Performance Analysis of Combined Transmit Selection Diversity and Receive Generalized Selection Combining in Rayleigh Fading channels

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***Abstract*— We analyze the average symbol error rate (SER) of M-PSK and M-QAM modulations with transmit antenna selection diversity (SD) and receive generalized selection combining (GSC) in Rayleigh fading channels. SER formulas are derived in closed form, and numerical results show that transmit SD and receive GSC are flexible to trade off performance for complexity.**

***Index Terms*— Generalized selection combining, Transmit diversity, Rayleigh fading.**

I. Introduction

Diversity is an effective means of combating fading in wireless communications. If several replicas of the same information bearing signal transmitted over uncorrelated fading channels are available, the receiver can exploit them to increase signal-to-noise ratio (SNR), and thereby reduce bit error rate (BER). Selection diversity (SD) is the simplest diversity technique in which the receiver chooses the diversity branch having the highest instantaneous SNR. In maximum-ratio combing (MRC), received signals from all diversity branches are combined; and the instantaneous SNR is maximized at the combiner output. Recently, generalized selection combining (GSC) has received considerable attention [1], [2], [5], [7], [8], [13], [14]. In GSC, signals from a subset of the available diversity branches are combined; changing the size of the subset, one can trade off performance for receiver complexity. As discussed in [14], reducing the number of branches in diversity combining can reduce the power consumption and cost of the RF electronics at the receiver. GSC is also more robust to channel estimation errors since the weak SNR branches are excluded from the combing process, as pointed out in [1].

Diversity can also be provided by multiple transmit antennas. If channel state information (CSI) is not available at the transmitter, space-time coding is an efficient technique to enable transmit diversity [11]. If CSI is available at the transmitter, the optimal weights for the transmit antennas can be found as the right singular vector of the channel matrix corresponding to the largest singular value [4]. However, feeding back the optimal weights to the transmitter consumes relatively large bandwidth. A simple transmit diversity using partial CSI is transmit antenna selection diversity, which chooses the transmit antenna having the strongest channel gain. Performance of transmit antenna SD with receive MRC was analyzed in [12]. The diversity system with antenna selection at both transmitter and receiver was studied in [10].

In this paper, we analyze the performance of transmit antenna SD with receive GSC over Rayleigh fading channels. We consider coherent detection of M-ary phase-shift keying (M-PSK) and quadrature amplitude modulation (M-QAM). Closed forms for the symbol error rate (SER) of M-PSK and M-QAM are derived. We also present numerical examples illustrating that with transmit SD, receive GSC bridges the performance gap between SD of [10] and MRC of [12].

II. Performance Analysis

Suppose that there are N_t transmit antennas, and the channel from each transmit antenna to the receiver has N paths. In flat fading channels, these N channel paths may arise if N receive antennas are employed; in frequency-selective fading channels, the receiver may receive N copies of the same signal transmitted over N different carrier frequencies, or may resolve the signal propagating from the same transmit antenna through N different paths. Let $h_{n,m}$ be the complex channel gain of the nth path from the mth transmit antenna. We assume that $\{h_{n,m}, n \in [1,N], m \in [1,N_t]\}$ are independent, identically distributed (i.i.d.), and circularly symmetric Gaussian random variables. Let $\alpha_{n,m} := |h_{n,m}|^2 E_s/N_o$, where E_s is the energy of the transmitted symbol, and N_0 is the one-side power spectral density of the white Gaussian noise. The average SNR per symbol per path is expressed as $\bar{\alpha} = E[|h_{n,m}|^2]E_s/N_0$, where $E[\cdot]$ denotes expectation with the random variables within the brackets.

Fig. 1 depicts the frame structure of the transmitted signals, where D denotes information-bearing (data) symbols, and P_i denotes pilot symbols transmitted from antenna i. Based on the time correlation of the channel gains, the receiver uses the pilot symbols to predict the channels for transmit antenna selection, as well as to estimate the channels for

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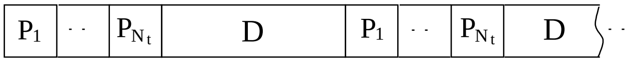

Fig. 1. The frame structure of the transmitted signals

coherent detection. Notice that pilot symbols are transmitted from each antenna in different time slots, which facilitates estimation and prediction of the channel gains associated with different antennas. We assume that the fading channels are slowly varying and the feedback delay is sufficiently small to render channel estimation and prediction errors negligible. We also assume that there is no feedback error. Based on these assumptions, we suppose that the receiver knows all channel gains $\{h_{n,m}, n \in [1,N], m \in [1,N_t]\}$ perfectly. Let $\alpha_{1:N,m} \geq \alpha_{2:N,m} \geq \ldots \geq \alpha_{N:N,m}$ be the order statistics obtained by arranging $\{\alpha_{n,m}\}_{n=1}^{N}$ in decreasing order of magnitude. Combining the first L ($1 \leq L \leq N$) variable(s) in the order statistics, the receiver obtains $\beta_m := \sum_{l=1}^{L} \alpha_{l:N,m}$, and then finds $\gamma := \max_m\{\beta_m\}$. Define $m_t := \arg\ \max_m\{\beta_m\}$. The receiver feeds back m_t to the transmitter, and the transmit antenna m_t is selected to transmit information bearing symbols. The decision variable for the transmitted symbol is the output of the generalized selection combiner which combines signals from the first L strongest paths, and thus, the instantaneous SNR of the decision variable is γ. We will next derive the moment generating function (MGF) of γ from the probability density function (pdf) of β_m. Based on the MGF of γ, we then find the average SER of M-PSK and M-QAM.

Let the pdf of β_m be $f_{\beta_m}(\beta_m)$, where for notational simplicity, we do not distinguish a random variable from its realization. The Laplace transform of $f_{\beta_m}(\beta_m)$ is found as [1]

$$\mathcal{L}\{f_{\beta_m}(\beta_m)\} = (1+s\bar{\alpha})^{-L} \prod_{L+1}^{N} (1+s\bar{\alpha}L/l)^{-1}. \quad (1)$$

The Laplace transform of the cumulative distribution function (cdf) of β_m, $F_{\beta_m}(\beta_m)$, is then given by $\mathcal{L}\{F_{\beta_m}(\beta_m)\} = \mathcal{L}\{f_{\beta_m}(\beta_m)\}/s$. Using partial fraction expansion, we can write $\mathcal{L}\{F_{\beta_m}(\beta_m)\}$ in the following equivalent form:

$$\mathcal{L}\{F_{\beta_m}(\beta_m)\} = \frac{c_0}{s} + \sum_{l=1}^{L} \frac{c_l}{(s+1/\bar{\alpha})^l} + \sum_{l=L+1}^{N} \frac{c_l}{s+l/L\bar{\alpha}}, \quad (2)$$

where

$$c_l = \begin{cases} 1 & l=0 \\ \bar{\alpha}^{-l+1}\left[-1+\sum_{k=L+1}^{N} \frac{(-1)^{k-l}\binom{N}{N-k}\binom{k-1}{k-L-1}}{(k/L-1)^{L-l+1}}\right] & 1 \leq l < L \\ -\bar{\alpha}^{-L+1}\binom{N}{N-L} & l=L \\ (-1)^l (l/L-1)^{-L}\binom{N}{N-l}\binom{l-1}{l-L-1} & L < l \leq N. \end{cases} \quad (3)$$

Taking inverse Laplace transform of $\mathcal{L}\{F_{\beta_m}(\beta_m)\}$, we obtain

$$\begin{aligned} F_{\beta_m}(\beta_m) =& c_0 + \sum_{l=1}^{L} c_l \beta_m^{l-1} \exp(-\beta_m/\bar{\alpha})/(l-1)! \\ &+ \sum_{l=L+1}^{N} c_l \exp(-l\beta_m/L\bar{\alpha}). \end{aligned} \quad (4)$$

An equivalent form of $F_{\beta_m}(\beta_m)$ was found in [7] by integrating the pdf $f_{\beta_m}(\beta_m)$, which is obtained from the inverse Laplace transform of (1). Since $\{\beta_m\}_{m=1}^{N_t}$ are i.i.d., and $\gamma = \max_m\{\beta_m\}$, the cdf of γ is given by $F_\gamma(\gamma) = [F_{\beta_m}(\gamma)]^{N_t}$. By using the multinomial theorem [3, p. 166], $F_\gamma(\gamma)$ can be written as

$$F_\gamma(\gamma) = \sum_{S_k \in \mathcal{S}} a_k \gamma^{b_k} e^{-d_k \gamma}, \quad (5)$$

where $\mathcal{S} = \{S_k \mid \sum_{n=0}^{N} n_{k,n} = N_t\}$ with non-negative integers $\{n_{k,n}\}$,

$$a_k = N_t! \prod_{l=1}^{L} (c_l/(l-1)!)^{n_{k,l}} \prod_{l=L+1}^{N} c_l^{n_{k,l}} \Big/ \prod_{n=0}^{N} n_{k,n}!\ , \quad (6)$$

$$b_k = \sum_{l=1}^{L} (l-1) n_{k,l}, \quad (7)$$

$$d_k = \left[\sum_{l=1}^{L} n_{k,l} + \sum_{l=L+1}^{N} l n_{k,l}/L\right] / \bar{\alpha}. \quad (8)$$

The Laplace transform of $F_\gamma(\gamma)$ can be found from (5) as $\mathcal{L}\{F_\gamma(\gamma)\} = \sum_{S_k \in \mathcal{S}} a_k b_k!/(s+d_k)^{b_k+1}$. Since $F_\gamma(0) = 0$, the Laplace transform of the pdf of γ, $f_\gamma(\gamma)$, can be written as $\Phi(s) := \mathcal{L}\{f_\gamma(\gamma)\} = s\mathcal{L}\{F_\gamma(\gamma)\}$. Let us now partition the set $\mathcal{S}$ into three subsets: $\mathcal{S}_0 = \{S_k | n_{k,0} = N_t; n_{k,l} = 0, l \neq 0\}$, $\mathcal{S}_1 = \{S_k | n_{k,l} = 0, l = 2, \ldots, L; n_{k,1} + \sum_{k,l=L+1}^{N} \neq 0\}$, and $\mathcal{S}_2 = \mathcal{S} - \mathcal{S}_0 - \mathcal{S}_1$. There is only one element S_0 in $\mathcal{S}_0$, and we have $a_0 = 1$, $b_0 = 0$, $d_0 = 0$. For $S_k \in \mathcal{S}_1$, we have $b_k = 0$, $d_k \neq 0$; for $S_k \in \mathcal{S}_2$, we have $b_k > 0$, $d_k \neq 0$. Since $f_\gamma(\gamma)$ is a continuous function, we must have $a_0 + \sum_{S_k \in \mathcal{S}_1} a_k = 0$; and then, the Laplace transform of $f_\gamma(\gamma)$ can be expressed as

$$\begin{aligned} \Phi(s) = & \sum_{S_k \in \mathcal{S}_1} -a_k d_k (s+d_k)^{-1} \\ & + \sum_{S_k \in \mathcal{S}_2} a_k b_k! [(s+d_k)^{-b_k} - d_k (s+d_k)^{-b_k-1}], \end{aligned} \quad (9)$$

with the MGF of γ given by $\mathcal{M}_\gamma(s) = \Phi(-s)$. While the average SNR and the variance of the SNR are now easily calculated from the MGF in (9), and the outage probability can be found from $F_\gamma(\gamma)$ in (5), we will use the MGF-based approach of [9, p. 268] to evaluate the SER of M-PSK and M-QAM modulations.

Using the MGF of γ, the SER of M-PSK is given by [9, p. 271]

$$P_{MPSK}(e) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \mathcal{M}_\gamma \left(-\frac{g_{PSK}}{\sin^2\theta}\right) d\theta, \quad (10)$$

where $g_{PSK} := \sin^2(\pi/M)$. For the MGF given by (9), since $\{b_k\}$ is a set of integers, the SER of M-PSK is found as

$$\begin{aligned} P_{MPSK}(e) = & \sum_{S_k \in \mathcal{S}_1} -a_k f_1(g_{PSK}/d_k, 1) \\ & + \sum_{S_k \in \mathcal{S}_2} a_k b_k! d_k^{-b_k} [f_1(g_{PSK}/d_k, b_k) \\ & - f_1(g_{PSK}/d_k, b_k+1)], \end{aligned} \quad (11)$$

where $f_1(c,m) := \pi^{-1} \int_0^{(M-1)\pi/M} [\sin^2\theta/(\sin^2\theta + c)]^m d\theta$, which has a closed form given in [9, p. 127]. For

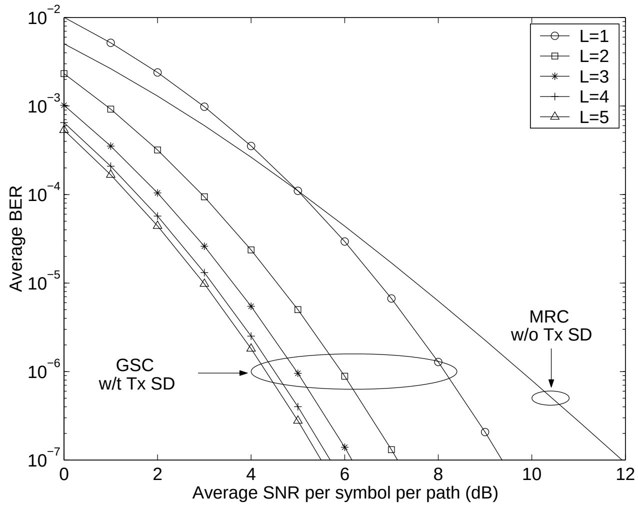

Fig. 2. Average BER of BPSK versus average SNR per path

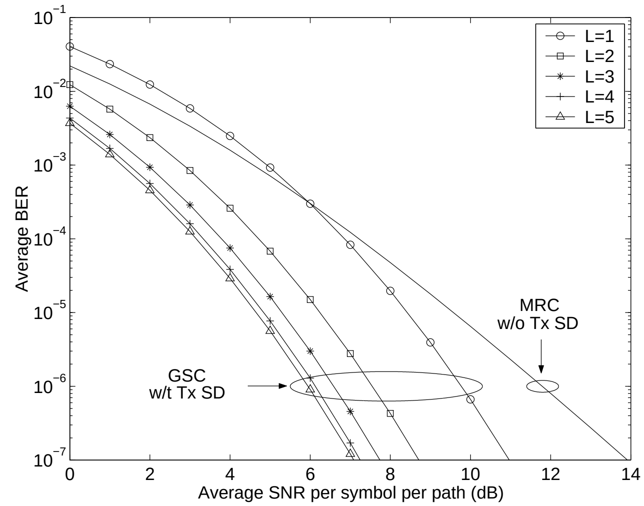

Fig. 3. Average SER of 8-PSK versus average SNR per path

BSPK, $f_1(c,m)$ is given by $f_1(c,m) = [(1-\mu)/2]^m \sum_{k=0}^{m-1} \binom{m-1+k}{k} [(1+\mu)/2]^k$ [9, p. 125][6, p. 781], where $\mu = \sqrt{c/(1+c)}$. We can also calculate the BER of BPSK as

$$\begin{aligned} P_{BPSK}(e) &= \int_0^\infty Q(\sqrt{2\gamma}) f_\gamma(\gamma) d\gamma \\ &= (2\sqrt{\pi})^{-1} \int_0^\infty \gamma^{-1/2} F_\gamma(\gamma) e^{-\gamma} d\gamma, \end{aligned} \tag{12}$$

where $Q(x) := \int_x^\infty \exp(-x^2/2)dx/\sqrt{2\pi}$, and we can evaluate (12) using numerical integration.

Using the MGF of γ, the SER of square M-QAM is given by [9, p. 273]

$$\begin{aligned} P_{MQAM}(e) =& \nu_1 \pi^{-1} \int_0^{\pi/2} \mathcal{M}_\gamma \left(-\frac{g_{QAM}}{\sin^2\theta} \right) d\theta \\ & - \nu_2 \pi^{-1} \int_0^{\pi/4} \mathcal{M}_\gamma \left(-\frac{g_{QAM}}{\sin^2\theta} \right) d\theta, \end{aligned} \tag{13}$$

where $\nu_1 := 4(1-1/\sqrt{M}\,)$, $\nu_2 := 4(1-1/\sqrt{M}\,)^2$, $g_{QAM} = 3/2(M-1)$. For the MGF given by (9), the SER of M-QAM is found as

$$\begin{aligned} P_{MQAM}(e) = \sum_{S_k \in \mathcal{S}_1} & -\nu_1 a_k f_2(g_{QAM}/d_k, 1) \\ & \times \nu_2 a_k f_3(g_{QAM}/d_k, 1) \\ + \nu_1 \sum_{S_k \in \mathcal{S}_2} & a_k b_k! d_k^{-b_k} [f_2(g_{QAM}/d_k, b_k) \\ & - f_2(g_{QAM}/d_k, b_k+1)] \\ - \nu_2 \sum_{S_k \in \mathcal{S}_2} & a_k b_k! d_k^{-b_k} [f_3(g_{QAM}/d_k, b_k) \\ & - f_3(g_{QAM}/d_k, b_k+1)], \end{aligned} \tag{14}$$

where $f_2(c,m) := \pi^{-1} \int_0^{\pi/2} [\sin^2\theta/(\sin^2\theta + c)]^m d\theta$, and $f_3(c,m) := \pi^{-1} \int_0^{\pi/4} [\sin^2\theta/(\sin^2\theta + c)]^m d\theta$. The closed forms of $f_2(c,m)$ and $f_3(c,m)$ are given in [9, p. 125, p. 128].

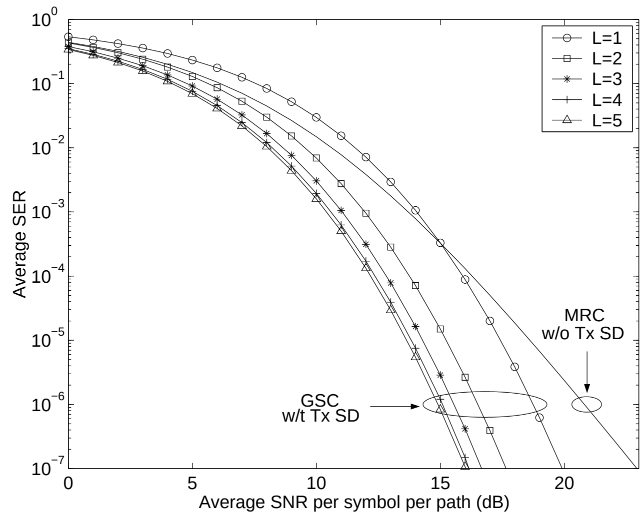

Fig. 4. Average SER of 16-QAM versus average SNR per path

III. NUMERICAL RESULTS

In this section, we present numerical results calculated from the analysis in Section II. We choose $N_t = 3$ transmit antennas, and $N = 5$ paths per transmit antenna. Fig. 2 depicts the average BER of BPSK versus average SNR per path. The BER of combined transmit-SD and receive-MRC of [12] corresponds to the curve with $L = 5$, and the BER of SD at both transmitter and receiver of [10] corresponds to the curve with $L = 1$. We see that when $L = 3$, GSC can collect most of the diversity, and the gap relative to MRC's BER curves is less than 1dB. We also plot the BER curve without transmit SD, but with receive MRC. When $L > 1$, GSC with transmit SD has better performance than MRC without transmit SD across the SNR region. Fig. 3 and Fig. 4 depict the average SER of 8-PSK and 16-QAM versus average SNR per path. Observations similar to those in Fig. 2 can be deduced from Fig. 3, and Fig. 4.

IV. CONCLUSIONS

We derived the average SER of M-PSK and M-QAM modulations with transmit SD and receive GSC in Rayleigh fading channels. With transmit SD, the receive GSC with $L > 1$ exhibits better performance than SD at both transmitter and receiver, or receive MRC without transmit SD. With a reasonable L, the performance gap between receive GSC and MRC (both with transmit SD) is very small.

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