PCC: Principal Components Combining for Dense Correlated Multipath Fading Environments*

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Abstract

Spread spectrum transmissions and RAKE receivers are known to alleviate the effects of random fading. In the context of future wideband/ultra-wideband systems, both estimation accuracy and receiver complexity are adversely affected when the number of channel parameters increases. As an alternative to generalized selection combining schemes, which have received a great deal of attention over the last couple of years, this work introduces a new class of diversity schemes that trade off optimally diversity gain with receiver complexity. The basic idea is to exploit the information on the channel statistics in selecting a linear mapping that reduces the channel order while minimizing the loss in terms of diversity gain. We prove that the optimal linear mapping amounts to projecting the received data onto the channel's principal components obtained by the eigenvectors of the channel correlation matrix corresponding to the Q strongest eigenvalues. We then derive closed-form expressions for the average combined signal-to-noise ratio and the average symbol error rate for various modulation schemes operating in dense Nakagami-m correlated multipath fading environments of practical interest.

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1 Introduction

The trend for next-generation wireless spread spectrum systems is towards extremely ultra wide-band transmission. One of the most promising features of such systems is their ability to resolve additional paths (compared, for example, to current "narrow-band" CDMA systems such as IS-95) resulting in an increased diversity which can be exploited by RAKE reception [1]-[4]. In this context, generalized selection combining (GSC) RAKE reception [5]-[21] was recently proposed and studied as an alternative to conventional maximal-ratio combining (MRC) or selection combining (SC). The idea behind GSC is to combine, as per the rules of MRC (or postdetection equal-gain combining (EGC) for noncoherent systems), contributions from the Q strongest paths. Although GSC approaches the performance of MRC with a reduced complexity receiver, it inherits many of the SC drawbacks. First and foremost, it requires continuous real-time monitoring as well as accurate estimation and ranking of *all* the channel taps that are often too many to handle in the case of wideband/ultra-wideband channels. Second, the maximum theoretical diversity gain promised by RAKE reception cannot be achieved if strong but correlated paths are selected. Finally, despite the many efforts devoted to analyzing GSC, analytical results are still limited to date to: (i) independent identically distributed Rayleigh paths [5, 6, 11, 14, 15, 16, 17, 18], (ii) independent but not identically distributed Rayleigh paths [5, 6, 7, 11, 11, 18] and; (iii) independent identically distributed Nakagami-m

paths [20, 19]. However, experimental measurements indicate that wideband multipath channels are typically characterized by a decaying power delay profile (a.k.a. multipath intensity profile (MIP)) as well as a strong fading correlation across the diversity paths [22, 23, 24, 25]. Yet the analytical performance evaluation of GSC over these typical channels of practical interest is very difficult (if not impossible).

This paper investigates an alternative combining scheme which alleviates the limitations of GSC while retaining its reduced complexity with a controllable performance loss. Rather than relying on *instantaneous* diversity path strengths, the proposed scheme exploits information on the channel *statistics* to optimally select a linear mapping that reduces the channel order. The optimality criteria that we consider here are based on the maximum average signal-to-noise ratio (SNR) at the combiner output as well as the minimum average symbol error rate (SER), both for a prescribed receiver complexity (measured in terms of fingers number for a given channel order). With the demand for spectrally efficient modulations over wireless channels in mind [26, 27], we show that our proposed scheme is optimal in the minimum average SER sense for any arbitrary two-dimensional (2D) M-ary constellation. To the best of the authors knowledge, little attention has been put in the vast literature on diversity combining [28, 29] on such schemes. A notable exception is [25], where thorough statistical analysis of several measured wideband impulse responses relied on *eigen analysis* to reduce the number of parameters to be treated for channel characterization purposes and effective multipath diversity assessment.

2 System Model

We describe the model in discrete time (DT), assuming that signals are sampled at the chip-rate and that the receiver is synchronous at the symbol level, except for a few chips (few compared to the spreading code $c(n)$ of length P in chips). Due to multipath propagation the signal experiences both intersymbol interference (ISI) and interchip interference (ICI). We model the channel $h(l)$ (which includes the effect of the pulse shaping receive-filter and chip-asynchronism) as an FIR filter of length L with random taps. Collecting in $\boldsymbol{r}(i)$ the $N = P + L - 1$ data samples received in the ith symbol period, which include the entire convolution between $h(l)$ and the spreading code $c(n)$ and neglecting ISI, we have

$$\boldsymbol{r}(i) = \boldsymbol{C}\,\boldsymbol{h}\,s(i) + \boldsymbol{n}(i), \qquad (1)$$

where $\boldsymbol{C}$ is the $N \times L$ Toeplitz matrix with $\{\boldsymbol{C}\}_{k+1,l+1} = c(k-l)$, $c(n)$ is the spreading code, $\boldsymbol{h} = (h(0), \ldots, h(L-1))^T$ is the random channel vector and $\boldsymbol{n}(i)$ is additive Gaussian noise vector $\sim \mathcal{N}(\boldsymbol{0}, \sigma_n^2\boldsymbol{I})$. Assuming that $c(n)$ is a pseudo-noise sequence the code correlation matrix $\boldsymbol{R}_{cc} = \boldsymbol{C}^H\boldsymbol{C} \approx 2E_s\boldsymbol{I}$, where E_s is the energy per symbol; thus, a first data reduction is obtained by correlating the received samples with $c(n)$, that yields the $L \times 1$ vector:

$$\begin{aligned} \boldsymbol{z}(i) &= \boldsymbol{C}^H\boldsymbol{r}(i) = \boldsymbol{C}^H\boldsymbol{C}\,\boldsymbol{h}\,s(i) + \boldsymbol{C}^H\boldsymbol{n}(i) \qquad (2)\\ &\approx E_s\boldsymbol{h}\,s(i) + \boldsymbol{v}(i), \end{aligned}$$

where ^{H} denotes the Hermitian transpose and $\boldsymbol{v}(i) \sim \mathcal{N}(\boldsymbol{0}, N_0\boldsymbol{I})$. Implementing the GSC as a Q-fingers RAKE, is equivalent to multiplying $\boldsymbol{z}(i)$ by a $Q \times L$ selection matrix $\boldsymbol{G}$. The entries of $\boldsymbol{G}$ assume zero-one values only and pick the strongest entries of $\boldsymbol{h}$ when projecting $\boldsymbol{G}\boldsymbol{z}(i)$ onto vector $\boldsymbol{w} := \boldsymbol{G}\boldsymbol{h}$. Hence, the receiver design consists of selecting matrix $\boldsymbol{G}$ and vector $\boldsymbol{w}$. When $\boldsymbol{v}(i)$ is white, $\boldsymbol{w} := \boldsymbol{G}\boldsymbol{h}$ is the optimal linear receiver for a given $\boldsymbol{G}$ and constitutes the MRC receiver on the *reduced* channel $\boldsymbol{G}\boldsymbol{h}$. Considering the structure of $\boldsymbol{G}$, which has only one non zero entry per row equal to one and in different positions depending on the row, we have that $\boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I}$. Thus, the combined SNR $\tilde{\gamma}_c$ per symbol at the MRC output is

$$\begin{aligned} \tilde{\gamma}_c(\boldsymbol{w}, \boldsymbol{G}, \boldsymbol{h}) &= \frac{2E_s|\boldsymbol{w}^H\boldsymbol{G}\boldsymbol{h}|^2}{N_0\,\boldsymbol{w}^H\boldsymbol{G}\boldsymbol{G}^H\boldsymbol{w}^H} \qquad (3)\\ &= \frac{2E_s\boldsymbol{w}^H\boldsymbol{G}\boldsymbol{h}\boldsymbol{h}^H\boldsymbol{G}^H\boldsymbol{w}}{N_0\,\boldsymbol{w}^H\boldsymbol{w}^H}\\ &\leq \frac{2E_s}{N_0}\boldsymbol{h}^H\boldsymbol{G}^H\boldsymbol{G}\boldsymbol{h} := \gamma_c(\boldsymbol{G}, \boldsymbol{h}) \end{aligned}$$

where equality is achieved for $\boldsymbol{w} \propto \boldsymbol{G}\boldsymbol{h}$. Thus,

$$\boldsymbol{w}_{opt} = \underset{\boldsymbol{w}}{\operatorname{argmax}}\,\tilde{\gamma}_c(\boldsymbol{w}, \boldsymbol{G}, \boldsymbol{h}) = \mu\boldsymbol{G}\boldsymbol{h}, \qquad (4)$$

where μ is a arbitrary complex number, and

$$\tilde{\gamma}_c(\boldsymbol{w}_{opt}, \boldsymbol{G}, \boldsymbol{h}) = \gamma_c(\boldsymbol{G}, \boldsymbol{h}) = \frac{2E_s}{N_0}\boldsymbol{h}^H\boldsymbol{G}^H\boldsymbol{G}\boldsymbol{h}. \qquad (5)$$

Maximizing $\tilde{\gamma}_c(\boldsymbol{w}, \boldsymbol{G}, \boldsymbol{h})$ with respect to $\boldsymbol{w}$ is equivalent to minimizing the SER; thus, for a given $\boldsymbol{G}$, $\boldsymbol{w}$ is optimal also in the minimum SER sense. The complete phase and amplitude information of only Q entries of $\boldsymbol{G}\boldsymbol{h}$ is necessary. However, to this point in our exposition, designing $\boldsymbol{G}$ still requires an estimate of the instantaneous amplitude $\boldsymbol{\alpha}$ of $\boldsymbol{h}$ (denoted by $\hat{\boldsymbol{\alpha}}$), ranking of these estimates, and updating both $\boldsymbol{G}$ and $\hat{\boldsymbol{\alpha}}$ fast enough to track small-scale channel variations. In the next section we propose our alternative design.

3 Optimal Design

Our idea is to let $\boldsymbol{G}$ be an arbitrary $Q \times L$ semi-unitary matrix (i.e., $\boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I}$), which preserves the noise whiteness, and maximize the average output SNR per symbol or minimize the average SER both with respect to $\boldsymbol{G}$.

3.1 Maximum Average Combined SNR Criterion

Average combined SNR is a typical performance measure of diversity systems [8, 10, 12, 13, 18, 28, 29]. From (5) we have that the average SNR is:

$$\overline{\gamma}_c(\boldsymbol{G}) = E\{\gamma_c(\boldsymbol{G},\boldsymbol{h})\} = \frac{2E_s}{N_0} tr(\boldsymbol{G}\boldsymbol{R}_{hh}\boldsymbol{G}^H), \quad (6)$$

where $tr(\boldsymbol{A})$ is the trace of the matrix $\boldsymbol{A}$.

The following lemma gives the optimal $\boldsymbol{G}$ that achieves maximum output SNR.

Lemma 1 *For any fading distribution, the maximum average SNR $\overline{\gamma}_c$ at the combiner output is reached by projecting the data vector $\boldsymbol{z}(i)$ on the principal components, i.e., the eigenvectors of the channel correlation matrix corresponding to the Q strongest eigenvalues. Denoting by $\boldsymbol{R}_{hh} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H$ the eigen value decomposition (EVD) of $\boldsymbol{R}_{hh}$ and by $\bar{\boldsymbol{U}}^H$ the matrix with columns the eigenvectors corresponding to the Q strongest diagonal entries of $\boldsymbol{\Lambda}$, i.e.*

$$\boldsymbol{G}_{opt} = \bar{\boldsymbol{U}}^H , \quad (7)$$

is the solution of:

$$\boldsymbol{G}_{opt} = \underset{\boldsymbol{G}}{\text{argmax}}\ \overline{\gamma}_c(\boldsymbol{G}) \ \text{ subject to } \ \boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I}. \quad (8)$$

Proof: The proof is given in Appendix A.

3.2 Minimum Average SER Criterion

Relying on Craig's method [30] and the moment generating function (MGF)-based approach for the performance evaluation of diversity systems, the average SER for any arbitrary 2D symbol constellations can always be expressed as [31, 27]

$$P_s(E) = \sum_{i=1}^{I} \frac{p_i}{2\pi} \int_0^{\theta_i} \mathcal{M}_{\gamma_c}\left(-\frac{a_i \sin^2\psi_i}{\sin^2(\phi+\psi_i)}\right) d\phi\,, \quad (9)$$

where

$$\mathcal{M}_{\gamma_c}(s) := E\{e^{s\gamma_c}\} \quad (10)$$

is the MGF of the total SNR per symbol at the diversity combiner output, γ_c, I is the number of decision regions or symbols, p_i is the a-priori probability of the ith symbol, a_i is a normalization factor and θ_i, ψ_i depend on the constellation geometry.

Starting from (9) and from the fact that the MGF of the total SNR per symbol can be shown to be given by [32]

$$\mathcal{M}_{\gamma_c}(s) = |\boldsymbol{I} - s\,\boldsymbol{G}^H\boldsymbol{G}\boldsymbol{R}_{hh}|^{-m}, \quad (11)$$

where m is the Nakagami-m fading parameter, we are able to derive the following lemma.

Lemma 2 *For Nakagami-m fading, the minimum average SER for any symbol constellation is also reached by projecting the data vector $\boldsymbol{z}(i)$ on the principal components; equivalently, using the same notations of Lemma 1, i.e.:*

$$\boldsymbol{G}_{opt} = \bar{\boldsymbol{U}}^H , \quad (12)$$

is the solution of

$$\boldsymbol{G}_{opt} = \underset{\boldsymbol{G}}{\text{argmin}}\ P_s(E) \ \text{ subject to } \ \boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I} \quad (13)$$

Proof: The proof is in Appendix A.

Remarkably, (7) and (12) are the same, and maximizing the average SNR with respect to $\boldsymbol{G}$ also minimizes the average SER. Using Lemma 2 and (11), we can derive the minimum average SER over Nakagami-m fading channels as stated in the following corrolary.

Corrolary 1 *Assuming that the eigenvalues of $\boldsymbol{R}_{hh}$ in $\boldsymbol{R}_{hh} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H$ are such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L$ then the minimum average SER is given by*

$$\min_{\boldsymbol{G}} P_s(E) = \sum_{i=1}^{I} \frac{p_i}{2\pi} \int_0^{\theta_i} \prod_{q=1}^{Q} \left(1 + \lambda_q \frac{a_i \sin^2\psi_i}{\sin^2(\phi+\psi_i)}\right)^{-m} d\phi\,. \quad (14)$$

3.3 Implementation Considerations

To derive the average output SNR we just need to estimate $\boldsymbol{R}_{hh}$. On the contrary, to derive the average SER we need to estimate the statistics of the random variable $\gamma_c(\boldsymbol{G},\boldsymbol{h})$ as a function of $\boldsymbol{G}$, which can be derived from the estimated joint probability density function (PDF) of $\boldsymbol{h}$. However, in the case of Nakagami-m fading the joint PDF is completely described by the channel correlation $\boldsymbol{R}_{hh}$ and by the fading parameter m. Assuming that $\boldsymbol{h}$ is ergodic and independent of $s(i)$ and $\boldsymbol{v}(i)$, the estimate of $\boldsymbol{R}_{hh}$ can be obtained from the data without training, using the sample averages of $\boldsymbol{z}(i)$

$$\hat{\boldsymbol{R}}_{zz} := \frac{1}{N} \sum_{n=0}^{N-1} \boldsymbol{z}(i)\boldsymbol{z}^H(i)\,, \quad (15)$$

and

$$\lim_{N\to\infty} \hat{\boldsymbol{R}}_{zz} = \boldsymbol{R}_{zz} = \sigma_s^2 \boldsymbol{R}_{hh} + \sigma_v^2 \boldsymbol{I}. \quad (16)$$

Noticeably, the knowledge of the fading parameter m is not required by the designs in Lemma 1 and 2.

It is also important to observe that for the two performance measures under consideration (average output SNR and average SER) the optimal design of $\boldsymbol{G}$ requires the estimation of $\bar{\boldsymbol{U}}^H$, which can be obtained performing an EVD on $\hat{R}_{hh}$ or, more efficiently implementing subspace tracking techniques [33] that track the dominant channel subspace. Performing the EVD of the correlation matrix to update G entails of course a certain computational complexity. However, in the proposed principal component combining (PCC) scheme, this needs to be done only when the channel statistics change (i.e., at the rate of large-scale channel variations). Furthermore, when used in conjunction with coherent MRC the PCC scheme requires instantaneous estimation of the Q components of Gh only. On the other hand, when used in conjunction with noncoherent post-detection EGC, instantaneous estimation of the Q components is not required. Hence unlike GSC, instantaneous amplitude information acquisition, estimation, and ranking of all channel taps is not required.

4 Performance Analysis

The choice of G in the PCC takes optimally into account the average power unbalance and the fading correlation across the diversity paths. Contrary to the GSC, it also yields simple closed form average output SNR and SER expressions in many encountered channels of practical interest, as we will discuss in more details next.

4.1 Input/Output Statistics

We assume that the entries of h have amplitudes $\{\alpha_l\}_{l=1}^{L}$ that are (not necessarily independent) Nakagami-m distributed with the same fading parameter m and with average fading powers $\{\Omega_l\}_{l=1}^{L}$. Note that

$$\{R_{hh}\}_{i,j} = E\{h(i)h^*(j)\}, \qquad (17)$$

is not equal to the correlation between the envelopes α_i and α_j but is the correlation between the channel samples. The average SNR per symbol for the l-th input path is given by $\overline{\gamma}_l = 2\Omega_l E_s/N_0$. After the linear mapping G, it can be shown that the entries of $w = Gh$ with amplitudes denoted by $\{\alpha_q'\}_{q=1}^{Q}$, are independent Nakagami-m distributed with the same fading parameter m and with average fading powers $\{\Omega_q'\}_{q=1}^{Q}$ equal to the Q strongest eigenvalues of R_{hh}. Furthermore, the matrix G does not change the statistics of the AWGN. Hence, the average SNR per symbol for the q-th output path is given by $\overline{\gamma}_q' = 2\Omega_q' E_s/N_0$. Two cases are of interest from the performance analysis perspective.

4.2 Case 1: $Q = L$

Although channel order reduction is not performed in this case, it is still of interest. Indeed, the fact that the output "mapped" paths are independent greatly simplifies the performance analysis of classical combining schemes (MRC, EGC, SC, and GSC) in a correlated fading environment. For example, it can be shown that the average BER performance of DPSK with conventional dual SC over correlated Nakagami-m fading channels is tightly lower bounded by

$$P_b(E)=\frac{1}{2m}\frac{(\overline{\gamma}_1'\overline{\gamma}_2')^m}{(\overline{\gamma}_1'+\overline{\gamma}_2')^{2m}}\frac{\Gamma(2m)}{(\Gamma(m))^2}\left(1+\frac{1}{m}\frac{\overline{\gamma}_1'\overline{\gamma}_2'}{(\overline{\gamma}_1'+\overline{\gamma}_2')}\right)^{-2m} \times\sum_{i=1}^{2}{}_2F_1\left(1,2m;m+1;\left[\overline{\gamma}_i'\left(\frac{1}{\overline{\gamma}_1'}+\frac{1}{\overline{\gamma}_2'}+\frac{1}{m}\right)\right]^{-1}\right), \qquad (18)$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function. This relatively simple lower bound has to be contrasted with the more complicated but exact formula given in [34, Eq. (32)]. Similarly, it can be shown that the average BER performance of binary differential phase-shift-keying (DPSK) with conventional SC over L equicorrelated identically distributed Rayleigh paths is tightly lower bounded by

$$\begin{aligned} P_b(E) &= \frac{1}{2}\sum_{l=0}^{L-1}(-1)^l\binom{L-1}{l} \\ &\times \frac{1}{1+\overline{\gamma}(1+(L-1)\rho)+l\frac{1+(L-1)\rho}{1-\rho}} \\ &+ \frac{L-1}{2}\sum_{l=0}^{L-2}(-1)^l\binom{L-2}{l}\left(\frac{1}{1+l+(1-\rho)\overline{\gamma}}\right. \\ &- \left.\frac{1}{(1-\rho)\overline{\gamma}+\frac{(1-\rho)}{1+(L-1)\rho}+1+l}\right), \end{aligned} \qquad (19)$$

where ρ is the fading correlation coefficient and $\overline{\gamma}$ is the average SNR per bit per path. Fig. 1 illustrates our claim for various values of the correlation coefficient ρ. Note that since no *exact* closed-form expressions are known to date for the average BER performance of DPSK with conventional SC over L equicorrelated Rayleigh paths, the exact results shown in Fig. 1 are obtained via Monte-Carlo simulations.

To generate the channel vector h in our simulations, we generated zero mean complex Gaussian random vectors with independent entries and colored the vectors by multiplying them by the Cholesky decomposition of R_{hh}. Our derivations involve the the correlation between $h(i)$ and $h(j)$ rather than the correlation between the corresponding envelopes α_i and α_j and we could have specified the fading parameters directly in terms of R_{hh}. However, in Fig. 1 the parameter ρ follows the traditional definition fading correlation coefficient and, thus, designates the correlation coefficient between the envelopes of the channel paths. To determine R_{hh} for our simulation we followed the same approach used in [35] and derived the correlation coefficients

$\rho_h(i,j)$ of the Gaussian complex random variables whose envelopes have specified correlation coefficients $\rho_\alpha(i,j)$. For every $\rho_\alpha(i,j)$ the value of $\rho_h(i,j)$ is obtained inverting numerically the equation

$$\rho_\alpha(i,j) = \frac{(1+|\rho_h(i,j)|)E_i\left(\frac{2\sqrt{|\rho_h(i,j)|}}{1+|\rho_h(i,j)|} - \frac{\pi}{2}\right)}{2-\frac{\pi}{2}} \quad (20)$$

where $E_i(\cdot)$ is the complete elliptic function of the second kind. In force of (20), we can also note that equicorrelated Rayleigh envelopes can be generated by generating equicorrelated complex Gaussian coefficients.

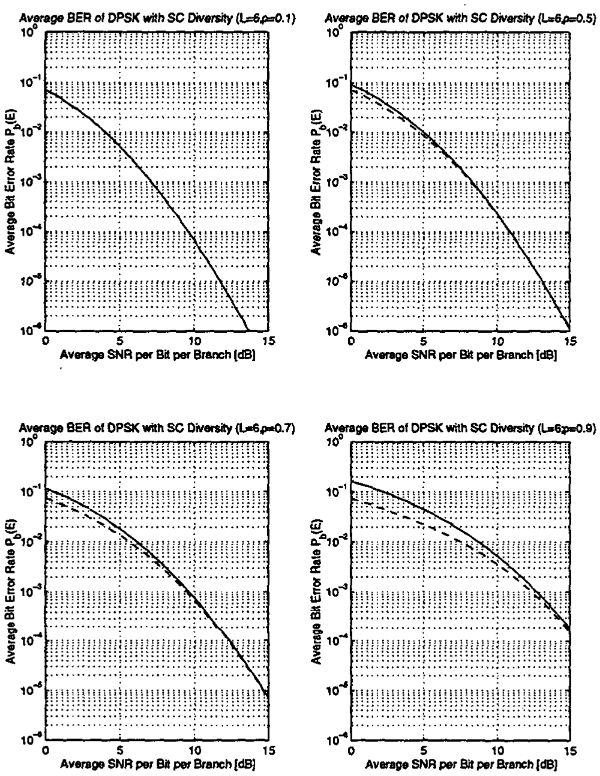

Figure 1. Average BER of DPSK with SC over 6 equi-correlated Rayleigh paths. The solid curves correspond to exact results obtained via Monte-Carlo simulations. The dashed curves correspond to the closed-form expression for the SC scheme over the mapped paths.

4.3 Case 2: $Q < L$

Because the Q output paths are independent the performance of PCC can be easily analyzed via the MGF-based approach [31, 36]. For example, the average SER performance of M-PSK with PCC over correlated Nakagami-m fading channels is given by

$$P_s(E) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \prod_{q=1}^{Q}\left(1+\frac{g_{\text{psk}}\,\overline{\gamma}'_q}{m\,\sin^2\phi}\right)^{-m} d\phi, \quad (21)$$

where $g_{\text{psk}} = \sin^2(\pi/M)$. As a numerical example, Fig. 2 shows the average BER performance of BPSK with PCC over a multipath Rayleigh fading channel with an exponentially decaying PDP and exponential correlation

$$\rho_h(i,j) = \rho^{i-j} \quad i,j = 1,\ldots,L \quad (22)$$

across the diversity paths, which for simplicity this time was specified in terms of the channel coefficients rather than the envelopes. When these performance plots are compared with the performance of GSC (which were obtained from Monte-Carlo simulations over the same channel conditions) we observed that PCC and GSC exhibit opposite behavior. PCC has more diminishing returns as the number of available paths L increases for a fixed number of combined paths Q. However, the performance gain with PCC is significant when we increase the number of combined paths Q for a fixed number of available paths L.

5 Conclusion

In this paper we proposed a novel combing scheme that reduces the complexity of the receiver in dense correlated multipath environments with respect to standard coherent MRC or noncoherent postdetection EGC. Compared to the GSC, the receiver structure replaces the selection matrix that extracts, on an instantaneous basis, the L strongest channel coefficients, with a linear operator $\boldsymbol{G}$ that projects the channel coefficients onto a space of reduced dimensionality. We have proved that, in order to maximize the average SNR or minimize the average SER in the case of Nakagami-m fading, the channel vector has to be projected onto the principal components of the channel covariance matrix. The decorrelating properties of our scheme allow us to derive simple closed form expressions for the average SER of the PCC that we compared with the performance of a GSC in correlated Rayleigh fading evaluated numerically through Monte-Carlo simulations. We have also illustrated how decorrelating the L channel paths with an $L \times L$ $\boldsymbol{G}$ matrix and analyzing the performance of the GSC applied to the uncorrelated channel can be useful as a theoretical tool to assess the average SER performance of GSC in correlated fading.

A *Proof of Lemma 1*

Lemma 1 is a direct consequence of the following [37]

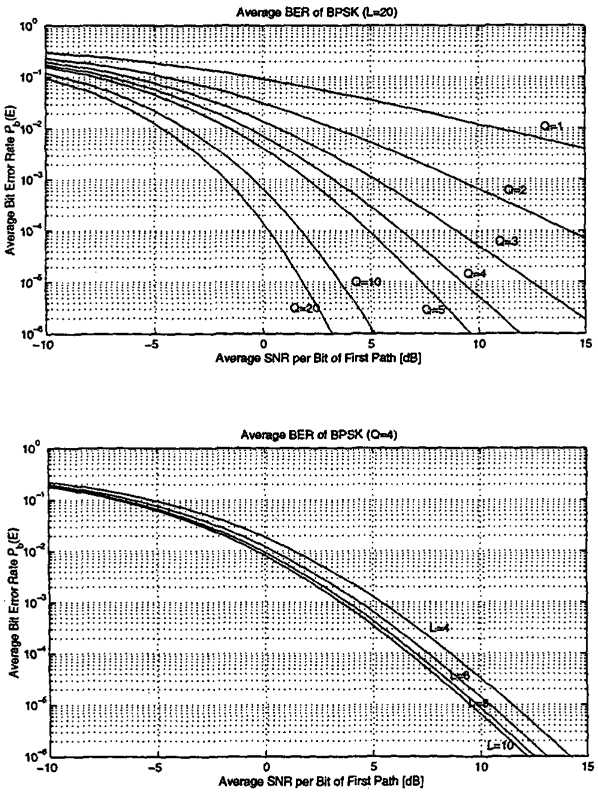

Figure 2. Average BER performance of BPSK with PCC over a multipath Rayleigh fading channel with an exponentially decaying PDP (power decay factor = 0.1) and exponential correlation across the paths.

Lemma 3 *For any semiunitary matrix* $\boldsymbol{G}$, *i.e. for any* $\boldsymbol{G}$ *with dimensionality* $Q \times L$ *with* $Q < L$ *and such that* $\boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I}$, *given an arbitrary positive semidefinite matrix* $\boldsymbol{A}$, *with eigenvalues* $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L$, *the following inequalities hold true:*

$$\sum_{l=Q+1}^{L} \lambda_l \leq tr(\boldsymbol{G}\boldsymbol{A}\boldsymbol{G}^H) \leq \sum_{l=1}^{Q} \lambda_l \qquad (23)$$

where the upper bound is achieved if and only if the columns of $\boldsymbol{G}^H$ *are the eigenvectors associated to the* Q *largest eigenvalues (or principal components) of* $\boldsymbol{A}$.

Recalling the definition of $\bar{\boldsymbol{U}}$ and replacing $\boldsymbol{A}$ by $\boldsymbol{R}_{hh}$ the solution given in (7) follows.

B *Proof of Lemma 2*

Because all the contributions to the average $P_s(E)$ in (9) are positive, minimizing each term with respect to $\boldsymbol{G}$ minimizes $P_s(E)$ too. The interesting aspect of this proof is that each of these components of $P_s(E)$ is minimized by the same matrix $\boldsymbol{G}$. In fact, under the constraint $\boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I}$, the $\boldsymbol{G}$ that minimizes the generic term $|\boldsymbol{I} + a\boldsymbol{G}^H\boldsymbol{G}\boldsymbol{R}_{hh}|^{-m}$, with $a \geq 0$, does not depend on s. To prove that $\boldsymbol{G}_{opt}$ is given by (12) we use the Hadamard inequality, which states that, for any $N \times N$ positive definite matrix $\boldsymbol{A}$

$$|\boldsymbol{A}| \leq \prod_{i=1}^{N} a_{ii} \qquad (24)$$

where the equality holds true if and only if $\boldsymbol{A}$ is diagonal.

For any pair of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ with compatible dimensions, the non null eigenvalues of $\boldsymbol{AB}$ and $\boldsymbol{BA}$ coincides. Hence from (11) we can write

$$\mathcal{M}_{\gamma_c}\left(-\frac{a_i \sin^2\psi_i}{\sin^2(\phi+\psi_i)}\right) = \left|\boldsymbol{I} + \frac{a_i \sin^2\psi_i}{\sin^2(\phi+\psi_i)}\boldsymbol{G}\boldsymbol{R}_{hh}\boldsymbol{G}^H\right|^{-m}. \qquad (25)$$

Therefore minimizing

$$\mathcal{M}_{\gamma_c}(-a_i \sin^2\psi_i / \sin^2(\phi+\psi_i)) \qquad (26)$$

with respect to $\boldsymbol{G}$ is equivalent to maximizing

$$|\boldsymbol{I} + a_i \sin^2\psi_i / \sin^2(\phi+\psi_i)\boldsymbol{G}\boldsymbol{R}_{hh}\boldsymbol{G}^H|. \qquad (27)$$

According to (24), in order to maximize this determinant we have to impose to $\boldsymbol{I} + a\boldsymbol{G}\boldsymbol{R}_{hh}\boldsymbol{G}^H$ a diagonal structure. Therefore, considering that $\boldsymbol{G}\boldsymbol{G}^H = \boldsymbol{I}$, the columns of $\boldsymbol{G}^H$ have to be the Q normalized eigenvectors of $\boldsymbol{R}_{hh}$. Finally, to reach the maximum the eigenvectors in $\boldsymbol{G}_{opt}^H$ have to be the ones associated to the strongest eigenvalues of $\boldsymbol{R}_{hh}$, and this leads to (12).

References

[1] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for next-generation mobile communications systems," *IEEE Commun. Mag.*, vol. 36, pp. 56–69, September 1998.

[2] M. Z. Win and R. A. Scholtz, "On the energy capture of ultrawide bandwidth signals in dense multipath environments," *IEEE Commun. Letters*, vol. 2, pp. 245–247, September 1998.

[3] M. Z. Win and R. A. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. COM-48, pp. 679–691, April 2000.

[4] M. Z. Win and R. A. Scholtz, "Infinite Rake and selective Rake receivers for ultra-wide bandwidth transmissions in multipath environments," *IEEE Commun. Letters*. To appear in 2000.

[5] N. Kong, T. Eng, and L. B. Milstein, "A selection combining scheme for RAKE receivers," in *Proc. IEEE Int. Conf. Univ. Personal Comm. ICUPC'95, Tokyo, Japan*, pp. 426–429, November 1995.

[6] T. Eng, N. Kong, and L. B. Milstein, "Comparison of diversity combining techniques for Rayleigh-fading channels," *IEEE Trans. on Commun.*, vol. COM-44, pp. 1117–1129, September 1996. See also "Correction to "Comparison of diversity combining techniques for Rayleigh-fading channels"", *IEEE Trans. Commun.*, vol. COM-46, p. 1111, September 1998.

[7] K. J. Kim, S. Y. Kwon, E. K. Hong, and K. C. Whang, "Comments on "Comparison of diversity combining techniques for Rayleigh-fading channels," *IEEE Trans. Commun.*, vol. COM-46, pp. 1109–1110, September 1998.

[8] N. Kong and L. B. Milstein, "Average SNR of a generalized diversity selection combining scheme," *IEEE Commun. Letters*, vol. 3, p. 5759, March 1999. See also *Proc. IEEE Int. Conf. on Commun. ICC'98, Atlanta, GA*, pp. 1556-1560, June 1998.

[9] N. Kong and L. B. Milstein, "A closed form expression for the average SNR when combining an arbitrary number of diversity branches with non-identical Rayleigh fading statistics," in *Proc. IEEE Int. Conf. on Commun. ICC'99, Vancouver, British Columbia, Canada*, pp. 1864–1868, June 1999.

[10] M. Z. Win and J. H. Winters, "Analysis of hybrid selection/maximal-ratio combining in Rayleigh fading," *IEEE Trans. Commun.*, vol. COM-47, pp. 1773–1776, December 1999. See also *Proceedings of IEEE International Conference on Communications (ICC'99)*, pp. 6-10, Vancouver, British Columbia, Canada, June 1999.

[11] M. Z. Win and J. H. Winters, "Analysis of hybrid selection/maximal-ratio combining of diversity branches with unequal SNR in Rayleigh fading," in *IEEE Veh. Technol. Conf. (VTC'99), Houston, TX*, pp. 215–220, May 1999.

[12] M. Z. Win and Z. A. Kostić, "Virtual path analysis of selective Rake receiver in dense multipath channels," *IEEE Commun. Letters*, vol. 3, pp. 308–310, November 1999. See also *Proceedings of 8th Communication Theory Mini-Conference (CTMC VIII) in conjunction with IEEE International Conference on Communications (ICC'99)*, pp. 78-82, Vancouver, British Columbia, Canada, June 1999.

[13] M. Z. Win and Z. A. Kostić, "Impact of spreading bandwidth on Rake reception in dense multipath channels," *IEEE J. Select. Areas Commun.*, vol. SAC-17, pp. 1794–1806, October 1999. See also *Proceedings of 8th Communication Theory Mini-Conference (CTMC VIII) in conjunction with IEEE International Conference on Communications (ICC'99)*, pp. 78-82, Vancouver, British Columbia, Canada, June 1999.

[14] M. Z. Win, R. K. Mallik, G. Chrisikos, and J. H. Winters, "Canonical expressions for the error probability performance for M-ary modulation with hybrid selection/maximal-ratio combining in Rayleigh fading," in *IEEE Wireless Commun. and Networking Conf. (WCNC'99), New Orleans, LA*, pp. 266–270, September 1999.

[15] M. Z. Win, G. Chrisikos, and N. R. Sollenberger, "Impact of spreading bandwidth and diversity order on the error probability performance of Rake reception in dense multipath channels," in *IEEE Wireless Commun. and Networking Conf. (WCNC'99), New Orleans, LA*, pp. 1558–1562, September 1999.

[16] M. Z. Win, G. Chrisikos, and J. H. Winters, "Error probability performance for M-ary modulation in Rayleigh fading using hybrid selection/maximal-ratio combining," in *Proc. IEEE Mil. Commun. Conf. (MILCOM'99)*, November 1999.

[17] M. Z. Win and J. H. Winters, "Exact error probability expressions for H-S/MRC in Rayleigh fading: A virtual branch technique," in *IEEE Global Commun. Conf. (GLOBECOM'99)*, pp. 537–541, December 1999.

[18] M. -S. Alouini and M. K. Simon, "An MGF-based performance analysis of generalized selective combining over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. COM-48, pp. 401–415, March 2000. See also *Proceedings of 8th Communication Theory Mini-Conference (CTMC VIII) in conjunction with IEEE International Conference on Communications (ICC'99)*, pp. 110-114, Vancouver, British Columbia, Canada, June 1999.

[19] M. -S. Alouini and M. K. Simon, "Performance of coherent receivers with hybrid SC/MRC over Nakagami-*m* fading channels," *IEEE Trans. Veh. Technol.*, vol. VT-48, pp. 1155–1164, July 1999.

[20] M. -S. Alouini and M. K. Simon, "Application of the Dirichlet transformation to the performance evaluation of generalized selection combining over Nakagami-m fading channels," *Int. J. of Commun. and Networks*, vol. 1, pp. 5–13, March 1999. See also *Proc. of the IEEE Vehicular Technology Conference (VTC'99-Fall)*, pp. 953-957, Amsterdam, The Netherlands, September 1999.

[21] J. A. Ritcey and M. Azizoğlu, "Performance analysis of generalized selection combining with switching constraints," *IEEE Commun. Letters*, vol. 4, pp. 152–154, May 2000. See also *Proc. Conf. on Information Sceinces and Systems (CISS'2000)*, Princeton, NJ, pp. WA8a-1-WA8a-6, March 2000.

[22] G. L. Turin, F. D. Clapp, T. L. Johnston, S. B. Fine, and D. Lavry, "A statistical model of urban multipath propagation," *IEEE Trans. Veh. Technol.*, vol. VT-21, pp. 1–9, February 1972.

[23] A. S. Bajwa, "UHF wideband statistical model and simulation of mobile radio multipath propagation effects," *IEE Proc.*, vol. 132, pp. 327–333, August 1985.

[24] F. Patenaude, J. Lodge, and J. -Y. Chouinard, "Temporal correlation analysis of frequency selective fading channels," in *Sixth Annual International Conference on Wireless Communications (Wireless'94), Calgary, Canada*, pp. 134–139, July 1994.

[25] F. Patenaude, J. Lodge, and J. -Y. Chouinard, "Eigenanalysis of wideband fading channel impulse responses," *IEEE Trans. Veh. Technol.*, vol. VT-48, pp. 593–606, March 1999.

[26] L. B. Milstein and M. S. Lim, "On the performance of a higher order alphabet size in CDMA," *IEEE Commun. Letters*, vol. 1, pp. 9–11, January 1997.

[27] X. Dong, N. C. Beaulieu, and P. H. Wittke, "Signaling constellations for fading channels," *IEEE Trans. Commun.*, vol. COM-47, pp. 703–714, May 1999. See also *Proc. Communication Theory Mini-Conference (CTMC-VII) in conjunction with IEEE Global Commun. Conf. GLOBECOM'98, Sydney, Australia*, pp. 22-27, November 1998.

[28] G. L. Stüber, *Principles of Mobile Communications*. Norwell, MA: Kluwer Academic Publishers, 1996.

[29] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ: PTR Prentice Hall, 1996.

[30] J. W. Craig, "A new, simple, and exact result for calculating the probability of error for two-dimensional signal constellations," in *Proc. IEEE Milit. Commun. Conf. MILCOM'91, McLean, VA*, pp. 571–575, October 1991.

[31] M. -S. Alouini and A. J. Goldsmith, "A unified approach for calculating the error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. COM-47, pp. 1324–1334, September 1999. See also *Proc. IEEE Int. Conf. on Commun. (ICC'98)*, Atlanta, GA, June 1998.

[32] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation," *IEEE Trans. Commun.*, vol. COM-47, pp. 44–52, January 1999.

[33] S. Hosur, A. H. Tewfik, and D. Boley, "ULV and generalized ULV subspace tracking adaptive algorithms," *IEEE Trans. Sig. Processing*, vol. SIG-46, pp. 1282–1297, May 1998.

[34] M. K. Simon and M. -S. Alouini, "A unified performance analysis of digital communications with dual selective combining diversity over correlated Rayleigh and Nakagami-m fading channels," *IEEE Trans. Commun.*, vol. COM-47, pp. 33–43, January 1999. See also *Proc. Communication Theory Mini-Conference (CTMC-VII) in conjunction with IEEE Global Commun. Conf. (GLOBECOM'98)*, pp. 28-33, Sydney, Australia, November, 1998.

[35] B. Natarajan, C. R. Nassar, and V. Chandrasekhar, "Generation of correlated Rayleigh fading envelopes for spread spectrum applications," *IEEE Commun. Letters*, vol. 4, pp. 9–11, January 2000.

[36] M. K. Simon and M. -S. Alouini, "A unified approach to the probability of error for noncoherent and differentially coherent modulations over generalized fading channels," *IEEE Trans. Commun.*, vol. COM-46, pp. 1625–1638, December 1998.

[37] P. Stoica and R. Moses, *Introduction to Spectral Analysis*. Prentice-Hall, Upper Saddle River, NJ, 1997.