Space-Time-Frequency Trellis Coding for Frequency-Selective Fading Channels*

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Abstract— **A novel space-time-frequency (STF) trellis coding scheme is developed for multi-antenna OFDM transmissions over frequency-selective Rayleigh fading channels. Incorporating subcarrier grouping and choosing appropriate system parameters, we first convert our system into a set of group STF (GSTF) systems. This enables simplification of STF block coding within each GSTF system. We derive design criteria for STF trellis coding, and exploit existing ST trellis coding techniques to construct STF trellis codes. The resulting codes are shown capable of achieving maximum diversity gains, while affording low-complexity decoding. The performance merits of our design is confirmed by corroborating simulations, and compared with existing alternatives.**

I. Introduction

Space-time (ST) coding relies on simultaneous coding across space and time to achieve diversity gain without necessarily sacrificing precious bandwidth. In ST coding, the maximum achievable diversity advantage is equal to the product of the number of transmit- and receive-antennas; and therefore, it is constrained by the size and cost a system can afford. The latter motivates exploitation of extra diversity dimensions, such as multipath (or frequency) diversity.

Multipath diversity becomes available when frequency selectivity is present. As proved in [6, 1, 8], multi-antenna transmissions over frequency selective fading channels can potentially provide a maximum diversity gain that is multiplicative in the number of transmit-antennas, receive-antennas, and the channel length. A number of coding schemes have been proposed recently to exploit frequency diversity. Because they offer low-complexity equalization-decoding, and facilitate the support of multirate services, multicarrier transmissions are typically adopted by those schemes [6, 1, 3, 8]. Among them, [3, 8] rely on combining ST codes with redundant or non-redundant linear precoders. Maximum diversity gain is achieved in [3, 8] at the expense of bandwidth efficiency [3], or, increased decoding complexity [3, 8]. On the other hand, [6, 1] are based on space-frequency (SF) coding, which amounts to simultaneously coding over space and frequency. However, due to the prohibitive complexity in constructing the codes, no SF codes have been designed in [6, 1]. Instead, [6, 1] simply adopt existing codes without maximum diversity gain guarantees. Moreover, issues pertaining to maximizing the coding gain have not been addressed so far.

Considering multi-antenna OFDM transmissions through frequency selective Rayleigh fading channels, this paper proposes a novel concept: joint space-time-frequency (STF) coding. Resorting to the subcarrier grouping we introduced in [4], and by choosing proper system parameters, we first divide our system into a set of what we term group STF (GSTF) sub-systems, within which STF coding is considered. We derive design criteria for STF codes, which provide a link between STF codes, and existing ST codes. We prove that subcarrier grouping does preserve maximum diversity gains, while simplifying not only the code construction, but also the decoding algorithm significantly. Aiming at maximum diversity and coding gains, we construct STF trellis (STFB) codes, whose performance is investigated both by theoretical analyses, and by corroborating simulations.

II. Preliminaries

A. System Model

We consider a wireless communication system with N_t transmit-antennas and N_r receive-antennas, where OFDM utilizing N_c subcarriers is employed per antenna transmission. The fading channel between the μth transmit-antenna and the νth receive-antenna is assumed to be frequency-selective, and is described by $\mathbf{h}_{\mu\nu} := [h_{\mu\nu}(0), \ldots, h_{\mu\nu}(L)]^T$, where L is the channel order.

Let $x_n^\mu(p)$ be the data symbol transmitted on the pth subcarrier from the μth transmit-antenna during the nth OFDM symbol interval. As defined, the symbols $\{x_n^\mu(p),\ \mu = 1, \ldots, N_t,\ p = 0, 1, \ldots, N_c - 1\}$ are transmitted in parallel on N_c subcarriers by N_t transmit-

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antennas. $x_n^\mu(p)$ can be viewed as a point in a 3-D STF parallelepiped. After FFT processing, the received data sample $y_n^\nu(p)$ at the νth receive-antenna can be expressed as:

$$y_n^\nu(p) = \sum_{\mu=1}^{N_t} H_{\mu\nu}(p) x_n^\mu(p) + w_n^\nu(p), \quad \nu = 1, \ldots, N_r, \tag{1}$$

where, $H_{\mu\nu}(p) := \sum_{l=0}^{L} h_{\mu\nu}(l) e^{-j\frac{2\pi}{P}lp}$; and the additive noise $w_n^\nu(p)$ is zero-mean, complex Gaussian with variance N_0.

Eq. (1) represents a general model for multi-antenna OFDM systems, including those considered in [8, 6, 1]. The difference among those systems lies in how $x_n^\mu(p)$'s are generated from the information symbols s_n, which eventually leads to corresponding tradeoffs among performance, decoding complexity, and transmission rate. In our system, the generation of $x_n^\mu(p)$ is performed via what we term STF coding that we describe next.

B. STF Coding

Recalling that each $x_n^\mu(p)$ is a point in 3-D, we define each STF codeword as the collection of transmitted symbols within the parallelepiped, spanned by N_t transmit-antennas, N_x OFDM symbol intervals, and N_c subcarriers. Mathematically, one STF codeword can be represented by a block matrix:

$$\mathbf{X} := [\mathbf{X}(0)\, \mathbf{X}(1) \, \cdots \mathbf{X}(N_c - 1)] \ \in \mathbb{C}^{N_t \times N_c N_x}, \tag{2}$$

where, $\mathbf{X}(p)$ is the $N_t \times N_x$ matrix with (μ, n)th entry $[\mathbf{X}(p)]_{\mu n} = x_n^\mu(p)$. Let us define the MIMO channel matrix $\mathbf{H}(p) \in \mathbb{C}^{N_r \times N_t}$ with $[\mathbf{H}(p)]_{\nu\mu} = H_{\nu\mu}(p)$; and the received sample matrix $\mathbf{Y}(p) \in \mathbb{C}^{N_r \times N_x}$ with $[\mathbf{Y}(p)]_{\nu n} = y_n^\nu(p)$. It follows from (1) that our 3-D STF system can be modeled as:

$$\mathbf{Y}(p) = \mathbf{H}(p)\mathbf{X}(p) + \mathbf{W}(p), \quad \forall\, p \in [0, N_c - 1]. \tag{3}$$

Suppose that $\mathbf{X}$ has been generated by $\bar{N}_I$ information symbols collected in the block $\mathbf{s} := [s_0, \ldots, s_{\bar{N}_I}]^T$. STF coding is then defined as an one-to-one mapping $\Psi : \mathbf{s} \rightarrow \mathbf{X}$.

Because $\mathbf{X}$ in (2) is described by three dimensions, STF coding simultaneously encodes information over space, time, and frequency, as its name reveals. Let $\mathcal{A}_s \ni s_n$ be the alphabet set to which the information symbol s_n belongs, and let $|\mathcal{A}_s|$ be the cardinality of $\mathcal{A}_s$. Since $\mathbf{X}$ is uniquely mapped from $\mathbf{s}$, the number of possible STF codewords $\mathbf{X}$ is $|\mathcal{A}_s|^{\bar{N}_I}$, which we collect into a finite set $\mathcal{A}_x$ with $|\mathcal{A}_x| = |\mathcal{A}_s|^{\bar{N}_I}$. From a conceptual point of view, STF coding is equivalent to constructing the finite set $\mathcal{A}_x$, as well as specifying the mapping Ψ. Based on (1) or (3), our goal is to achieve maximum diversity and coding advantages, by carefully designing Ψ, and properly choosing system parameters.

III. SUBCARRIER GROUPING

Our design of STF coding Ψ involves designing the set $\mathcal{A}_x$ with codewords $\mathbf{X}$ of size $N_t \times N_x N_c$. However, N_c is typically large in practice. Thinking of the difficulties already encountered in designing ST codes of a much smaller size, it can be expected that this design will be far more challenging, without any effort to alleviate the "curse of dimensionality". The tool we will use to reduce the dimensionality, and thus facilitate design and decoding is subcarrier grouping.

Subcarrier grouping was originally suggested in [4] to reduce design and decoding complexity, while preserving both diversity and coding advantages, for *single-antenna* linear constellation precoded OFDM systems. For STF coding, the first step towards subcarrier grouping is to choose:

$$N_c = N_g(L+1), \tag{4}$$

for a certain positive integer N_g denoting the number of groups. We assume that: **as1)** the channel taps $h_{\mu\nu}(l)$'s are i.i.d., zero-mean, complex Gaussian with variance $1/(2L+2)$ per dimension. The generalization to correlated channel taps has been treated in [5].

Under as1), it can be verified that: $\forall\, \mu, \mu', \nu, \nu'$, and $p_1 \neq p_2$,

$$E\big[H_{\mu\nu}(p_1) H^*_{\mu'\nu'}(p_2)\big] = 0, \quad \text{if} \quad \mathrm{mod}\,(p_1 - p_2, N_g) = 0, \tag{5}$$

from which, it is deduced that $H_{\mu\nu}(p_1)$ and $H_{\mu'\nu'}(p_2)$ are statistically independent. The second step is to split the $N_t \times N_c N_x$ STF codeword $\mathbf{X}$ into N_g group STF (GSTF) codewords $\mathbf{X}_g$:

$$\mathbf{X}_g = [\mathbf{X}_g(0), \mathbf{X}_g(1), \ldots, \mathbf{X}_g(L)], \quad \forall\, g \in [0, N_g - 1], \tag{6}$$

where $\mathbf{X}_g(l) := \mathbf{X}(N_g l + g)$. Accordingly, we divide the STF system (3) into N_g GSTF subsystems, which we describe through the input-output relationships:

$$\mathbf{Y}_g(l) = \mathbf{H}_g(l)\mathbf{X}_g(l) + \mathbf{W}_g(l), \quad \forall\, l \in [0, L], \tag{7}$$

where $\mathbf{Y}_g(l) := \mathbf{Y}(N_g l + g)$, and $\mathbf{H}_g(l) := \mathbf{H}(N_g l + g)$.

It is important to recognize the following property that will be used to simplify our design criteria.

Property 1 *Under as1) and with the choice of parameters in* (4), *all subchannels within* $\{\mathbf{H}_g(l)\}_{l=0}^{L}$ *of each GSTF subsystem are statistically independent.*

Each GSTF subsystem is nothing but a simplified STF system with a much smaller size in the frequency dimension, as compared to the original STF system. To take advantage of subcarrier grouping, we will consider STF coding within each GSTF subsystem; i.e., we will perform STF coding to generate $\mathbf{X}_g$'s individually, rather than generating $\mathbf{X}$ as a whole. As we will show in the next subsection, doing so will not incur any reduction in the diversity advantage, while it will reduce the design complexity considerably. To distinguish GSTF from the STF coding, we hereafter name the STF coding for each GSTF subsystem as GSTF coding, and denote it by the unique mapping $\Psi_g : \mathbf{s}_g \to \mathbf{X}_g$, where, $\mathbf{s}_g \in \mathbb{C}^{N_I \times 1}$ is the information symbol block used to generate $\mathbf{X}_g$. It is clear that we have $\bar{N}_I = N_g N_I$.

So far, we have converted the design of Ψ into the design of the set $\{\Psi_g\}_{g=0}^{N_g-1}$. Since all Ψ_g's are basically uniform, we will only focus on one of them in the ensuing discussion.

IV. Design Criteria

We derive here design criteria for the GSTF codes $\mathbf{X}_g$. In addition to as1), we further assume that: **as2)** maximum likelihood (ML) detection is performed with perfect channel state information at the receiver; and **as3)** high SNR is observed at the receiver. It is noted that assumptions as1)-as3) are also made in [1, 6]. Under as1)-as3), our derivations rely on analyzing the diversity and coding advantages of GSTF transmissions modeled in (7). Due to the lack of space, we omit details, and state the main results only.

Let $\mathcal{A}_{x_g}$ be the set of all possible $\mathbf{X}_g$'s. We are interested in designing $\mathcal{A}_{x_g}$ such that both diversity and coding advantages are maximized. The design criteria are summarized as follows.

C1) (Sum-of-ranks criterion) Design $\mathcal{A}_{x_g}$ such that: $\forall\, \mathbf{X}_g \neq \mathbf{X}'_g \in \mathcal{A}_{x_g}$, the matrices

$$\mathbf{\Lambda}_e(l) = [\mathbf{X}_g(l) - \mathbf{X}'_g(l)][\mathbf{X}_g(l) - \mathbf{X}'_g(l)]^{\mathcal{H}}, \quad \forall\, l \in [0, L] \tag{8}$$

have full rank.

C2) (Product-of-determinants criterion) For the set of matrices satisfying C1), design $\mathcal{A}_{x_g}$ such that: $\forall\, \mathbf{X}_g \neq \mathbf{X}'_g \in \mathcal{A}_{x_g}$, the minimum of $\prod_{l=0}^{L} \det[\mathbf{\Lambda}_e(l)]$ is maximized.

Two remarks are due at this point:

Remark 1 Checking the dimensionality of $\mathbf{\Lambda}_e(l)$ reveals that the maximum diversity advantage is $G_d^{\max} = N_t N_r (L+1)$, which coincides with that of STF systems without subcarrier grouping [8]. Thus, our subcarrier grouping does not sacrifice the diversity order. It is not difficult to show that this result holds true even with arbitrary subcarrier grouping (instead of (6)) as long as each GSTF subsystem contains $L+1$ subcarriers. However, arbitrary subcarrier grouping generally involves correlated subchannels per GSTF system [c.f. Property 1], which will decrease the coding advantage. Thus, our subcarrier grouping scheme in (6) is optimal in the sense of maximizing coding advantage for a GSTF system of a given size.

Remark 2 Because codeword size affects directly the design complexity, our scheme enjoys lower design complexity relative to [1, 6], since $N_c > N_t(L+1)$ in typical applications. More important, $\mathbf{\Lambda}_e(l)$ in both C1) and C2) is related to $\mathbf{X}_g(l)$ in a much simpler way, as compared to that in [1, 6]. This will lead to a much simpler construction of our codes relative to those in [1, 6][1].

Having obtained the design criteria, we proceed to design GSTF trellis codes. GSTF book codes are designed in [5].

V. STF Trellis Coding

In this section, we design STF trellis codes by applying multiple trellis coded modulation (M-TCM) that has been used for developing ST trellis codes [7]. To enable application of M-TCM, we first build a link between our STF system, and the conventional ST system.

A. Space Virtual-Time Transmissions

Recalling that (6) represents 3-D GSTF codes $\mathbf{X}_g$ as a set of 2-D codes, we have implicitly suggested that the transmission of $\mathbf{X}_g$ can be thought of as being carried out in an equivalent 2-D (what we term) space-virtual-time (SVT) system with N_t transmit-antennas and N_r receive-antennas, where the virtual-time dimension corresponds to the joint dimension of time and frequency in a way we detail next.

First, let us define $\mathbf{x}_t := [x_t^1, \ldots, x_t^{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ as the symbol block transmitted by N_t transmit-antennas during the tth virtual time interval, and $\mathbf{y}_t := [y_t^1, \ldots, y_t^{N_r}]^T \in \mathbb{C}^{N_r \times 1}$ as the corresponding block of received samples. The SVT system is modeled as

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{w}_t, \qquad t = 0, \ldots, N_x(L+1) - 1, \tag{9}$$

[1] Indeed, no code construction is offered by [1, 6] due to its difficulty.

where, $\mathbf{H}_t \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix, and $\mathbf{w}_t \in \mathbb{C}^{N_r \times 1}$ is the noise vector. To link the SVT system to the GSTF system, let us define [c.f. (7)]

$$\begin{aligned} \mathbf{Y}_g &:= [\mathbf{Y}_g(0), \ldots, \mathbf{Y}_g(L)] &\in \mathbb{C}^{N_r \times N_x(L+1)} \\ \mathbf{W}_g &:= [\mathbf{W}_g(0), \ldots, \mathbf{W}_g(L)] &\in \mathbb{C}^{N_r \times N_x(L+1)} , \end{aligned}$$

and specify $\mathbf{y}_t$, $\mathbf{x}_t$, $\mathbf{w}_t$, and $\mathbf{H}_t$ as:

$$\begin{aligned} \mathbf{y}_t &= [\mathbf{Y}_g]_{\zeta(t)} & \mathbf{x}_t &= [\mathbf{X}_g]_{\zeta(t)} \\ \mathbf{w}_t &= [\mathbf{W}_g]_{\zeta(t)} & \mathbf{H}_t &:= \mathbf{H}_g(\tau(t)) . \end{aligned} \quad (10)$$

The index functions $\zeta(t)$ and $\tau(t)$ in (10) are given by (with $\lfloor\cdot\rfloor$ denoting integer floor):

$$\begin{aligned} \zeta(t) &:= N_x t - \lfloor \frac{t}{L+1} \rfloor (N_x L + N_x - 1) + 1 , \\ \tau(t) &:= t - \lfloor \frac{t}{L+1} \rfloor (L+1) , \end{aligned} \quad (11)$$

respectively. It is not difficult to recognize that except for the difference in the ordering of transmissions, the model in (9) is mathematically equivalent to that in (7). It is important to point out that in modeling our STF system as an SVT system we have exploited the property that STF transmissions are separable in both frequency, and time, but not in space.

Although the underlying fading channels are time-invariant, we notice that $\mathbf{H}_t$ varies with virtual-time, due to the fact each GSTF codeword is transmitted over different subcarriers. Therefore, (9) can be thought of as an ST system transmitting over time-selective (but frequency-flat) fading channels. This provides an explicit link between each GSTF subsystem and the well-developed ST system. Furthermore, (9) implies that: i) due to the presence of time diversity (or precisely, virtual-time diversity), the SVT (or STF) system can potentially achieve diversity advantage higher than $N_t N_r$, which corroborates the results in Sections IV; and ii) it is possible to take advantage of existing ST coding techniques in designing STF trellis (STFT) codes.

The so-called "smart-greedy" ST trellis codes have already been designed in [7] to achieve acceptable performance for both flat fading, and time-varying channels. Unlike [7], we are dealing with a different channel situation. Because the time-varying channel $\mathbf{H}_t$ herein is artificially created and its time-variations are well structured, we do not have to design ST codes that are "smart". Instead, we only need them to be "greedy" in order to take advantage of time-selectivity. Therefore, it suffice for us to stay with the design criteria C1) and C2).

B. Code Construction

Based on (9), the design of STFT codes is equivalent to building a trellis to generate $\mathbf{x}_t$'s continuously. Before pursuing their design, we state an important property of STFT codes:

Theorem 1 *Suppose that each transmitted symbol x_t^{μ} belongs to the constellation set $\mathcal{A}_t$ with $|\mathcal{A}_t| = 2^b$ elements. If the maximum diversity advantage $G_d^{\max} = N_t N_r (L+1)$ is achieved, then the transmission rate is at most $R_t^{\max} = \log_2 |\mathcal{A}_t| / (L+1) = b/(L+1)$ bits per second per hertz (bps/Hz).*

Proof: The proof can be readily extended from [7, Theorem 3.3.1, Corollary 3.3.1]. □

Because $R_t^{\max}$ is related to R by $R_t^{\max} = R \log_2 |\mathcal{A}_s|$, Theorem 1 implicitly suggests two possible design strategies:

Strategy 1 Design an STF trellis code with rate $R = 1$, where the trellis outputs a single block $\mathbf{x}_t$ corresponding to each information symbol s_t, and has cardinality $|\mathcal{A}_t| = |\mathcal{A}_s|^{(L+1)}$.
Strategy 2 Design an STF trellis code with code rate $R = 1/(L+1)$, where the trellis outputs $L+1$ blocks $\mathbf{x}_t$ corresponding to each s_t, and has cardinality $|\mathcal{A}_s| = |\mathcal{A}_t|$.

These two design strategies are equivalent in the sense that the resulting codes achieve the same transmission rate $R_t^{\max}$. Moreover, both strategies expand either the constellation of the transmitted symbols, or, that of the information symbols. However, their implementations are drastically different. According to the first strategy, one looks for a trellis involving constellation expansion, which is not the case for most existing ST trellis codes. Because we wish to take advantage of existing techniques developed for ST trellis codes, we will construct our STFT codes using the second strategy.

Let us denote with $\mathcal{T}_{\mathrm{ST}}(\cdot)$ an ST trellis encoder with $R = 1$, and $|\mathcal{A}_s| = |\mathcal{A}_t| = 2^{R_t^{\max}(L+1)}$. According to Strategy 2, our STF trellis encoder with $R = 1/(L+1)$ is constructed from $\mathcal{T}_{\mathrm{ST}}(\cdot)$ by simply repeating its output $L+1$ times. In other words, going back to each GSTF subsystem, we basically repeat transmissions over $L+1$ subcarriers. Therefore, the design of STFT codes is reduced to designing conventional ST trellis codes with repeated transmissions. A similar approach was also taken in designing the "smart-greedy" ST trellis codes in [7]. For example, when $N_t = 2$, $L = 1$, and $R_t = 2$ bps/Hz, we can use the ST trellis depicted in [7, Fig. 19] to generate STFT codes.

C. Decoding of GSTF Trellis Codes

Because GSTF trellis codes are generated by a trellis, their decoding can be efficiently implemented by using Viterbi decoding. According to (9), the branch metric for $\mathbf{x}_t$ is given by

$$\sum_{\nu=1}^{N_r} |y_t^\nu - \sum_{\mu=1}^{N_t} [\mathbf{H}_t]_{\nu\mu} x_t^\mu|^2 , \quad (12)$$

where $[\mathbf{H}_t]_{\nu\mu}$ denotes the (ν, μ)th element of $\mathbf{H}_t$. Similar to ST trellis codes [7], the decoding complexity of GSTF trellis codes is exponential in the number of trellis states, and the transmission rate.

Having designed STF trellis codes, we next use simulation to test their performance.

VI. SIMULATIONS

Here, we present one simulation to investigate the performance of our designs with $N_t = 2$ and $N_r = 1$. Our figure of merit is OFDM symbol error rate (OFDM-SER), which we average over 100,000 channel realizations. In our simulations, we will let L_{real} denote the physical channel order, and L the channel order assumed in designing STF block codes.

Example 1 We compare our STFT coding schemes to the SF coding schemes in [1, 6]. The 16-state TCM code with effective length 3, and the 16-state ST trellis code [7, Fig. 5] are used to generate the SF codes of [6] and [1], respectively. We choose QPSK modulation for the two SF coding scheme, and 16-QAM for our STFT coding. $N_c = 64$ is chosen for all schemes. The random channels ($L_{\text{real}} = 8$) are based on the HiperLan 2 channel model A [2]. The design of STFT codes is based on $L = 1$. Fig. 1 shows that STFT codes outperform SF codes at high SNR. Note that our STFT codes are designed regardless of the real channel order $L_{\text{real}} = 8$, channel correlation, and power profile in HiperLan 2 channels, which speaks for the robustness of our design.

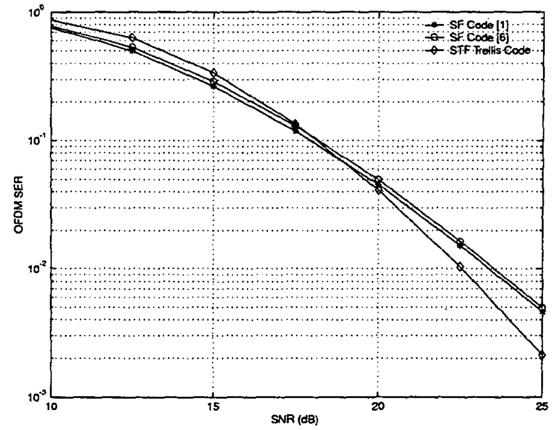

Fig. 1. Comparison with SF codes (HiperLan 2 channels)

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