SPACE-TIME CODING FOR IMPULSE RADIO

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ABSTRACT

Impulse Radio (IR) ultrawide band systems have well documented advantages for low-power peer-to-peer, and multiple access communications. Space-Time (ST) coding on the other hand, has gained popularity as an effective means of boosting rates and performance. Existing IR transmitters rely on a single antenna, while ST coders have so far focused on digital linearly modulated transmissions. In this paper, we develop an orthogonal ST coding scheme for the analog nonlinearly (pulse-position) modulated multi-antenna IR system. We show that the resulting analog non-coherent detector is equivalent to an existing digital ST decoder possessing maximum likelihood optimality. Simulations testing peer-to-peer and multi-access IR scenarios confirm considerable increase in both bit-error-rate performance, and number of users that can be accommodated, when wedding ST coding with IR.

1. INTRODUCTION

Impulse Radio (IR) is an ultrawide band communication system with attractive features for baseband multiple access, tactical wireless communications, and multimedia services [4]. An IR transmission consists of a pseudo-randomly shifted train of very short pulses, where the information is encoded in the shift via pulse position modulation (PPM). With the help of random time-hopping (TH) codes, multiple users can be accommodated [4, 5]. The random shifts combined with the short pulse shaper, and the data modulation result in a transmitted signal with low power spectral density spread across the ultra-wide bandwidth.

Multi-antenna Space-Time (ST) coding is a very effective technique to increase channel capacity and performance [1]. Existing IR transmitters rely on a single antenna, while ST coders have so far focused primarily on digital linearly modulated transmissions.

In this paper, we develop an ST coding scheme for the analog nonlinearly (pulse-position) modulated multi-antenna IR system. Both peer-to-peer and multiple access (MA) scenarios are addressed. For simplicity, we focus on the two-transmit one-receive antennae setup of [1]. Different from [1] however, Channel State Information (CSI) is not required for our analog ST decoder. The resulting non-coherent detector is equivalent to an existing digital ST decoder possessing maximum likelihood optimality [2], but can be implemented in analog fashion with conventional IR receivers. Simulations testing peer-to-peer and multi-access IR scenarios confirm considerable increase in both bit-error-rate performance, and number of users that can be accommodated, when wedding ST coding with IR.

The rest of the paper is organized as follows. In Section 2, we introduce our M-ary PPM-IRMA model that starts from the conventional continuous-time model of [5], and relies on PPM interpretation given in [3]. The STIR system is derived in Section 3, where both the case of a single user without TH, and the case of multiple users with TH, and possibly asynchronous transmissions, are addressed. In Section 4, simulations are performed to compare Bit Error Rate (BER) performance of the STIR system against the conventional IR system that deploys a single transmit antenna. Conclusions are given in Section 5.

Notation: Bold upper (lower) case letters denote matrices (column vectors); $(\cdot)^T$ denotes transpose; $\delta(\cdot)$ stands for Kronecker's delta; $E\{\cdot\}$ for expectation, $p(\cdot)$ for probability, and $\lfloor\cdot\rfloor$ for integer floor.

2. PPM AS A SUM OF LINEAR MODULATIONS

In this section, we present our M-ary PPM Impulse Radio signal model with TH, starting from the conventional continuous-time model [5]. To facilitate the transition from the conventional continuous-time PPM to our ST coded model, we first briefly review the PPM-IR setup in the single-user case [3].

In M-ary PPM-IRMA, for each, e.g., the u-th user, every transmitted M-ary PPM symbol is repeated over N_f frames each having duration T_f. During a signalling interval of duration $T_s = N_f T_f$ seconds, $k_b = \log_2 M$ message bits of a user, having a bit rate R_b, are loaded in a k_b-bit buffer. Hence, the k_b-bit output symbol rate is $R_s = R_b/k_b$. We denote the u-th user's i-th symbol as $I_u(i)$, where $I_u(i) \in [0, M-1]$. It follows that the information bearing symbol transmitted during the k-th frame is $I_u(\lfloor k/N_f \rfloor)$.

Each frame contains N_c chips. With T_c denoting chip duration, we have $T_f = N_c T_c + T_g$, where T_g is a guard time introduced to account for processing delay at the receiver between two successively received frames. The u-th user's transmitted waveform is given by:

$$\nu_u(t) = \mathcal{P}_u \sum_{k=-\infty}^{+\infty} w\left(t - kT_f - c_u(k)T_c - \tau_{I_u(\lfloor k/N_f \rfloor)}\right), \quad (1)$$

where $\mathcal{P}_u$ is the u-th user's transmit power, $w(t)$ denotes the ultra-short monocycle that we normalize to have unit energy, and $c_u(k) \in [0, N_c - 1]$ is a periodic TH pseudo-random sequence with period P_c. The role of $c_u(k)$ is to enable multiple access, and security in e.g., military communications. Each signalling interval of duration T_s contains N_f copies of a single symbol (one per frame), with the monocycle time-shifted in each frame according to the symbol value; i.e., it is shifted by τ_m if $I_u(\lfloor k/N_f \rfloor) = m$, for $m \in [0, M-1]$. Let T_w denote the monocycle duration. In order to ensure orthogonal modulation, PPM delays τ_m must satisfy $\tau_m - \tau_{m-1} \geq T_w$, $\forall m \in [1, 2, \ldots, M-1]$. Thus, the chip duration is chosen to satisfy: $T_c \geq MT_w$.

It is possible to express $\nu_u(t)$ in (1) using M parallel branches each realizing a shifted version of the pulse stream. In order to generate the signal $\nu_u(t)$, we then only need to select one branch (out of M) depending on the symbol value. Adopting this point of

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view, we can re-express (1) as:

$$\nu_u(t) = \sum_{m=0}^{M-1} \nu_{u,m}(t) \tag{2}$$

with

$$\nu_{u,m}(t) = \mathcal{P}_u \sum_{k=-\infty}^{+\infty} s_{u,m}(k) w\left(t - kT_f - c_u(k)T_c - \tau_m\right), \tag{3}$$

where $s_{u,m}(k) := \delta(I_u(\lfloor k/N_f \rfloor) - m)$, $\forall m \in [0, M-1]$. Let the time-shifted pulses be defined as

$$w_m(t) := w(t - \tau_m), \quad m \in [0, M-1]. \tag{4}$$

Notice that when orthogonal PPM is used, we have

$$\int_0^{T_f} w_m(t) w_{m'}(t) = \delta(m - m'). \tag{5}$$

Using the definition in (4), we can rewrite (3) as

$$\nu_{u,m}(t) = \mathcal{P}_u \sum_{k=-\infty}^{+\infty} s_{u,m}(k) w_m\left(t - kT_f - c_u(k)T_c\right). \tag{6}$$

Notice that (2) and (6) express the nonlinear M-ary PPM in IR(MA) as a superposition of M linearly modulated transmissions.

3. ANALOG ST CODING FOR IR

Let us consider an IR system with $N_t = 2$ transmit antennas, and $N_r = 1$ receive antenna. We first parse frames into frame pairs. Each (say the n-th) frame pair consists of the $2n$-th, and the $(2n+1)$-st frames. Taking N_f to be even, it follows that each symbol is transmitted over $N_f' := N_f/2$ such frame pairs.

We assume that the channel coefficients (from transmit antennas to the receive antenna) are real Gaussian random variables with zero-mean and unit variance. We also assume that the channel coefficients remain invariant at least over one frame pair, i.e., over $2T_f$; but are allowed to vary from pair to pair. The fading channel coefficients are denoted as:

$$\mathbf{h}(n) := [h_0(n) \quad h_1(n)]^T, \tag{7}$$

where $h_l(n)$ is the quasi-static coefficient of the channel from the l-th transmit antenna to the receive antenna during the n-th frame pair. At the receiver, white Gaussian noise $\eta(t)$ with zero mean and variance $N_0/2$ is added to the received signal.

3.1. Single User

Here we deal with a peer-to-peer link, and suppose that synchronization has been acquired successfully. Each frame contains only one monocycle, and TH is not necessary. We thus omit the user-specific subscript u from (2), and the delay term in (6) which corresponds to the TH code. The ST coded signal transmitted from the two antennas over the n-th frame pair can be expressed as:

$$\boldsymbol{\mathcal{V}}(n;t) = \begin{bmatrix} \nu^{00}(n;t) & \nu^{01}(n;t) \\ \nu^{10}(n;t) & \nu^{11}(n;t) \end{bmatrix}, \tag{8}$$

where $\nu^{kl}(n;t)$ denotes the waveform transmitted from the l-th transmit antenna during the $(2n+k)$-th frame duration, and can be expressed explicitly as:

$$\begin{aligned}
\nu^{00}(n;t) &= \frac{\mathcal{P}}{\sqrt{2}} \sum_{m=0}^{M-1} s_m(2n) w_m(t - 2nT_f) \\
\nu^{01}(n;t) &= \frac{\mathcal{P}}{\sqrt{2}} \sum_{m=0}^{M-1} s_m(2n+1) w_m(t - 2nT_f) \\
\nu^{10}(n;t) &= \frac{\mathcal{P}}{\sqrt{2}} \sum_{m=0}^{M-1} s_m(2n+1) w_m(t - (2n+1)T_f) \\
\nu^{11}(n;t) &= -\frac{\mathcal{P}}{\sqrt{2}} \sum_{m=0}^{M-1} s_m(2n) w_m(t - (2n+1)T_f).
\end{aligned}$$

Notice that the monocycle from each antenna is transmitted with amplitude $\mathcal{P}/\sqrt{2}$, which yields a total power of $\mathcal{P}^2$ for the two antennas during each frame duration. This ensures identical transmit power as in a single antenna transmitter. Because N_f is even, we have $\lfloor 2n/N_f \rfloor = \lfloor (2n+1)/N_f \rfloor$, $\forall n$; i.e., $s_m(2n) = s_m(2n+1)$ $\forall m, n$. Hence, (8) can be written as:

$$\boldsymbol{\mathcal{V}}(n;t) = \frac{\mathcal{P}}{\sqrt{2}} \sum_{m=0}^{M-1} s_m(2n) \cdot \begin{bmatrix} w_m(t - 2nT_f) & w_m(t - 2nT_f) \\ w_m(t - (2n+1)T_f) & -w_m(t - (2n+1)T_f) \end{bmatrix}.$$

Notice that our ST transmission can be viewed as the *analog* frame-counterpart of Alamouti's *digital* orthogonal code [1].

The n-th frame pair at the receive antenna, $\mathbf{x}(n;t) := [x(2n;t) \; x(2n+1;t)]^T$, can then be expressed as $\mathbf{x}(n;t) = \boldsymbol{\mathcal{V}}(n;t)\mathbf{h}(n) + \boldsymbol{\eta}(t)$, or, componentwise, as:

$$\begin{aligned}
x(2n;t) &= \frac{\mathcal{P}}{\sqrt{2}}[h_0(n) + h_1(n)] \sum_{m=0}^{M-1} s_m(2n) w_m(t - 2nT_f) \\
&\quad + \eta(t) \\
x(2n+1;t) &= \frac{\mathcal{P}}{\sqrt{2}}[h_0(n) - h_1(n)] \sum_{m=0}^{M-1} s_m(2n) w_m(t - (2n+1)T_f) \\
&\quad + \eta(t).
\end{aligned}$$

We now pass each received frame through a bank of filters each matched to a shifted monocycle $\{w_{m'}(t)\}_{m'=0}^{M-1}$, and sample the resulting outputs at the frame rate. The pair of samples corresponding to each pair of frames can be written as:

$$\begin{aligned}
y_{m'}(2n) &= \int_{2nT_f}^{(2n+1)T_f} w_{m'}(t - 2nT_f) x(2n;t) dt \\
&= \frac{\mathcal{P}}{\sqrt{2}}[h_0(n) + h_1(n)] s_{m'}(2n) + \tilde{\eta}_{m'}(2n) \\
y_{m'}(2n+1) &= \int_{(2n+1)T_f}^{(2n+2)T_f} w_{m'}(t - (2n+1)T_f) x(2n+1;t) dt \\
&= \frac{\mathcal{P}}{\sqrt{2}}[h_0(n) - h_1(n)] s_{m'}(2n) + \tilde{\eta}_{m'}(2n+1),
\end{aligned} \tag{9}$$

where we used the orthogonality among time-shifted monocycles in (5); and $\tilde{\eta}(2n) := \int_{2nT_f}^{(2n+1)T_f} w_{m'}(t - 2nT_f)\eta(t) dt$ is the fil-

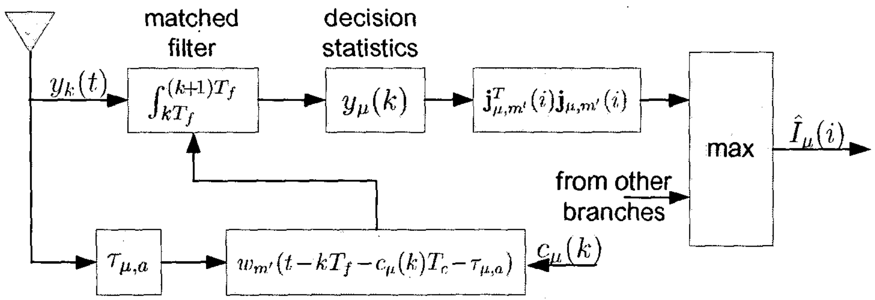

Fig. 1. Receiver block diagram corresponding to the m'-th branch of the μ-th user.

tered noise. Clearly, the noise components have mean values

$$\mathrm{E}\{\tilde{\eta}_m(k)\} = \int_{kT_f}^{(k+1)T_f} \mathrm{E}\{\eta(t)\} w_m(t-kT_f)dt = 0,$$

for all m and k, and covariances

$$\mathrm{E}\{\tilde{\eta}_m(k)\tilde{\eta}_{m'}(k')\} = \frac{N_0}{2}\delta(m-m')\delta(k-k').$$

Therefore, the noise components are zero-mean, uncorrelated Gaussian random variables, with variance $\sigma^2 = N_0/2$.

We will use as decision statistics the entries of the vector:

$$\mathbf{y}_{m'}(n) = \begin{bmatrix} y_{m'}(2n) \\ y_{m'}(2n+1) \end{bmatrix} = \frac{\mathcal{P}}{\sqrt{2}} \begin{bmatrix} h_0(n)+h_1(n) \\ h_0(n)-h_1(n) \end{bmatrix} s_{m'}(2n) + \begin{bmatrix} \tilde{\eta}_{m'}(2n) \\ \tilde{\eta}_{m'}(2n+1) \end{bmatrix}$$

for all m'. Let us now consider the sequence of N_f' decision statistics corresponding to N_f' frame pairs that contain the i-th symbol $I(i)$; and collect them in the vector:

$$\bar{\mathbf{y}}_{m'}(i) := [\mathbf{y}^T_{m'}(iN_f'), \ldots, \mathbf{y}^T_{m'}((i+1)N_f'-1)]^T.$$

Further noticing that $\mathbf{y}^T_{m'}(n)\mathbf{y}_{m'}(n) = \mathcal{P}(|h_0(n)|^2 + |h_1(n)|^2) \cdot s_{m'}(2n)$ in the absence of noise, and that $s_{m'}(2n)$ is identical for $n \in [iN_f', (i+1)N_f')$, we arrive at:

$$\bar{\mathbf{y}}^T_{m'}(i)\bar{\mathbf{y}}_{m'}(i) = \mathcal{P} s_{m'}(iN_f) \sum_{n=iN_f'}^{(i+1)N_f'-1} (|h_0(n)|^2 + |h_1(n)|^2). \quad (10)$$

Relative to a single-antenna IR transmission, (10) discloses that our STIR system has the ability to double the diversity gain with two transmit antennas. In fact, it is possible to show that with N_t transmit -antennas, and N_r receive antennas, the diversity order of STIR transmissions over flat fading channels can be as high as $N_t N_r$.

Recalling that $s_{m'}(iN_f) := \delta(I(i) - m')$, the decoding of $I(i)$ can be carried out according to [c. f. (10)]:

$$\hat{I}(i) = \arg\max_{m'}\{\|\bar{\mathbf{y}}_{m'}(i)\|^2\}, \quad (11)$$

which gives a unique estimate of the i-th transmitted symbol. In the Appendix, we show that such a decoding scheme is optimum in the maximum likelihood sense.

It is not difficult to verify that our ST code matric in (8) is unitary. As a matter of fact, our proposed ST coded IR belongs to the class of unitary ST coded systems, and our decoding scheme turns out to be equivalent to the optimum receiver of [2], when the channel is unknown. In addition to allowing for analog STIR transmitters, a unique feature of our orthogonal STIR coder is that it does not suffer rate loss when $N_t > 2$ transmit antennas are deployed, simply because the IR transmissions are real by design.

3.2. Multiple Users

In order to enable multiple access, TH is employed in this subsection. We allow our multiple transmissions to be asynchronous. Over the n-th frame pair, the u-th user transmits the ST coded waveform:

$$\mathcal{V}(n;t) = \frac{\mathcal{P}_u}{\sqrt{2}} \sum_{m=0}^{M-1} s_{u,m}(2n) \begin{bmatrix} \nu^{00}_{u,m}(n;t) & \nu^{01}_{u,m}(n;t) \\ \nu^{10}_{u,m}(n;t) & \nu^{11}_{u,m}(n;t) \end{bmatrix}, \quad (12)$$

where $\nu^{kl}_{u,m}(n;t)$ denotes the waveform transmitted from the l-th transmit antenna during the $(2n+k)$-th frame duration, and can be expressed explicitly as:

$$\begin{aligned}
\nu^{00}_{u,m}(n;t) &= w_m(t - 2nT_f - c_u(2n)T_c - \tau_{u,a}) \\
\nu^{01}_{u,m}(n;t) &= w_m(t - 2nT_f - c_u(2n)T_c - \tau_{u,a}) \\
\nu^{10}_{u,m}(n;t) &= w_m(t - (2n+1)T_f - c_u(2n+1)T_c - \tau_{u,a}) \\
\nu^{11}_{u,m}(n;t) &= -w_m(t - (2n+1)T_f - c_u(2n+1)T_c - \tau_{u,a}),
\end{aligned}$$

where $\tau_{u,a}$ denotes the asynchronism of the u-th user. This uplink model subsumes the synchronous downlink setup as a special case, where $\tau_{u,a} = \tau_{\mu,a}, \forall u$. With the μ-th user being the desired user, the received signal, which is the aggregate signal from all N_u users, can be expressed as:

$$\begin{aligned}
x(2n;t) &= \frac{\mathcal{P}_\mu}{\sqrt{2}}[h_0(n)+h_1(n)]s_{\mu,m}(2n)w_m(t-2nT_f \\
&\quad - c_\mu(2n)T_c - \tau_{\mu,a}) + \eta_{MU}(t) + \eta(t) \\
x(2n+1;t) &= \frac{\mathcal{P}_\mu}{\sqrt{2}}[h_0(n)-h_1(n)]s_{\mu,m}(2n)w_m(t-(2n+1)T_f \\
&\quad - c_\mu(2n+1)T_c - \tau_{\mu,a}) + \eta_{MU}(t) + \eta(t),
\end{aligned}$$

where $\eta_{MU}(t)$ captures the Multiuser Interference (MUI).

At the receiver, the demodulator knows the TH pattern of the desired μ-th user, and acquires his/her asynchronism. The frame-rate sampled output of the receive filter that is matched to $w_{m'}(t)$

is now given by:

$$\begin{aligned} y_{\mu,m'}(2n) &= \int_{\tau_{\mu,a}+2nT_f}^{\tau_{\mu,a}+(2n+1)T_f} w_{m'}(t-2nT_f-c_\mu(2n)T_c)x(2n;t)dt \\ &= \frac{\mathcal{P}_\mu}{\sqrt{2}}[h_0(n)+h_1(n)]s_{\mu,m'}(2n)+\tilde{\eta}_{MU}(2n)+\tilde{\eta}_{m'}(2n), \end{aligned}$$

where the filtered additive noise $\tilde{\eta}_{m'}(k)$ is the same as in (9), and the MUI term $\tilde{\eta}_{MU}(k)$ will be approximated as a Gaussian random process with zero-mean and variance [5]

$$\sigma_a^2 := \frac{1}{T_f}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} w(x-s)w(x)dx\right]^2 ds. \tag{13}$$

Similarly, during the $(2n+1)$-st frame duration, we have

$$\begin{aligned} y_{\mu,m'}(2n+1) &= \frac{\mathcal{P}_\mu}{\sqrt{2}}[h_0(n)-h_1(n)]s_{\mu,m'}(2n) \\ &\quad +\tilde{\eta}_{MU}(2n+1)+\tilde{\eta}_{m'}(2n+1). \end{aligned}$$

As in the single user case, we may concatenate N_f' decision statistics $\mathbf{y}_{\mu,m'}(k)$ for the i-th symbol $I_\mu(i)$, form the vector $\bar{\mathbf{y}}_{\mu,m'}(i)$, and perform decoding via:

$$\hat{I}_\mu(i) = \arg\max_{m'}\{\|\bar{\mathbf{y}}_{\mu,m'}(i)\|^2\}. \tag{14}$$

In the following section, we will test (11) and (14) using simulations, and compare their performance against conventional single-antenna IR systems.

4. SIMULATIONS AND COMPARISONS

We select the monocycle to be the normalized unit-energy second derivative of the Gaussian pulse:

$$w(t) = \sqrt{\frac{8}{3}}\left(1-4\pi\left(\frac{t}{T_0}\right)^2\right)\exp\left(-2\pi\left(\frac{t}{T_0}\right)^2\right),$$

where $T_0 = 0.2877ns$ yields a pulse width of $T_w = 0.7ns$. The frame duration is chosen to be $T_f = 100ns$, as in [5].

First, we consider the downlink setup with flat fading real channels. We further assume that the TH sequences are orthogonal, i.e., $c_u(k) \neq c_\mu(k)$, $\forall k$, and for any two users u, μ. The multiuser detection problem boils down to a single user detection problem. We compare our ST coded IR scheme with the conventional IR system that employs a single transmit and a single receive antenna. The BER performance versus Signal-to-Noise Ratio (SNR) for $N_f = 8$, and modulation sizes $M = 2, 4, 8, 16$, is depicted in Figure 2. In all cases, our proposed ST coded IR scheme outperforms its single antenna counterpart, which confirms the diversity gained with the two transmit antennas.

Next, we consider the uplink setup, where N_u users transmit asynchronously, but with equal power, as in [5]. The MUI is treated as Gaussian noise with zero-mean and variance σ_a^2 as in (13), which can be computed using numerical integration. With the selected monocycle parameters, it turns out that $\sigma_a^2 = 0.0014835$.

With a fixed number of users, we first plot the BER performance with increasing SNR. As shown in Figure 3, an error floor exists whenever the number of users $N_u > 1$, due to the presence of

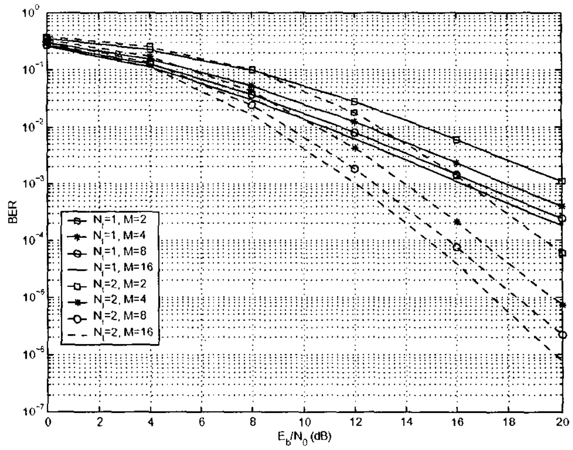

Fig. 2. Single user, $N_f = 8$, $M = 2, 4, 8, 16$.

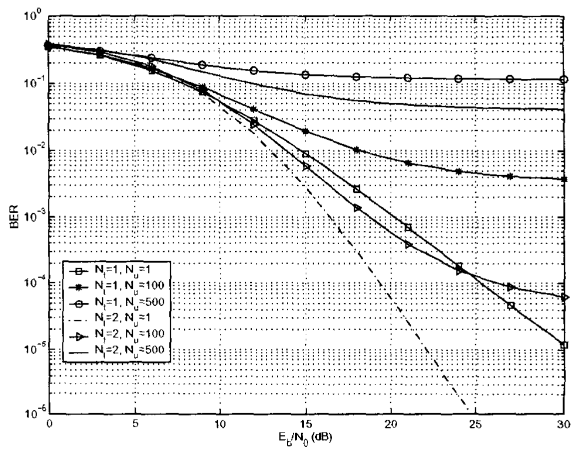

Fig. 3. Multiple users, $N_f = 8$, $M = 2$.

MUI. Performance degradation together with a higher error floor can be observed when increasing the number of users. Notice that our proposed STIR scheme outperforms its counterpart in all scenarios.

With the SNR fixed, the performance of both schemes degrades when the number of users increases. This agrees with [5], where it is asserted that additional power is required to achieve invariant BER performance with an increasing number of users. Once again, Figure 4 confirms that our STIR outperforms the single-antenna IR system in flat fading channels.

5. CONCLUSIONS AND DISCUSSION

In this paper, we proposed a Space-Time coded Impulse Radio system for flat fading channels. The scheme can be implemented with low complexity and analog electronics. Decoding can be performed using conventional analog receivers without channel knowledge. Simulation results show improved performance compared to the conventional single transmit antenna IR scheme.

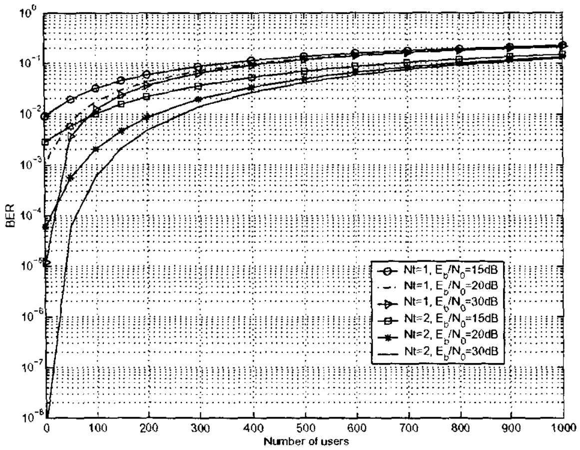

Fig. 4. Multiple users, $N_f = 8$, $M = 2$.

When IRMA systems operate in dense multipath environments, both Multiple User Interference (MUI) and Intersymbol Interference (ISI) emerge, and adversely affect system performance. It is possible for our STIR system to cope with such effects by combining the MUI/ISI-resilient techniques we developed in [6] with the ST coder put forth in [7] for ISI channels.

Appendix

For simplicity, we consider binary PPM signals. During the i-th symbol period $t \in [iN_fT_f, (i+1)N_fT_f)$, the symbol $I(i)$ is transmitted, and the detector observes the decision vector $\bar{\mathbf{y}}(i) := [\bar{\mathbf{y}}_0^T(i), \bar{\mathbf{y}}_1^T(i)]^T$. Hereafter, we shall confine ourselves to the i-th symbol, and omit the index (i) for clarity. The optimum decision rule is based on the *likelihood ratio*:

$$\Lambda(\bar{\mathbf{y}}) = \frac{p(\bar{\mathbf{y}}|I=0)}{p(\bar{\mathbf{y}}|I=1)} \underset{\hat{I}=1}{\overset{\hat{I}=0}{\gtrless}} 1, \qquad (15)$$

when the possible symbols '0' and '1' are equiprobable.

The probability density function $p(\bar{\mathbf{y}}|I=0)$ can be obtained by averaging the pdfs $p(\bar{\mathbf{y}}|I=0,\mathbf{h})$ over the pdfs of the random channel coefficients $\mathbf{h} := [h_0(iN_f'), h_1(iN_f'), h_0(iN_f'+1), \ldots, h_0(iN_f'+N_f'-1), h_1(iN_f'+N_f'-1)]^T$.

When the symbol '0' is transmitted, we have

$$y_0(2n) = \tfrac{\mathcal{P}}{\sqrt{2}}(h_0+h_1)+\tilde{\eta}_0(2n), \qquad y_1(2n)=\tilde{\eta}_1(2n)$$
$$y_0(2n+1) = \tfrac{\mathcal{P}}{\sqrt{2}}(h_0-h_1)+\tilde{\eta}_0(2n+1), \quad y_1(2n+1)=\tilde{\eta}_1(2n+1)$$

for even and odd frames, respectively. Recall that the noise components are statistically independent in both time and modulation dimensions. Hence, the joint pdf of $\bar{\mathbf{y}}$ may be expressed as the product of the marginal pdfs:

$$p(\bar{\mathbf{y}}_0|I=0,\mathbf{h}) = \prod_{n=iN_f'}^{(i+1)N_f'-1} p(\mathbf{y}_0(n)|I=0, h_0(n), h_1(n))$$

$$p(\bar{\mathbf{y}}_1|I=0) = \prod_{n=iN_f'}^{(i+1)N_f'-1} p(\mathbf{y}_1(n)|I=0),$$

where

$$p(\mathbf{y}_0(n)|I=0, h_0(n), h_1(n)) = \frac{1}{2\pi\sigma^2} \times \exp\left[\frac{-(y_0(2n) - \frac{\mathcal{P}}{\sqrt{2}}(h_0(n)+h_1(n))^2}{2\sigma^2}\right] \times \exp\left[\frac{-(y_0(2n+1) - \frac{\mathcal{P}}{\sqrt{2}}(h_0(n)-h_1(n))^2}{2\sigma^2}\right],$$

and

$$p(\mathbf{y}_1(n)|I=0) = \frac{1}{2\pi\sigma^2}\exp\left[\frac{-(y_1^2(2n)+y_1^2(2n+1))}{2\sigma^2}\right].$$

Furthermore, recalling that the fading channel coefficients are also statistically independent, the averaging can be carried out independently with respect to each channel use, which implies that

$$\Lambda(\bar{\mathbf{y}}) = \exp\left[\frac{\mathcal{P}^2(\bar{\mathbf{y}}_0^T\bar{\mathbf{y}}_0 - \bar{\mathbf{y}}_1^T\bar{\mathbf{y}}_1)}{2\sigma^2(\mathcal{P}^2+\sigma^2)}\right].$$

Therefore, the optimum decision rule in the maximum likelihood sense is equivalent to

$$\hat{I}(i) = \arg\max_m\{\|\bar{\mathbf{y}}_m(i)\|^2\}. \qquad (16)$$

Generalization to the M-ary case is straightforward.

6. REFERENCES

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