On Regularity and Identifiability of Blind Source Separation Under Constant-Modulus Constraints

Yingwei Yao, *Member, IEEE,* and Georgios B. Giannakis, *Fellow, IEEE*

***Abstract*—We investigate the information regularity and identifiability of the blind source separation (BSS) problem with constant modulus (CM) constraints on the sources. We establish for this problem the connection between the information regularity [existence of a finite Cramér–Rao bound (CRB)] and local identifiability. Sufficient and necessary conditions for local identifiability are derived. We also study the conditions under which unique (global) identifiability is guaranteed within the inherently unresolvable ambiguities on phase rotation and source permutation. Both sufficient and necessary conditions are obtained.**

***Index Terms*—Blind source separation (BSS), constant modulus (CM), Cramér–Rao bound (CRB), identifiability, information regularity.**

I. Introduction

THE problem of recovering multiple signals from a linear mixture of them is frequently encountered in wireless communications and signal processing applications. In direct-sequence code-division multiple-access (DS-CDMA) systems, the received signals are the superposition of signals from multiple users, where the mixing matrix is the users' signature sequences. In systems with multiple transmit and receive antennas, each receive antenna receives a mixture of signals from all transmit antennas. Usually, the mixing matrix is known at the receiver (e.g., in some DS-CDMA systems), or it is estimated by sending pilot signals from transmitters to receivers. With knowledge of the mixing matrix, standard detection techniques can then be used to retrieve the source symbols [1], [2].

In some cases, however, the mixing matrix is unknown, and the training is either undesirable due to spectral efficiency concerns or unavailable due to security reasons. The problem of extracting the source symbols without knowledge of the mixing matrix or the training symbols is referred to as blind source separation (BSS). BSS algorithms typically take advantage of certain properties of the source data, such as source independence [3], [4], constant modulus (CM) [5], [6], or finite alphabet [7], [8].

One of the most widely used BSS algorithms is the constant modulus algorithm (CMA). Originally proposed for blind equalization [9]–[11], it has also been applied to blind beamforming and BSS [12], [5], [6], [13]. While extensive literature exists on the CMA and various uses of the CM criterion (see, e.g., [14] and the references therein), identifiability issues have not been resolved fully.

Closely related to identifiability is the regularity of the Fisher information matrix (FIM). The Cramér–Rao bound (CRB) has been widely used to investigate the performance limit of unbiased parameter estimators. Since the existence of a useful CRB depends on the regularity of the FIM, it is also of interest to study the conditions of information regularity.

Information regularity and identifiability of blind identification of multi-input multi-output (MIMO) finite impulse response (FIR) systems have been investigated in [15]–[17], where no constraints were placed on the source signals. Different from these works, our focus in this paper is on the problem of separating blindly multiple CM sources from an instantaneous mixture. We will briefly describe the source separation problem in Section II. In Section III, we will investigate the existence of a finite CRB for the BSS problem under CM constraints. To accommodate the constraints imposed on the source symbols, we will adopt the constrained CRB expression of [18], which will be shown to be equivalent to the CRB obtained by reparameterizing the constrained parameters. The link between information regularity (a.k.a. existence of CRB) and local identifiability will be established, and the conditions for local identifiability will be derived. The issue of global identifiability for the BSS problem under CM constraints will be studied in Section IV. We will derive a sufficient and necessary condition for this problem to be globally identifiable up to a phase rotation and source permutation ambiguity, both of which are inherently unresolvable by any blind algorithm. Conclusions will be presented in Section V.

A. Notation

Upper and lowercase bold symbols will be used to denote matrices and column vectors, respectively; $(\cdot)^*$ will denote conjugation, $(\cdot)^{\mathcal{H}}$ Hermitian transpose, and $(\cdot)^T$ transpose. $\mathbf{I}_N$ will stand for the $N \times N$ identity matrix; $\bar{x}$ and $\tilde{x}$ will denote the real and imaginary parts of x, respectively; $\otimes$ will represent the Kronecker product; $\text{diag}(\boldsymbol{x})$ will denote a diagonal matrix whose diagonal elements are the entries of the vector $\boldsymbol{x}$. Finally, $r(\mathbf{A})$ and $\|\mathbf{A}\|$ will be used to denote the rank and the Frobenius norm of the matrix $\mathbf{A}$, respectively.

Manuscript received August 7, 2003; revised April 12, 2004. This work was prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U.S. Army Research Laboratory under the Collaborative Technology Alliance Program under Cooperative Agreement DAAD19-01-2-0011. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Carlos H. Muravchik.

Y. Yao was with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA. He is now with the Department of Electrical and Computer Engineering, University of Illinois, Chicago, IL 60607 USA (e-mail: yyao@ece.uic.edu).

G. B. Giannakis is with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: georgios@ece.umn.edu).

Digital Object Identifier 10.1109/TSP.2005.843718

II. Problem Formulation

Consider the following input-output matrix-vector model:

$$\boldsymbol{x}(i) = \mathbf{H}\boldsymbol{b}(i) + \boldsymbol{w}(i) \tag{1}$$

where $\mathbf{H}$ is an $N \times K$ mixing matrix, $\boldsymbol{x}(i)$ is the received vector in the ith symbol interval, and $\boldsymbol{b}(i)$ and $\boldsymbol{w}(i)$ are the source symbol vector and the noise vector, respectively. Equation (1) appears in many problems related to wireless communications and signal processing. A few examples follow.

A. DS-CDMA Systems

After chip-matched filtering and chip-rate sampling, the received waveform can be written in the form of (1), where N is the spreading gain, K is the number of users, and the columns of $\mathbf{H}$ are signature sequences of all users scaled by their received amplitudes. Signatures are spreading codes possibly convolved with frequency-selective channels.

B. MIMO Systems

Here, N is the number of receive antennas, K is the number of transmit antennas, and H_{ij} stands for the flat fading channel coefficient between the ith receive antenna and the jth transmit antenna.

C. Beamforming

Equation (1) can also model an antenna array with N sensors receiving K signals, where the ith column of $\mathbf{H}$ is the array response vector corresponding to the ith signal.

In this paper, we will assume that the mixing matrix $\mathbf{H}$ is of full column rank, the additive noise $\boldsymbol{w}(i)$ is zero-mean, white Gaussian with covariance matrix $\sigma^2\mathbf{I}_N$, and the source symbols have constant modulus, namely, $\boldsymbol{b}(i) := [b_1(i), \ldots, b_K(i)]^T$ satisfies $|b_k(i)| = 1$ for $k = 1, \ldots, K$.

III. Information Regularity and Local Identifiability

The CRB is the lower bound of the covariance of unbiased estimators of unknown parameters. It indicates the fundamental performance limit of unbiased estimators. The nonexistence of a finite CRB often signifies the lack of identifiability of a particular parameter estimation problem. Thus, it is of interest to investigate the existence of a finite CRB for the BSS problem.

A. CRB for BSS Under CM Constraints

We will first compute the CRB of estimators $\hat{\mathbf{H}}$, $\hat{\boldsymbol{b}}(i)$ in the BSS model (1) under CM constraints. Since a small amount of training data is needed to resolve the inherent phase ambiguity of the CM constraint, we will suppose that a sufficient number of (say the first T) symbols of every source $b_k(i)$ are known.

Traditionally, the CRB for a constrained parameter estimation problem is obtained by first eliminating the redundancy in the parameters through a reparameterization and then computing the FIM. In [19], Gorman and Hero derived a CR-type lower bound from the Barankin bound, which can be computed directly from the unconstrained CRB and the constraint equations. A more accessible proof was given by Marzetta in [20], while Stoica and Ng generalized this bound to cases where the unconstrained FIM is singular [18]. Compared with the traditional approach, the approach of [18] is more convenient and provides more insight to the effects of the constraints. In [21]–[24], this constrained CRB was used to investigate the effects of various constraints, including CM, semi-blind, and source independence, in the context of both calibrated and uncalibrated array processing.

1) Constrained CRB Formulation: As mentioned previously, we will rely on the constrained CRB expression of [18], which can be restated as follows. Let $\boldsymbol{y}$ be the vector of observations and $\boldsymbol{\phi} \in \mathbb{R}^n$ be the $n\times 1$ real parameter vector to be estimated. For a set of k ($k < n$) constraints $\boldsymbol{f}(\boldsymbol{\phi}) = \mathbf{0}$, define the gradient matrix of the constraints as

$$\mathbf{F}(\boldsymbol{\phi}) = \frac{\partial \boldsymbol{f}(\boldsymbol{\phi})}{\partial \boldsymbol{\phi}^T} = \begin{pmatrix} \frac{\partial f_1}{\partial \phi_1} & \frac{\partial f_1}{\partial \phi_2} & \cdots & \frac{\partial f_1}{\partial \phi_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_k}{\partial \phi_1} & \frac{\partial f_k}{\partial \phi_2} & \cdots & \frac{\partial f_k}{\partial \phi_n} \end{pmatrix}. \tag{2}$$

Assume that $\mathbf{F}(\boldsymbol{\phi})$ has full row rank for $\boldsymbol{\phi}$ in the constrained parameter space [rank deficiency of $\mathbf{F}(\boldsymbol{\phi})$ usually indicates that certain constraints are redundant and can be eliminated]. Let $\mathbf{U}(\boldsymbol{\phi})$ be the $n \times (n-k)$ matrix whose columns form an orthonormal basis for the null space of $\mathbf{F}(\boldsymbol{\phi})$ and denote the unconstrained FIM as $\mathbf{J}_\phi$. If $\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}$ (for notational brevity, we will omit the parameter $\boldsymbol{\phi}$ of $\mathbf{U}$ from now on) is nonsingular, then any unbiased estimator $\hat{\boldsymbol{\phi}}$ must satisfy

$$E\{(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi})(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi})^T\} \geq \mathbf{U}(\mathbf{U}^T\mathbf{J}_\phi\mathbf{U})^{-1}\mathbf{U}^T. \tag{3}$$

A finite CRB for the constrained parameter estimation exists if and only if $|\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}| \neq 0$ [18].

2) Relating Constrained With Reparameterized Unconstrained CRB: Unlike bounds such as the modified CRB [25], the constrained CRB is tight. In the following, we will present an alternative derivation of (3). Compared with the proof given in [18], our derivation is less general (we need the additional assumption that a global reparameterization exists), but it illuminates the link between this new CRB formula and the CRB obtained by reparameterization rather nicely.

Since $\mathbf{F}(\boldsymbol{\phi})$ has full row rank k for all $\boldsymbol{\phi}$ satisfying $\boldsymbol{f}(\boldsymbol{\phi}) = \mathbf{0}$, the constrained parameter space is a submanifold of $\mathbb{R}^n$ with dimension $n-k$ [26]. Suppose now that a global reparameterization exists, such that each $\boldsymbol{\phi}$ in the constrained parameter space $\mathcal{U} := \{\boldsymbol{\phi} | \boldsymbol{f}(\boldsymbol{\phi}) = \mathbf{0}\}$ can be written as $\boldsymbol{\phi} = \boldsymbol{g}(\boldsymbol{\alpha})$, and $\boldsymbol{g}^{-1}(\mathcal{U})$ is an open subset of $\mathbb{R}^{n-k}$. It then follows from the chain rule of differentiation that

$$\mathbf{0} = \frac{\partial \boldsymbol{f} \circ \boldsymbol{g}(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^T} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\phi}^T}\mathbf{G} \tag{4}$$

where $\circ$ denotes composition, and $\mathbf{G} := \partial \boldsymbol{g}/\partial \boldsymbol{\alpha}^T$. Equation (4) implies that $\mathbf{G}$ belongs to the null space of $\mathbf{F}$, which allows us to write

$$\mathbf{G} = \mathbf{U}\mathbf{V} \tag{5}$$

where $\mathbf{U}$ is the $n \times (n-k)$ matrix in (3). Denote the probability density function (pdf) before and after the reparameterization as

$p(\boldsymbol{y},\boldsymbol{\phi})$ and $q(\boldsymbol{y},\boldsymbol{\alpha})$, respectively. The FIM after reparameterization can be written as

$$\begin{aligned}\mathbf{J}_\alpha &:= E_\alpha\left[\frac{\partial \ln q(\boldsymbol{y},\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}}\frac{\partial \ln q(\boldsymbol{y},\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^T}\right]\\ &= E_\alpha\left[\mathbf{G}^T\frac{\partial \ln p(\boldsymbol{y},\boldsymbol{g}(\boldsymbol{\alpha}))}{\partial \boldsymbol{g}(\boldsymbol{\alpha})}\frac{\partial \ln p(\boldsymbol{y},\boldsymbol{g}(\boldsymbol{\alpha}))}{\partial \boldsymbol{g}^T(\boldsymbol{\alpha})}\mathbf{G}\right]\\ &= \mathbf{G}^T E_\phi\left[\frac{\partial \ln p(\boldsymbol{y},\boldsymbol{\phi})}{\partial \boldsymbol{\phi}}\frac{\partial \ln p(\boldsymbol{y},\boldsymbol{\phi})}{\partial \boldsymbol{\phi}^T}\right]\mathbf{G}\\ &= \mathbf{V}^T\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}\mathbf{V} \end{aligned}\tag{6}$$

where $\mathbf{J}_\phi$ is the unconstrained FIM. If $\mathbf{J}_\alpha$ is nonsingular, then $\mathbf{V}$ must be of full rank, and the reparameterized CRB can be expressed as

$$E\left\{(\hat{\boldsymbol{\alpha}}-\boldsymbol{\alpha})(\hat{\boldsymbol{\alpha}}-\boldsymbol{\alpha})^T\right\} \geq (\mathbf{J}_\alpha)^{-1} = \mathbf{V}^{-1}\left(\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}\right)^{-1}\mathbf{V}^{-T}. \tag{7}$$

Translating back to the original parameter $\boldsymbol{\phi}$ using the CRB expression for transformations [27], we have

$$\begin{aligned}E\left\{(\hat{\boldsymbol{\phi}}-\boldsymbol{\phi})(\hat{\boldsymbol{\phi}}-\boldsymbol{\phi})^T\right\} &\geq \mathbf{G}\mathbf{J}_\alpha^{-1}\mathbf{G}^T\\ &= \mathbf{U}\mathbf{V}\mathbf{V}^{-1}\left(\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}\right)^{-1}\mathbf{V}^{-T}\mathbf{V}^T\mathbf{U}^T\\ &= \mathbf{U}\left(\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}\right)^{-1}\mathbf{U}^T \end{aligned}\tag{8}$$

which coincides with (3).

3) Constrained CRB for BSS: Given M (generally complex) observed vectors $\mathbf{X} = \{\boldsymbol{x}(0),\ldots,\boldsymbol{x}(M-1)\}$, the likelihood function is

$$p(\mathbf{X};\boldsymbol{\phi}) = \prod_{i=0}^{M-1}\frac{1}{(\pi\sigma^2)^N}\exp\left[-\frac{1}{\sigma^2}|\boldsymbol{x}(i)-\mathbf{H}\boldsymbol{b}(i)|^2\right] \tag{9}$$

where

$$\begin{aligned}\boldsymbol{\phi} := \Big[&\bar{\boldsymbol{h}}_0^T,\tilde{\boldsymbol{h}}_0^T,\ldots,\bar{\boldsymbol{h}}_{N-1}^T,\tilde{\boldsymbol{h}}_{N-1}^T,\bar{\boldsymbol{b}}^T(0)\\ &\times\tilde{\boldsymbol{b}}^T(0),\ldots,\bar{\boldsymbol{b}}^T(M-1),\tilde{\boldsymbol{b}}^T(M-1)\Big]^T\end{aligned}$$

is the set of parameters to be estimated (expressed in terms of their real and imaginary parts), and we have denoted the nth row of $\mathbf{H}$ as $\boldsymbol{h}_n^T$. The unconstrained FIM for this problem has been derived by Sadler and Kozick in the context of uncalibrated array processing [21], shown in (10) at the bottom of the page, where

$$\mathcal{A} := \begin{pmatrix}\mathrm{Re}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right] & \mathrm{Im}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right]\\ -\mathrm{Im}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right] & \mathrm{Re}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right]\end{pmatrix} \tag{11}$$

$$\mathcal{D} := \begin{pmatrix}\mathrm{Re}\left[\mathbf{H}^{\mathcal{H}}\mathbf{H}\right] & -\mathrm{Im}\left[\mathbf{H}^{\mathcal{H}}\mathbf{H}\right]\\ \mathrm{Im}\left[\mathbf{H}^{\mathcal{H}}\mathbf{H}\right] & \mathrm{Re}\left[\mathbf{H}^{\mathcal{H}}\mathbf{H}\right]\end{pmatrix} \tag{12}$$

$$\mathcal{M}_{n,i} := \begin{pmatrix}\mathrm{Re}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right] & \mathrm{Im}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right]\\ -\mathrm{Im}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right] & \mathrm{Re}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right]\end{pmatrix} \tag{13}$$

$$\mathbf{B} := \begin{pmatrix} b_1(0) & \cdots & b_1(M-1)\\ \vdots & \ddots & \vdots\\ b_K(0) & \cdots & b_K(M-1)\end{pmatrix} \tag{14}$$

and the empty spaces indicate that the corresponding matrix entries are zero. Note that, due to the difference in the order of the parameters, (10)–(14) are slightly different from those in [21]. The matrix $\mathbf{J}_\phi$ is singular because

$$\sum_{i=0}^{N-1}\mathrm{col}(i+1)\begin{pmatrix}\bar{\boldsymbol{h}}_i\\ \tilde{\boldsymbol{h}}_i\end{pmatrix} - \sum_{i=0}^{M-1}\mathrm{col}(N+i+1)\begin{pmatrix}\bar{\boldsymbol{b}}(i)\\ \tilde{\boldsymbol{b}}(i)\end{pmatrix} = \mathbf{0} \tag{15}$$

where $\mathrm{col}(i)$ is the ith block column of $\mathbf{J}_\phi$ (e.g., $\mathrm{col}(1) = (\mathcal{A}^T,\mathbf{0},\ldots,\mathbf{0},\mathcal{M}_{0,0},\ldots,\mathcal{M}_{0,M-1})^T$). To verify (15), let us examine the $(n+1)$st $(0 \leq n \leq N-1)$ block row of the left-hand side of (15):

$$\begin{aligned}&\mathcal{A}\begin{pmatrix}\bar{\boldsymbol{h}}_n\\ \tilde{\boldsymbol{h}}_n\end{pmatrix} - \sum_{i=0}^{M-1}\mathcal{M}_{n,i}\begin{pmatrix}\bar{\boldsymbol{b}}(i)\\ \tilde{\boldsymbol{b}}(i)\end{pmatrix}\\ &= \begin{pmatrix}\mathrm{Re}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right] & \mathrm{Im}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right]\\ -\mathrm{Im}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right] & \mathrm{Re}\left[\mathbf{B}\mathbf{B}^{\mathcal{H}}\right]\end{pmatrix}\begin{pmatrix}\bar{\boldsymbol{h}}_n\\ \tilde{\boldsymbol{h}}_n\end{pmatrix}\\ &\quad - \sum_{i=0}^{M-1}\begin{pmatrix}\mathrm{Re}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right] & \mathrm{Im}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right]\\ -\mathrm{Im}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right] & \mathrm{Re}\left[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\right]\end{pmatrix}\begin{pmatrix}\bar{\boldsymbol{b}}(i)\\ \tilde{\boldsymbol{b}}(i)\end{pmatrix}\\ &= \begin{pmatrix}\mathrm{Re}\left[\mathbf{B}^*\mathbf{B}^T\boldsymbol{h}_n\right]\\ \mathrm{Im}\left[\mathbf{B}^*\mathbf{B}^T\boldsymbol{h}_n\right]\end{pmatrix} - \sum_{i=0}^{M-1}\begin{pmatrix}\mathrm{Re}\left[\boldsymbol{b}^*(i)\boldsymbol{b}^T(i)\boldsymbol{h}_n\right]\\ \mathrm{Im}\left[\boldsymbol{b}^*(i)\boldsymbol{b}^T(i)\boldsymbol{h}_n\right]\end{pmatrix}\\ &= \mathbf{0}.\end{aligned}\tag{16}$$

$$\mathbf{J}_\phi = \frac{2}{\sigma^2}\left(\begin{array}{ccc|ccc} \mathcal{A} & & & \mathcal{M}_{0,0} & \cdots & \mathcal{M}_{0,M-1}\\ & \ddots & & \cdots & \cdots & \cdots\\ & & \mathcal{A} & \mathcal{M}_{N-1,0} & \cdots & \mathcal{M}_{N-1,M-1}\\ \hline \mathcal{M}_{0,0}^T & \cdots & \mathcal{M}_{N-1,0}^T & \mathcal{D} & & \\ \cdots & \cdots & \cdots & & \ddots & \\ \mathcal{M}_{0,M-1}^T & \cdots & \mathcal{M}_{N-1,M-1}^T & & & \mathcal{D}\end{array}\right) := \left(\begin{array}{c|c}\mathbf{J}_{11} & \mathbf{J}_{12}\\ \hline \mathbf{J}_{12}^T & \mathbf{J}_{22}\end{array}\right) \tag{10}$$

and the case of $N \le n \le N+M-1$ can be verified similarly. Therefore, without additional constraints, no meaningful CRB exists.

Under the assumption that the sources are CM with amplitude 1 and the first T symbols are known, the parameter set $\boldsymbol{\phi}$ should satisfy the following set of constraint equations:

$$f_{2iK+k}(\boldsymbol{\phi}) := \bar{b}_k(i) - \bar{b}_{ki} = 0 \qquad i=0,\ldots,T-1;\ k=1,\ldots,K \tag{17}$$

$$f_{(2i+1)K+k}(\boldsymbol{\phi}) := \tilde{b}_k(i) - \tilde{b}_{ki} = 0 \qquad i=0,\ldots,T-1;\ k=1,\ldots,K \tag{18}$$

$$f_{(T+i)K+k}(\boldsymbol{\phi}) := \bar{b}_k(i)^2 + \tilde{b}_k(i)^2 - 1 = 0 \qquad i=T,\ldots,M-1;\ k=1,\ldots,K. \tag{19}$$

The gradient matrix for these constraints is an $(M+T)K \times (2N+2M)K$ matrix

$$\mathbf{F}(\boldsymbol{\phi}) = \left(\begin{array}{c|cccc} & \mathbf{I}_{2TK} & & & \\ & & \mathbf{F}_T & & \\ \mathbf{0}_{(M+T)K\times 2NK} & & & \ddots & \\ & & & & \mathbf{F}_{M-1} \end{array}\right) \tag{20}$$

where $\mathbf{F}_i := [2\bar{\mathbf{B}}_i, 2\tilde{\mathbf{B}}_i]$, $\bar{\mathbf{B}}_i := \mathrm{diag}(\bar{\boldsymbol{b}}(i))$, $\tilde{\mathbf{B}}_i := \mathrm{diag}(\tilde{\boldsymbol{b}}(i))$, and the empty spaces indicate zero entries. The basis for its null space is

$$\mathbf{U} = \left(\begin{array}{c|cccc} \mathbf{I}_{2NK\times 2NK} & & * & & \\ \hline & \mathbf{0}_{2TK\times(M-T)K} & & & \\ & -\tilde{\mathbf{B}}_T & & & \\ * & \bar{\mathbf{B}}_T & & & \\ & & \ddots & & \\ & & & -\tilde{\mathbf{B}}_{M-1} & \\ & & & \bar{\mathbf{B}}_{M-1} & \end{array}\right) := \left(\begin{array}{c|c} \mathbf{U}_{11} & * \\ \hline * & \mathbf{U}_{22} \end{array}\right) \tag{21}$$

where $*$ stands for all-zero matrices of appropriate dimensions. Using the inversion formula for block matrices, we obtain the CRB in estimating vec$(\mathbf{H}^T)$ as

$$\left[\mathbf{J}_{11} - \mathbf{J}_{12}\mathbf{U}_{22}\left(\mathbf{U}_{22}^T\mathbf{J}_{22}\mathbf{U}_{22}\right)^{-1}\mathbf{U}_{22}^T\mathbf{J}_{12}^T\right]^{-1} := [\mathbf{J}_{11} - \mathcal{K}]^{-1} \tag{22}$$

where $\mathcal{K}$ denotes the block matrix

$$\mathcal{K} = \begin{pmatrix} \mathcal{K}_{0,0} & \cdots & \mathcal{K}_{0,N-1} \\ \cdots & \cdots & \cdots \\ \mathcal{K}_{N-1,0} & \cdots & \mathcal{K}_{N-1,N-1} \end{pmatrix} \tag{23}$$

whose entries can be shown (after some algebra) to be

$$\mathcal{K}_{m,n} = \frac{2}{\sigma^2}\sum_{i=T}^{M-1}\begin{pmatrix} \mathrm{Im}[\boldsymbol{b}(i)\boldsymbol{h}_m^{\mathcal{H}}\mathbf{B}_i^{\mathcal{H}}] \\ \mathrm{Re}[\boldsymbol{b}(i)\boldsymbol{h}_m^{\mathcal{H}}\mathbf{B}_i^{\mathcal{H}}] \end{pmatrix}\left(\mathrm{Re}[\mathbf{B}_i^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{H}\mathbf{B}_i]\right)^{-1} \times \left(\mathrm{Im}[\boldsymbol{b}(i)\vec{h}_n^{\mathcal{H}}\mathbf{B}_i^{\mathcal{H}}]^T \quad \mathrm{Re}[\boldsymbol{b}(i)\boldsymbol{h}_n^{\mathcal{H}}\mathbf{B}_i^{\mathcal{H}}]^T\right). \tag{24}$$

Having expressed in closed form the constrained CRB for our BSS parameter estimation problem, we proceed to use it in linking local identifiability with information regularity.

B. Local Identifiability and Information Regularity

The regularity (invertibility) of the FIM is related to parameter identifiability. To investigate the link between them in the BSS context, we first introduce several relevant definitions [28].

Definition 1: Suppose a family of probability measures $\{F(\cdot,\boldsymbol{\phi}), \boldsymbol{\phi} \in \Phi\}$ on a measurable space $(\Omega, \mathcal{F})$ is dominated by a σ-finite measure μ[1]. Two parameter values $\boldsymbol{\phi}_0$ and $\boldsymbol{\phi}_1$ are said to be observationally equivalent if

$$\frac{dF(\mathbf{x},\boldsymbol{\phi}_0)}{d\mu} = \frac{dF(\mathbf{x},\boldsymbol{\phi}_1)}{d\mu}\text{a.e.}[\mu]. \tag{25}$$

Definition 1 basically means that two parameter values can be seen as equivalent if the probability densities indexed by them are the same except on a set of measure 0.

Definition 2: Suppose the family $\{F(\cdot,\boldsymbol{\phi}), \boldsymbol{\phi} \in \Phi\}$ is dominated by a σ-finite measure μ. A parameter value $\boldsymbol{\phi}_0 \in \Phi$ is said to be locally identifiable if there exists an open neighborhood of $\boldsymbol{\phi}_0$ containing no other $\boldsymbol{\phi} \in \Phi$ that is observationally equivalent to $\boldsymbol{\phi}_0$; it is said to be (globally) identifiable if there exists no other $\boldsymbol{\phi} \in \Phi$ that is observationally equivalent to $\boldsymbol{\phi}_0$.

Due to the permutation ambiguity present in the BSS problem, $[\mathbf{H}, \mathbf{B}]$ is not globally identifiable, in general. In this subsection, we will focus on local identifiability. The link between local identifiability and information regularity is given by the following theorem [29]:

Theorem 1: Assume that i) the unconstrained parameter space Φ is an open set in $\mathbb{R}^n$; ii) the likelihood function p is non-negative, and the equation $\int p(\mathbf{y}|\boldsymbol{\phi})d\mathbf{y} = 1$ holds for all $\boldsymbol{\phi} \in \Phi$; iii) the support of p is the same for all $\boldsymbol{\phi} \in \Phi$; iv) p is smooth in $\boldsymbol{\phi}$; v) the elements of the unconstrained FIM $\mathbf{J}_\phi$ exist and are continuous functions of $\boldsymbol{\phi}$ everywhere in Φ; and vi) $\boldsymbol{\phi}$ satisfies a set of constraint equations $f_i(\boldsymbol{\phi}) = 0, \quad i = 1,\ldots,k$, where each f_i possesses continuous partial derivatives. Denote Φ' as the constrained parameter space. If the matrix

$$\mathbf{V}(\boldsymbol{\phi}) = \begin{bmatrix} \mathbf{J}_\phi \\ \mathbf{F}(\boldsymbol{\phi}) \end{bmatrix} \tag{26}$$

has full rank at $\boldsymbol{\phi} = \boldsymbol{\phi}_0 \in \Phi'$, then $\boldsymbol{\phi}_0$ is locally identifiable. Furthermore, if both $\mathbf{V}(\boldsymbol{\phi})$ and $\mathbf{F}(\boldsymbol{\phi})$ do not change ranks in an open neighborhood of $\boldsymbol{\phi}_0$, then the converse is also true.

In the following, we will show that in our BSS context, information regularity is closely related to local identifiability. To this end, we will need the following lemma.

Lemma 1: If $\mathbf{F}(\boldsymbol{\phi}_0)$ has full rank, then $\mathbf{V}(\boldsymbol{\phi}_0)$ has full rank if and only if $\mathbf{U}^T(\boldsymbol{\phi}_0)\mathbf{J}_{\phi_0}\mathbf{U}(\boldsymbol{\phi}_0)$ has full rank, where $\mathbf{U}(\boldsymbol{\phi}_0)$ is a matrix whose columns form an orthonormal basis of the null space of $\mathbf{F}(\boldsymbol{\phi}_0)$.

Proof: See Appendix A.

Recall that a sufficient and necessary condition for the constrained CRB to exist is $|\mathbf{U}^T\mathbf{J}_\phi\mathbf{U}| \ne 0$. Checking (9)–(13), it is easy to see that conditions i)–vi) of Theorem 1 are satisfied by the BSS problem under CM constraints; therefore, local identifiability is a necessary condition for information regularity. Establishing the sufficiency part is more difficult. We can verify that $\mathbf{F}(\boldsymbol{\phi})$ has constant rank on the constrained parameter space

[1] A measure μ_1 is said to be dominated by μ_0, if $\mu_0(F) = 0$ implies that $\mu_1(F) = 0$, where $F \in \mathcal{F}$.

by checking (17)–(20). While we can also prove that $\mathbf{J}_\phi$ has constant rank using results from [17] and [23], we are not able to establish the constant rank property for $\mathbf{V}(\boldsymbol{\phi})$. Numerical study seems to suggest, however, that local identifiability in general leads to information regularity.

C. Conditions of Local Identifiability

From (9), we can see that a set of parameters $\boldsymbol{\phi}$ is not locally identifiable if and only if given any $\epsilon > 0$, there exists another set of parameters $\boldsymbol{\phi}'$ such that $\boldsymbol{\phi}'$ belongs to the constrained parameter space $\|\boldsymbol{\phi} - \boldsymbol{\phi}'\| < \epsilon$, and

$$\mathbf{HB} = \mathbf{H}'\mathbf{B}'. \tag{27}$$

With this fact, we can obtain the following result:

Theorem 2: Assume that i) the data symbols of different sources are independent; ii) the kth source's symbols $b_k(i)$, $i = 0, \ldots, M-1$ are independent and identically distributed (i.i.d.) random variables drawn from the CM constellation $\mathbb{A}_k$ such that $|b_k(i)| = 1$ and $E[b_k(i)] = 0$; iii) the mixing matrix $\mathbf{H}$ has full column rank; and iv) at most one source has binary antipodal constellation. Then, the true parameter set $[\mathbf{H}, \mathbf{B}]$ is locally identifiable (with probability 1, as $M \to \infty$), if and only if one data symbol from each source is known (i.e., the only local ambiguity is the phase ambiguity).

Proof: Local identifiability of $\boldsymbol{\phi}$ means that there exists an open neighborhood of $\boldsymbol{\phi}$ in which we cannot find $\boldsymbol{\phi}'$ such that (27) holds true. Establishing necessity is easy because of the existence of phase ambiguity. To prove sufficiency, suppose that $[\mathbf{H}, \mathbf{B}]$ is not locally identifiable. Consider $\boldsymbol{\Psi} := [\mathbf{X}, \mathbf{Y}]$ such that $\|\boldsymbol{\Psi}\| = 1$, where $\mathbf{X}$ and $\mathbf{Y}$ are matrices with the same dimensions as $\mathbf{H}$ and $\mathbf{B}$, respectively. Denote $\zeta(\boldsymbol{\Psi}) := \|\mathbf{XB} + \mathbf{HY}\|$. Due to the lack of local identifiability, we can find for any $\epsilon > 0$ a parameter variation $\delta\boldsymbol{\phi}$ such that $\|\delta\boldsymbol{\phi}\| = r < \epsilon$ and

$$\mathbf{HB} = (\mathbf{H} + \delta\mathbf{H})(\mathbf{B} + \delta\mathbf{B}). \tag{28}$$

From (28), we have

$$r^2 \geq \|(\delta\mathbf{H})(\delta\mathbf{B})\| = \|\mathbf{H}(\delta\mathbf{B}) + (\delta\mathbf{H})\mathbf{B}\| = r\zeta\left(\frac{\delta\boldsymbol{\phi}}{\|\delta\boldsymbol{\phi}\|}\right). \tag{29}$$

As r goes to zero, we have that if $\delta\boldsymbol{\phi}$ satisfies (28), then it must satisfy the following equation:

$$\zeta\left(\frac{\delta\boldsymbol{\phi}}{\|\delta\boldsymbol{\phi}\|}\right) = \|(d\mathbf{H})\mathbf{B} + \mathbf{H}(d\mathbf{B})\| = 0 \tag{30}$$

where $d\mathbf{H} := \delta\mathbf{H}/\|\delta\boldsymbol{\phi}\|$, and $d\mathbf{B} := \delta\mathbf{B}/\|\delta\boldsymbol{\phi}\|$. Therefore,

$$(d\mathbf{H})\mathbf{B} + \mathbf{H}(d\mathbf{B}) = \mathbf{0}. \tag{31}$$

Similarly, from the constant modulus constraints on the data symbols, we can show that

$$d|b_k(i)|^2 = \bar{b}_k(i)d\bar{b}_k(i) + \tilde{b}_k(i)d\tilde{b}_k(i) = 0. \tag{32}$$

Since $\mathbf{H}$ and $\mathbf{B}$ have full rank, we have from (31)

$$d\mathbf{H} = -\mathbf{H}(d\mathbf{B})\mathbf{B}^T\left(\mathbf{BB}^T\right)^{-1} := \mathbf{H}\mathbf{\Pi}_{K\times K} \tag{33}$$

and

$$d\mathbf{B} = -\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T(d\mathbf{H})\mathbf{B} := \Xi_{K\times K}\mathbf{B}. \tag{34}$$

Substituting (33) and (34) into (31), we obtain

$$\mathbf{H}(\mathbf{\Pi} + \Xi)\mathbf{B} = \mathbf{0} \tag{35}$$

which leads to

$$\mathbf{\Pi} = -\Xi \tag{36}$$

because $\mathbf{H}$ and $\mathbf{B}$ have full rank. Therefore, (31) has a nontrivial solution only if we can find a matrix Ξ such that $d\mathbf{B} = \Xi\mathbf{B} \neq \mathbf{0}$, and the elements of $d\mathbf{B}$ satisfy (32). Writing (34) in the following form:

$$\begin{pmatrix} d\bar{\mathbf{B}} & d\tilde{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} \bar{\Xi} & \tilde{\Xi} \end{pmatrix}\begin{pmatrix} \bar{\mathbf{B}} & \tilde{\mathbf{B}} \\ -\tilde{\mathbf{B}} & \bar{\mathbf{B}} \end{pmatrix} \tag{37}$$

we can see that

$$\begin{pmatrix} d\bar{b}_1(m) \\ d\tilde{b}_1(m) \end{pmatrix} = \begin{pmatrix} \bar{b}_1(m) & \cdots & \bar{b}_K(m) & -\tilde{b}_1(m) & \cdots & -\tilde{b}_K(m) \\ \tilde{b}_1(m) & \cdots & \tilde{b}_K(m) & \bar{b}_1(m) & \cdots & \bar{b}_K(m) \end{pmatrix}\boldsymbol{\xi}_1 \tag{38}$$

where $\boldsymbol{\xi}_1^T$ is the first row of $[\bar{\Xi}, \tilde{\Xi}]$. Left multiplying (38) by $[\bar{b}_1(m)\ \tilde{b}_1(m)]$, and considering all m, we obtain (39), shown at the bottom of the page, where $\rho_{1k}(m) := b_1(m)b_k^*(m)$. Permuting the columns of (39), we obtain (40), shown at the bottom of the next page. Defining

$$\mathbf{B}_{mk} := \begin{pmatrix} -\tilde{b}_k(m) & \bar{b}_k(m) \\ \bar{b}_k(m) & \tilde{b}_k(m) \end{pmatrix} \tag{41}$$

$$\mathbf{0} = \begin{bmatrix} 1 & \bar{\rho}_{12}(0) & \cdots & \bar{\rho}_{1K}(0) & 0 & \tilde{\rho}_{12}(0) & \tilde{\rho}_{1K}(0) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \bar{\rho}_{12}(M-1) & \cdots & \bar{\rho}_{1K}(M-1) & 0 & \tilde{\rho}_{12}(M-1) & \tilde{\rho}_{1K}(M-1) \end{bmatrix}\boldsymbol{\xi}_1 := \mathbf{\Lambda}\boldsymbol{\xi}_1 \tag{39}$$

from (40), we can see that $\left(\mathbf{\Lambda}'\right)^T \mathbf{\Lambda}'/M$ is a block matrix, with (i,j)th block

$$\frac{1}{M}\sum_{m=0}^{M-1}\mathbf{B}_{mi}^T \begin{bmatrix}\bar{b}_1(m)\\ \tilde{b}_1(m)\end{bmatrix}\begin{bmatrix}\bar{b}_1(m) & \tilde{b}_1(m)\end{bmatrix}\mathbf{B}_{mj} := \mathbf{\Sigma}_{ij}. \quad (42)$$

We can verify that if $i=j=1$, then $\mathbf{\Sigma}_{ij} = \mathrm{diag}(0,1)$, whereas if $i \neq j$, then $\mathbf{\Sigma}_{ij}$ converges to the zero matrix as $M \to \infty$; finally, if $i = j \neq 1$, then as M goes to infinity, $\mathbf{\Sigma}_{ij}$ converges to

$$\mathbf{\Sigma}_{ii}^0 = \begin{pmatrix}\mathbf{\Sigma}_{ii}^0(1,1) & \mathbf{\Sigma}_{ii}^0(1,2)\\ \mathbf{\Sigma}_{ii}^0(2,1) & \mathbf{\Sigma}_{ii}^0(2,2)\end{pmatrix} \quad (43)$$

where $\mathbf{\Sigma}_{ii}^0(1,1) = E[\bar{b}_1^2\tilde{b}_i^2 + \tilde{b}_1^2\bar{b}_i^2 - 2\bar{b}_1\tilde{b}_1\bar{b}_i\tilde{b}_i]$, $\mathbf{\Sigma}_{ii}^0(2,2) = E[\bar{b}_1^2\bar{b}_i^2 + \tilde{b}_1^2\tilde{b}_i^2 + 2\bar{b}_1\tilde{b}_1\bar{b}_i\tilde{b}_i]$, and $\mathbf{\Sigma}_{ii}^0(1,2) = \mathbf{\Sigma}_{ii}^0(2,1) = E[(-\bar{b}_1^2+\tilde{b}_1^2)\bar{b}_i\tilde{b}_i + (\bar{b}_i^2 - \tilde{b}_i^2)\bar{b}_1\tilde{b}_1]$. The determinant of $\mathbf{\Sigma}_{ii}^0$ can then be shown to be

$$\begin{aligned}|\mathbf{\Sigma}_{ii}^0| = &\left[\left(E[\bar{b}_1^2]\right)^2 + \left(E[\tilde{b}_1^2]\right)^2 + 2\left(E[\bar{b}_1\tilde{b}_1]\right)^2\right]\\ &\times\left[E[\bar{b}_i]^2E[\tilde{b}_i]^2] - \left(E[\bar{b}_i\tilde{b}_i]\right)^2\right]\\ &+\left[\left(E[\bar{b}_i^2]\right)^2 + \left(E[\tilde{b}_i^2]\right)^2 + 2\left(E[\bar{b}_i\tilde{b}_i]\right)^2\right]\\ &\times\left[E[\bar{b}_1]^2E[\tilde{b}_1^2] - \left(E[\bar{b}_1\tilde{b}_1]\right)^2\right] \end{aligned}\quad (44)$$

which is equal to 0 if and only if $\tilde{b}_1 = a_1\bar{b}_1$ and $\tilde{b}_i = a_i\bar{b}_i$ with probability 1 for some constants a_1 and a_i. Due to the CM constraint, the condition $\tilde{b}_1 = a_1\bar{b}_1$ leads to $\bar{b}_1 = \pm 1/\sqrt{a_1^2+1}$, meaning that b_1 is binary and antipodal. The same arguments hold for b_i. Therefore, for $\mathbf{\Sigma}_{ii}^0$ to be singular, both b_1 and b_i must be binary and antipodal. Since we have assumed that at most one user has binary and antipodal modulation (condition iv), $\mathbf{\Sigma}_{ii}^0$ is nonsingular, and hence, $\mathbf{\Lambda}$ has rank $2K-1$. We can see that for $\boldsymbol{\xi}_1$ satisfying (39), only the $(K+1)$st entry of $\boldsymbol{\xi}_1$ can be nonzero, i.e., $\boldsymbol{\xi}_1 = [\mathbf{0}_K\ c_1\ \mathbf{0}_{K-1}]^T$, for some c_1. Applying similar arguments to the other columns of $\mathbf{\Xi}$, we obtain that

$$\bar{\mathbf{\Xi}} = \mathbf{0} \quad (45)$$

and

$$\tilde{\mathbf{\Xi}} = \mathrm{diag}\{c_1,\ldots,c_K\}. \quad (46)$$

Combining with (33) and (36), we have

$$d\boldsymbol{h}_k = -c_k j\boldsymbol{h}_k. \quad (47)$$

Therefore, the admissible local variation $\|\delta\boldsymbol{\phi}\| = r < \epsilon$ should satisfy $\delta\boldsymbol{h}_k \cong -rc_k j\boldsymbol{h}_k$, which leads to

$$\boldsymbol{h}_k + \delta\boldsymbol{h}_k \cong \boldsymbol{h}_k(1 - jrc_k). \quad (48)$$

Denoting the angle of $\delta\boldsymbol{h}_k$ as $\angle\delta\boldsymbol{h}_k$, we have

$$\lim_{r\to 0}\frac{\|\boldsymbol{h}_k + \delta\boldsymbol{h}_k\| - \|\boldsymbol{h}_k\|}{\angle\delta\boldsymbol{h}_k} = \lim_{r\to 0} -\frac{1}{2}rc_k\|\boldsymbol{h}_k\| = 0. \quad (49)$$

Therefore, the only local ambiguity is the phase ambiguity, which can be eliminated by assuming that we know $[b_1(0),\ldots,b_K(0)]$. ■

Next, we examine the case where all sources have binary antipodal constellations. We first prove the following lemma.

Lemma 2: Assume that $\mathbf{B}_K$ is a $K \times 2^K$ matrix with entries $b_{ij} \in \{1,-1\}$ and that no two columns of $\mathbf{B}_K$ are identical. In addition, assume that $\mathbf{y}$ is a $2^K \times 1$ vector whose ith element is of the form $b_{ki}(e^{j\theta_i} - 1)$, where $\theta_i \in (-\pi/2, \pi/2)$. Then, the solution $\mathbf{x} := [x_1,\ldots,x_K]^T$ to the following equation:

$$\mathbf{x}^T\mathbf{B}_K = \mathbf{y}^T \quad (50)$$

has at most one nonzero element besides x_k.

Proof: See Appendix B.

With Lemma 2, we can now derive the condition for local identifiability of systems with binary antipodal source symbols.

Theorem 3: Assuming that all sources use binary antipodal constellations and that data symbols from different sources and in different time slots are independent from each other, the first T symbols of all sources $[b_1(m),\ldots,b_K(m)]$, $m = 0,\ldots,T-1$ are known, and $\mathbf{H}$ has full column rank. Then, the true parameter set $[\mathbf{H},\mathbf{B}]$ is locally identifiable (with probability 1, as $M \to \infty$) if and only if no two rows of the matrix $\mathbf{B}_T$ are dependent, where

$$\mathbf{B}_T = \begin{pmatrix} b_1(0) & \cdots & b_1(T-1)\\ \vdots & \ddots & \vdots\\ b_K(0) & \cdots & b_K(T-1)\end{pmatrix}. \quad (51)$$

Proof: If all sources adopt the same binary antipodal constellation, we can assume without loss of generality that $b_k(m) \in \{-1,+1\}$. This problem is locally identifiable if and only if there exists $\epsilon > 0$ such that there is no $\delta\boldsymbol{\phi} \neq \mathbf{0}$ satisfying $\|\delta\boldsymbol{\phi}\| < \epsilon$

$$\mathbf{HB} = (\mathbf{H} + \delta\mathbf{H})(\mathbf{B} + \delta\mathbf{B}) \quad (52)$$

$$\mathbf{0} = \begin{bmatrix} 0 & 1 & \tilde{\rho}_{12}(0) & \bar{\rho}_{12}(0) & \cdots & \tilde{\rho}_{1K}(0) & \bar{\rho}_{1K}(0)\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 1 & \tilde{\rho}_{12}(M-1) & \bar{\rho}_{12}(M-1) & \cdots & \tilde{\rho}_{1K}(M-1) & \bar{\rho}_{1K}(M-1)\end{bmatrix}\boldsymbol{\xi}_1' := \mathbf{\Lambda}'\boldsymbol{\xi}_1'. \quad (40)$$

all elements of $\mathbf{B}+\delta\mathbf{B}$ have magnitude 1, and $\delta\mathbf{B}_T=\mathbf{0}$, where

$$\delta\mathbf{B}_T := \begin{pmatrix} \delta b_1(0) & \cdots & \delta b_1(T-1) \\ \vdots & \ddots & \vdots \\ \delta b_K(0) & \cdots & \delta b_K(T-1) \end{pmatrix}. \quad (53)$$

Therefore, a lack of local identifiability is equivalent to the existence of nonzero $\delta\mathbf{H}$ and $\delta\mathbf{B}$, satisfying

$$(\mathbf{H}+\delta\mathbf{H})(\delta\mathbf{B})+(\delta\mathbf{H})\mathbf{B}=\mathbf{0} \quad (54)$$

$$\delta\mathbf{B}_T=\mathbf{0} \quad (55)$$

and the CM constraint for the symbols. Since $\mathbf{H}$ has full column rank, for ϵ small enough, $\mathbf{H}+\delta\mathbf{H}$ also has full column rank. From (54), we deduce that there exists Ξ such that

$$\delta\mathbf{B}=\Xi\mathbf{B}. \quad (56)$$

Substituting (56) into (54), we obtain

$$\delta\mathbf{H}(\mathbf{I}+\Xi)=-\mathbf{H}\Xi. \quad (57)$$

Therefore, the kth row $\boldsymbol{\xi}_k^T$ of Ξ should satisfy

$$\delta\boldsymbol{b}_k^T=\boldsymbol{\xi}_k^T\mathbf{B} \quad (58)$$

where the ith entry of $\delta\boldsymbol{b}_k$ is $\delta b_k(i)=b_k(i)(e^{j\theta_k(i)}-1)$ for some $\theta_k(i)\in(-\pi/2,\pi/2)$. As $M\to\infty$, the columns of $\mathbf{B}$ contain all possible values of $[b_1,\ldots,b_K]^T$ with probability 1. From Lemma 2, at most one element of $\boldsymbol{\xi}_k$ is nonzero besides ξ_{kk}. If no two rows of $\mathbf{B}_T$ are dependent, then $\boldsymbol{\xi}_k^T\mathbf{B}_T=\mathbf{0}^T$ dictates that $\boldsymbol{\xi}_k=\mathbf{0}$. Since this holds for all k, the local identifiability is established. On the other hand, suppose two rows of $\mathbf{B}_T$ are dependent. Without loss of generality, we assume that the first two rows $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ are dependent. We have either $\boldsymbol{b}_1=\boldsymbol{b}_2$ or $\boldsymbol{b}_1=-\boldsymbol{b}_2$. If $\boldsymbol{b}_1=\boldsymbol{b}_2$, then we can verify that letting

$$\Xi=\begin{pmatrix} \frac{1}{2}(e^{j\theta}-1) & -\frac{1}{2}(e^{j\theta}-1) & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad (59)$$

where $|\theta|\neq 0$ is small, we can obtain nonzero $\delta\mathbf{H}$ and $\delta\mathbf{B}$ that satisfy all the constraints. Similar arguments hold when $\boldsymbol{b}_1=-\boldsymbol{b}_2$. ■

Remark: If no two rows of $\mathbf{B}_T$ are dependent, then the minimum value of T is $T_{\min}=\lceil\log_2 K\rceil+1$. To see this, first note that we can assume without loss of generality that $b_1(0)=\cdots=b_K(0)=1$. If $T\geq T_{\min}$, then we can let each of $\boldsymbol{b}'_k:=[b_k(1),\ldots,b_k(T-1)]^T$ $(k=1,\ldots,K)$ take one distinct value among all 2^{T-1} $(T-1)$-vectors with ± 1 entries. Apparently, no two rows of the resulting matrix are dependent. If, on the other hand, $T<T_{\min}$, then there exist k_1 and k_2 such that $\boldsymbol{b}'_{k_1}=\boldsymbol{b}'_{k_2}$ and row k_1 and row k_2 are dependent.

IV. Unique Identifiability

A definition of identifiability that is particularly relevant to the BSS problem with CM constraints is the global (unique) identifiability that is defined as follows [30].

Definition 3: The BSS problem under CM constraints is uniquely identifiable if any set of parameters $\boldsymbol{\phi}'$ that is observationally equivalent to the true parameter set $\boldsymbol{\phi}$ satisfies

$$\mathbf{H}'=\mathbf{H}\mathbf{T}^{-1} \quad (60)$$

and

$$\mathbf{B}'=\mathbf{T}\mathbf{B} \quad (61)$$

where $\mathbf{T}$ is an admissible transformation, i.e., $\mathbf{T}=\mathrm{diag}(\alpha_1,\ldots,\alpha_K)\mathbf{P}$, in which $\mathbf{P}$ is a permutation matrix, and $|\alpha_k|=1,\forall k=1,\ldots,K$.

In [30], it is shown that for CM signals, if the number of observed samples is large enough and if the signals are rich in phase, then these signals are uniquely identifiable. More recently, necessary and sufficient conditions for unique identifiability were obtained in [31] based on Kruskal's permutation lemma. Both the phase richness condition of [30], as well as the one in [31], are difficult to verify. In [32], persistently exciting sources were shown to guarantee unique identifiability, and a lower bound of the finite sample identifiability was given for i.i.d. circularly symmetric CM sources. In this section, we will show that sources with finite CM constellations are uniquely identifiable as long as all except one source constellation have more than two elements.

Theorem 4: Suppose the data symbols from the kth CM source belong to the finite alphabet set $\mathbb{A}_k$, and the probability assigned to each vector of $\mathbb{A}_1\times\cdots\times\mathbb{A}_K$ is nonzero. When the number of observed samples is large enough, the transmitted data and the mixing matrix can be uniquely identified if and only if the mixing matrix $\mathbf{H}$ has full column rank, and at most one alphabet set is binary and antipodal.

Proof: Assume that $\mathbf{B}'=\mathbf{T}\mathbf{B}$ and all entries of $\mathbf{B}$ and $\mathbf{B}'$ have amplitude 1. Let us examine the ith row of $\mathbf{T}$, which we denote as $\boldsymbol{t}_i:=[t_{i1},\ldots,t_{iK}]^T$. Apparently, $\boldsymbol{t}_i\neq\mathbf{0}$. Assume without loss of generality that $t_{i1}\neq 0$. Since the alphabet set is finite, given enough samples, we can pick four columns of $\mathbf{B}$ that have the following form:

$$(\,\boldsymbol{b}(0)\quad\boldsymbol{b}(1)\quad\boldsymbol{b}(2)\quad\boldsymbol{b}(3)\,):=\begin{pmatrix} 1 & e^{j\theta} & 1 & e^{j\theta} \\ 1 & 1 & e^{j\psi} & e^{j\psi} \\ b_3 & b_3 & b_3 & b_3 \\ \cdots & \cdots & \cdots & \cdots \\ b_K & b_K & b_K & b_K \end{pmatrix} \quad (62)$$

where $\theta,\psi\in(0,2\pi)$. Since all entries of $\mathbf{B}$ and $\mathbf{B}'$ have amplitude 1, $\boldsymbol{t}_i$ must satisfy

$$\boldsymbol{t}_i^T\boldsymbol{b}(0)=t_{i1}+t_{i2}+\sum_{k=3}^{K}t_{ik}b_k=e^{j\phi_0} \quad (63)$$

$$\boldsymbol{t}_i^T\boldsymbol{b}(1)=t_{i1}e^{j\theta}+t_{i2}+\sum_{k=3}^{K}t_{ik}b_k=e^{j\phi_1} \quad (64)$$

$$\boldsymbol{t}_i^T\boldsymbol{b}(2)=t_{i1}+t_{i2}e^{j\psi}+\sum_{k=3}^{K}t_{ik}b_k=e^{j\phi_2} \quad (65)$$

$$\boldsymbol{t}_i^T\boldsymbol{b}(3)=t_{i1}e^{j\theta}+t_{i2}e^{j\psi}+\sum_{k=3}^{K}t_{ik}b_k=e^{j\phi_3} \quad (66)$$

for some $\phi_n \in [0, 2\pi)$, $n = 0$, 1, 2, and 3. Therefore

$$e^{j\phi_1} - e^{j\phi_0} = e^{j\phi_3} - e^{j\phi_2} \tag{67}$$

which is equivalent to

$$e^{j((\phi_1+\phi_0)/2)} \sin\frac{\phi_1-\phi_0}{2} = e^{j((\phi_3+\phi_2)/2)} \sin\frac{\phi_3-\phi_2}{2}. \tag{68}$$

Therefore, we have either

$$\sin\frac{\phi_1-\phi_0}{2} = \sin\frac{\phi_3-\phi_2}{2} = 0 \tag{69}$$

or

$$\frac{\sin\frac{\phi_1-\phi_0}{2}}{\sin\frac{\phi_3-\phi_2}{2}} = \frac{e^{j((\phi_1+\phi_0)/2)}}{e^{j((\phi_3+\phi_2)/2)}} = \pm 1. \tag{70}$$

If (69) holds, then $\phi_1 - \phi_0 = \phi_2 - \phi_3 = 0$. From (63) and (64), it is straightforward to see that $t_{i1} = 0$, which contradicts our assumption. Therefore, (70) holds. If

$$\sin\frac{\phi_1-\phi_0}{2} = \sin\frac{\phi_3-\phi_2}{2} \neq 0 \tag{71}$$

then

$$e^{j((\phi_1+\phi_0)/2)} = e^{j((\phi_3+\phi_2)/2)}. \tag{72}$$

From (71) and (72), we deduce that

$$\phi_1 + \phi_0 = \phi_2 + \phi_3 \tag{73}$$

and

$$\phi_1 - \phi_0 = \pm 2\pi - (\phi_3 - \phi_2) \tag{74}$$

which in turn leads to

$$\phi_1 - \phi_2 = \phi_3 - \phi_0 = \pm\pi. \tag{75}$$

Combining (64), (65) and (75), we have that

$$t_{i1}(1 + e^{j\theta}) + t_{i2}(1 + e^{j\psi}) + 2\sum_{k=3}^{K} t_{ik} b_k = 0. \tag{76}$$

Since (76) holds for all possible values of $\{b_3, \ldots, b_K\}$, we have $t_{ik} = 0$, $k = 3, \ldots, K$. If at most one source has binary antipodal constellation, then $e^{j\psi} \neq -1$; otherwise, (76) does not hold. From (76), we have

$$t_{i2} = -\frac{1 + e^{j\theta}}{1 + e^{j\psi}} t_{i1}. \tag{77}$$

Substituting (77) into (63) and (64), after some algebra, we can obtain

$$e^{j(\phi_0-\phi_1)} = \frac{e^{j\psi} - e^{j\theta}}{e^{j(\theta+\psi)} - 1} = \frac{\sin\frac{\psi-\theta}{2}}{\sin\frac{\psi+\theta}{2}}. \tag{78}$$

Therefore, $\phi_0 - \phi_1 = m\pi$ for some integer m. If m is odd, from (75), we deduce that $e^{j\phi_0} = e^{j\phi_2}$. Combining with (63) and (65), we have $t_{i2} = 0$. If m is even, from (63), (64), and the assumption that $t_{i1} \neq 0$, we have $e^{j\theta} = 1$, which also leads to $t_{i2} = 0$ because of (77). For the case of

$$\sin\frac{\phi_1-\phi_0}{2} = -\sin\frac{\phi_3-\phi_2}{2} \neq 0 \tag{79}$$

we can obtain identical results using similar arguments. Therefore, only one element of $\boldsymbol{t}_i$ is nonzero, and this nonzero element has amplitude 1. Since i is arbitrary, any row of $\mathbf{T}$ has at most one nonzero entry. If any column of $\mathbf{T}$ has more than one nonzero entry, then at least one column of $\mathbf{T}$ has all zero entries. Supposing that column k is all zero, then since $\mathbf{H}$ has full column rank, we have

$$\mathbf{B} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{H}'\mathbf{T}\mathbf{B}. \tag{80}$$

If the kth row of $\mathbf{B}$ is changed, the left-hand side of (80) will change, whereas the right-hand side will stay the same, which is clearly impossible. Therefore, any column of $\mathbf{T}$ has exactly one nonzero entry, and hence, it must be an admissible transformation if at most one source constellation is binary and antipodal.

If, on the other hand, at least two source constellations are binary and antipodal, we can assume without loss of generality that $\mathbb{A}_1 = \mathbb{A}_2 = \{1, -1\}$. It is then straightforward to see that we can find θ, ψ such that

$$\mathbf{T}_0 := \begin{pmatrix} \mathbf{T}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{K-2} \end{pmatrix} \tag{81}$$

is not admissible, where

$$\mathbf{T}_2 := \frac{1}{2}\begin{pmatrix} 1 + e^{j\theta} & 1 + e^{j\psi} \\ 1 - e^{j\theta} & 1 - e^{j\psi} \end{pmatrix}. \tag{82}$$

Since all the entries of $\mathbf{T}_0\mathbf{B}$ have magnitude 1, this problem is not uniquely identifiable. ■

V. Conclusions

In this paper, we investigated the information regularity and unique identifiability of the BSS problem under CM constraints. We derived the sufficient and necessary conditions for local identifiability, which is closely related to information regularity. We also studied the unique identifiability of blindly separating multiple CM sources with finite alphabet. Sufficient and necessary conditions under which this problem has a unique (up to phase rotations and source permutation) solution were obtained.[2]

Appendix A
Proof of Lemma 1

We will first prove necessity. For notational brevity, we will omit the parameter $\boldsymbol{\phi}_0$ here. If $\mathbf{V}$ has full rank, then $r(\mathbf{V}) = n$ and $r(\mathbf{F}) = k$. Using the following inequality for the rank of matrix products [33]:

$$r(\mathbf{VU}) \geq r(\mathbf{V}) + r(\mathbf{U}) - n \tag{83}$$

[2]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

we have that $r(\mathbf{VU}) = n - k$, which implies that $\mathbf{VU}$ has full column rank. Since

$$\begin{pmatrix} \mathbf{J}_\phi \\ \mathbf{F} \end{pmatrix} \mathbf{U} = \begin{pmatrix} \mathbf{J}_\phi \mathbf{U} \\ \mathbf{0} \end{pmatrix} \tag{84}$$

$\mathbf{J}_\phi \mathbf{U}$ has full column rank, and there exists no $\boldsymbol{x} \neq \mathbf{0}$ such that $\mathbf{U}\boldsymbol{x}$ is in the null space of $\mathbf{J}_\phi$. Since $\mathbf{J}_\phi$ is a correlation matrix, and thus positive semidefinite, there exists no $\boldsymbol{x} \neq \mathbf{0}$ such that $\boldsymbol{x}^T \mathbf{U}^T \mathbf{J}_\phi \mathbf{U} \boldsymbol{x} = 0$, which means that $\mathbf{U}^T \mathbf{J}_\phi \mathbf{U}$ is of full rank.

To establish sufficiency, assume that there exists $\boldsymbol{y} \neq \mathbf{0}$ that satisfies

$$\mathbf{V}\boldsymbol{y} = \begin{pmatrix} \mathbf{J}_\phi \\ \mathbf{F} \end{pmatrix} \boldsymbol{y} = \mathbf{0}. \tag{85}$$

Apparently, $\boldsymbol{y}$ is in the null space of $\mathbf{F}$ and can be written as $\boldsymbol{y} = \mathbf{U}\boldsymbol{x}$ for some vector $\boldsymbol{x} \neq \mathbf{0}$. Hence, we have $\mathbf{J}_\phi \mathbf{U} \boldsymbol{x} = \mathbf{0}$, and $\mathbf{U}^T \mathbf{J}_\phi \mathbf{U}$ is rank deficient.

APPENDIX B
PROOF OF LEMMA 2

For clarity, consider the special case where $K = 3$ and $k = 1$. From (50), the solution $\boldsymbol{x} = [x_1, x_2, x_3]^T$ should satisfy

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} e^{j\theta_1} - 1 \\ e^{j\theta_2} - 1 \\ e^{j\theta_3} - 1 \\ e^{j\theta_4} - 1 \end{pmatrix}. \tag{86}$$

It is easy to see that

$$2x_2 = e^{j\theta_1} - e^{j\theta_3} = e^{j\theta_2} - e^{j\theta_4} \tag{87}$$

and

$$2x_3 = e^{j\theta_1} - e^{j\theta_2} = e^{j\theta_3} - e^{j\theta_4}. \tag{88}$$

Since

$$e^{j\theta_m} - e^{j\theta_n} = 2je^{j((\theta_m+\theta_n)/2)} \sin \frac{\theta_m - \theta_n}{2} \tag{89}$$

and $|\theta_n| < \pi/2$ ($n = 1, \ldots, 4$), from (88), we have that either

$$\begin{cases} \theta_1 - \theta_2 = \theta_3 - \theta_4 \\ \theta_1 + \theta_2 = \theta_3 + \theta_4 \end{cases} \tag{90}$$

which leads to $\theta_1 = \theta_3$ and $x_2 = 0$, or

$$\theta_1 - \theta_2 = \theta_3 - \theta_4 = 0 \tag{91}$$

which leads to $x_3 = 0$. It is straightforward to generalize this procedure to $K > 3$ sources.

ACKNOWLEDGMENT

The authors would like to thank Prof. V. Poor for helpful discussions.

REFERENCES

[1] S. Verdú, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.
[2] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multielement antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–49, 1996.
[3] J. F. Cardoso, "Blind signal separation: Statistical principles," *Proc. IEEE*, vol. 86, no. 10, pp. 2009–2025, Oct. 1998.
[4] P. Comon, "Independent component analysis, a new concept?," *Signal Process.*, vol. 36, no. 3, pp. 287–314, Apr. 1994.
[5] J. J. Shynk and R. P. Gooch, "The constant modulus array for cochannel signal copy and direction finding," *IEEE Trans. Signal Process.*, vol. 44, no. 3, pp. 652–660, Mar. 1996.
[6] A. J. van der Veen and A. Paulraj, "An analytical constant modulus algorithm," *IEEE Trans. Signal Process.*, vol. 44, no. 5, pp. 1136–1155, May 1996.
[7] S. Talwar, M. Viberg, and A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array—Part I: Algorithms," *IEEE Trans. Signal Process.*, vol. 44, no. 5, pp. 1184–1197, May 1996.
[8] Y. Yao and H. V. Poor, "Eavesdropping in the synchronous CDMA channel: An EM-based approach," *IEEE Trans. Signal Process.*, vol. 49, no. 8, pp. 1748–1756, Aug. 2001.
[9] Y. Sato, "A method of self-recovering equalization for multilevel amplitude-modulation systems," *IEEE Trans. Commun.*, vol. COM-23, pp. 679–682, Jun. 1975.
[10] D. N. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication systems," *IEEE Trans. Commun.*, vol. COM-28, pp. 1867–1875, Nov. 1980.
[11] J. R. Treichler and B. G. Agee, "A new approach to multipath correction of constant modulus signals," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-31, pp. 459–471, Apr. 1983.
[12] J. R. Treichler and M. G. Larimore, "New processing techniques based on constant modulus adaptive algorithm," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-33, pp. 420–431, Apr. 1985.
[13] C. B. Papadias and A. J. Paulraj, "A constant modulus algorithm for multiuser signal separation in presence of delay spread using antenna arrays," *IEEE Signal Process. Lett.*, vol. 4, no. 6, pp. 178–181, Jun. 1997.
[14] R. Johnson, P. Schniter, T. J. Endres, J. D. Behm, D. R. Brown, and R. A. Casas, "Blind equalization using the constant modulus criterion: A review," *Proc. IEEE*, vol. 86, no. 10, pp. 1927–1950, Oct. 1998.
[15] K. Abed-Meraim and Y. Hua, "Strict identifiability of multichannel FIR systems: Further results and developments," in *Proc. Intl. Conf. Telecomm.*, Melbourne, Australia, Apr. 1997, pp. 1029–1032.
[16] T. J. Moore, B. M. Sadler, and R. J. Kozick, "Regularity and strict identifiability in MIMO systems," *IEEE Trans. Signal Process.*, vol. 50, no. 8, pp. 1831–1842, Aug. 2002.
[17] T. J. Moore and B. M. Sadler, "Sufficient conditions for regularity and strict identifiability in MIMO systems," *IEEE Trans. Signal Process.*, vol. 52, no. 9, pp. 2650–2655, Sep. 2004.
[18] P. Stoica and B. C. Ng, "On the Cramér-Rao bound under parametric constraints," *IEEE Signal Process. Lett.*, vol. 5, no. 6, pp. 177–179, Jul. 1998.
[19] J. D. Gorman and A. O. Hero, "Lower bounds for parametric estimation with constraints," *IEEE Trans. Inf. Theory*, vol. 26, pp. 1285–1301, Nov. 1990.
[20] T. L. Marzetta, "A simple derivation of the constrained multiple parameter Cramer-Rao bound," *IEEE Trans. Signal Process.*, vol. 41, no. 6, pp. 2247–2249, Jun. 1993.
[21] B. M. Sadler and R. J. Kozick, "Bounds on uncalibrated array signal processing," in *Proc. 10th IEEE Workshop Stat. Signal Array Process.*, Pocono Manor, PA, Aug. 2000, pp. 73–77.
[22] B. M. Sadler, R. J. Kozick, and T. Moore, "Bounds on bearing and symbol estimation with side information," *IEEE Trans. Signal Process.*, vol. 49, no. 4, pp. 822–834, Apr. 2001.
[23] B. M. Sadler, R. J. Kozick, T. Moore, and A. Swami, "Bounds on SIMO and MIMO channel estimation and equalization with side information," *J. VLSI Signal Process.*, vol. 30, pp. 107–126, Jan. 2002.
[24] B. M. Sadler, R. J. Kozick, and T. J. Moore, "On the performance of source separation with constant modulus signals," in *Proc. ICASSP*, Orlando, FL, May 2002, pp. 2373–2376.
[25] A. N. D'Andrea, U. Mengali, and R. Reggiannini, "The modified Cramér-Rao bound and its application to synchronization problems," *IEEE Trans. Commun.*, vol. 42, pp. 1391–1399, Feb./Mar./Apr. 1994.
[26] P. W. Michor. Topics in Differential Geometry: Lecture Notes of a Course in Vienna. [Online]. Available: http://www.mat.univie.ac.at/~michor
[27] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.
[28] B. L. S. P. Rao, *Identifiability in Stochastic Models*. San Diego, CA: Academic, 1992.

[29] T. J. Rothenberg, "Identification in parametric models," *Econometrica*, vol. 39, no. 3, pp. 577–591, May 1971.
[30] Y. Wang, Y. C. Pati, Y. M. Cho, A. Paulraj, and T. Kailath, "A matrix factorization approach to signal copy of constant modulus signals arriving at an antenna array," in *Proc. 28th Conf. Inf. Sci. Syst.*, Princeton, NJ, Mar. 1994, pp. 178–183.
[31] T. Jiang and N. D. Sidiropoulos, "Kruskal's permutation lemma and the identification of CANDECOMP/PARAFAC and bilinear models with constant modulus constraints," *IEEE Trans. Signal Processing*, vol. 52, no. 9, pp. 2625–2636, Sep. 2004.
[32] A. Leshem, N. Petrochilos, and A. J. van der Veen, "Finite sample identifiability of multiple constant modulus sources," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2314–2319, Sept. 2003.
[33] W. J. Rugh, *Linear System Theory*, Second ed. Upper Saddle River, NJ: Prentice-Hall, 1996.

Yingwei Yao (S'98–M'03) received the Ph.D. degree in electrical engineering from Princeton University, Princeton, NJ, in 2002.

He was a Postdoctoral Researcher with the University of Minnesota, Minneapolis, from 2002 to 2004. Currently, he is an Assistant Professor of electrical and computer engineering with the University of Illinois at Chicago. His research interests include statistical signal processing, wireless communication theory, and wireless networks.

Georgios B. Giannakis (F'97) received the Diploma in electrical engineering from the National Technical University of Athens, Athens, Greece, in 1981 and the M.Sc. degree in electrical engineering in 1983, the MSc. degree in mathematics in 1986, and the Ph.D. degree in electrical engineering in 1986, all from the University of Southern California (USC), Los Angeles.

After lecturing for one year at USC, he joined the University of Virginia, Charlottesville, in 1987, where he became a Professor of electrical engineering in 1997. Since 1999, he has been a professor with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, where he now holds an ADC Chair in Wireless Telecommunications. His general interests span the areas of communications and signal processing, estimation and detection theory, time-series analysis, and system identification—subjects on which he has published more than 160 journal papers, 300 conference papers, and two edited books. Current research focuses on transmitter and receiver diversity techniques for single- and multiuser fading communication channels, complex-field and space-time coding, multi-carrier, ultrawide band wireless communication systems, cross-layer designs, and distributed sensor networks.

Dr. Giannakis is the (co-) recipient of four best paper awards from the IEEE Signal Processing (SP) Society in 1992, 1998, 2000, and 2001. He also received the Society's Technical Achievement Award in 2000. He served as Editor in Chief for the IEEE SIGNAL PROCESSING LETTERS, as Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING and the IEEE SIGNAL PROCESSING LETTERS, as secretary of the SP Conference Board, as member of the SP Publications Board, as member and vice-chair of the Statistical Signal and Array Processing Technical Committee, as chair of the SP for Communications Technical Committee, and as a member of the IEEE Fellows Election Committee. He is currently a member of the the IEEE-SP Society's Board of Governors, the Editorial Board for the PROCEEDINGS OF THE IEEE, and chairs the steering committee of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.