on the MUI-free receiver output (13); channel estimation will thus be performed separately for each user. For this reason, we concentrate only on the user of interest and drop subscript  $m$  (denoting the  $m$ th user) for notational brevity.

For space limitations, we will only present basic results for blind channel estimation method based on the finite-alphabet property of the source symbols. Considering a general signal constellation with points  $\{z_q\}_{q=1}^Q$  on the complex plane, channel identifiability based on the noise-free receiver output (13) is established in the following proposition:

**Proposition 2:** *If the code period is selected to satisfy  $R \geq QL + 1$  and  $\{\rho_r\}_{r=0}^{R-1}$  are distinct, identifiability of the  $L$ th-order channel  $H(z)$  is guaranteed from the received data  $y(i; r) = H(\rho_r)s(i; r)$ , for symbols  $s(i; r)$  drawn from any constellation of size  $Q$ .*

Additional blind channel estimators can be found in [9,10].

## IV. FAST-HOPPING

The signature points  $\{\rho_{m,k}\}_{m=1}^M$  we selected to design our MUI-elimination codes in Section III were allowed to vary from symbol to symbol. We will focus in this section on fast-hopping systems.

Thanks to signature point hopping, users in our GFH-OFDMA system will not suffer from severe fading persistently because no single transmission will be hit consistently by channel nulls. In addition, changing distributed errors to bursty errors, one expects system performance to improve significantly through channel coding. To illustrate the advantage of GFH-OFDMA over conventional coded OFDM, we specify here the channel code to be the simplest repetition code; namely, we adopt symbol transmission via several signature points (or frequencies), which corresponds to nothing but a fast-hopping system. Suppose that we use an  $(1, T)$  repetition code and let  $R$  be a multiple of  $T$ , say  $R = WT$ , which means that there are  $W$  symbols transmitted during one long code period  $RP$ . For clarity, we express  $r = wT + t$ ,  $r \in [0, R - 1]$ ,  $w \in [0, W - 1]$ ,  $t \in [0, T - 1]$  and denote  $x(i; r) := x(i; w; t)$ . Proposition 2 still holds true and the blind channel identification methods of [9] are directly applicable. Once the channels have been estimated, symbols

can be recovered using a maximum ratio combining (matched filter) decoder as:

$$\begin{aligned} \hat{s}_m(i; w) &= \sum_{t=0}^{T-1} \hat{H}_m^*(\rho_{m,w,t})y_m(i; w; t) \\ &= \sum_{t=0}^{T-1} |\hat{H}_m(\rho_{m,w,t})|^2 s_m(i; w) + \sum_{t=0}^{T-1} \hat{H}_m^*(\rho_{m,w,t})\eta_m(i; w; t). \end{aligned}$$

Note that channels of order  $L$  have at most  $L$  roots, and if we choose  $T \geq L + 1$ , symbol recovery is always guaranteed, which is never the case for conventional OFDMA, no matter how large  $T$  is chosen. Although bandwidth is expanded when  $T \geq L + 1$ , improved performance may justify such an expansion in e.g., military applications. Even though symbol recovery is not guaranteed with  $1 < T \leq L$ , symbol error rate performance improves significantly with fast hopping because the larger  $T$  is, the less likely it becomes to have at the same time all  $T$  signature points coincide with channel nulls.

## V. PERFORMANCE ANALYSIS AND SIMULATIONS

Due to space limitations, we omit our theoretical performance analysis, that can be found in [9], and only present several simulation results here.

**Test Case 1:** Adopting code design in (12), we compare our

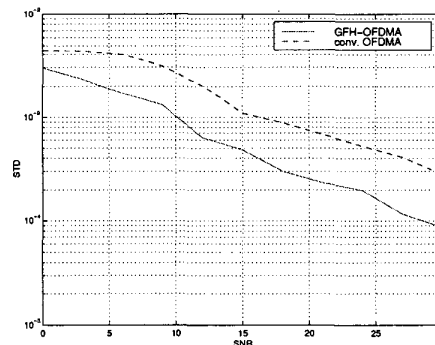


Fig. 3. Comparison based on std-BER

GFH-OFDMA with conventional OFDMA on an  $M = 16$  user system transmitting through Rayleigh fading multipath channels of order  $L = 2$  (3 rays). For GFH-OFDMA, we select  $R = 16$ , and perform 500 Monte Carlo realizations. Although for a specific channel realization the Bit Error Rate (BER) averaged over all users may be significantly lower in GFH-OFDMA relative to that in conventional OFDMA, the improvement disappears when we also average over all possible channel realizations. However, the advantage of GFH-OFDMA over standard OFDMA can be appreciated when evaluating how far the real performance would be from its average, which measures reliability of the transmission link. Fig. 3 shows that GFH-OFDMA exhibits much lower standard deviation compared to conventional OFDMA, which demonstrates the advantage of FH.

**Test Case 2:** To illustrate the advantage of structured GFH-

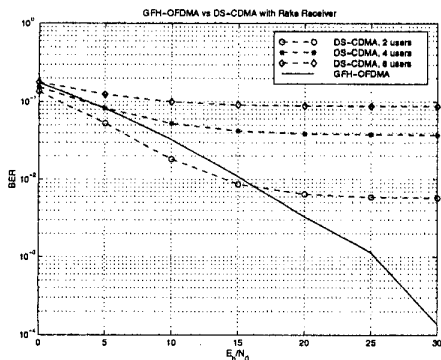


Fig. 4. DS-CDMA based on W-H codes

OFDMA over long code DS-CDMA with RAKE reception, we simulate GFH-OFDMA with  $R = M = 16$  signature points  $\{\rho_{k,m} = [1 + 0.1\cos(\pi\bar{k}_m/2)] \exp(j2\pi\bar{k}_m/M)\}$ , where  $\bar{k}_m = (k+m) \bmod M$  (without hopping ( $\bar{k}_m = m$ ) these codes were also used in [5] to show resilience to Doppler effects). We also implement a long code DS-CDMA system employing Walsh-Hadamard codes with lengths  $P = 16$  scrambled by random  $\{\pm 1\}$  sequences over 100 Rayleigh fading channels with order  $L = 2$ . Fig. 4 shows that the long-code CDMA system suffers from MUI severely when the system load increases. MUI-free GFH-OFDMA clearly outperforms long-code DS-CDMA under moderate system load and SNR, as confirmed by the average BER curves depicted in Fig. 4.

**Test Case 3:** To check the blind channel estimation algorithms detailed in [9], we adopt the code design (12), set  $R = M = 16$ ,  $L = 2$ , and simulate over 500 randomly generated channels. We here only show the result for BPSK signaling and adopt  $I = 30$  blocks; thus,  $N = IR = 480$  symbols are used for channel estimation. Fig. 5 depicts the BER based on channel estimates obtained from the Minimum Distance (MD), the Root Selection (RS), the RS followed by one step Decision Directed iteration (RS-DD), and the RS followed by root optimization (RS-RO) algorithms detailed in [9]. Fig. 5 shows that BERs relying on estimated channels are close to the ideal scenario where perfect CSI is available.

**Test Case 4:** To check how much fast hopping improves our GFH-OFDMA system over conventional OFDMA, we adopt the code design in (12) with  $M = R = 16$ ,  $L = 2$  and simulate 500 random channels. Fig. 6 illustrates that the average performance is much better for GFH-OFDMA than for the conventional OFDMA when  $T = 2$ , even though symbol recovery is not guaranteed for GFH-OFDMA in this case.

## VI. CONCLUSIONS AND DISCUSSION

We developed in this paper a general CDMA system utilizing structured time-varying long codes. Superior to long code DS-CDMA with Rake reception, GFH-OFDMA achieves MUI elimination deterministically by design, and relative to conventional OFDMA, it exhibits improved performance because no user suffers persistently from channel nulls. GFH-OFDMA system performs significantly better than conventional OFDMA when the same amount of redundancy is added through channel coding in the form of fast hopping.

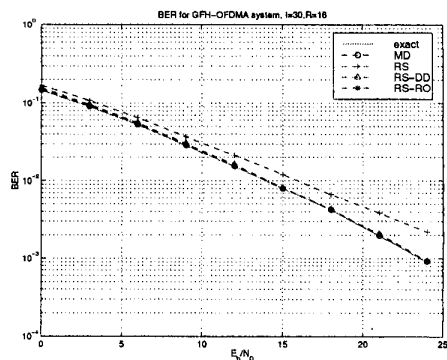


Fig. 5. Blindly estimated channels (BPSK)

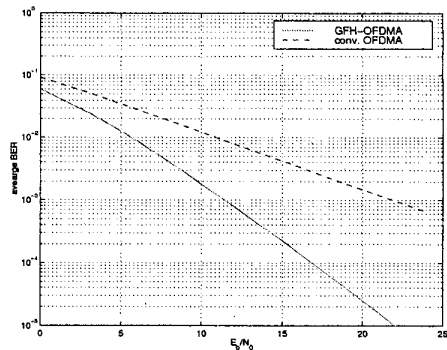


Fig. 6. Fast frequency hopping,  $T = 2$

Novel blind channel estimation methods based on the MUI-free received data guarantee channel identifiability for any finite signal constellation.

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