

# A SPECTRAL MOMENT APPROACH TO VELOCITY ESTIMATION IN MOBILE COMMUNICATIONS

Cihan Tepedelenlioğlu and Georgios B. Giannakis<sup>1</sup>

Dept. of Elect. and Comp. Eng.

University of Minnesota

Minneapolis, MN 55455

e-mail: cihan{georgios}@ece.umn.edu

**Abstract** — Estimation of the maximum Doppler spread, or equivalently the vehicle velocity is useful in improving handoff algorithms, and necessary for the optimal tuning of parameters for systems that adapt to changing channel conditions. We provide a novel velocity estimator based on the spectral moments of the in-phase and the quadrature-phase components of the squared envelope of the received signal. We characterize the joint effects of the Ricean  $K$ -factor, the directivity and the angle of non-isotropic scattering, and the effects of additive white noise on our estimator and other covariance-based velocity estimators analytically. Simulations illustrate our approach and compare with existing techniques.

## I. INTRODUCTION

In mobile communication systems, the received signal strength varies significantly in space due to constructive and destructive interference arising due to multipath components, a phenomenon also known as small scale fading [17]. The mobile velocity dictates how fast in time the fading is experienced by the receiver, and its knowledge at the base station can be utilized for handoff purposes. This is because the average signal strength is often the parameter that dictates handoff, and its accurate but quick estimation introduces a tradeoff which necessitates the appropriate choice of a temporal averaging window length, which in turn is dependent on the mobile velocity [5]. The base stations' knowledge of the vehicle velocity is also useful in overlaid cell architectures where slow mobiles are assigned to microcells and fast ones to 'umbrella' macrocells.

The mobile velocity  $v$  is proportional to the maximum Doppler spread  $\omega_D$  through  $\omega_D = (2\pi v f_c)/c$ , where  $f_c$  is the carrier frequency and  $c$  is the speed of propagation. Hence, given the system parameters, estimating  $v$  and estimating  $\omega_D$  are equivalent. In many communication applications such as adaptive coding/modulation/antenna diversity/power control, knowledge of  $\omega_D$  is of paramount importance in adapting the system parameters to new channel conditions. In fact, when the angle of arrival (AOA) distribution is uniform and a line of sight (LOS) component is absent,  $\omega_D$  is the sole parameter that determines the Doppler spectrum [10]. But, as the need for higher spectral efficiency increases, directional antennas are used to avoid interference, making it necessary to

characterize the effects of the LOS component and directional scattering on velocity estimators.

In this paper, we propose novel velocity (Doppler) estimators, and characterize analytically, the robustness of our estimator and other covariance based velocity estimators to non-uniform AOA distributions, the presence of a LOS component, and additive white noise. We illustrate by simulation that when the sampling rate is high and the estimation window is small, the covariance-based estimators have significantly smaller sample variances than their level crossing rate (LCR)-based counterparts [5, 17], and show that among the covariance-based estimators, the proposed method has the least sample variance in this regime.

In Section II we adopt a intuitive model for the received narrow band process, and provide a parametric model for the AOA distribution, by using the von Mises probability density function (pdf), also used in [1], [2]. In Section III we introduce the proposed velocity estimator and in Section IV we delineate its relationship with other covariance-based estimators. In Section V we provide the joint effect of the LOS factor, scattering directivity and their angles of arrival, on the covariance-based velocity estimators. Section VI briefly outlines estimation of the Ricean  $K$ -factor, and Section VII analyzes the effect of additive white noise on covariance-based velocity estimators. In Section VIII, simulations illustrate the results.

A few words on notation: We will use  $*$  for conjugate,  $\mathcal{R}\{\cdot\}$  for real part and  $\mathcal{I}\{\cdot\}$  for imaginary part of a complex number;  $E[\cdot]$  will denote mathematical expectation with respect to all the random variables within the brackets;  $\delta_K[n]$  denotes the Kronecker's delta function; Superscript  $(n)$  will denote  $n^{\text{th}}$  derivative, but if  $n = 1$  or  $2$ , we will also use  $'$  or  $''$  respectively.

## II. CHANNEL MODEL

We will assume the following model for the complex envelope of the narrow band (NB) random channel process in baseband:

$$\begin{aligned} h(t) &= \frac{\sigma_h}{\sqrt{K+1}} \lim_{M \rightarrow \infty} \frac{1}{\sqrt{M}} \sum_{m=1}^M a_m e^{j(\omega_D \cos(\theta_m)t + \phi_m)} \\ &+ \sigma_h \sqrt{\frac{K}{K+1}} e^{j(\omega_D \cos(\theta_0)t + \phi_0)} \\ &:= x(t) + y(t), \end{aligned} \quad (1)$$

where  $\theta_m \sim p(\theta)$ ,  $m = 1, \dots, M$  are independent and identically distributed (i.i.d.) angles that the incoming waves make with the mobile direction;  $\phi_m$  are i.i.d. phases, uniformly distributed on  $(-\pi, \pi]$ ;  $\theta_0$  and  $\phi_0$  are *deterministic* constants,  $K$  is the Ricean factor, which is the ratio of the specular component  $y(t)$ 's power to the diffuse component  $x(t)$ 's power;  $\theta_0$  is

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the angle that the LOS component makes with the mobile direction;  $\sigma_h^2 := E|h(t)|^2$  is the power of the received signal, and  $a_m$  are deterministic complex constants normalized to satisfy  $\lim_{M \rightarrow \infty} M^{-1} \sum_{m=1}^M |a_m|^2 = 1$ , so that  $\sigma_h^2 = E|h(t)|^2$  holds. Note that making  $M$  arbitrarily large ensures  $x(t)$  to be a Gaussian process due to the central limit theorem, resulting in  $|h(t)|$  being Ricean distributed. In the absence of a LOS component ( $K = 0$ ),  $|h(t)|$  has a Rayleigh density. Because the phases  $\phi_m$  are uniformly distributed on  $(-\pi, \pi]$ ,  $E[x(t)] = 0$  and hence  $E[h(t)] = y(t)$ . Notice that  $y(t)$  depends on time if the LOS component is not perpendicular to the direction of motion ( $\theta_0 \neq \pi/2$ ); hence, similar to [4], and [13], we allow for a sinusoidally time varying specular component.

It follows from (1) that the correlation function of  $h(t)$  is given by

$$r_h(\tau) := E[h(t)h^*(t+\tau)] = \frac{\sigma_h^2}{K+1} \int_{-\pi}^{\pi} p(\theta) e^{-j\omega_D \cos(\theta)\tau} d\theta + \frac{K\sigma_h^2}{K+1} e^{-j\omega_D \cos(\theta_0)\tau}, \quad (2)$$

by direct substitution and using the assumptions on  $\phi_m$ ,  $\theta_m$ , and  $a_m$ .

We see from (2) that  $r_h(\tau)$  depends on the probability distribution of the angle of arrival  $p(\theta)$ . In order to capture the effects of directional scattering on  $r_h(\tau)$  in a parametric fashion, we will use the von Mises distribution:

$$p(\theta) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\theta - \alpha)}, \quad \theta \in (-\pi, \pi], \quad (3)$$

where  $I_n(\kappa)$  is the  $n^{\text{th}}$  order modified Bessel function of the first kind,  $\kappa$  denotes the beamwidth, and  $\alpha$  denotes the angle that the average scattering direction makes with the mobile direction. Figure 1 illustrates the von Mises distribution for different values of  $\kappa$  and  $\alpha = 0$  (notice that  $\kappa = 0$  reduces (3) to a uniform distribution and that an  $\alpha \neq 0$  merely rotates the plot by  $\alpha$  radians). The von Mises distribution is widely

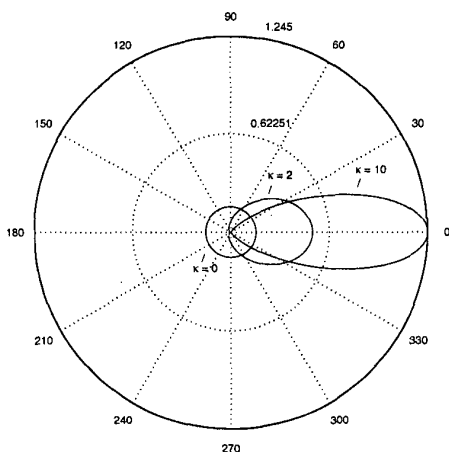


Figure 1: The von Mises PDF

used in directional statistics [12], and has been justified empirically to be an accurate model for the AOA distribution of

NB channels [2]. In addition, the von Mises pdf enables us to relate, in closed form, the effect of  $\psi := [K, \theta_0, \kappa, \alpha]$  to the Doppler spectrum, the correlation function, and  $r_h^{(n)}(0)$  [2]. Since velocity estimators that rely on the covariances can be expressed as a function of  $r_h^{(n)}(0)$ ,  $n = 0, 1, 2$ , we will use the closed form expressions relating  $\psi$  to  $r_h^{(n)}(0)$  in evaluating how velocity estimators designed assuming<sup>1</sup>  $\psi = \mathbf{0}$  are affected in environments for which  $\psi \neq \mathbf{0}$ .

The model in (1) also enables us to derive the autocovariance of  $|h(t)|^2$  in closed form, which is given by [14]

$$c_{|h|^2}(\tau) = \frac{\sigma_h^4}{(K+1)^2} \left[ \left| E[e^{j\omega_D \cos(\theta)\tau}] \right|^2 + 2K\mathcal{R}\{E[e^{-j\omega_D \cos(\theta)\tau}]e^{j\omega_D \cos(\theta_0)\tau}\} \right], \quad (4)$$

and is a generalization of the result derived in [4] where  $r_x(\tau)$  was assumed to be real. Notice that when  $K = \kappa = 0$ , (4) reduces to  $c_{|h|^2}(\tau) = J_0^2(\omega_D\tau)$ .

### III. NOVEL VELOCITY ESTIMATOR

In this section, we propose novel velocity estimators that use  $\mathcal{R}\{h(t)\}, \mathcal{I}\{h(t)\}$  when the I/Q components are measured, or utilize  $|h(t)|^2$ , when only the channel envelope is available. It is well-known that  $\omega_D$  is directly proportional to the second spectral moment, which is a measure of the Doppler bandwidth, and a decreasing function of  $r_h''(0)$ , the curvature of  $r_h(\tau)$  at zero. This notion can be made more precise with the following:

$$\omega_D = \sqrt{\frac{-2r_h''(0)}{r_h(0)}}, \quad (5)$$

which is valid when  $K = \kappa = 0$ , i.e., when  $r_h(\tau) = \sigma_h^2 J_0(\omega_D\tau)$ . We will examine the effects of  $\psi \neq \mathbf{0}$  on covariance based velocity estimators in Section V. Our velocity estimator aims at estimating  $r_h(0)$  and  $r_h''(0)$  separately, and uses (5) to get an estimate for  $\omega_D$ . To this end, we will fit a parabola to the  $L$  points of the sample correlations  $\{\hat{r}_h(lT_s)\}_{l=0}^L$ , where  $T_s$  is the sampling period. We will assume that the sampling rate  $1/T_s$  is sufficiently high to insure  $LT_s \ll 1$ , which is satisfied, for example, if channel samples used to estimate  $r_h(0)$  and  $r_h''(0)$  are provided by channel estimators of narrow band TDMA systems. The steps for estimating  $\omega_D$  from the I/Q components of  $h(t)$  are as follows:

**Step 1:** Find the correlation estimates  $\{\hat{r}_h(lT_s)\}_{l=0}^L$  by sample averaging;

**Step 2:** Find  $\hat{a}_k = \operatorname{argmin}_{a_k} \sum_{l=0}^L |\hat{r}_h(lT_s) - \sum_{k=0}^2 a_k l^k|^2$ ;

**Step 3:** Obtain  $\hat{r}_h^{(n)}(0) = n! \hat{a}_n / T_s^n$ ,  $n = 0, 2$ ;

**Step 4:** Substitute  $\hat{r}_h^{(n)}(0)$ ,  $n = 0, 2$ , in (5).

Note that the mapping from the correlation estimates to the Taylor's series coefficient estimates  $(\{\hat{r}_h(lT_s)\}_{l=0}^L \rightarrow \{\hat{r}_h^{(n)}(0)\}_{n=0}^2)$  that solves the least squares problem in Step 2 is a linear transformation (a  $3 \times (L+1)$  matrix multiplication) which can be precomputed once  $T_s$  is known. Unlike [11], to estimate  $\omega_D$ , we will only use a second order polynomial, which in view of the fact that  $\omega_D$  can be estimated from

<sup>1</sup>Note that  $\psi = \mathbf{0}$  implies uniform AOA with no LOS component

$r_h^{(n)}(0)$  for  $n = 0, 2$ , obviates unnecessary approximations and the need for finding the roots of a higher than second-order polynomial. Notice that, in practice, steps 1-4 will be applied to  $\mathcal{R}\{h(t)\}$  or  $\mathcal{I}\{h(t)\}$  which are obtained from the in-phase and quadrature-phase components of the NB channel.

If we only have the envelope  $|h(t)|$  available, we can use  $|h(t)|^2$  to estimate  $\omega_D$  by exploiting the fact that  $c_{|h|^2}(\tau)|_{\psi=0} = \sigma_h^4 J_0^2(\omega_D \tau)$  (c.f. (4)), which can be used to show  $\omega_D = [-c_{|h|^2}''(0)/c_{|h|^2}(0)]^{1/2}$ , when  $\psi = 0$ . Hence to estimate  $\omega_D$  from  $|h(t)|^2$ , we would still use steps 1-4 except  $\{\hat{c}_{|h|^2}(lT_s)\}_{l=0}^L$  would be estimated from  $|h(t)|^2$ , and  $\hat{c}_{|h|^2}^{(n)}(0)$ ,  $n = 0, 2$  would be estimated and substituted into  $[-c_{|h|^2}''(0)/c_{|h|^2}(0)]^{1/2}$ .

#### IV. RELATIONSHIP WITH OTHER COVARIANCE-BASED ESTIMATORS

In [9], Holtzman and Sampath proposed a covariance-based Doppler estimator using the formula

$$\hat{\omega}_D^{HS} := \frac{C}{lT_s} \sqrt{\frac{V(lT_s)}{\hat{r}_z(0)}}, \quad (6)$$

where  $V(lT_s) := N^{-1} \sum_{n=0}^{N-1} [z((n+l)T_s) - z(nT_s)]^2$ ,  $\hat{r}_z(0) := N^{-1} \sum_{n=0}^{N-1} z^2(nT_s)$ , and  $C$  is a constant depending on whether  $z(t) = h(t)$ ,  $|h(t)|$ , or  $\log|h(t)|$ . Anim-Appiah has analyzed this class of covariance-based estimators in great detail in a recent paper [3], where he considered the cases  $z(t) = |h(t)|^n$  and  $z(t) = \mathcal{R}\{h(t)\}^n + \mathcal{I}\{h(t)\}^n$ .

In order to relate the estimator in (6) to the proposed estimator, it is important to realize that  $E[V(lT_s)] = 2[r_z(0) - r_z(lT_s)]$  and recall the following result from [5]:

$$\lim_{T_s \rightarrow 0} \frac{C}{lT_s} \sqrt{\frac{E[V(lT_s)]}{r_z(0)}} = \frac{C}{l} \sqrt{\frac{-r_z''(0)}{r_z(0)}}, \quad (7)$$

which is a constant multiple of (5). The limit in (7) can be shown by substituting  $2[r_z(0) - r_z(lT_s)]$  for  $E[V(lT_s)]$ , moving the  $1/T_s$  inside the square root, and applying L'Hôpital's rule twice. Equation (7) illustrates that the covariance-based estimator in [9] is affected by a  $\psi \neq 0$  the same way as the one proposed in Section III, when  $N$  is large and  $T_s$  is small.

#### V. EFFECT OF $\psi$ ON VELOCITY ESTIMATORS

Since it is often unfeasible to estimate  $\psi$  accurately, a common strategy is to assume  $\psi = 0$  in designing velocity estimators, and subsequently deriving the effect of  $\psi \neq 0$  on the estimator [3], [5]. Expressions relating  $\psi$  and  $r_z^{(n)}(0)$ ,  $n = 0, 1, 2$  can be derived using (2) [14, 15]. All velocity estimators relying on (5), or the square of the envelope that are designed for  $\psi = 0$ , get scaled as a function of  $K, \kappa, \alpha$  and  $\theta_0$ . We will briefly mention the corresponding scale functions  $S(\psi)$  that were derived in [15, 14], where, for the first time the effect of directional scattering was incorporated analytically.

Consider first the covariance-based estimators that approximate (5) (see e.g. [3], [5]). Using (2), we can obtain,

$$S_1(\psi) := \frac{1}{\omega_D} \sqrt{\frac{-2r_h''(0)}{r_h(0)}}$$

$$= \left[ \frac{1}{(K+1)} \left( 1 + \frac{\cos(2\alpha)I_2(\kappa)}{I_0(\kappa)} \right) + \frac{K}{K+1} (1 + \cos(2\theta_0)) \right]^{\frac{1}{2}}. \quad (8)$$

Notice that  $S_1(0) = 1$  as expected. Examining (8) more closely, we realize that for large values of  $K$ , the scale factor is not influenced by  $\kappa$  and  $\alpha$ , and depends solely on the LOS direction  $\theta_0$ . An extreme case would be  $\theta_0 = \pi/2$  for large  $K$ , which would yield a Doppler estimate of  $\hat{\omega}_D = 0$ . This is because  $h(t)$  in (1) would not be time-dependent, a situation where the received signal contains no information about  $\omega_D$ .

Using (4) it can be shown that [14, 15] the estimator that relies on  $|h(t)|^2$  (described right before Section IV) gets scaled by a factor of  $S_2(\psi)$ , where

$$S_2(\psi) = \left[ \left( 1 + \frac{\cos(2\alpha)I_2(\kappa)}{I_0(\kappa)} \right) - 2 \left( \frac{\cos(\alpha)I_1(\kappa)}{I_0(\kappa)} \right)^2 + K \left( 2 + \frac{\cos(2\alpha)I_2(\kappa)}{I_0(\kappa)} + \cos(2\theta_0) - 4 \cos\theta_0 \frac{\cos(\alpha)I_1(\kappa)}{I_0(\kappa)} \right) \right]^{\frac{1}{2}} (1+2K)^{-1/2}. \quad (9)$$

Observe that the scale factor  $S_2(0) = 1$ . Note also that for  $\kappa = \alpha = 0$ , (9) reduces to  $S_2(K, \theta_0, 0) = [1 + K \cos(2\theta_0)/(1+2K)]^{1/2}$ , which is the result derived in [9] for uniform AOA. Hence, (9) generalizes [9, eqn. (6)] to when the angle of arrival distribution  $p(\theta)$  is not uniform. Similar expressions can be derived for velocity estimators that rely on the level crossing rates of the envelope and zero crossing rates [14].

#### VI. ESTIMATION OF THE RICEAN $K$ -FACTOR

As we mentioned before,  $|h(t)|$  is a Ricean process with parameter  $K$ . It is well-known that the value of  $K$  is a measure of the severity of fading, with  $K = 0$  being the most severe Rayleigh fading, and  $K = \infty$  representing no fading. Hence, the knowledge of the  $K$ -factor is important in link budget calculations [7]. Also, as we have seen in the previous sections, the presence of the LOS factor introduces a modeling error in velocity estimation, which may be compensated for by estimating  $K$ . Greenwood and Hanzo in [7] have proposed techniques that fit the distribution of the envelope  $|h(t)|$  to estimate  $K$ , but these techniques are not well suited for on-line estimation. More recently Greenstein et. al. introduced a moment based method for estimating  $K$  under the assumption that the LOS angle of arrival  $|\theta_0| = \pi/2$  (i.e.,  $E[h(t)]$  is a constant) [6]. In this section, we will show that as a simple consequence of our framework, the estimator in [6] works also when  $\theta_0 \neq \pi/2$  (i.e., when  $y(t)$  depends on time). Consider (4) with  $\tau = 0$  to obtain:  $c_{|h|^2}(0) = [\sigma_h^4/(K+1)^2](1+2K)$ . By solving the resulting quadratic equation for  $K$  in terms of  $\sigma_h^4$  and  $c_{|h|^2}(0)$  we find

$$K = \frac{\sigma_h^4 - c_{|h|^2}(0) + \sigma_h^2 \sqrt{\sigma_h^4 - c_{|h|^2}(0)}}{c_{|h|^2}(0)}, \quad (10)$$

which is the result in [6]. Since  $\sigma_h^4$  and  $c_{|h|^2}(0)$  can be estimated from  $|h(t)|^2$ , (10) provides us with an estimate of  $K$ . It

is important to notice that  $y(t)$  need not be constant for (10) to hold, as was assumed in [6].

## VII. EFFECT OF ADDITIVE WHITE NOISE

In this section, we will discuss the effect of additive white noise that is assumed to be independent of  $h(t)$ , on the proposed estimator and the estimator in (6) originally proposed in [9]. The effect of additive noise on velocity estimators was addressed in [5] and [9], where the noise was spectrally flat over a finite bandwidth, which results in colored noise in continuous time. We evaluate the covariance-based velocity estimators in a framework where  $\hat{\omega}_D$  is estimated using noisy channel samples obtained with pilot sequences. In such a setup, if the cascade of the transmit and receive filters are designed to have Nyquist properties to avoid inter-symbol interference, the channel estimates will be corrupted by uncorrelated noise in discrete-time.

First we will discuss the effect of white noise on the covariance-based estimator given in (6) when  $z(nT_s) = h(nT_s) + v(nT_s)$ . As mentioned in Section IV, for large  $N$ , (6) is given by,  $\hat{\omega}_D \approx (C/lT_s)[2(r_z(0) - r_z(lT_s)/r_z(0))^{1/2}]$ , which, using the independence of  $h(nT_s)$  and  $v(nT_s)$ , assuming a small  $T_s$  and using (7) can be written as

$$\hat{\omega}_D \approx \frac{C}{l} \sqrt{\frac{-r_h''(0)}{[r_h(0) + r_v(0)]} + \frac{2r_v(0)}{T_s^2[r_h(0) + r_v(0)]}} \quad (11)$$

where we used the fact that  $r_v(nT_s) = 0$  for  $n \neq 0$ . We see that if the SNR :=  $r_h(0)/r_v(0)$  is moderate-low, with  $T_s$  being very small, the second term in (11) will cause  $\hat{\omega}_D$  to deviate from  $\omega_D$  in a pronounced manner due to the presence of noise ( $r_v(0) \neq 0$ ). We also corroborate this in the simulations.

One way to overcome this limitation is to avoid  $r_v(0)$  by adopting the following variation on  $\hat{\omega}_D^{HS}$  in (6):  $\hat{\omega}_D = T_s^{-1}[-(2/3)[V(T_s) - V(2T_s)]/\hat{r}_z(0)]^{1/2}$ . In this case, for large  $N$  and small  $T_s$ , we have  $\hat{\omega}_D = [-2r_h''(0)/(r_h(0) + r_v(0))]^{1/2}$ , which like (11) is also influenced by  $r_v(0)$ , but not nearly as much because for moderate SNRs, the denominator  $r_h(0) + r_v(0) \approx r_h(0)$ , so  $\hat{\omega}_D$  will not deviate from  $\omega_D$  significantly. This 'denoising' approach is similar to the one suggested in [5] for robustness against cochannel interference.

To circumvent the effect of white noise on our proposed scheme we will modify our algorithm as follows. Given noisy channel estimates  $z(nT_s) = h(nT_s) + v(nT_s)$ , we first obtain the estimates of  $r_z(lT_s) = r_h(lT_s) + r_v(lT_s) = r_h(lT_s) + \sigma_v^2 \delta_K[l]$  via sample averaging. We can then fit a polynomial to  $\hat{r}_z(lT_s) \approx \hat{a}_2 l^2 + \hat{a}_0$  for  $l = 1, 2, \dots, L-1$ , discarding the  $l = 0$  lag to obtain estimates  $\hat{r}_h''(0) = 2\hat{a}_2/T_s^2$ ,  $\hat{r}_h(0) = \hat{a}_0$ . In contrast to the 'denoised' estimator mentioned in the last paragraph where the noise variance affects the estimator even for large  $N$ , the proposed estimator is asymptotically unaffected by the SNR. We also illustrate this effect in the simulations.

Notice that for the aforementioned method, we do not even need to know the noise variance  $\sigma_v^2$  since we can ignore the white noise altogether by not using the lag  $l = 0$ . When the noise is colored with known color, (as might be induced with transmitter/receive filters without Nyquist properties) we can still reduce the noise effects by subtracting the noise correlation  $r_v(\tau)$  from the estimate of  $r_h(\tau) + r_v(\tau)$ , and proceed to estimate  $\hat{r}_h^{(n)}(0)$ . This is a feature that  $\hat{\omega}_D^{HS}$  cannot

be equipped with straightforwardly, since the correlations are only implicitly calculated in (6).

All the estimators mentioned so far rely on accurate covariance estimates. We have shown in [14] that assuming the model in (1), it is possible to show that the sample correlations of (1) are mean-square consistent if the AOA distribution is bounded, which also establishes the consistency of covariance based estimators that rely on the I/Q components.

## VIII. SIMULATIONS

In our simulations, we generated 100 different realizations of  $h(t)$  using the simulator in [8] which approximates Jakes's spectrum, and plotted the histogram for the 100 velocity estimates normalized by the true velocity, corresponding to each realization. The performance of each estimator can be judged by how closely its histogram is clustered around 1. In all experiments  $v = 100$  km/hr, and  $T_s = 4.12 \times 10^{-5}$ , which is the symbol period for the IS-54 TDMA standard adopted in North America. The carrier frequency is  $f_c = 900$  MHz implying  $\omega_D/2\pi \approx 83.3$  Hz. For the proposed method we chose  $L = 15$  correlation lags.

In Figure 2 we observe that for a time duration of 20 milliseconds ( $N = 485$  samples) the sample variances of the covariance-based estimators ( $\hat{\omega}_D^{HS}$  and the proposed) are an order of magnitude smaller than their LCR-based counterpart, and that the proposed estimator has a slightly smaller sample variance than  $\hat{\omega}_D^{HS}$ . This illustrates that as far as convergence of estimators to their ensemble values is concerned, at high sampling rates and small estimation windows, covariance-based estimators are more reliable than their LCR-based counterparts. This is because, for such small data lengths, the signal does not experience many level crossings as can also be seen from the top-left plot in Figure 2. This large gap in the maximum Doppler spread estimator variance for small window lengths between the LCR-based schemes and those that rely on the covariances, to the best of the authors' knowledge, have not been reported in the literature, and is important to know in contexts where quick estimates of  $\hat{\omega}_D$  are needed, such as adaptive modulation/coding.

In Figure 3 we look at the effect of white noise at SNR = 20 dB. We observe that  $\hat{\omega}_D^{HS}$  performs very poorly as explained by equation (11). Also, the proposed estimator has an order of magnitude less variance than the one obtained by 'denoising'  $\hat{\omega}_D^{HS}$ , mentioned in the previous section, in the presence of noise.

In conclusion, we see that as far as the convergence of sample estimates and sensitivity to white noise is concerned, the proposed estimator (whether it uses the envelope or the I/Q components) is always the best alternative, when  $T_s \ll 1$ . The intuitive explanation for this is that the proposed estimator involves more correlation lags and hence it is more reliable as compared to  $\hat{\omega}_D^{HS}$ . Naturally, utilizing more correlation lags to improve performance comes at the expense of more computational complexity.

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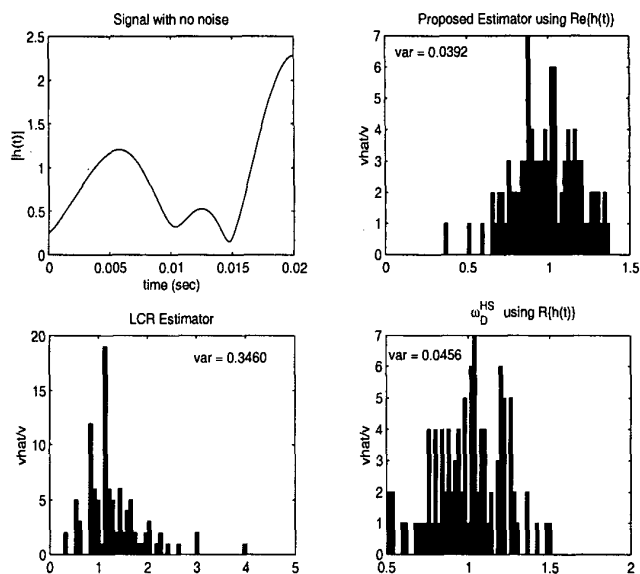


Figure 2: Histogram of velocity estimates for  $N = 485$  (0.02 seconds)

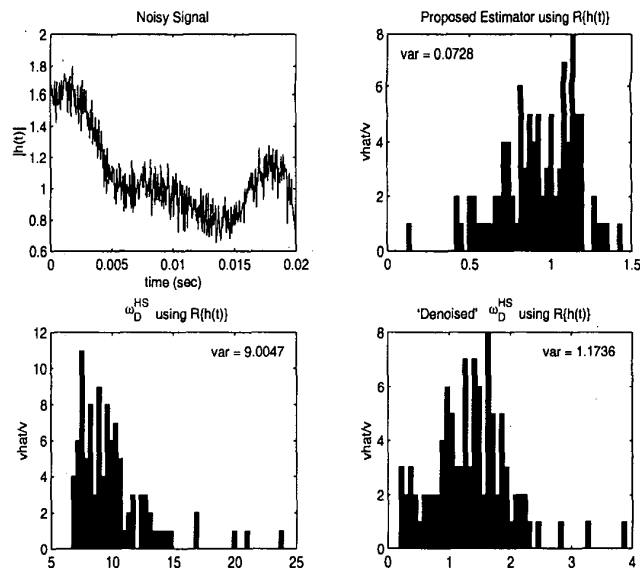


Figure 3: Histogram of velocity estimates for  $N = 485$ , SNR = 20 dB