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On the Instability of Slotted Aloha with Capture

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***Abstract*— We analyze the stability properties of slotted Aloha with capture for random access over fading channels with infinitely-many users. We assume that each user node knows only its own uplink channel gain, and uses this decentralized channel state information (CSI) to perform power control and/or probability control. The maximum stable throughput (MST) for a general capture model is obtained by means of drift analysis on the backlog Markov chain. We then specialize our general result to a signal-to-interference-plus-noise ratio (SINR) capture model. Our analysis shows that if the channels of all users are identical and independently distributed (*i.i.d.*) with finite means, the system is unstable under any kind of power and probability control mechanism that is based only on decentralized CSI.**

***Index Terms*— Slotted Aloha, stability, maximum stable throughput, SINR capture model.**

I. Introduction

IT IS well-known that the slotted Aloha system with an infinite user population under the conventional collision channel model is unstable in the sense that its maximum stable throughput (MST) is zero [1]. In this collision channel model, whenever more than one packet is transmitted at the same time, collision occurs and all the packets are destructed. In order to stabilize the system, retransmission probabilities must adapt in accordance with the backlog state of the system, when the backlog is known to (or can be estimated by) the users [1], [3], [5]. However, it is difficult to obtain the backlog information in practice. It has been shown that the MST achievable by slotted Aloha with such decentralized stabilizing techniques is $1/e$ [1].

It turns out that this collision model is too pessimistic. With the development of multiuser detection, space-time processing, spread spectrum and other emerging techniques, it is possible to resolve some packets when collision happens. Relying on a more general, so called multipacket reception (MPR) model, it has been shown that with infinitely many users the MST approaches the limit (C_∞) of the average number (C_n) of packets resolved in a collision of size n, as n goes to infinity [6], [7]. This result shows that MPR alone is capable of stabilizing the slotted Aloha, if C_∞ is nonzero. For a special MPR model, the SINR capture model, a lot of efforts have been devoted to computing C_n as well as C_∞ in the presence of slow or fast fading, shadowing, as well as near-far effects [4], [10].

To further exploit the capture effect and improve the MST, both power and probability control for the SINR capture model are pursued in [11], when users have no access to their own uplink channel gain information. It turns out that such decentralized information is very useful for Aloha systems. A variation of conventional Aloha, known as channel-aware Aloha, is introduced in [12] to exploit multiuser diversity for medium access. Probability control based on decentralized channel state information (CSI) is considered in [8] and [9].

In this paper, we deal with both power *and* probability control issues for the SINR capture model in a system with infinitely-many users, where each user only has knowledge of its own uplink channel gain information. We will show that for any physically meaningful fading channel and any system with maximum transmit-power limitations, the system is always unstable. At first sight, our result seems to contradict some existing results, for example [4], [9]. The answer to this discrepancy lies in the received model used in [4], [9], where the path loss is modelled as $Kr^{-\alpha}$ with K and α being positive constants, and r denoting the distance from the base station (BS) to the user. In this case, it turns out that the path loss variation is the governing factor of the channel. However, this path loss model is only suitable for the far field radiation of transmit antennas. If a user is close to the BS ($r \to 0$), this model implies infinite mean power received at the BS, and is thus not physically realistic. Unfortunately, this hidden problem with the path model $Kr^{-\alpha}$ leading to a physically meaningless conclusion is generally overlooked by [4], [9], [10].

The organization of the paper is as follows. In Section II, we describe the system model. In Section III, we derive the MST for a general receiver model when both power and probability control based on decentralized CSI are employed. In Section

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IV, we compute the MST for the SINR capture model. Finally, we present our conclusions in Section VI.

II. System Model

We consider a slotted Aloha random access system consisting of a single BS and an infinite number of users. We assume that new packet arrivals are Poisson distributed with an overall rate λ. As in [1], [2] and [3], we consider a system with infinitely-many users, in which each arriving packet is allocated to a new user that has never previously been assigned a packet; thus, every user has at most one packet to transmit. This model precludes queueing of incoming packets at individual users. For the new arriving packets, we consider the delayed first transmission (DFT) access rule, in which new packets are treated the same way as backlogged packets [7].

Throughout, we shall adopt a block-fading model for the uplink channel between any user and the BS, where the channel propagation coefficient is invariant per slot but is allowed to change to a new independent value in the next slot. The uplink CSI of user k is quantified by the channel gain γ_k between the user k and the BS. All $\{\gamma_k\}_{k=1}^{\infty}$ variables are assumed to be *i.i.d.* with a CDF $F(\cdot)$. Further, we assume that each user node only has knowledge of its own CSI at the beginning of a slot. In addition, each user is informed when its packet is successfully received by the BS.

Among the objectives of this paper is to exploit the possible benefits of decentralized control based on CSI. Unlike [8], we consider a model with infinitely-many users and employ both probability and power control. Specifically, the transmission probability and transmit power of user k with CSI γ_k, is updated according to two respective functions, $s(\gamma_k)$ and $P_t(\gamma_k)$.

Suppose n users transmit packets in slot t and their instantaneous CSI variables are $\{\gamma_k\}_{k=1}^{n}$. The received signal at the BS $y(t)$ is given by

$$y(t) = \sum_{k=1}^{n} \sqrt{\gamma_k P_t(\gamma_k)} e^{-j\phi_k} x_k(t) + z(t), \tag{1}$$

where $x_k(t)$ denotes the transmitted signal of user k with unit average power, ϕ_k the received signal phase, and $z(t)$ stands for additive white noise. The received power level of user k at the BS is $\rho_k = \gamma_k P_t(\gamma_k)$. Clearly, $\{\rho_k\}_{k=1}^{n}$ are also *i.i.d.* random variables. If the CDF of $\{\rho_k\}_{k=1}^{n}$ is $G(\cdot)$, it follows that

$$G(\rho) = \int_{P_t(\gamma)\gamma \le \rho} dF(\gamma). \tag{2}$$

The reception model we consider is similar to the one used in [9] and is in fact a special case of the MPR model. For each user k transmitting a packet, we define a random variable for the outcome as follows

$$\theta_k = \begin{cases} 1, & \text{if user } k \text{ succeeds,} \\ 0, & \text{if user } k \text{ fails.} \end{cases}$$

Let $C_n(G(\cdot))$ be the average number of packets correctly received by the BS. Given that n packets are transmitted and the received average power levels of n users at the BS, $\{\rho_k\}_{k=1}^{n}$, are *i.i.d.* with CDF $G(\cdot)$, we have

$$C_n(G(\cdot)) = \int_0^{\infty} \cdots \int_0^{\infty} \sum_{k=1}^{n} \mathrm{E}\{\theta_k | \{\rho_i\}_{i=1}^{n}\} \times dG(\rho_1) \cdots dG(\rho_n). \tag{3}$$

III. Probability and Power Control based on CSIs

In this section, we assume that each user node knows only its own CSI at the beginning of slot t. As described in Section II, we let both the transmission power and the transmit probability of each user node be functions of its CSI. Our first goal is to determine the MST for a given probability control function $s(\cdot)$ and a given power control function $P_t(\cdot)$. Subsequently, we will explore the possible benefits of probability control and power control.

If we denote the number of backlogged packets in the system at the beginning of slot t as X_t, then $\{X_t\}_{t=0}^{\infty}$ is a discrete homogenous Markov chain with time evolution function

$$X_{t+1} = X_t + A_t - \pi_t, \tag{4}$$

where A_t is the number of newly arrived packets in slot t, and π_t is the number of successfully resolved packets in slot t. We assume that the Markov chain $\{X_t\}_{t=0}^{\infty}$ is aperiodic and irreducible, which is valid for most nontrivial models. A sufficient condition for aperiodicity and irreducibility is given in [6]. We define the system to be stable if $\{X_t\}_{t=0}^{\infty}$ is ergodic, and unstable otherwise [6].

To analyze stability, we compute the drift of the system at state n

$$d_n = \mathrm{E}\{X_{t+1} - X_t | X_t = n\}. \tag{5}$$

From (4) and the fact that the mean arriving rate is $\mathrm{E}\{A_t\} = \lambda$, we have

$$d_n = \lambda - \mathrm{E}\{\pi_t | X_t = n\}. \tag{6}$$

Since the CSI variables in slot t for the n users are $\{\gamma_k\}_{k=1}^{n}$, we obtain

$$\mathrm{E}\{\pi_t | X_t = n, \{\gamma_k\}_{k=1}^{n}\} = \sum_{(j_1 \cdots j_k j_{k+1} \cdots j_n)} s(\gamma_{j1}) \cdots s(\gamma_{j_k}) \times (1 - s(\gamma_{j_{k+1}})) \cdots (1 - s(\gamma_{j_n}) \sum_{m=1}^{k} \mathrm{E}\{\theta_{j_m} | \{P_t(\gamma_{j_i})\gamma_{j_i}\}_{i=1}^{k}\}, \tag{7}$$

where $(j_1 \cdots j_k j_{k+1} \cdots j_n)$ are the all combinations such that $(j_1 \cdots j_k) \cup (j_{k+1} \cdots j_n) = (1 \cdots n)$, and $(j_1 \cdots j_k) \cap (j_{k+1} \cdots j_n) = \emptyset$, for $1 \le k \le n$.

Now integrating $\{\gamma_{j_i}\}_{i=1}^{n}$ over $[0, \infty)$ and combining equivalent terms, we obtain

$$\mathrm{E}\{\pi_t | X_t = n\} = \sum_{k=1}^{n} \binom{n}{k} \left(1 - \int_0^{\infty} s(\gamma) dF(\gamma)\right)^{n-k} \times \int_0^{\infty} \cdots \int_0^{\infty} s(\gamma_1) \cdots s(\gamma_k) \sum_{m=1}^{k} \mathrm{E}\{\theta_m | \{P_t(\gamma_i)\gamma_i\}_{i=1}^{k}\} \times dF(\gamma_1) \cdots dF(\gamma_k). \tag{8}$$

If we define a new distribution function $H(\cdot)$ as

$$H(\gamma) := \frac{1}{P_s}\int_0^{\gamma} s(\widetilde{\gamma})dF(\widetilde{\gamma}), \tag{9}$$

where $P_s = \int_0^\infty s(\gamma)dF(\gamma) \neq 0$ for any nontrivial function $s(\gamma)$, then

$$\mathrm{E}\{\pi_t|X_t = n\} = \sum_{k=1}^{n}\binom{n}{k}(1-P_s)^{n-k}P_s^k \times \int_0^\infty \cdots \int_0^\infty \sum_{m=1}^{k}\mathrm{E}\{\theta_m|\{P_t(\gamma_i)\gamma_i\}_{i=1}^k\}dH(\gamma_1)\cdots dH(\gamma_k). \tag{10}$$

Changing variables from $\{\gamma_i\}_{i=1}^k$ to $\{\rho_i\}_{i=1}^k$ through the transformation $\rho_i = P_t(\gamma_i)\gamma_i$, and letting $T(\cdot)$ denote the distribution function of $\{\rho_i\}_{i=1}^k$, we arrive at

$$T(\rho) = \int_{P_t(\gamma)\gamma\le\rho} dH(\gamma) = \frac{\int_{P_t(\gamma)\gamma\le\rho} s(\gamma)dF(\gamma)}{\int_0^\infty s(\gamma)dF(\gamma)}. \tag{11}$$

Then we have

$$\mathrm{E}\{\pi_t|X_t = n\} = \sum_{k=1}^{n}\binom{n}{k}(1-P_s)^{n-k}P_s^k \times \int_0^\infty \cdots \int_0^\infty \sum_{m=1}^{k}\mathrm{E}\{\theta_m|\{\rho_i\}_{i=1}^k\}dT(\rho_1)\cdots dT(\rho_k). \tag{12}$$

From (3), (6) and (12), we deduce that the drift is

$$d_n = \lambda - \sum_{k=1}^{n}\binom{n}{k}(1-P_s)^{n-k}P_s^k C_k(T(\cdot)),\ n \ge 1. \tag{13}$$

Now we can use Pakes' and Kaplan's criteria as in [6] to establish the following result:

Theorem 1: Suppose $F(\gamma)$ is the channel gain distribution function, $s(\gamma)$ the transmission probability control function and $P_t(\gamma)$ the transmit power control function. If $C_k(T(\cdot))$ has a limit $C_\infty(T(\cdot)) = \lim_{k\to\infty} C_k(T(\cdot))$, where $T(\cdot)$ is given by (11), then the system is stable for all $\lambda \le C_\infty(T(\cdot))$, and is unstable for $\lambda > C_\infty(T(\cdot))$. In other words, the MST η is $C_\infty(T(\cdot))$.

As observed in [9], the power of decentralized control based on CSI is that it allows one to manipulate the equivalent a posteriori average received power distribution. Without control, the MST is given by $C_\infty(F(\cdot))$. With the help of the control functions $s(\gamma)$ and $P_t(\gamma)$, we steer the distribution from $F(\cdot)$ to $T(\cdot)$, and accordingly change the MST from $C_\infty(F(\cdot))$ to $C_\infty(T(\cdot))$. Therefore, it is possible to find control functions that result in higher MST.

In the next section however, we will see that for the SINR capture model there is no benefit of either power or probability control, if the channel gain has finite mean. This result excludes the use of both power control and probability control in practice, if we consider a system with a large number of users without using alternative stabilizing techniques.

IV. SINR Capture Model

In the SINR capture model, a packet is successfully decoded by the BS, if its SINR is large enough. Supposing that n packets are transmitted simultaneously, the packet of user 1 is captured if

$$\frac{\rho_1}{\sigma^2 + \sum_{k=2}^n \rho_k} > z, \tag{14}$$

where σ^2 is the power of additive noise, z is a positive "capture threshold" and $\{\rho_k\}_{k=1}^n$ are the received power levels of the n users.

As shown by Theorem 1, we need to compute $C_\infty(T(\cdot))$, namely the average number of successfully received packets under the SINR model, to determine the MST. For AWGN channels, where all $\{\rho_k\}_{k=1}^n$ are positive constants, if we take n to infinity, Eq. (14) does not hold since the left hand side approaches 0; i.e., $C_\infty(T(\cdot)) = 0$. However, this argument does not carry over when we consider fading channels, since now $\{\rho_k\}_{k=1}^n$ are *i.i.d.* random variables. In this case, $C_\infty(T(\cdot))$ is given by the following integral form [c. f. (3)]

$$\begin{aligned}C_n(T(\cdot)) &= \int_0^\infty \cdots \int_0^\infty \sum_{k=1}^{n}\mathrm{P_r}\{\frac{\rho_k}{\sigma^2 + \sum_{j\neq k}\rho_j} > z|\{\rho_i\}_{i=1}^n\} \times dT(\rho_1)\cdots dT(\rho_n)\\ &= n\int_0^\infty \mathrm{Pr}\{\frac{\rho_1}{\sigma^2+\sum_{k=2}^n \rho_k} > z|\rho_1\}dT(\rho_1).\end{aligned} \tag{15}$$

We have the following proposition.

Proposition 1: If the received power levels of users at the BS are *i.i.d.* with an equivalent a posteriori distribution function $T(\cdot)$ having finite mean, then for the SINR capture model, $C_n(T(\cdot))$ has zero limiting value, i.e.,

$$C_\infty(T(\cdot)) = \lim_{k\to\infty} C_k(T(\cdot)) = 0. \tag{16}$$

Proof: see Appendix I. □

For any physically meaningful fading channel model, the mean gain should be finite; i.e., $\mathrm{E}_{F(\cdot)}\{\gamma\} = \int_0^\infty \gamma dF(\gamma) < \infty$. Since $s(\gamma)$ is a probability control function, we have $0 \le s(\gamma) \le 1$, and thus

$$\begin{aligned}\mathrm{E}_{H(\cdot)}\{\gamma\} &= \int_0^\infty \gamma dH(\gamma) = \frac{1}{P_s}\int_0^\infty \gamma s(\gamma)dF(\gamma)\\ &\le \frac{1}{P_s}\int_0^\infty \gamma dF(\gamma) < \infty.\end{aligned} \tag{17}$$

In practice, the transmit power is always upper bounded; i.e., $P_t(\gamma) \le P_0$. Therefore,

$$\begin{aligned}\mathrm{E}_{T(\cdot)}\{\rho\} &= \int_0^\infty \rho dT(\rho) = \int_0^\infty P_t(\gamma)\gamma dH(\gamma)\\ &\le P_0\int_0^\infty \gamma dH(\gamma) = P_0\mathrm{E}_{H(\cdot)}\{\gamma\} < \infty.\end{aligned} \tag{18}$$

Now we are ready to state our main result.

Theorem 2: If the fading channel gain distribution function $F(\gamma)$ has finite mean, the transmission probability control function $s(\gamma)$ satisfies $0 \le s(\gamma) \le 1$ and the transmit power control function $P_t(\gamma)$ is upper bounded as $P_t(\gamma) \le P_0$, then the Aloha system with infinitely-many users under the SINR capture model is unstable.

Proof: Under the given conditions, we have $\mathrm{E}_{T(\cdot)}\{\rho\} < \infty$; so, from Proposition 1 we find that $C_\infty(T(\cdot)) = 0$. But from Theorem 1, we know that the MST $\eta = 0$; hence, the system is unstable. □

Theorem 2 implies that the SINR capture capability alone cannot stabilize slotted Aloha with an infinite number of users for any physically reasonable fading channel. This result discourages the use of any kind of power as well as probability control to ensure stability. The potential of exploiting the capture effect to improve the performance of the slotted Aloha system has long been noted, but as we show here, without involving other stabilizing techniques this is impossible for the SINR capture model. Therefore, we still have to rely on the knowledge of the backlog to stabilize the system. In this case, selecting the retransmission probability and the transmission power should be based not only on the CSI, but also on the number of users waiting for transmission, namely the backlog information. However, the optimal control functions maximizing throughput in this case are generally unknown.

The statement of Proposition 1 seems in contradiction with existing claims, e.g., [4], [9]. But actually they are consistent. In Section I, we pointed out that the discrepancy is due to the fact that the path loss model $Kr^{-\alpha}$ is not applicable to the near field of the radiation of transmit antennas, since $Kr^{-\alpha} \to \infty$ as $r \to 0$. Specifically, let us consider a channel distribution function $G(\cdot)$ with roll-off parameter $\delta > 0$, defined as follows [4]

$$\exists \text{ a constant } c \in (0, \infty) \ s.t. \ \lim_{\gamma \to \infty} (1 - G(\gamma))\gamma^\delta = c. \quad (19)$$

If we assume that users are randomly distributed in a circle around the BS and that the channel model obeys $Kr^{-\alpha}$, the distribution function of the received user power will obey (19), [4].

It has been shown in [4] that

$$\lim_{n\to\infty} C_n(G(\cdot)) = \begin{cases} \beta^{-\delta} \frac{\sin(\pi\delta)}{\pi\delta}, & 0 < \delta < 1 \\ 0, & \delta \geq 1 \end{cases}. \quad (20)$$

We can see that only when $\delta \in (0, 1)$, $G(\cdot)$ will result in a nonzero $C_\infty(G(\cdot))$ [1], and for $\delta \geq 1$, $C_\infty(G(\cdot))$ is zero. We can easily check that for $0 < \delta < 1$, the mean of γ is infinite. Therefore, the finite mean condition in Proposition 1 does not hold.

V. Conclusions

We investigated the stability of slotted Aloha with infinitely-many users under the SINR capture model. Our main conclusion is that the system is always unstable whether or not power and probability control mechanisms are employed based only on CSI. To ensure stability, we still have to rely on traditional stabilizing techniques. Nevertheless, our result neither excludes the use of capture to improve the throughput for a system with a finite population, nor it ignores the throughput gain due to capture for a system with infinitely-many users that employs stabilizing schemes. For these two cases, an interesting and equally challenging problem is to explore which kind of pragmatic control functions yields the best achievable stable throughput for a given SINR capture model. [2]

[1]A different notation $\phi(z, \delta)$ instead of $C_\infty(G(\cdot))$ is used in [4].

Appendix I
Proof of Proposition 1

Proposition 1 can be proved by mimicking the steps in [9] to prove Proposition 6. For completeness, we provide the proof as follows. Let us rewrite the capture condition (14) as

$$\frac{\sigma^2}{\rho_1} + \sum_{k=2}^{n} \frac{\rho_k}{\rho_1} < \frac{1}{z}, \quad (21)$$

and substitute it into (15) to find

$$\begin{aligned} C_n(F(\cdot)) &= n \int_0^\infty \Pr\{\frac{\sigma^2}{\rho_1} + \sum_{k=2}^{n} \frac{\rho_k}{\rho_1} < \frac{1}{z}|\rho_1\} dF(\rho_1) \\ &\leq n \int_0^\infty \Pr\{\sum_{k=2}^{n} \frac{\rho_k}{\rho_1} < \frac{1}{z}|\rho_1\} dF(\rho_1) \quad (22) \\ &\leq n \int_0^\infty \Pr\{\sum_{k=2}^{n} \frac{\rho_k}{\rho_1 + \rho_k} < \frac{1}{z}|\rho_1\} dF(\rho_1). \end{aligned}$$

Now, we can use Chernoff's bound to obtain an upper bound on the inner probability

$$\begin{aligned} \Pr\{\sum_{k=2}^{n} \frac{\rho_k}{\rho_1 + \rho_k} < \frac{1}{z}|\rho_1\} &\leq \mathrm{E}\{\exp(\frac{\rho}{z} - \sum_{k=2}^{n} \frac{\rho\rho_k}{\rho_1 + \rho_k})|\rho_1\} \\ &= \exp(\frac{\rho}{z})\mathrm{E}^{n-1}\{\exp(-\frac{\rho\rho}{\rho_1 + \rho})|\rho_1\}. \end{aligned} \quad (23)$$

Let $\rho = 1$, and define $\mu(\rho_1)$ as

$$\begin{aligned} \mu(\rho_1) &= \mathrm{E}\{\exp(-\frac{\rho}{\rho_1 + \rho})|\rho_1\} \\ &= \int_0^\infty \exp(-\frac{\rho}{\rho_1 + \rho}) dF(\rho). \end{aligned} \quad (24)$$

Using (22) and (24), it follows that

$$C_m(F(\cdot)) \exp(-\frac{1}{z}) \leq m \int_0^\infty \mu^{n-1}(\rho_1) dF(\rho_1). \quad (25)$$

Considering $\rho_1(1 - \mu(\rho_1))$ as $\rho_1 \to \infty$, we find

$$\begin{aligned} \rho_1(1 - \mu(\rho_1)) &= \int_0^\infty \rho_1(1 - \exp(-\frac{\rho}{\rho_1 + \rho})) dF(\rho) \\ &\geq \frac{1}{2} \int_0^\infty \frac{\rho_1\rho}{\rho_1 + \rho} dF(\rho), \end{aligned} \quad (26)$$

where the last inequality comes from the fact that $1 - e^{-x} \geq x/2$ with $0 \leq x \leq 1$. Taking the limit of (26) and using the monotone convergence theorem, we arrive at

$$\begin{aligned} \lim_{\rho_1\to\infty} \rho_1(1 - \mu(\rho_1)) &\geq \frac{1}{2} \int_0^\infty \lim_{\rho_1\to\infty} \frac{\rho_1\rho}{\rho_1 + \rho} dF(\rho) \\ &= \frac{1}{2}\mathrm{E}(\rho). \end{aligned} \quad (27)$$

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Eq. (27) implies that there exists a constant ρ^* such that for $\rho_1 > \rho^*$ we have

$$\rho_1(1-\mu(\rho_1) > \frac{1}{2}\mathrm{E}(\rho), \text{ or } \mu(\rho_1) < 1 - \frac{\mathrm{E}(\rho)}{2\rho_1}. \quad (28)$$

It follows from (25) that

$$\begin{aligned} & C_n(F(\cdot))\exp(-\frac{1}{z}) \\ & \le \int_0^{\rho^*} n\mu^{n-1}(\rho_1)dF(\rho_1) + \int_{\rho^*}^{\infty} n(1-\frac{\mathrm{E}(\rho)}{2\rho_1})^{n-1}dF(\rho_1) \\ & \le \int_0^{\rho^*} n\mu^{n-1}(\rho^*)dF(\rho_1) + \int_{\rho^*}^{\infty} n(1-\frac{\mathrm{E}(\rho)}{2\rho_1})^{n-1}dF(\rho_1). \end{aligned} \quad (29)$$

The first term in the *r.h.s.* of (29) goes to zero as $n \to \infty$, since $0 < \mu(\rho^*) < 1$. For the second term, we have

$$n(1-\frac{\mathrm{E}(\rho)}{2\rho_1})^{n-1} \le \rho_1, \quad (30)$$

and

$$\int_{\rho^*}^{\infty} \rho_1 dF(\rho_1) \le \mathrm{E}(\rho) < \infty. \quad (31)$$

From the dominated convergence theorem, we know that the second term also vanishes as $n \to \infty$; therefore, we obtain $\lim_{n\to\infty} C_n(F(\cdot)) = 0$.
□

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