Opportunistic Medium Access for Wireless Networking Adapted to Decentralized CSI

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***Abstract*— Relative to a centralized operation, opportunistic medium access capitalizing on decentralized multiuser diversity in a channel-aware homogeneous slotted Aloha system with analog-amplitude channels has been shown to incur only partial loss in throughput due to contention. In this context, we provide sufficient conditions for stability as well as upper bounds on average queue sizes, and address three equally important questions. The first one is whether there exist decentralized scheduling algorithms for homogeneous users with higher throughputs than available ones. We prove that binary scheduling maximizes the sum-throughput. The second issue pertains to heterogeneous systems where users may have different channel statistics. Here we establish that binary scheduling not only maximizes the sum of the logs of the average throughputs, but also asymptotically guarantees fairness among users. The last issue we address is extending the results to finite state Markov chain (FSMC) channels. We provide a convex formulation of the corresponding throughput optimization problem, and derive a simple binary-like access strategy.**

***Index Terms*— slotted Aloha, decentralized multiuser diversity, stability, maximum stable throughput, scheduling, fairness.**

I. INTRODUCTION

IN THE traditional paradigm of wired networks, the medium access control (MAC) and the physical as well as higher layers are designed separately - an approach simplifying protocol design and network maintenance. For wireless networks however, this methodology needs to be revised primarily because of the inherent time-varying fading behavior of wireless links. Nowadays, cross layer designs are pursued to improve the overall system performance of wireless networks. In this new paradigm, knowledge of the physical channel, the queue status and QoS requirements are shared across layers and used jointly for scheduling purposes [6], [22].

Another traditional view is that channel fading impairs link reliability and should be mitigated. However, in the context of multiuser communications, this is not always the case. One important example emerges with multiuser diversity in a centralized downlink system, where the base station (BS) schedules the user with the best instantaneous channel to transmit, and thereby increases the maximum sum-throughput as the number of users increases [7], [8]. Such an approach has been incorporated in the design of Qualcomm's High Data Rate (HDR) system (1xEV-DO) for downlink packet scheduling [9].

In this paper, we consider a random access setup in the uplink, where multiple users are communicating with a single BS over flat fading channels and transmission time is slotted. We assume that each user node has only available its own uplink fading channel coefficient - what is often termed *decentralized channel state information* (D-CSI). For such a setting, multiuser diversity can be effected by jointly designing the physical and MAC layers [1], [4]. Assuming that all users always have data to transmit in a homogeneous slotted Aloha system with D-CSI available (a.k.a. *channel-aware Aloha*), a binary distributed scheduling scheme has been derived to asymptotically achieve a fraction $(1/e)$ of the centralized system's throughput [1]. Considering a n-user system with randomly arriving packets and employing both transmission probability control and adaptive rate transmission, a decentralized protocol has also been developed to achieve a fraction $(1-1/n)^{n-1}$ of the centralized system's throughput, where $(1-1/n)^{n-1}$ is a factor due to the inherent contention present in any finite-user slotted Aloha protocol [4].

In this paper, we provide a general approach for decentralized opportunistic medium access in a finite-user slotted Aloha system with randomly arriving traffic, where transmissions adapt to D-CSI by adjusting both rates and transmission probabilities. We derive sufficient conditions for system stability as well as upper bounds on average queue sizes using the dominant system approach. We further address three important open questions. The first one is whether there exist decentralized scheduling algorithms with higher throughput than [1], [4]. To this end, we prove that the binary scheduling of [1] maximizes the sum-throughput when user links are *homogeneous*; i.e., the corresponding channel statistics are identical. Another issue pertains to *heterogeneous* systems, where users may have different channel statistics. We show that if users behave as if they were in a homogeneous system, the binary scheduling algorithm maximizes the sum of the logs of the average throughputs while asymptotically guaranteeing fairness among users - two desirable properties of the centralized proportional fair (PF) scheduling algorithm [8]. The last issue we address is extending the results of analog-amplitude channels to quantized-amplitude channels that are typically modeled as a finite state Markov chain (FSMC) [21]. We provide a convex MAXDET formulation as well as a

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Linear Programming (LP) formulation of the corresponding throughput-maximizing scheduler for FSMC channels, and derive a simple binary-like access strategy.

There have been two more approaches taking advantage of D-CSI. One aims for enhancing the capture effect by adjusting transmission probabilities and power based on D-CSI [10], [11], [12], [5]. It has been shown that use of population-dependent transmission control improves throughput [10]; however, capture alone cannot stabilize slotted Aloha in a population-independent network when the number of users is very large [5]. The other approach to exploiting D-CSI relies on splitting algorithms to resolve collisions over a sequence of mini-slots, and determines the user with the best channel to transmit [2], [3].

The rest of the paper is organized as follows. In Section II, we introduce the system model. In Section III, we obtain sufficient stability conditions and provide bounds on average queue lengths. In Section IV, we consider fading channels with gains assuming arbitrary values and prove the optimality of binary scheduling in both homogeneous and heterogeneous systems. In Section V, we develop a MAXDET formulation of the throughput-maximizing problem for FSMC channels and obtain an analytical solution. Numerical examples are given in Section VI and conclusions are drawn in Section VII.

II. System Model

We consider a discrete time slotted Aloha random access system with a single BS and n users transmitting data to the BS over flat fading channels. Time is slotted into intervals of equal length T and the t-th slot refers to the time interval $[tT, (t+1)T)$, where $t = 0, 1, \ldots$; i.e., transmission attempts are made at discrete time instances t. Each user has an infinite buffer for storing incoming data. The arrival process $A_k(t)$ represents the number of bits that arrive during slot t of user k and is assumed to be stationary and independent of $A_l(t)$ for $l \neq k$. If the first- and second-order moments of $A_k(t)$ are λ_k and $\overline{A_k^2}$, the total arrival rate for the system is $\sum_{k=1}^{n} \lambda_k$.

Throughout, we shall adopt a flat block fading model for the uplink propagation between any user and the BS, where the channel fading coefficient is invariant per slot but is allowed to change from slot to slot. The channel magnitude-square process $\gamma_k(t) \in \mathbb{R}^+ = [0, +\infty)$ models user k's link condition as a function of time and is assumed stationary, has finite mean, and is assumed independent of $\gamma_l(t)$ for $l \neq k$. Let the function $F_k(\cdot)$ denote the cummulative distribution function (CDF) of $\gamma_k(t)$, $k = 1, \ldots, n$. Further, we will assum that at the beginning of each slot, each user node knows only its own uplink channel.

Based on such D-CSI, we adopt a variation of the conventional slotted Aloha protocol as in [1] and [10], where each user has an intelligent scheduler taking into account the available D-CSI. The general function of the scheduler is to adapt rate and transmission probability to the corresponding D-CSI in order to effect higher throughput. We denote the rate control function as $R_k(\gamma)$ and the probability control function as $s_k(\gamma)$, where γ refers to the magnitude-square channel realization over the current slot. The operation of $R_k(\gamma)$ in practice is implementation dependent. For example, if the duration of a slot is long enough for the system to afford sufficiently high encoding and decoding complexity, it is possible to transmit at a rate approaching Shannon's capacity of the channel. In general, we assume that the rate control function $R_k(\gamma)$ of each user is fixed and continuously increasing over $[0, +\infty)$. Note that our model is quite general, since it includes the conventional slotted Aloha, the SNR threshold model with fixed rate transmission [10], the fixed power transmission model [4] and the power control model [1].

In this paper, for simplicity we only consider a collision model for multiple access, in which a packet is successfully received if and only if there is one packet transmission.[1] Due to the time-varying channel and the time-varying user contention, the service process $\mu_k(t)$ (representing the number of bits that can be served during slot t) is also time-varying: $\mu_k(t) = R(\gamma_k(t))T$, if no other user transmits; and $\mu_k(t) = 0$, otherwise.

III. Stability and Upper Bounds on Average Queue Sizes

In this section, we will derive sufficient conditions for the stability of the system we outlined in the previous section. Note that even for the conventional slotted Aloha it is difficult to obtain the exact stability region except for homogeneous users. To establish stability conditions as well as upper bounds on average queue sizes, we will rely on the notion of the dominant system and stochastic ordering arguments. For clarity, we will first consider the simplest case where $A_k(t), \mu_k(t), k = 1, \ldots, n$ are independent and identically distributed (*i.i.d.*), and later extend our results to more general cases.

A. Stability

Let $U_k(t)$ denote the number of unprocessed bits in user k's queue at the beginning of slot t. The time evolution of the Markov chain $\{U_k(t)\}_{t=0}^{\infty}$ corresponding to user k is given by

$$U_k(t+1) = [U_k(t) - \mu_k(t)]^+ + A_k(t), \tag{1}$$

where $[x]^+ := \max\{x, 0\}$. We define the following *overflow* function $g_k(V)$:

$$\begin{aligned} g_k(V) : &= \limsup_{t\to\infty} \mathbb{E}\{\frac{1}{t}\int_0^t 1_{[U_k(\tau)>V]} d\tau\} \\ &= \limsup_{t\to\infty} \frac{1}{t}\int_0^t \mathbb{P}_r[U_k(\tau) > V] d\tau, \end{aligned} \tag{2}$$

where the indicator function $1_E = 1$, whenever event E is satisfied; and $1_E = 0$, otherwise.

Based on $g_k(V)$, we can define stability as follows:

Definition 1 ([14], [15]): The system is *stable* if $g_k(V) \to 0$ as $V \to \infty$ for all $k = 1, \ldots, n$.

[1] However, many ideas can be extended to the capture model, or more generally, to the multi-packet reception model of [19].

B. Stochastic Ordering and Dominant System

A tool frequently used in the paper is *stochastic ordering*, which is defined below:

Definition 2 ([16]): Suppose F and G are two CDFs. We say that F is strongly stochastically smaller than G, and write it as $F \leq_{st} G$, if their complementary CDFs satisfy $F^c(x) \leq G^c(x)$ for all $x \in \mathbb{R}$. Also for random variables (r.v.'s) X, Y with distributions F and G respectively, we will write $X \leq_{st} Y$ as synonym for $F \leq_{st} G$.

It is apparent that $F \leq_{st} F$. The following lemma characterizes the stochastic ordering $\leq_{st}$.

Lemma 1 ([16]): It holds that $F \leq_{st} G$, if and only if

$$\int_{-\infty}^{\infty} \phi(x) dF(x) \leq \int_{-\infty}^{\infty} \phi(x) dG(x), \tag{3}$$

for all increasing functions $\phi(\cdot)$, for which the integrals exist.

Let us now introduce a *dominant system* corresponding to the original system, which was implicitly used in [17] and later explicitly exploited by [18]; see also [20]. The dominant system behaves the same way as the original system except that in every slot each user node transmits a dummy packet using the same rate and transmission probability even when its queue is empty. It can be proved that if both the dominant and the original system start from the same initial state and encounter the same arrivals and channels, then it holds that [2] $U_k(t) \leq_{st} \widetilde{U}_k(t), k = 1, \ldots, n$ for $t \geq 0$ [17], [18]. In addition, if the dominant system is stable, the original system is also stable.

Using a 1-step Lyapunov function analysis as in [15, Lemma 3], we can easily obtain a stability condition and an upper bound on the queue size of the dominant system, which also apply to the original system. With $\overline{\mu}_k$ and $\overline{\mu_k^2}$ denoting the first- and second-order moments of $\widetilde{\mu}_k(t)$ of the dominant system, respectively, we find that

$$\overline{\mu}_k := \mathbb{E}\{\widetilde{\mu}_k(t)\} = \prod_{j=1, j\neq k}^{n} [1 - \int_0^{\infty} s_j(\gamma) dF_j(\gamma)] \cdot \int_0^{\infty} s_k(\gamma) R_k(\gamma) T dF_k(\gamma). \tag{4}$$

Now we can summarize our result in the following theorem:

Theorem 1: If $\lambda_k < \overline{\mu}_k, k = 1, \ldots, n$, then the original system is stable and its average queue size is upper bounded by

$$\overline{U_k} \leq \frac{\overline{\mu_k^2} + \overline{A_k^2}}{2(\overline{\mu}_k - \lambda_k)}. \tag{5}$$

Remark 1: Theorem 1 can be extended to non-*i.i.d.* stationary arrival and channel processes, as long as the arrival and channel processes for different users are independent. Suppose that the time average of the first- and second-order moments of $A_k(t)$ ($\widetilde{\mu}_k(t)$) over K consecutive slots approximate closely λ_k ($\overline{\mu}_k$) and $\overline{A_k^2}$ ($\overline{\mu_k^2}$). Then, by using a K-step Lyapunov function analysis as in [15], we can prove that the sufficient conditions for stability coincide with these in Theorem 1, but the upper bound in (5) on the average queue sizes increases by a multiplicative factor K. This is not surprising, since K quantifies the memory effect of the channel. The longer the channel memory is, the longer the backlog will be. Nonetheless, the sufficient conditions for stability are the same. The fact that the channel memory does not affect the long-term throughput but the packet delay has been already pointed out in [1, Sec. 5] for the power control model.

Remark 2: Another reason why channel memory does not affect the MST is that users do not cooperate and each user node makes its decision to transmit based on the expected contention in the current slot not on previous channel history. Consider the dominant system approach we used earlier. Since users in the dominant system cannot inform each other about their status, all queues are decoupled and can influence the throughput of each other only through steady state statistics. For this reason, a specific user node per slot faces on the average the same contention effect from other users. Whether its channel is "good" or "bad" in earlier slots is irrelevant to the expected contention in the current slot.

Remark 3: Theorem 1 does not imply that we cannot take advantage of the known channel memory. For example, in a reservation-type protocol over a channel with a long memory, once a user node is successful experiencing a good channel, it can reserve the channel for immediately following slots until the channel becomes bad. Such a scheme reduces collisions for users with good channel conditions, leading to higher throughput. However, in this paper we do not consider reservation-type protocols and only focus on Aloha-type protocols. Nonetheless, the schemes developed in this paper can be used for the channel reservation phase in any reservation-type protocols.

IV. Fading Channels with Analog Amplitudes

In this section, we consider user channels with gains assuming arbitrary values and assume that all the CDFs are continuous, which we refer to as analog-amplitude channels. We first focus on homogeneous systems, and then extend our results to heterogeneous systems. In a homogenous system, since all users have the same arrival rate $\lambda_k = \lambda/n$ and identical channel statistics $F_k(\cdot) = F(\cdot)$, they adopt the same scheduling policy $s_k(\cdot) = s(\cdot), \forall k$. If for simplicity we omit the user index k, it follows from (4) that

$$\overline{\mu} = T \cdot [1 - \int_0^{\infty} s(\gamma) dF(\gamma)]^{n-1} \cdot \int_0^{\infty} s(\gamma) R(\gamma) dF(\gamma). \tag{6}$$

Theorem 1 implies that the system is stable if the system throughput satisfies

$$\lambda < \eta := n\overline{\mu} = T \cdot n[1 - \int_0^{\infty} s(\gamma) dF(\gamma)]^{n-1} \cdot \int_0^{\infty} s(\gamma) R(\gamma) dF(\gamma). \tag{7}$$

We have not been able to prove, but we conjecture that the system is unstable for $\lambda > \eta$, which holds at least for some special cases [17]. Nevertheless, our goal is to maximize η by judiciously selecting the scheduling function $s(\cdot)$. In the following, we will first investigate a heuristic scheme we introduced in [4] as well as the binary scheduling put forth

[2] Hereafter, the tilde denotes quantities corresponding to the dominant system.

by [1], and then prove that the latter is optimal in terms of throughput for a more system model considered in Section II. In the end of this section, we will revisit the throughput and fairness issues for heterogeneous systems.

A. Multiuser Diversity

First, let us suppose that at the beginning of slot t, each user has available full CSI; i.e., each user knows all $\{\gamma_k(t)\}_{k=1}^n$. Then each user, say user k, knows the maximum of other users' CSI, which we define as

$$\beta_k(t) := \max\{\gamma_j(t)\}_{j=1,j\neq k}^n. \tag{8}$$

Based on $\beta_k(t)$, a simple binary transmission strategy for each user k operates as follows: if $\gamma_k(t) \geq \beta_k(t)$, transmit with probability 1; if $\gamma_k(t) < \beta_k(t)$, do not transmit with probability 1. This protocol amounts to having a centralized scheduler which allows the user with the best channel to transmit in every slot. The CDF of $\beta_k(t)$ is given by

$$\begin{aligned} G_{\beta_k(t)}(\beta) &= \mathbb{P}_r\{\beta_k(t) \leq \beta\} \\ &= \prod_{j=1,j\neq k}^{n-1} \mathbb{P}_r\{\gamma_j(t) \leq \beta\} = F^{n-1}(\beta). \end{aligned} \tag{9}$$

It is not to difficult to obtain the throughput of such a centralized scheduling scheme:

$$\begin{aligned} \eta_{central} &= n \int_0^\infty R(\gamma) T F^{n-1}(\gamma) dF(\gamma) \\ &= T \cdot \mathbb{E}_{F^n(\cdot)}\{R(\gamma)\}. \end{aligned} \tag{10}$$

The second term in (10) is exactly the capacity of a centralized multiuser diversity system, when there are n independent homogeneous users and only the best user is allowed to transmit in every slot. This can be seen from the fact that $F^n(\gamma)$ is the CDF of the maximum of n *i.i.d.* CSI variables with distribution $F(\gamma)$. Therefore, we expect that in fading channels $\eta_{central}$ increases as the number of users n increases, which manifests the notion of multiuser diversity [8].

Let us now return to our original problem where each user node knows only its own CSI. Even though the exact value of $\beta_k(t)$ at the beginning of slot t is unknown, the distribution of $\beta_k(t)$ is known, since we are considering a homogeneous system and $\{\gamma_j(t)\}_{j=1,j\neq k}^n$ are *i.i.d.*. Relying on this D-CSI, a heuristic choice for $s(\gamma)$ is:

$$\begin{aligned} s(\gamma_k(t)) &= \mathbb{P}_r\{\gamma_k(t) \geq \beta_k(t) | \gamma_k(t)\} \\ &= F^{n-1}(\gamma_k(t)). \end{aligned} \tag{11}$$

Such a scheme results in average transmission probability $P := \int_0^\infty s(\gamma) dF(\gamma) = 1/n$. Substituting (11) into (7), we obtain the throughput of this heuristic scheduler as

$$\begin{aligned} \eta_{heur} &= T \cdot (1 - \frac{1}{n})^{n-1} \cdot \int_0^\infty R(\gamma) n F^{n-1}(\gamma) dF(\gamma) \\ &= T \cdot (1 - \frac{1}{n})^{n-1} \cdot \mathbb{E}_{F^n(\cdot)}\{R(\gamma)\}. \end{aligned} \tag{12}$$

An immediate consequence of (12) is that $\eta_{heur} = (1 - 1/n)^{n-1} \cdot \eta_{central}$. The only difference between η_{heur} and $\eta_{central}$ is the factor $(1 - 1/n)^{n-1}$, which represents the contention effect. This factor decreases as n increases and converges to $1/e$ as $n \to \infty$. Except for this contention factor, which is inherent to all slotted Aloha systems, η_{heur} shares the same multiuser diversity as $\eta_{central}$, which we refer to as *decentralized multiuser diversity*.

Note that our idea of decentralized opportunistic scheduling is related to but constitutes an interesting extension of centralized opportunistic scheduling [8]. One may expect that this heuristic algorithm is optimal; however, we will prove in the next subsection that it is suboptimal.

B. Throughput-Optimality of Binary Scheduling

In this section we prove that a suitable binary scheduling is throughput-optimal. Since $P = \int_0^\infty s(\gamma) dF(\gamma)$ is each user's average transmission probability, we can rewrite η in (7) as

$$\begin{aligned} \eta &= T \cdot nP(1-P)^{n-1} \cdot \frac{1}{P} \int_0^\infty s(\gamma) R(\gamma) dF(\gamma) \\ &= T \cdot nP(1-P)^{n-1} \cdot \mathbb{E}_{G(\cdot)}\{R(\gamma)\}, \end{aligned} \tag{13}$$

where $G(\gamma) = (1/P) \int_0^\gamma s(\gamma) dF(\gamma)$ is a distribution function.

To find the optimal transmission control function that maximizes η, we first fix P and seek the optimal choice for $s(\cdot)$ under the constraint $P = \int_0^\infty s(\gamma) dF(\gamma)$, which amounts to finding $s(\cdot)$ that maximizes $\mathbb{E}_{G(\cdot)}\{R(\gamma)\}$ under the same constraint. Since $s(\gamma) \in [0,1]$ for $\gamma \in \mathbb{R}^+$, we have

$$\begin{aligned} G^c(\gamma) &= \frac{1}{P} \int_\gamma^\infty s(\gamma) dF(\gamma) \\ &\leq \frac{1}{P} \int_\gamma^\infty dF(\gamma) \\ &= P^{-1} F^c(\gamma). \end{aligned} \tag{14}$$

On the other hand, $G^c(\gamma)$ cannot exceed 1, which implies that

$$G^c(\gamma) \leq \min\{P^{-1} F^c(\gamma), 1\}. \tag{15}$$

Now consider the following binary scheduling function (introduced by [1]):

$$s_B(\gamma) = \begin{cases} 1, & \gamma \geq \gamma_B \\ 0, & \gamma < \gamma_B \end{cases}, \tag{16}$$

where γ_B is the threshold determined by $P = \int_{\gamma_B}^\infty dF(\gamma) = F^c(\gamma_B)$. The corresponding $G_B^c(\gamma)$ is given by

$$\begin{aligned} G_B^c(\gamma) &= \begin{cases} P^{-1} F^c(\gamma), & \gamma \geq \gamma_B \\ 1, & \gamma < \gamma_B \end{cases} \\ &= \min\{P^{-1} F^c(\gamma), 1\}. \end{aligned} \tag{17}$$

Comparing (15) with (17), we deduce that $G^c(\gamma) \leq G_B^c(\gamma)$ for $\forall \gamma \in \mathbb{R}^+$; therefore, from Definition 2, we have $G \leq_{st} G_B$. In addition, it follows from Lemma 2 that $\mathbb{E}_{G(\cdot)}\{R(\gamma)\} \leq \mathbb{E}_{G_B(\cdot)}\{R(\gamma)\}$, since $R(\gamma)$ is an increasing function. So far, we have proved that for a fixed P, $s_B(\gamma)$ maximizes η, and the maximum equals

$$\eta_{binary} = T \cdot n[1 - \int_{\gamma_B}^\infty dF(\gamma)]^{n-1} \cdot \int_{\gamma_B}^\infty R(\gamma) dF(\gamma). \tag{18}$$

To find the throughput-maximizing γ_B, we need to solve the equation $\partial\eta_{binary}/\partial\gamma_B = 0$, yielding

$$(n-1)\int_{\gamma_B}^{\infty} R(\gamma)dF(\gamma) = F(\gamma_B)R(\gamma_B). \tag{19}$$

However, (19) may have no solution. Assume that both $F(\gamma)$ and $R(\gamma)$ are continuous functions with $\int_0^\infty R(\gamma)dF(\gamma) < \infty$ and define $h_{\min} := \inf_{\gamma_B \in [0,\infty)} h(\gamma_B)$, where

$$h(\gamma_B) := \frac{F(\gamma_B)R(\gamma_B)}{\int_{\gamma_B}^{\infty} R(\gamma)dF(\gamma)}. \tag{20}$$

It is apparent that $h(\gamma_B)$ is a continuous and increasing function over $[h_{\min}, \infty)$. If $n-1 \geq h_{\min}$, the optimal threshold can be obtained from (19), even though solving for γ_B may be tedious; if $n-1 \geq h_{\min}$, it is not clear how to compute the optimal threshold. Note that if $R(0) = 0$ as in most reasonable models, then $h_{min} = 0$ and the condition $n-1 \geq h_{min}$ will always be satisfied. In practice, a suboptimal choice is to select γ_B to satisfy $P_B = 1/n$, which is asymptotically optimal for the power control model considered in [1].

For notational brevity, we keep using γ_B to denote this optimal threshold, and summarize our optimality claim in the following theorem.

Theorem 2: If $F(\gamma)$ is continuous and strictly increasing over $[0, \infty]$, and $R(\gamma)$ is continuous and increasing over $[0, \infty]$ with $\int_0^\infty R(\gamma)dF(\gamma) < \infty$ and $n-1 \geq h_{\min}$, then the scheme maximizing throughput is the binary scheduling in (16) with the optimal threshold γ_B chosen to satisfy (19).

Noting that $\int_{\gamma_B}^\infty R(\gamma)dF(\gamma) \geq R(\gamma_B)[1-F(\gamma_B)]$, we have $F(\gamma_B)R(\gamma_B) \geq (n-1)R(\gamma_B)[1-F(\gamma_B)]$, which leads to:

Corollary 1: Under the same conditions as in Theorem 2, it holds that $P_B := \int_{\gamma_B}^\infty dF(\gamma) \leq 1/n$.

Although it has been shown in [1] that the binary scheduling in (16) can exploit multiuser diversity and the average transmission probability satisfies $P_B \leq 1/n$ for the power control model, we prove here that these claims are true for the more general models mentioned in Section II, where $F(\gamma)$ and $R(\gamma)$ do not have to take the specific forms in [1]. Compared with the heuristic algorithm discussed in the previous subsection, (16) requires a user node to stop competing for the channel when its link condition is below a critical value γ_B, but allows it to transmit deterministically when the channel condition exceeds that threshold.

Also note that Theorem 2 does not apply to the classic slotted Aloha over AWGN channels, where the optimal transmission probability is always $1/n$ for an n-user system. The reason is that we assume the channel CDF $F(\cdot)$ is continuously increasing in Theorem 2, while an AWGN channel has constant gain and thus adheres to a discrete distribution. We will elaborate further on this issue in Corollary 3 of Section V-B.

Finally, we consider a specific rate control function $R^0(\gamma) = 1$ for $\gamma \geq \gamma_0$ and $R^0(\gamma) = 0$ for $\gamma < \gamma_0$, which is the SNR threshold model in [10, eq. (1)]. We observe that if $\gamma_B \leq \gamma_0$, (18) can always be maximized by setting $\gamma_B = \gamma_0$; therefore, we deduce that $\gamma_B \geq \gamma_0$. Under $\gamma_B \geq \gamma_0$, (19) can be rewritten as

$$\eta_{binary} = T \cdot n[1 - \int_{\gamma_B}^{\infty} dF(\gamma)]^{n-1} \cdot \int_{\gamma_B}^{\infty} dF(\gamma). \tag{21}$$

Based on (21), we can readily deduce the following optimal solution:

Corollary 2: For the SNR threshold model, $\gamma_B = \gamma_0$ if $F^c(\gamma_0) \leq 1/n$, and $\gamma_B = F^{-1}(1-1/n)$ if $F^c(\gamma_0) > 1/n$.

Another optimal binary solution is given by [10, eq. (12)]: $s^*(\gamma) = 0$ for $\gamma < \gamma_0$, and $s^*(\gamma) = \min((nF^c(\gamma_0))^{-1}, 1)$ for $\gamma \geq \gamma_0$. It is not difficult to show that this scheduler attains the same throughput as the binary one provided by Corollary 2.

C. Heterogeneous Systems

In the previous section, we dealt with homogeneous users. In practice however, users may have non-identical arrival rates and channel statistics. At first sight, it seems difficult to generalize the homogeneous system analysis to a heterogeneous setting, since heterogeneous users have no prior knowledge of others users' channel statistics. Another challenge is maintaining fairness among users with different channel statistics. As shown in a centralized system, simply maximizing the total throughput would incur long delays for users with poor channel statistics while overly favoring users with better channel statistics.

Let us look closer at how fair scheduling is pursued in a centralized system. An attractive approach, known as PF scheduling [8], can harness channel variations to improve throughput while at the same time maintaining fairness among users. The basic idea is to schedule the user whose corresponding instantaneous channel quality is the highest *relative* to the average channel condition over a given time scale. Under certain statistical assumptions, it has been established that with infinitely large averaging time PF maximizes the sum of the logs of the average throughputs of the various users [8, app. A], while allocating each user roughly the same number of time slots.

A revelation from the PF scheme in *centralized* operation is that maximizing the sum of the logs (or equivalently, the product) of throughputs results in multiuser diversity and indirectly effects fairness among users. Notice that maximizing the product-throughput in the homogeneous case reduces to maximizing the sum-throughput. The reason behind maximizing the product in the heterogeneous case is that it prevents users from having low throughputs; and thereby it mitigates unfairness which could emerge when maximizing the sum-throughput (recall that maximizing the sum-throughput is not fair for users with "poor quality" channels). Motivated by these considerations, we first seek a *decentralized* scheduler that maximizes the product of users' throughputs, and check fairness issues afterwards. It follows from (4) that

$$\prod_{k=1}^{n} \overline{\mu}_k = \prod_{k=1}^{n} \prod_{j=1, j\neq k}^{n} [1 - \int_0^\infty s_j(\gamma)dF_j(\gamma)] \cdot \int_0^\infty s_k(\gamma)R_k(\gamma)TdF_k(\gamma). \tag{22}$$

Through combining identical terms, we have

$$\prod_{k=1}^{n} \overline{\mu}_k = T^n \cdot \prod_{k=1}^{n} [1 - \int_0^{\infty} s_k(\gamma) dF_k(\gamma)]^{n-1} \cdot \int_0^{\infty} s_k(\gamma) R_k(\gamma) dF_k(\gamma). \quad (23)$$

It is clear that in order to maximize $\prod_{k=1}^{n} \overline{\mu}_k$, we only need to maximize independently each individual factor, $[1-\int_0^{\infty} s_k(\gamma)dF_k(\gamma)]^{n-1} \int_0^{\infty} s_k(\gamma)R_k(\gamma)dF_k(\gamma)$, in the right hand side of (23). This maximization can be carried out as in Section IV-B, and its solution suggests selecting the following transmission probabilities:

$$s_{B,k}(\gamma) = \begin{cases} 1, & \gamma \geq \gamma_{B,k} \\ 0, & \gamma < \gamma_{B,k} \end{cases}, \quad k = 1, \ldots, n, \quad (24)$$

where the thresholds $\gamma_{B,k}$ are chosen to maximize $[1 - \int_{\gamma_{B,k}}^{\infty} dF_k(\gamma)]^{n-1} \cdot \int_{\gamma_{B,k}}^{\infty} R_k(\gamma) dF_k(\gamma)$. For large n, they are determined by

$$(n-1) \int_{\gamma_{B,k}}^{\infty} R_k(\gamma) dF_k(\gamma) = F_k(\gamma_{B,k}) R_k(\gamma_{B,k}), \quad (25)$$

for $k = 1, \ldots, n$. With regards to the optimality of (23), we summarize our main result in the following theorem:

Theorem 3: The scheduling algorithm described by (24) maximizes $\sum_{k=1}^{n} \log(\overline{\mu}_k)$.

An interesting observation is that with this algorithm each user node does not need to worry if the system is heterogeneous or not, and just acts as if it was in a homogeneous system. This is intuitively justifiable since with each user node having no a priori knowledge about others, the best thing is to assume that all have identical channel statistics.

Let us now turn our attention to fairness issues. Given that user k has a packet ready for transmission, the probability of a successful transmission is

$$P_{succ,k} = [1 - F_k(\gamma_{B,k})] \cdot \prod_{j=1, j \neq k}^{n} F_j(\gamma_{B,j}). \quad (26)$$

Therefore, each user will get some chance of successful transmission. If we choose an suboptimal $\gamma_{B,k}$ such that $P_{B,k} := 1 - F_k(\gamma_{B,k}) = 1/n$, then every user gets the same chance of successful transmission, that is, with probability $P_{B,k} = (1 - 1/n)^{n-1}/n$. The algorithm implied by (24) not only maximizes the sum of the logs of the average user throughputs, but also ensures fairness – the two main advantages of the centralized PF scheduling. Further, (24) with $P_{B,k} = 1/n$ achieves the following sum-throughput for large n

$$\begin{aligned} \eta'_{binary} &= \sum_{k=1}^{n} T \cdot \frac{1}{n} (1 - \frac{1}{n})^{n-1} \cdot \mathbb{E}_{G_{B,k}(\cdot)}\{R(\gamma)\} \\ &= T \cdot (1 - \frac{1}{n})^{n-1} \cdot \mathbb{E}_{\frac{1}{n}\sum_{k=1}^{n} G_{B,k}(\cdot)}\{R(\gamma)\}, \end{aligned} \quad (27)$$

where $(1/n) \sum_{k=1}^{n} G_{B,k}$ is a new CDF with $G_{B,k}(\gamma) := n \int_0^{\gamma} s_{B,k}(\gamma) dF_k(\gamma)$.

For comparison purposes, it is of interest to analyze the throughput of the heuristic algorithm presented in Section IV-A in the heterogeneous setting. It follows easily that

$$\begin{aligned} \overline{\mu}_{k,heuris} &= T \cdot (1 - \frac{1}{n})^{n-1} \cdot \int_0^{\infty} \{F_k(\gamma)\}^{n-1} R(\gamma) dF_k(\gamma) \\ &= T \cdot (1 - \frac{1}{n})^{n-1} \cdot \frac{1}{n} \mathbb{E}_{\{F_k(\cdot)\}^n}\{R(\gamma)\}, \\ \eta'_{heur} &= \sum_{k=1}^{n} \overline{\mu}_{k,heuris} = T \cdot (1 - \frac{1}{n})^{n-1} \\ &\quad \cdot \mathbb{E}_{\frac{1}{n}\sum_{k=1}^{n} \{F_k(\cdot)\}^n}\{R(\gamma)\}. \end{aligned} \quad (28)$$

V. FSMC Channels

Until now, we have dealt with channels having analog amplitudes which require knowledge of the channel CDF. However, estimating the latter in practice is challenging. Moreover, it is hard to continuously adapt the transmission rate to each channel realization in practice. Therefore, the results obtained so far offer analytical insights but fall short when it comes to practical implementation.

Typically, a fading channel with analog amplitudes can be approximated by a quantized one that can be modeled as a finite-state Markov chain [21]. This requires quantizing the received instantaneous SNR into a finite number of levels, with each level representing a specific state of the channel. And for each channel state, there is a corresponding steady-state probability of being in this state, and a corresponding fixed feasible transmission rate supported by this state. For the block fading model considered in this paper, the channel remains in the same state during a slot and the state transitions from slot to slot are determined by the probability transition matrix of the corresponding Markov chain. To maximize throughput, it suffices to consider the case where channels are time-independent (see also Remarks 1, 2 and 3 of Theorem 1).

Similar to channels with analog amplitudes, our results for homogeneous users with FSMC channels can be extended to the heterogeneous case too. For this reason, we will focus only on homogeneous users supposing that each user's channel states take values from a common finite state space $\{1, \ldots, J\}$. Let p_j denote the stationary probability that the channel is in state j. Note that it is much easier to obtain $\{p_j\}_{j=1}^{J}$ than the CDF in practice. For each channel state j, the feasible rate is R_j. Further, we assume that the feasible rates are strictly ordered: $0 < R_1 < R_2 < \ldots < R_J$. Under these assumptions, arguing as in Section IV we can prove that the throughput is given by

$$\eta = T \cdot n[1 - \sum_{j=1}^{J} p_j s_j]^{n-1} \cdot [\sum_{j=1}^{J} p_j R_j s_j], \quad (29)$$

where $0 \leq s_j \leq 1$ is the transmission probability when the channel is in state $j = 1, \ldots, J$. Notice that the summation here replaces the integral in (7). Now, our goal is to find transmission probabilities $\{s_j\}_{j=1}^{J}$ that maximize η in (29), either numerically or analytically. Since η is generally not concave over its arguments s_j, $j = 1, \ldots, J$, maximizing the throughput is not a convex optimization problem; therefore, existing efficient algorithms for convex optimization cannot be used directly [23], [27].

A simple observation is that the term $P_{avg} := \sum_{j=1}^{J} p_j s_j$ in (29) is actually the average transmission probability. Recall that with analog-amplitude channels the average transmission probability approaches $1/n$ when the number of users grows large. This suggests a suboptimal algorithm for FSMC channels that maximizes $T \cdot n(1-\frac{1}{n})^{n-1} \cdot (\sum_{j=1}^{J} p_j R_j s_j)$ subject to the constraint $\sum_{j=1}^{J} p_j s_j = 1/n$. It turns out that this is a linear programming (LP) problem, which can be solved efficiently by existing software [24].

A better formulation than LP is possible if we take a $(-\log)$ transform of η. This results in the following optimization setup [3] (after omitting the constant term $-\log(nT)$)

$$\begin{aligned}\text{minimize} \quad & -\log[(1-\sum_{j=1}^{J} p_j s_j)^{n-1}] - \log(\sum_{j=1}^{J} p_j R_j s_j) \\ \text{subject to} \quad & 0 \le s_j \le 1,\ j = 1, \ldots, J,\end{aligned} \tag{30}$$

which is a convex program since the objective in $-\log$ form is now a convex function of $\{s_j\}_{j=1}^{J}$.

A. Structure of the Optimal Solution

To identify the structure of the optimal solution, let us consider the Lagrangian of (30)

$$\begin{aligned}\mathbb{L}(s, \lambda, \mu) = & -\log(1-\sum_{j=1}^{J} p_j s_j)^{n-1} - \log(\sum_{j=1}^{J} p_j R_j s_j) \\ & + \sum_{j=1}^{J} \lambda_j (s_j - 1) + \sum_{j=1}^{J} \mu_j (-s_j),\end{aligned} \tag{31}$$

where $\{\lambda_j\}_{j=1}^{J}$ and $\{\mu_j\}_{j=1}^{J}$ are the Lagrange multipliers. Let the optimal primal solution be $\{s_j^*\}_{j=1}^{J}$, and the optimal dual solution be $(\{\lambda_j^*\}_{j=1}^{J}, \{\mu_j^*\}_{j=1}^{J})$. Since (30) is a convex optimization program, the Karush-Kuhn-Tucker (KKT) conditions which are both sufficient and necessary for optimality [27], are given by:

$$\lambda_j^* \ge 0, \tag{32}$$

$$\mu_j^* \ge 0, \tag{33}$$

$$\lambda_j^* (s_j^* - 1) = 0, \tag{34}$$

$$\mu_j^* (-s_j^*) = 0, \tag{35}$$

$$\frac{(n-1)p_j}{1-\sum_{j=1}^{J} p_j s_j^*} - \frac{p_j R_j}{\sum_{j=1}^{J} p_j R_j s_j^*} + \lambda_j^* - \mu_j^* = 0, \tag{36}$$

for all $j = 1, \ldots, J$. From (32)-(36), we will be able to identify the structure of the optimal scheme. We will distinguish among three cases depending on s_j^*:

[3]Upon constructing a diagonal $n \times n$ matrix $\mathbf{A} = \text{diag}\{1 - \sum_{j=1}^{J} p_j s_j, \ldots, 1 - \sum_{j=1}^{J} p_j s_j, \sum_{j=1}^{J} p_j R_j s_j\}$, the objective function becomes $\log\det \mathbf{A}^{-1}$, which is exactly a determinant maximization (MAXDET) problem [25], and can be solved efficiently; e.g., using a software tool called sdpsol [26].

Case 1 ($s_j^* = 0$): Condition (34) requires $\lambda_j^* = 0$. Using the fact that $\mu_j^* \ge 0$ and $\lambda_j^* = 0$, it follows from (36) that

$$\frac{(n-1)p_j}{1-\sum_{j=1}^{J} p_j s_j^*} \ge \frac{p_j R_j}{\sum_{j=1}^{J} p_j R_j s_j^*}. \tag{37}$$

Since $p_j > 0$, after straightforward manipulations we obtain

$$R_j \le \frac{(n-1)\sum_{j=1}^{J} p_j R_j s_j^*}{1-\sum_{j=1}^{J} p_j s_j^*}. \tag{38}$$

Case 2 ($s_j^* = 1$): Condition (35) implies that $\mu_j^* = 0$. Using the fact that $\lambda_j^* \ge 0$ and $\mu_j^* = 0$, similar to Case 1 it follows from (36) that

$$R_j \ge \frac{(n-1)\sum_{j=1}^{J} p_j R_j s_j^*}{1-\sum_{j=1}^{J} p_j s_j^*}. \tag{39}$$

Case 3 ($0 < s_j^* < 1$): From conditions (34) and (35), we obtain that $\lambda_j^* = \mu_j^* = 0$, which leads to

$$R_j = \frac{(n-1)\sum_{j=1}^{J} p_j R_j s_j^*}{1-\sum_{j=1}^{J} p_j s_j^*}. \tag{40}$$

We can draw two conclusions here. One is that the right hand side of (38), (39) and (40) are identical and independent of the index j. The other is that there exists at least one j for which either (39) or (40) holds, because there exists at least one j such that $s_j^* > 0$. Therefore, there exists a k so that

$$\begin{aligned}k :&= \min\{j : R_j \ge \frac{(n-1)\sum_{j=1}^{J} p_j R_j s_j^*}{1-\sum_{j=1}^{J} p_j s_j^*}, 1 \le j \le J\} \\ &= \min\{j : s_j^* > 0, 1 \le j \le J\}.\end{aligned} \tag{41}$$

Since the feasible rates are strictly ordered as $0 < R_1 < R_2 < \ldots < R_J$, the optimal solution should have a binary-like structure as shown in the proposition next:

Proposition 1: The optimal solution to (29) has a binary-like structure as follows:

$$s_j^* = \begin{cases} 0, & \text{if } j < k, \\ 1, & \text{if } j > k. \end{cases} \tag{42}$$

B. Analytical Solution

Once the structure of the optimal solution is known, we can find an analytical form for it. To verify this, let us first compute s_k^*. There are two cases: If $0 < s_k^* < 1$, it follows from (40) and Proposition 1 that

$$R_k = \frac{(n-1)(p_k R_k s_k^* + \sum_{j=k+1}^{J} p_j R_j)}{1 - p_k s_k^* - \sum_{j=k+1}^{J} p_j}, \tag{43}$$

which reduces to

$$s_k^* = t_k := \frac{(\sum_{j=1}^{k} p_j) R_k - (n-1)\sum_{j=k+1}^{J} p_j R_j}{n p_k R_k}. \tag{44}$$

On the other hand, if $s_k^* = 1$, it follows from (39) and Proposition 1 that

$$R_k \ge \frac{(n-1)(p_k R_k s_k^* + \sum_{j=k+1}^{J} p_j R_j)}{1 - p_k s_k^* - \sum_{j=k+1}^{J} p_j}, \tag{45}$$

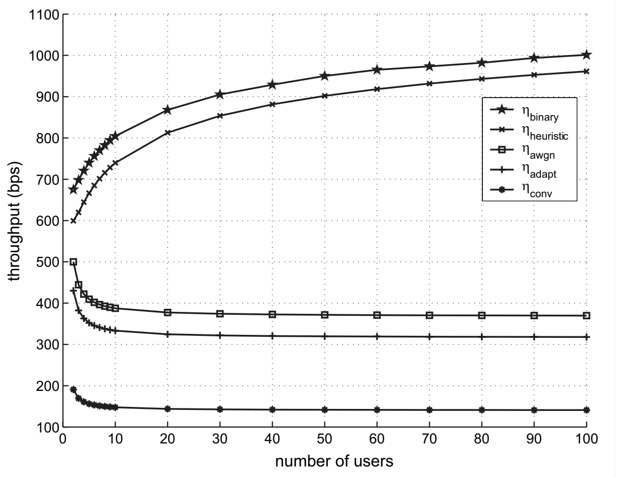

Fig. 1. Homogeneous system: Throughput vs. Number of users, SNR=$0dB$, Rayleigh fading channel

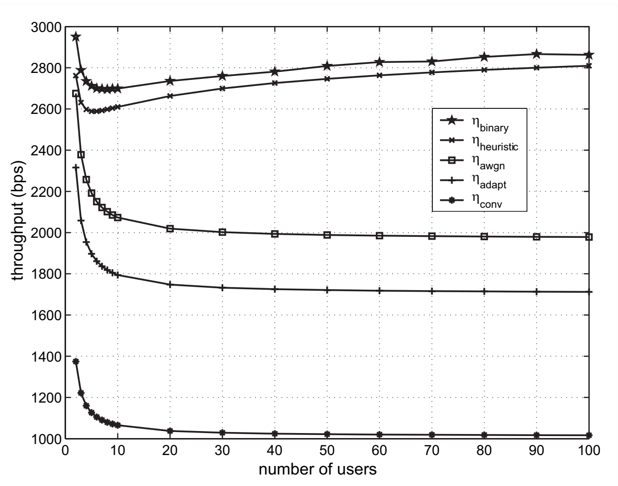

Fig. 2. Homogeneous system: Throughput vs. Number of users, SNR=$16dB$, Rayleigh fading channel

which leads to $s_k^* \leq t_k$. Combing the two, we have

$$s_k^* = \min\{t_k, 1\}. \tag{46}$$

Note that s_k^* is uniquely determined by t_k. We can generalize this result to arbitrary j, by defining t_j similar to t_k:

$$t_j := \frac{(\sum_{i=1}^{j} p_i)R_j - (n-1)\sum_{i=j+1}^{J} p_i R_i}{n p_j R_j}. \tag{47}$$

We maintain that t_j uniquely determines s_j^*. To prove that, we first look at the properties of t_j.

Proposition 2: It holds that $t_j < 0$ for $1 \leq j < k$, and $t_j > 1$ for $k < j \leq J$.

Proof: See Appendix I. ■

Combining (46) and Proposition 2, we obtain the following theorem:

Theorem 4: The optimal solution to (30) is given by

$$s_j^* = \begin{cases} 0, & \text{if } t_j \leq 0, \\ t_j, & \text{if } 0 < t_j < 1, \\ 1, & \text{if } t_j \geq 1. \end{cases} \tag{48}$$

For the special case when there is only one channel state $J = 1$, we have $t_j = 1/n$. From Theorem 4 we can easily prove the following result, which is consistent with that of the conventional slotted Aloha over AWGN channels.

Corollary 3: If $J = 1$, it holds that $s_1^* = 1/n$, and the MST is $\eta = (1 - 1/n)^{n-1} \cdot R_1$.

Except for t_J, it follows from (47) that the numerator in t_j decreases as n increases. Therefore, for large n, all $t_j \leq 0$ except for $t_J = (np_J)^{-1}$, which indicates that a user node will have a positive transmission probability $\min(1, (np_J)^{-1})$ only when its channel is in the "best condition", namely, in state J. It is now easy to prove the following result:

Corollary 4: If $n \geq 1 + (1-p_J)R_{J-1}/(p_J R_J)$, we have $s_J^* = \min(1, (np_J)^{-1})$, while $s_j^* = 0$ for $1 \leq j \leq J-1$.

VI. Numerical Results

In this section we will test the performance of our scheduling schemes through numerical examples.

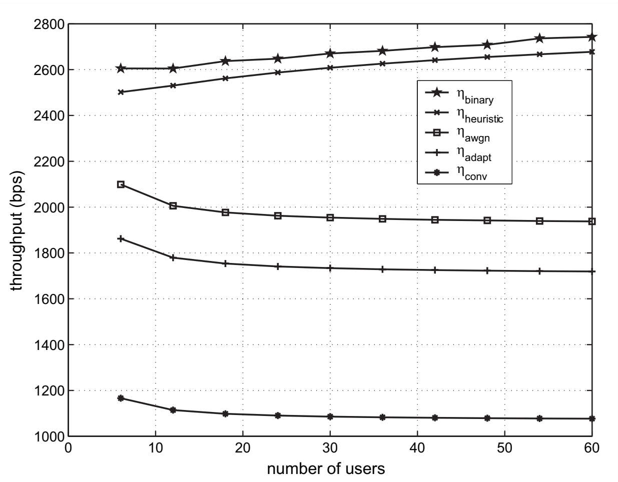

Fig. 3. Heterogeneous system: Throughput vs. Number of users, SNR range: $0 \sim 30dB$, Rayleigh fading channel

Analog-amplitude Channels: Throughout, we assume that users have independent Rayleigh fading channels. We take the slot duration $T = 1$, bandwidth $W = 1KHz$ and $R_k(\gamma) = W\log(1 + \frac{P_0\gamma}{N_0 W})$. Figs. 1 and 2 compare throughput versus the number of homogeneous users for SNR values $0\ dB$ and $16\ dB$, respectively. We see that η_{binary} is always higher than η_{heur}, and both increase as the number of users increases when $n \geq 5$. The difference between the two becomes smaller as the number of users grows. On the other hand, η_{awgn}, η_{adapt} and η_{conv} (the subscripts refer to the AWGN channel, Aloha with adaptive rate transmission but without probability control and the conventional Aloha in fading channels, respectively) all decrease as the number of users increases. Notice that η_{adapt} is higher than η_{conv}, since it takes advantage of adaptive rate transmission; however, it is still smaller than η_{awgn}, which is a result of the fact that Shannon capacity of a block fading channel is always smaller than that of an AWGN channel for the same SNR. By taking advantage of decentralized multiuser diversity, both η_{binary} and η_{heur} outperform η_{awgn}.

For the heterogeneous system, we choose six SNR values evenly spaced over the SNR range $0dB - 30dB$. For each

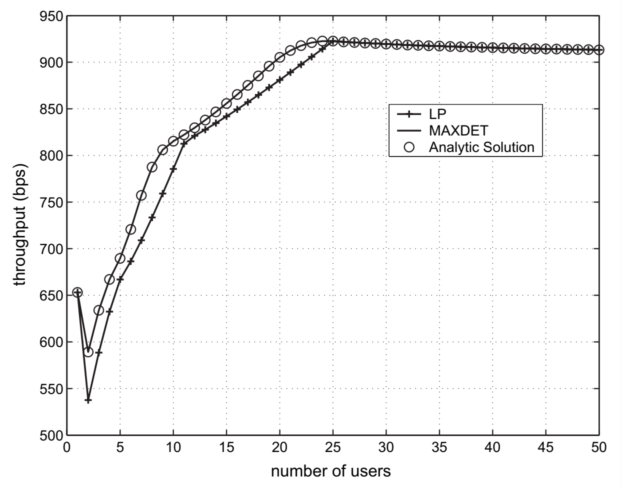

Fig. 4. Homogeneous system: Throughput vs. Number of users, FSMC channel (see Table I)

SNR value, we vary the number of users from $n = 1$ to $n = 10$ simultaneously, so that the total number of users varies from 6 to 60. Fig. 3 compares throughput versus the number of heterogeneous users for several scheduling policies. Here we observe a behavior similar to the homogeneous case. Furthermore, Fig. 3 illustrates that η'_{binary} and η'_{heur} take advantage of decentralized multiuser diversity, while η'_{adapt} and η'_{conv} do not.

FSMC Channels: Table I gives the channel parameters. With $T = 1$, we vary the number of users from 1 to 50 and run simulations using the suboptimal LP program, the optimal MAXDET program, and compare the results with the optimal analytical solution in (48). Fig. 4 shows throughput versus the number of homogeneous users. Observe that the optimal MAXDET program and the analytical solution in (48) yield identical results as expected; but the computational complexity of the latter is much lower than that of the former. Fig. 4 further illustrates that the suboptimal LP program is optimal when the number of users exceeds 25. We also notice that the throughput curves become flat when n is large, which is reasonable since an FSMC channel can only provide finite amount of multiuser diversity. This behavior is quite different from that of, e.g., a Rayleigh channel, where the throughput increases without bound as the number of users increases. Table II shows the results obtained by (48) with $n = 1 - 30$. We observe that as the number of users grows, the number of states having positive transmission probabilities drops quickly to 1 starting from $n = 19$.

VII. Conclusions

In this paper, we considered homogeneous as well as heterogeneous channel-aware slotted Aloha systems with decentralized CSI for channels with analog and quantized amplitudes. For channels with analog amplitudes, we proved that binary scheduling maximizes sum-throughput for homogeneous systems, and maximizes the sum of the logs of the average throughputs for heterogeneous systems. For channels with quantized amplitudes, which are widely accepted in practice, we have provided an optimal convex MAXDET formulation as well as a suboptimal LP formulation for the corresponding throughput-maximization problem, and developed a simple analytical solution exhibiting a binary-like structure similar to that we found optimal for analog-amplitude channels. In our future work, we will investigate delay aspects. Although the upper bounds on average queue sizes we derived here can be used to calculate upper bounds on the average delays, it will be nice to develop tighter delay bounds.[4]

Appendix I
Proof of Proposition 2

Proof: We will show first that for $j \neq k$, it holds that $t_j \notin (0, 1)$. The proof proceeds via a contradiction argument based on the fact that the KKT conditions for a convex program are sufficient and necessary, and a feasible convex program has one and only one optimal solution. If there exists one $t_j \in (0, 1), j \neq k$, then we can construct a new solution as follows

$$s'_i = \begin{cases} 0, & i < j, \\ t_j, & i = j, \\ 1, & i > j. \end{cases} \tag{49}$$

It is not difficult to check that this solution satisfies conditions (38), (39) and (40); therefore, we can find suitable $\lambda'_i, \mu'_i, i = 1, \ldots, J$ such that the KKT conditions (32)–(36) are all satisfied, resulting in another optimal solution. This contradicts that a feasible convex program has one and only one optimal solution. Therefore, it must be true that $t_j \notin (0, 1)$ for $j \neq k$.

Next, notice that the variable $x_j := (\sum_{i=1}^{j} p_i)R_j - (n-1)\sum_{i=j+1}^{J} p_i R_i$ is increasing as j increases. Since $x_k = t_k \cdot np_j R_j > 0$, we have $x_j \geq x_k > 0$ for $j > k$, leading to $t_j > 1$, since $t_j \notin (0, 1)$.

Finally, we assert that $t_j \leq 0$ for $1 \leq j < k$. If $k = 1$, then the assertion is trivial. Suppose that $k > 1$. Since $s^*_{k-1} = 0$, it follows from (38) that

$$R_{k-1} \leq \frac{(n-1)(p_k R_k s^*_k + \sum_{i=k+1}^{J} p_i R_i)}{1 - p_k s^*_k - \sum_{i=k+1}^{J} p_i}, \tag{50}$$

which yields

$$\begin{aligned}(\sum_{i=1}^{k} p_i)R_{k-1} &\leq R_{k-1}p_k s^*_k + (n-1)(p_k R_k s^*_k + \sum_{i=k+1}^{J} p_i R_i) \\ &\leq R_{k-1}p_k + (n-1)\sum_{i=k}^{J} p_i R_i.\end{aligned} \tag{51}$$

Therefore, we have $x_{k-1} = (\sum_{i=1}^{k-1} p_i)R_{k-1} - (n-1)\sum_{i=k}^{J} p_i R_i \leq 0$. Since x_j is increasing with j, it is also true that $x_j \leq 0$ for $j \leq k-1$, which implies that $t_j \leq 0$ for $j \leq k-1$. Because $t_j \notin (0, 1)$, we have that $t_j \leq 0$ for $1 \leq j < k$, which completes the proof. ∎

[4]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

TABLE I

FSMC CHANNEL PARAMETERS

Channel State j	1	2	3	4	5	6	7	8	9	10
Rate(kbps) R	76.8	102.6	153.6	204.8	307.2	614.4	921.6	1228.8	1843.2	2457.6
Probability p	0.01	0.04	0.08	0.15	0.24	0.18	0.09	0.12	0.05	0.04

TABLE II

OPTIMAL SOLUTIONS FOR THE FSMC CHANNEL OF TABLE I

n	η^*	P^*_{avg}	$1/n$	s^*_1	s^*_2	s^*_3	s^*_4	s^*_5	s^*_6	s^*_7	s^*_8	s^*_9	s^*_{10}
1	653.06	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	589.28	0.3075	0.5000						0.0417	1.0000	1.0000	1.0000	1.0000
3	633.85	0.2289	0.3333							0.2099	1.0000	1.0000	1.0000
4	666.90	0.2012	0.2500								0.9271	1.0000	1.0000
5	689.59	0.1480	0.2000								0.4833	1.0000	1.0000
6	720.56	0.1125	0.1667								0.1875	1.0000	1.0000
7	757.11	0.0900	0.1429									1.0000	1.0000
8	787.39	0.0900	0.1250									1.0000	1.0000
9	806.10	0.0900	0.1111									1.0000	1.0000
10	815.23	0.0880	0.1000									0.9600	1.0000
11	822.09	0.0788	0.0909									0.7758	1.0000
12	829.71	0.0711	0.0833									0.6222	1.0000
13	837.93	0.0646	0.0769									0.4923	1.0000
14	846.65	0.0590	0.0714									0.3810	1.0000
15	855.80	0.0542	0.0667									0.2844	1.0000
16	865.32	0.0500	0.0625									0.2000	1.0000
17	875.18	0.0463	0.0588									0.1255	1.0000
18	885.35	0.0430	0.0556									0.0593	1.0000
19	895.79	0.0400	0.0526										1.0000
20	905.22	0.0400	0.0500										1.0000
21	912.46	0.0400	0.0476										1.0000
22	917.68	0.0400	0.0455										1.0000
23	921.01	0.0400	0.0435										1.0000
24	922.62	0.0400	0.0417										1.0000
25	922.62	0.0400	0.0400										1.0000
26	921.89	0.0385	0.0385										0.9615
27	921.21	0.0370	0.0370										0.9259
28	920.59	0.0357	0.0357										0.8929
29	920.01	0.0345	0.0345										0.8621
30	919.47	0.0333	0.0333										0.8333

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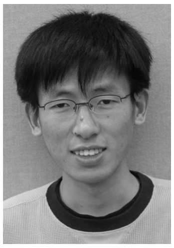

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