Local ML Detection for Multicarrier DS-CDMA Downlink Systems with Grouped Linear Precoding

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***Abstract*— A multicarrier direct-sequence code-division multiple-access (MC-DS-CDMA) downlink system with linear precoding over a group of subcarriers is considered. This scheme preserves user orthogonality independently of the underlying frequency-selective channel, collects the channel diversity and enables low-complexity decoding. In this context, we examine a local maximum-likelihood (LML) detection technique that searches for the maximum-likelihood (ML) solution in the neighborhood of the output provided by the minimum mean-squared error (MMSE) detector. By exploiting the soft information of the MMSE detector output and the precoder structure, we introduce useful criteria to reduce the computational complexity of the LML search. Simulations illustrate that the LML-MMSE detector with minimum neighborhood size yields considerable BER improvement with respect to MMSE, and outperforms a block decision-feedback equalization (DFE) approach at comparable complexity.**

***Index Terms*— Multicarrier communications, MC-DS-CDMA, linear precoding, maximum likelihood detection, MMSE detection.**

I. INTRODUCTION

AMONG the multicarrier spread-spectrum schemes proposed in the literature, we can distinguish between two different approaches: frequency-domain spreading, as in multicarrier code-division multiple-access (MC-CDMA) systems, and time-domain spreading, as in multicarrier direct-sequence code-division multiple-access (MC-DS-CDMA) systems [5]. In frequency-selective channels, MC-CDMA is able to exploit the multipath diversity through multiuser reception which can handle the loss of user orthogonality. On the other hand, uncoded MC-DS-CDMA maintains user orthogonality, but does not provide diversity. In order to overcome these limitations, different variants of MC-CDMA and MC-DS-CDMA have been proposed. Cai, Zhou, and Giannakis proposed a reduced-complexity MC-CDMA system where spreading is applied only on a group of subcarriers [3]. In this system, the data of users belonging to different groups are orthogonal, while the data of users within the same group can be recovered by low-complexity multiuser detection. However, since the receiver requires the spreading codes of all the users in the group, [3] is more suitable for the uplink rather than the downlink. On the other hand, Petré, Leus, and Moonen incorporate a linear precoding approach to introduce diversity in MC-DS-CDMA systems [16]. Although [16] is able to collect multipath diversity and maintains user orthogonality, linear precoding is performed over all the available subcarriers, and hence the computational complexity at the decoding stage is relatively high when the number of subcarriers is large. In this paper, we consider an MC-DS-CDMA system that weds the linear precoding approach of [16] with the subcarrier grouping method of [3]. Differently from [16], we focus on non-redundant precoding [10], so that the frequency diversity gain is obtained without sacrificing the data rate, as firstly proposed in [2] for single-carrier flat-fading links. Moreover, subcarrier grouping allows us to reduce computational complexity at the decoding stage, as a direct consequence of the precoder size reduction. In addition, the scheme we consider does not require the spreading codes of the other users, thus enabling low-complexity detectors that are suitable also for downlink scenarios.

To recover the precoded data, various detectors can be applied, each one offering a different BER-complexity tradeoff. The maximum-likelihood (ML) detector is able to collect both diversity and coding gains, with an exponential complexity in the precoder size. Near-ML techniques such as sphere decoding (SD) [20], semi-definite programming (SDP) [13], and probabilistic data association (PDA) [12], approach the ML performance, but their complexity may be still large for downlink applications. In addition, the worst-case complexity can be much higher than the average complexity [6], thus complicating real-time communications. On the contrary, linear detectors and decision-directed schemes exhibit lower complexity, but suffer from BER performance loss. In this contribution, we look at local ML (LML) detection techniques, which perform a complexity-constrained ML search in the neighborhood of an initial estimate. We show that the output of the MMSE detector is a convenient choice for such an initial estimate. Specifically, we show that, by adjusting the neighborhood size, the LML-MMSE detector can nicely trade performance for complexity, filling the gap between the MMSE and the ML detectors. Simulation results in typical urban channels show that the LML-MMSE detectors outperform a block decision-feedback equalization (DFE) approach [1] [18], with a similar complexity.

II. MC-DS-CDMA WITH GROUPED LINEAR PRECODING

We consider the downlink of an MC-DS-CDMA system with N subcarriers and U active users. In MC-DS-CDMA systems with grouped linear precoding (GLP), either frequency-domain separation or user code despreading is employed to separate different users. We assume that the N subcarriers

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are divided in B groups with K subcarriers per group, i.e., $N = BK$. We further suppose that subcarriers in the same group are maximally separated in frequency. Let us divide the U users in B groups, with U_b users in the bth group, and assume that K symbols of a generic user are linearly precoded over the K subcarriers associated with its group, similarly to [3] for MC-CDMA and [10] for orthogonal frequency-division multiplexing (OFDM). Since users' data in different groups are orthogonal, we will focus on a specific group, e.g., the first group. A more detailed system model can be found in [17]. To distinguish the data of users in the same group, time-domain spreading is employed, with orthogonal codes characterized by the same processing gain G. Throughout the paper, we assume that the multipath channel is time invariant, with each path gain being Rayleigh distributed, and with maximum delay spread not exceeding the cyclic prefix duration. We also assume perfect time and frequency synchronization.

Due to the time-domain spreading, in order to decode K data symbols, the receiver has to collect G consecutive OFDM-like blocks. Without loss of generality, we assume that the user of interest is the first of the first group. By selecting only the K subcarriers of interest after FFT, the $K \times G$ received matrix can be expressed as

$$\bar{\mathbf{Z}}[l] = \mathbf{D}\boldsymbol{\Theta}\mathbf{S}_1[l]\mathbf{C}_1 + \bar{\mathbf{W}}[l], \tag{1}$$

where $\mathbf{D}$ is the $K \times K$ diagonal matrix that contains the frequency-domain channel gains, $\boldsymbol{\Theta}$ is the non-redundant $K \times K$ precoder, designed as in [10], $\mathbf{S}_1[l]$ is the $K \times U_1$ matrix that contains the K uncoded symbols of the U_1 users, $\mathbf{C}_1$ is the $U_1 \times G$ matrix that contains the unit-norm spreading codes of the first group, $\bar{\mathbf{W}}[l]$ stands for the additive white Gaussian noise (AWGN), and l is the index of the data block. The data symbols in $\mathbf{S}_1[l]$, drawn from a constellation of size M, are assumed to be independent and identically distributed (i.i.d.) with power $\sigma_{\mathrm{S}}^2 = 1$. The received signal, after despreading, is expressed as: $\mathbf{y}[l] = \bar{\mathbf{Z}}[l]\mathbf{c}_{1,1}^{*} = \mathbf{D}\boldsymbol{\Theta}\mathbf{s}[l] + \mathbf{w}[l]$, where the $G \times 1$ vector $\mathbf{c}_{1,1}$ is the spreading code of the user of interest, $\mathbf{s}[l]$ is the uncoded data block of the user of interest, and $\mathbf{w}[l] = \bar{\mathbf{W}}[l]\mathbf{c}_{1,1}^{*}$. To simplify notation, we drop the index l, thus obtaining

$$\mathbf{y} = \mathbf{D}\boldsymbol{\Theta}\mathbf{s} + \mathbf{w} = \mathbf{H}\mathbf{s} + \mathbf{w}, \tag{2}$$

where $\mathbf{H} = \mathbf{D}\boldsymbol{\Theta}$ represents the aggregate effect of the channel and the precoder on the uncoded data vector $\mathbf{s}$.

In order to exploit all the performance advantages of linear precoding, ML detection should be performed at the receiver side [21]. In this case, due to the AWGN nature of $\mathbf{w}$, the decision rule can be formulated as $\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\max_{\mathbf{s}\in S} \{\Lambda(\mathbf{s})\}$, where

$$\Lambda(\mathbf{s}) = 2\mathrm{Re}(\mathbf{s}^H\mathbf{H}^H\mathbf{y}) - \mathbf{s}^H\mathbf{H}^H\mathbf{H}\mathbf{s} \tag{3}$$

is the log-likelihood function (LLF), and S is the set of all possible transmitted data vectors, with cardinality equal to M^K. Albeit the precoder size K is reduced by a factor B with respect to [16], the computational complexity involved in the evaluation of M^K LLFs can still be too high. Linear detection techniques allow to obtain a soft estimate of the transmitted symbol vector by a simple matrix multiplication, expressed as $\tilde{\mathbf{s}} = \mathbf{G}\mathbf{y}$. By the zero-forcing (ZF) or the MMSE criterion, $\mathbf{G}$ is given by $\mathbf{G}_{\mathrm{ZF}} = \mathbf{H}^{-1}$, or, $\mathbf{G}_{\mathrm{MMSE}} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \sigma_w^2\mathbf{I}_K)^{-1}$, respectively, where σ_w^2 is the variance of the elements of the AWGN vector $\mathbf{w}$.

III. Local ML Detection for MC-DS-CDMA with GLP

The key idea behind LML detection is to perform the ML search by exploring only a subset of S. Indeed, if an accurate first estimate $\hat{\mathbf{s}}$ is available at the receiver, there is a high probability to refine this estimate by restricting the ML search only to those vectors that are close to $\hat{\mathbf{s}}$. Given a symbol vector $\hat{\mathbf{s}}$ and an integer $P \in \{0, \ldots, K\}$, we let $S_P(\hat{\mathbf{s}})$ denote the *neighborhood* of $\hat{\mathbf{s}}$ of size P, which is defined as

$$S_P(\hat{\mathbf{s}}) = \{\mathbf{s} \in S | d_H(\mathbf{s}, \hat{\mathbf{s}}) \le P\}, \tag{4}$$

where $d_H(\cdot)$ denotes the Hamming distance. We define the LML detector of size P associated with $\hat{\mathbf{s}}$ as

$$\hat{\mathbf{s}}_{\mathrm{LML}}(P) = \arg\max_{\mathbf{s}\in S_P(\hat{\mathbf{s}})} \{\Lambda(\mathbf{s})\}, \tag{5}$$

that is, the ML detector constrained to the restricted set $S_P(\hat{\mathbf{s}})$. In other words, the LML detector evaluates all the LLFs associated with the vectors that differ in at most P entries from $\hat{\mathbf{s}}$, and selects the symbol vector $\hat{\mathbf{s}}_{\mathrm{LML}}$ that produces the highest likelihood among them. The Hamming distance allows for an exact prediction of the cardinality of $S_P(\hat{\mathbf{s}})$, expressed by

$$C(P, M, K) = \sum_{i=0}^{P} \binom{K}{i} (M-1)^i, \tag{6}$$

which turns out to be independent of $\hat{\mathbf{s}}$. Hence, the number of LLFs to be evaluated in (5) can be easily controlled by a convenient choice of P. One of the key properties of the LML detectors is the following.

Property 1: For any fixed initial estimate $\hat{\mathbf{s}}$, it holds true that

$$\Pr\{\hat{\mathbf{s}}_{\mathrm{LML}}(P) \ne \mathbf{s}\} \leqslant \Pr\{\hat{\mathbf{s}}_{\mathrm{LML}}(i) \ne \mathbf{s}\}, \quad \forall i < P. \tag{7}$$

Proof: Since $S_P(\hat{\mathbf{s}}) \supset S_{P-1}(\hat{\mathbf{s}})$, it holds true that $\Lambda(\hat{\mathbf{s}}_{\mathrm{LML}}(P)) \geqslant \Lambda(\hat{\mathbf{s}}_{\mathrm{LML}}(P-1))$, and hence $\Pr\{\hat{\mathbf{s}}_{\mathrm{LML}}(P) = \mathbf{s}\} \geqslant \Pr\{\hat{\mathbf{s}}_{\mathrm{LML}}(P-1) = \mathbf{s}\}$, which easily leads to (7).

In particular, for $i = 0$, Property 1 states that applying an LML search to the output $\hat{\mathbf{s}}$ of any suboptimal detector does not produce a block-error probability increase, thus motivating the LML approach. It should be pointed out that, in single-carrier DS-CDMA systems, the LML approach is often applied iteratively, i.e., the output of an LML detector is used as the initial estimate of another LML decoding stage [7] [19] [11]. Indeed, in DS-CDMA the number of users, which plays the same role as the precoder size K in our case, can be very high, and therefore the neighborhood size is forced to a value $P = 1$ to limit complexity. As a consequence, instead of increasing P, LML detectors for DS-CDMA try to improve the BER performance by iterating the LML detection with $P = 1$. On the contrary, in multicarrier systems, the precoder size may be very small, because the maximum diversity gain is limited by the number of channel paths, and consequently the LML detectors with $P = 2$ are not very complex.

A. LML-MMSE Detector

When maximizing non-convex functions, initialization is often critical in avoiding local maxima. For an LML decoder of size P, we would like an initial estimate with at most P errors, which is impossible even when using the ML detector. Therefore, as an alternative criterion, we could look for a detector whose soft output vector contains at least $K - P$ entries that are close (in some sense) to the transmitted ones. This way we can force the LML detector to confine its search to those vectors that differ from the estimated one only in the remaining P entries. If we select the mean squared-error (MSE) as the measure of closeness, and we restrict the choice among the linear detectors for complexity reasons, the detector we are looking for is the MMSE. Indeed, the MMSE detector minimizes the MSE of each symbol in the data vector $\mathbf{s}$, independently of the others.

Instead of the MMSE detector, [7] adopts the ZF detector as a first stage. However, in multicarrier systems with linear precoding, this detector performs poorly in the presence of deep fades. Alternatively, [11] suggests to use a DFE approach. Although the BER for DFE is typically smaller than for MMSE detection, the DFE suffers from error propagation, a phenomenon that concentrates errors in the same blocks. For this reason, LML techniques could be less effective when decision-directed detectors are used for initialization.

B. Effect of a Pilot-Aided Channel Estimation Technique

So far, we have implicitly assumed that the diagonal channel matrix $\mathbf{D}$ is known to the receiver. In practice, only an estimated version of $\mathbf{D}$ is available, and therefore the LML detector should be obtained by replacing the exact $\mathbf{H}$ with its estimate $\hat{\mathbf{H}} = \hat{\mathbf{D}}\mathbf{\Theta}$. In this subsection, our aim is to modify the MMSE detector of the first stage to take into account channel estimation errors. We assume that N_{pil} pilot subcarriers, equally spaced [15], are inserted in the first G transmitted blocks. Consequently, the $N - N_{pil}$ data subcarriers are divided so that $N - N_{pil} = BK$. We also assume the same power for pilot and data subcarriers. At the receiver, we assume ML channel estimation, which achieves the Cramér-Rao lower bound (CRLB) [14]. In this case, it can be shown that if $K \in \{N_{pil}/2^n, n \in \mathbb{N}\}$, the covariance matrix of the channel estimation error vector is $\sigma_w^2 \mathbf{I}_K$ [17]. This implies that the estimation error can be interpreted as an additive white Gaussian error with power identical to the thermal AWGN. Hence, we can define the modified MMSE (MMMSE) detector as $\mathbf{G}_{\text{MMMSE}} = \hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + 2\sigma_w^2\mathbf{I}_K)^{-1}$, and, by similar considerations, the modified DFE (MDFE) and the modified SD (MSD).

C. Complexity Reduction by Excluding Improbable Vectors

For $M > 2$, $C(P, M, K)$ in (6) may be high even with moderate values of P and K. A possibility is to further reduce the cardinality of the search set by excluding those vectors whose entries are not adjacent to those of $\hat{\mathbf{s}}$. Since the excluded symbols are less likely to be correct, the performance loss should be small. Moreover, we can exploit the soft output $\tilde{\mathbf{s}}$ of the MMSE detector to exclude other improbable vectors. By focusing on the kth entry $\tilde{s}_k$, we can use the quantity $r_k^{(m)} = \left|s^{(m)} - \tilde{s}_k\right|^{-2}$ to measure the *reliability* of the symbol $s^{(m)}$ that belongs to the adopted constellation. Consequently, for a fixed k, we can rank the symbols $\{s^{(m)}, m = 1, ..., M\}$ depending on their reliability $\{r_k^{(m)}, m = 1, ..., M\}$, and check only the LLFs associated with the $\bar{M}$ symbols having highest reliability. This approach will be denoted as soft reduced constellation (SRC).

Another possibility is to keep K_{F} symbols of $\hat{\mathbf{s}}$ fixed, and allow for the variation of at most P of the other $K_{\text{V}} = K - K_{\text{F}}$ symbols. It is reasonable that the fixed symbols should be those with the highest reliability, where the reliability of the kth symbol $\hat{s}_k$ can be expressed by $r_k = |\hat{s}_k - \tilde{s}_k|^{-2}$. The number of vectors in the new search set becomes $C(P, \bar{M}, K_V)$, which can be controlled by the design parameters P, $\bar{M}$, and K_{V}. We would like to point out that this approach is reminiscent of what is proposed in [9], where the reliability is measured by using the log-likelihood ratio. However, different from [9], our approach can be applied not only to BPSK but also to higher order constellations.

D. Complexity Reduction by Exploiting the Precoder Structure

A further reduction in decoding complexity is achieved by exploiting the specific structure of some precoders designed for cyclic-prefixed multicarrier systems. In this case, rather than reducing the number of vectors in the neighborhood set, we simplify the computation of the LLF. For instance, we may assume that $\mathbf{\Theta}$ is unitary, and that all its entries have modulus equal to $1/\sqrt{K}$. This class of precoders includes those designed for linear MMSE detection [8], and those designed for ML detection expressed by $\mathbf{\Theta} = \mathbf{F}_K\mathbf{A}$ [10] [2], $\mathbf{F}_K$where $\mathbf{F}_K$ is the $K \times K$ unitary FFT matrix, K is a power of two, $\mathbf{A} = \text{diag}(1, \alpha, \cdots, \alpha^{K-1})$ and α satisfies the equation $\alpha^K = \sqrt{-1}$. In this case, the complexity of the LML detector with $P = 1$ can be significantly reduced. This fact is explained in the following for BPSK, assuming $K_{\text{V}} = K$. Letting $\hat{\mathbf{s}} + \mathbf{e}_k$ denote the vector obtained by flipping the kth entry of $\hat{\mathbf{s}}$, where $\mathbf{e}_k = [0, ..., 0, -2\hat{s}_k, 0, ..., 0]^T$ is non-zero only in its kth position, it holds true that

$$\Lambda(\hat{\mathbf{s}}+\mathbf{e}_k) = \Lambda(\hat{\mathbf{s}}) + 2\text{Re}(\mathbf{e}_k^T\mathbf{H}^H\mathbf{y} - \mathbf{e}_k^T\mathbf{H}^H\mathbf{H}\hat{\mathbf{s}}) - \mathbf{e}_k^T\mathbf{H}^H\mathbf{H}\mathbf{e}_k. \tag{8}$$

Since $\mathbf{e}_k$ is non-zero only in the kth position, $\mathbf{e}_k^T\mathbf{H}^H\mathbf{H}\mathbf{e}_k$ turns out to be equal to $4[\mathbf{H}^H\mathbf{H}]_{k,k}$. However, since $\mathbf{H} = \mathbf{D}\mathbf{\Theta}$ and $|[\mathbf{\Theta}]_{i,j}| = 1/\sqrt{K}$, $[\mathbf{H}^H\mathbf{H}]_{k,k}$ does not depend on k, and $\mathbf{e}_k^T\mathbf{H}^H\mathbf{H}\mathbf{e}_k = 4\text{tr}(\mathbf{D}^H\mathbf{D})/K$. Hence, in order to find the most likely among the vectors $\{\hat{\mathbf{s}}+\mathbf{e}_k, k = 1, ..., K\}$, it is sufficient to look for the maximum value assumed by $2\text{Re}(\mathbf{e}_k^T\mathbf{H}^H(\mathbf{y} - \mathbf{H}\hat{\mathbf{s}}))$, for $k = 1, ..., K$. By defining the $K \times K$ diagonal matrix $\mathbf{E} = -2\text{diag}(\hat{\mathbf{s}}) = [\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_K]^T$, the LML detector has only to find the maximum value among the elements of the vector

$$\mathbf{v} = 2\mathbf{E}\text{Re}(\mathbf{H}^H(\mathbf{y} - \mathbf{H}\hat{\mathbf{s}})), \tag{9}$$

and successively, if $\max(\mathbf{v}) > 4\text{tr}(\mathbf{D}^H\mathbf{D})/K$, it has to flip the symbol of $\hat{\mathbf{s}}$ corresponding to the position of $\max(\mathbf{v})$. As a consequence of (9), the complexity of this LML detector is comparable to that of decision-directed detectors. For other

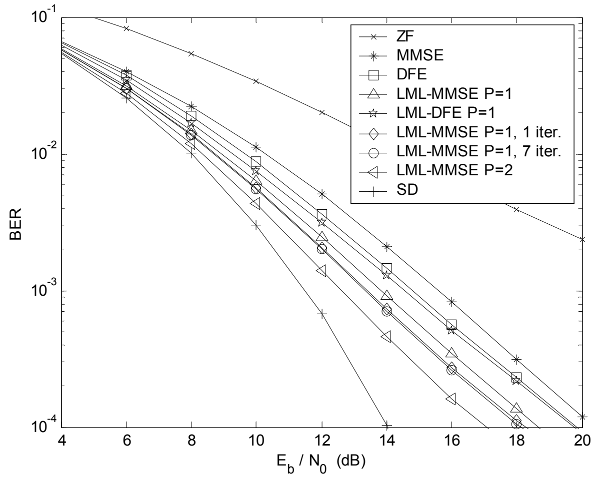

Fig. 1. BER comparison among various detection schemes.

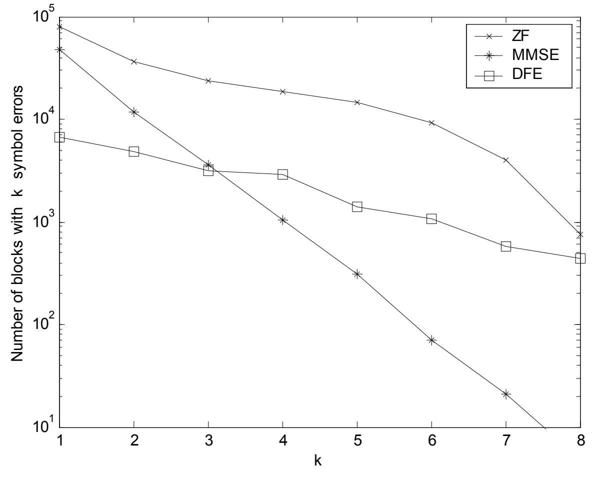

Fig. 2. Distribution of the number of errors within the same data block.

constellations, similar considerations hold true with minor modifications.

Before elaborating further on the computational complexity of the LML-MMSE detector, we first highlight that a unitary precoder also simplifies the MMSE detector computation, because of the diagonal matrix inversion. Moreover, we also point out that the LML detector can equivalently maximize the *relative LLF* $\Lambda(\hat{\mathbf{s}}+\mathbf{e})-\Lambda(\hat{\mathbf{s}})$, which is easier to be evaluated than $\Lambda(\hat{\mathbf{s}}+\mathbf{e})$, where $\mathbf{e}$ represents an error vector containing at most P non-zero values. Therefore, by plugging $\mathbf{e}$ in (8) and exploiting $\mathbf{H}=\mathbf{D}\boldsymbol{\Theta}=\mathbf{D}\mathbf{F}_K\mathbf{A}$, the number of complex multiplications required per received block $\mathbf{y}$ can be reduced to [17]

$$N_{mult}=4K\log_2 K+8K+\sum_{i=0}^{P}(i+i^2)(\bar{M}-1)^i\begin{pmatrix}K_{\rm V}\\ i\end{pmatrix}. \tag{10}$$

The number of complex multiplications N_{mult} can be further reduced when $\bar{M}=2$, if the LML search is performed by exploiting a $K_{\rm V}$-ary tree structure having the vector $\hat{\mathbf{s}}$ as the root, the vectors $\{\hat{\mathbf{s}}+\mathbf{e}_k, k=1,...,K_{\rm V}\}$ as leaves, and so on. In this case, N_{mult} can be obtained as in (10) by replacing $i+i^2$ with $3i+1$ [17]. Moreover, $K\log_2 K+3K-1$ additional complex multiplications are required at the beginning to compute $\mathbf{H}^H\mathbf{H}$, which has to be updated only when the channel changes. From (10), by assuming $\bar{M}=2$ and $P\leqslant K/2$, the computational complexity increases as $O(K_{\rm V}^P)$. Thus, for $P=2$, the *fixed* complexity of the LML-MMSE detector stays below the *average* complexity of SD, which is roughly $O(K^3)$ [6], and below the complexities of PDA and SDP, which are $O(K^3)$ [12], and $O(K^{3.5})$ [13], respectively. This fact motivates the usefulness of the proposed algorithm with $P\leqslant 2$ for multicarrier systems. Moreover, when $P>2$, the complexity of the LML-MMSE detector can be shrunk by reducing $K_{\rm V}$.

IV. Simulation Results

In this section, we present simulation results in order to assess the BER performance of the LML-MMSE detectors. As an example, which is not exhaustive of the several scenarios a designer could be faced with, we consider an MC-DS-CDMA system with cyclic prefix of length $L=128$ and $N=1024$ subcarriers, whose $N_{pil}=128$ are reserved for information broadcasting. The sampling frequency is $f_S=1/T_S=20$ MHz, and hence the subcarrier separation is $\Delta_f=f_S/N\approx 19.5$ kHz. The $N-N_{pil}$ data subcarriers are divided in $B=112$ groups of $K=8$ subcarriers. The precoder is the one of [10]. We assume QPSK with Gray mapping, processing gain $G=16$, Walsh-Hadamard spreading codes and a fully loaded system with $U=BG=1792$ *virtual* users, each one with bit rate roughly equal to 17.36 kbps. This bit rate can be increased by assigning to each active user more than one code or group of subcarriers, e.g., a single active user can correspond to many virtual users. As an example, if we want to increase the user bit rate by a factor R, it is preferable to assign R groups to that user rather than increasing the precoder size by a factor R. In fact, computational complexity in the first case increases linearly with R, whereas in the second case it increases more than linearly, depending on the decoding algorithm (e.g., it increases as R^3 when the decoding complexity is cubic in the precoder size).

As far as the channel model is concerned, we use the 12 tap typical urban (TU) model of the COST 207 standard [4]. In this model, each tap undergoes independent Rayleigh fading, with a maximum delay spread of 5 μs. Simulations are performed by assuming that each channel realization is time invariant within each data block. This assumption, which preserves user orthogonality, is quite realistic in several scenarios. For the simulation scenario we considered ($B=112$, $K=8$ and $G=16$) , if the mobile receiver has velocity $V\leqslant 30$ Km/h, and a carrier frequency of $f_c=2$ GHz, the maximum Doppler frequency is $f_D\approx 55.6$ Hz. Since the duration of a data block is $T_B=G(N+L)T_S\approx 922$ μs, Clarke's autocorrelation function is $J_0(2\pi f_D T_B)\approx 0.974$, and hence the channel can be supposed constant. Anyway, higher speeds can be supported by reducing the processing gain G of certain groups, thus reducing the number of virtual users in these groups while increasing their bit rates.

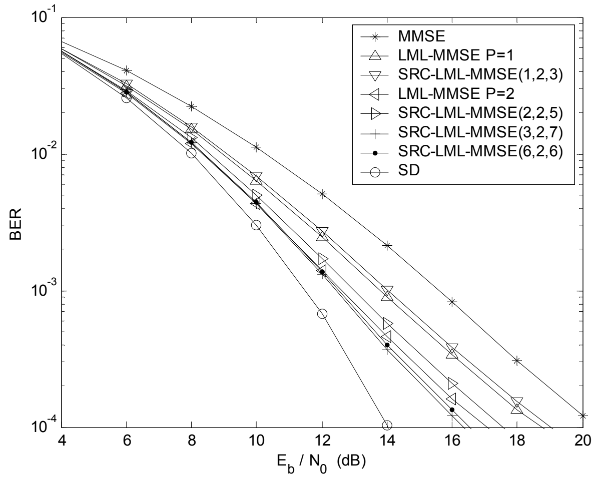

Fig. 3. BER comparison among different SRC-LML-MMSE detectors.

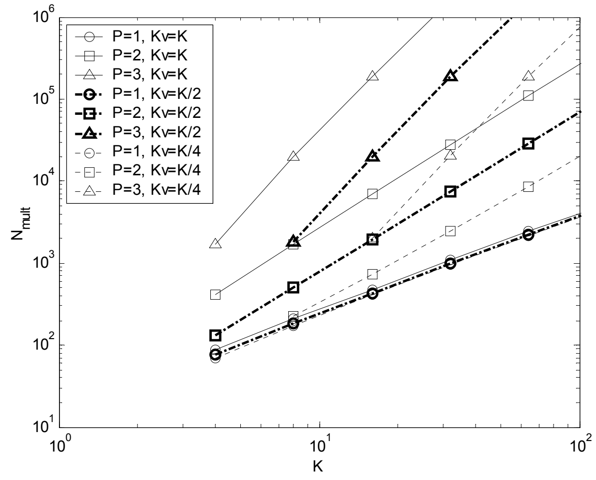

Fig. 5. Complexity of the LML detectors vs. precoder size K when $\bar{M} = 4$.

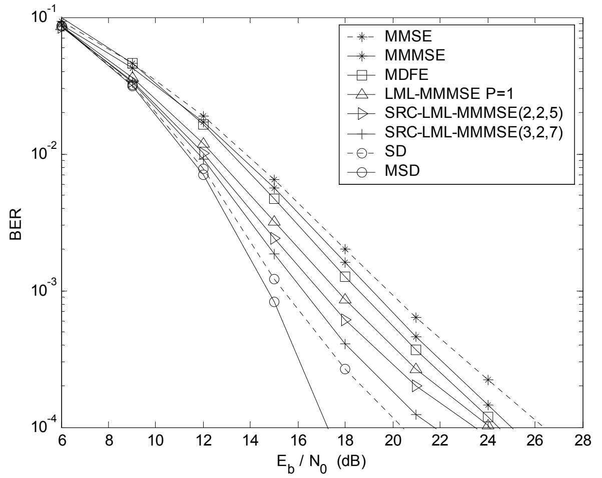

Fig. 4. BER of the LML detectors in the presence of channel estimation errors.

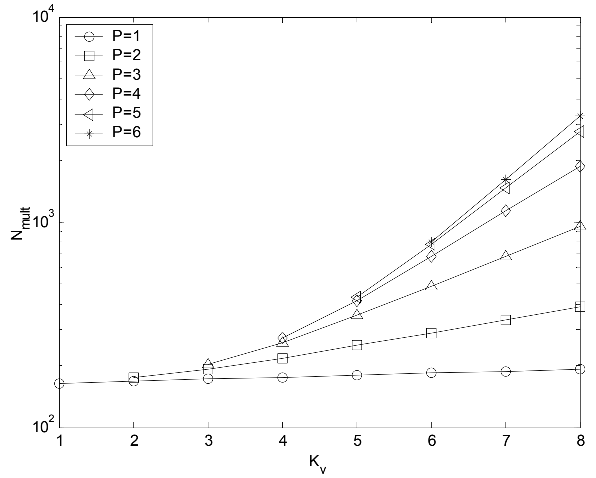

Fig. 6. Complexity of the SRC-LML-MMSE detector when $K = 8$ and $\bar{M} = 2$.

In all figures, performance of the SD algorithm in [22] is shown instead of the true ML performance, which is not feasible to simulate in a reasonable computational time. Fig. 1 depicts BER performance of various detectors versus E_b/N_0, averaged over 100 channel realizations. At BER $= 10^{-3}$, the performance gain of the LML-MMSE detector with $P = 2$ with respect to DFE and MMSE detector is roughly 2.2 dB and 3 dB, respectively, while the loss with respect to SD is approximately 1.1 dB. Fig. 1 also shows that the LML-MMSE detector with $P = 1$ outperforms the DFE detector (1 dB gain at BER $= 10^{-3}$), while presenting a comparable complexity. It is worth noting that 1 iteration of the LML search gives small performance improvement (0.4 dB when BER $= 10^{-3}$) with respect to the non-iterative LML-MMSE detector, at the expense of doubling the complexity. More iterations are not effective. Therefore, it seems to be more convenient to increase the neighborhood size P instead of iterating the LML-MMSE detector with $P = 1$ as in [7].

Fig. 1 also suggests that, although the DFE outperforms the MMSE detector, their LML counterparts behave differently. Indeed, when $P = 1$, the LML-MMSE detector outperforms the LML detector with DFE initialization (LML-DFE), with a gain of roughly 0.8 dB when BER $= 10^{-3}$. This fact is clearly explained by Fig. 2, which plots versus k the number of detected blocks with k errors, when $1.84 \cdot 10^6$ blocks are transmitted at $E_b/N_0 = 14$ dB. Due to error propagation, the DFE produces a significant number of blocks with several symbol errors, which are not recovered by a subsequent LML approach. On the contrary, most of the erroneous blocks of the MMSE detector contain only one error, and therefore in this case the LML approach with $P = 1$ is quite effective.

Fig. 3 illustrates BER performance of the LML-MMSE detectors that use the SRC approach. It is evident that the SRC-LML-MMSE detector with $(P, \bar{M}, K_{\rm V}) = (1, 2, 3)$, which evaluates only $C = 4$ LLFs, provides almost the same performance as the full LML-MMSE detector with $P = 1$, characterized by $(P, \bar{M}, K_{\rm V}) = (1, 4, 8)$ and $C = 25$. Moreover, the SRC-LML-MMSE with $(P, \bar{M}, K_{\rm V}) = (2, 2, 5)$ and $C = 16$ incurs a performance loss of 0.4 dB with respect to the full LML-MMSE detector characterized by $P = 2$ and

$C = 277$. This loss can be recovered by a SRC-LML-MMSE with $(P, \bar{M}, K_{\mathrm{V}}) = (3, 2, 7)$ or with $(P, \bar{M}, K_{\mathrm{V}}) = (6, 2, 6)$, with an increased complexity $C = 64$.

Fig. 4 depicts the BER of the modified (e.g., LML-MMMSE) detectors in the presence of channel estimation errors. The ML channel estimation technique described in [14] is employed. It can be observed that the LML-MMMSE approach is effective also in this case. Fig. 5 illustrates the number of multiplications N_{mult} required by the LML detector to decode a block of K symbols. The plot is obtained by evaluating (10) with $\bar{M} = 4$. It is clear that complexity can be controlled by both P and K_{V} parameters. Moreover, the complexity can be reduced further by adopting the SRC approach and setting $\bar{M} = 2$, as described by Fig. 6.

V. Conclusions

We have considered an MC-DS-CDMA scheme that maintains user orthogonality in frequency-selective downlink channels and collects the multipath diversity by a group-wise linear precoding technique. Low-complexity decoding schemes based on the LML approach have been investigated. We have shown that the output of the MMSE detector offers a convenient initialization for the LML detector. We have also clarified how performance and complexity of the proposed LML-MMSE detector, which fall between those of MMSE and ML detectors, can be nicely adjusted by controlling the neighborhood size and by exploiting the soft information of the MMSE detector. Simulation results in typical urban channels have demonstrated that the LML-MMSE detector with minimum neighborhood size outperforms a DFE approach, while exhibiting comparable complexity.

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