Resource Allocation for Wireless Multiuser OFDM Networks

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***Abstract*—Resource allocation issues are investigated in this paper for multiuser wireless transmissions based on orthogonal frequency division multiplexing (OFDM). Relying on convex and stochastic optimization tools, the novel approach to resource allocation includes: i) development of jointly optimal subcarrier, power, and rate allocation for weighted sum-average-rate maximization; ii) judicious formulation and derivation of the optimal resource allocation for maximizing the utility of *average* user rates; and iii) development of the stochastic resource allocation schemes, and rigorous proof of their convergence and optimality. Simulations are also provided to demonstrate the merits of the novel schemes.**

***Index Terms*—convex optimization, OFDM, resource allocation and scheduling, stochastic approximation.**

I. Introduction

The emerging demand for high-rate wireless connectivity under diverse quality-of-service (QoS) requirements motivates intelligent multiuser scheduling designs for next-generation wireless networks. Orthogonal frequency division multiplexing (OFDM) facilitates high-speed wireless communications over the emergent frequency-selective links because it copes efficiently with inter-symbol interference, which limits the achievable data rates. Furthermore, OFDM subchannels can be allocated dynamically among multiple users, providing an extra degree of freedom in multiuser scheduling. For these reasons, OFDM has become the workhorse for broadband wireless applications and has been adopted by current and future standards including IEEE802.11a/g [1], IEEE 802.16 standards [2] and 3GPP-Long Term Evolution (LTE) [3].

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Resource allocation for multiuser OFDM networks has attracted a lot of interest [26], [12], [9], [18], [11], [27]. The goal is to jointly allocate subcarriers, rate and power in order to maximize (respectively minimize) the weighted sum of user rates (powers) under a prescribed power (rate) budget. For these problems, a low-complexity yet provably optimal solution is not available. In this context, [26], [9], [12], [11], [27] reported suboptimal algorithms which tradeoff complexity for (sub)optimality. Recently, there has also been interest to expand the scope of resource allocation/scheduling to: i) ensure fairness among users, ii) provide QoS guarantees, and iii) cope with mobility and network dynamics, both of which render the wireless channel uncertain. Fairness and QoS guarantees can be effected by maximizing a suitable utility function of average user rates and introducing minimum rate constraints per user [6], [4], [21], [13], [5], [15], [23]. Channel uncertainty on the other hand, can be accommodated through on-line channel-adaptive scheduling schemes that essentially learn the underlying channel distribution on-the-fly [13], [21], [21]. The resultant "opportunistic" schedulers, however, are mainly developed for single-carrier, time-division multiplexing systems, and the few extensions to OFDM networks are only limited to best-effort traffic without rate requirements [20]. In addition, existing approaches pertain to either deterministic links [20], or if random fading effects are accounted for, the channel links are confined to obey a finite-state Markov chain model [13], [21]. This is not the case in wireless propagation where fading coefficients take on a continuum of values.

To overcome limitations of existing approaches, this paper takes a fresh look at the analytical approach and algorithmic development of resource allocation and scheduling problems for multiuser wireless OFDM systems. Relying on convex optimization tools, the ergodic rate region is specified first, and the corresponding optimal subcarrier, rate, and power allocation is developed afterwards. It is shown that almost surely optimal resource allocation can be obtained in closed-form by a greedy water-filling approach with linear complexity in the number of users and subcarriers, provided that the distribution function of the random fading channel is continuous. General utility-maximizing schedulers are further developed for multiuser OFDM systems with minimum average rate guarantees when the fading distribution is known and when it is unknown. Designed and analyzed using stochastic-averaging tools popular in adaptive signal processing theory, merits of the proposed novel schedulers relative to [6], [21], [5] and [20], include reduced complexity, faster convergence, and provable optimality for wireless channels with continuous fading distributions.

The rest of the paper is organized as follows. Section II describes the system and channel models. Section III derives the ergodic rate region and the corresponding optimal resource allocation for OFDM channels. Section IV is devoted to utility based scheduling schemes, which are capable of dynamically learning the underlying channel distribution and asymptotically converging to the optimal benchmark with average rate guarantees. The novel schedulers are tested in Section V with simulations; and Section VI concludes the paper.

II. Modeling Preliminaries

Consider OFDM-based communication over wireless fading links between an access point (AP) and J wireless users. An intelligent scheduler at the AP relies on the channel state information (CSI) of the wireless links to allocate the available resources, namely, subcarriers, rate, and power.

Slotted transmissions take place over bandwidth B, which is divided into K orthogonal narrow-band sub-channels, each with bandwidth $\Delta = B/K$ small enough for each subcarrier to experience only flat fading. With $h_{j,l}[n]$ denoting the complex channel taps, and $\{\tau_l\}_{l=1}^{L_j}$ the corresponding delays per slot n, the square amplitude of the discrete-time Fourier transform on subcarrier k is given by

$$\gamma_{j,k}[n] = \left| \sum_{l=1}^{L_j} h_{j,l}[n] e^{-j2\pi\tau_l k\Delta} \right|^2. \quad (1)$$

The following operational conditions are assumed.

oc-1) Fading coefficients $\boldsymbol{\gamma} = \{\gamma_{j,k},\ j = 1,\ldots,J, k = 1,\ldots,K\}$ obey a block fading model; i.e., they are fixed per slot n but can vary from slot to slot according to a random process which is assumed stationary and ergodic with cumulative distribution function (cdf) $F(\boldsymbol{\gamma})$.

oc-2) During a training phase entailing sufficiently long pilot sequences, the AP acquires $\boldsymbol{\gamma}$ and each terminal j acquires $\gamma_{j,k}$ $\forall k$ per slot n.

Whereas the block-fading model in oc-1) is commonly assumed, the full CSI assumption in oc-2) is reasonable for time-division duplex (TDD) systems where link reciprocity holds. Based on $\boldsymbol{\gamma}$, the scheduler at the AP wishes to optimally allocate subcarriers, rate, and power per slot n, to all users.

Notation: Boldface letters denote column vectors and inequalities for vectors are defined element-wise; $\mathbb{E}_{\boldsymbol{\gamma}}[\cdot]$ denotes the expectation operator over fading states $\boldsymbol{\gamma}$, $\mathbf{0}$ the all-zero vector, ^{T} transposition, $\|\boldsymbol{x}\|$ the vector norm, $|x|$ the absolute value, $\mathbf{int}\ (S)$ the interior of a set S, $\nabla f(\boldsymbol{x})$ the gradient of $f(\boldsymbol{x})$ with respect to vector $\boldsymbol{x}$, and $[x]^+ := \max(x, 0)$.

III. Ergodic Rate Region and Optimal Resource Allocation

This section characterizes the ergodic region of achievable rates, and develops optimal resource allocation schemes for OFDM-based wireless networks.

A. Ergodic Rate Region

Consider for now that per time slot, each subcarrier can be shared by multiple users over nonoverlapping time fractions. Let $\alpha_{j,k} \geq 0$ denote the nonnegative time-sharing fraction of a slot and $p_{j,k} \geq 0$ the average transmit-power allocated for transmission to user j on subcarrier k. Supposing without loss of generality (w.l.o.g.) that the slot duration is unity, fractions per subcarrier must obey $\sum_{j=1}^{J} \alpha_{j,k} \leq 1, \forall k = 1,\ldots,K$. Since only a fraction $\alpha_{j,k} > 0$ of the slot is designated for terminal j, its transmit-power on subcarrier k during the active time fraction is $p_{j,k}/\alpha_{j,k}$. Assuming w.l.o.g. that the additive white Gaussian noise (AWGN) at the receiver has unit variance and the sub-bandwidth $\Delta = 1$, the maximum achievable rate of user j on subcarrier k is then provided by Shannon's capacity formula

$$c_{j,k}(\alpha_{j,k}, p_{j,k}) = \begin{cases} \alpha_{j,k} \log_2\left(1 + \frac{\gamma_{j,k} p_{j,k}}{\alpha_{j,k}}\right), & \alpha_{j,k} > 0 \\ 0, & \alpha_{j,k} = 0. \end{cases} \quad (2)$$

The allocation scheme sought should specify resources per fading channel realization $\boldsymbol{\gamma}$. Since the rate in (2) depends on $\alpha_{j,k}$ and $p_{j,k}$, the optimization variables are $\boldsymbol{\alpha} := \{\alpha_{j,k}(\boldsymbol{\gamma}), \forall j, k, \forall \boldsymbol{\gamma}\}$ and $\boldsymbol{p} := \{p_{j,k}(\boldsymbol{\gamma}), \forall j, k, \forall \boldsymbol{\gamma}\}$. Once the optimal $\boldsymbol{\alpha}^*$ and $\boldsymbol{p}^*$ are found, it follows from (2) that the optimal rates are determined as $r_{j,k}^*(\boldsymbol{\gamma}) = c_{j,k}(\alpha_{j,k}^*(\boldsymbol{\gamma}), p_{j,k}^*(\boldsymbol{\gamma}))$, $\forall j, k$. For notational brevity, consider the set of "trivial" constraints $\mathcal{A} := \{\boldsymbol{\alpha}, \boldsymbol{p} \mid \alpha_{j,k}(\boldsymbol{\gamma}) \geq 0,\ p_{j,k}(\boldsymbol{\gamma}) \geq 0,\ \forall j, k;\ \sum_{j=1}^{J} \alpha_{j,k}(\boldsymbol{\gamma}) \leq 1, \forall k\}$. Given a specific allocation policy $(\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{A}$, it is clear that the maximum average rate per user j is

$$\bar{r}_j(\boldsymbol{\alpha}, \boldsymbol{p}) := \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))\right] \quad (3)$$

and the maximum average rate vector is $\bar{\boldsymbol{r}}(\boldsymbol{\alpha}, \boldsymbol{p}) := [\bar{r}_1(\boldsymbol{\alpha}, \boldsymbol{p}), \ldots, \bar{r}_J(\boldsymbol{\alpha}, \boldsymbol{p})]^T$.

Consider for specificity the OFDM-based downlink where the AP has an average sum-power budget $\check{P}$ for transmissions. Let $\mathcal{F}$ denote the set of all feasible allocation policies $(\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{A}$ satisfying also the average sum-power constraint $\mathbb{E}_{\boldsymbol{\gamma}}[\sum_{j=1}^{J} \sum_{k=1}^{K} p_{j,k}(\boldsymbol{\gamma})] \leq \check{P}$. With $\bar{\boldsymbol{r}} := [\bar{r}_1, \ldots, \bar{r}_J]^T$, the region of achievable average rates is clearly given by

$$\mathcal{C} := \bigcup_{(\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{F}} \{\bar{\boldsymbol{r}} \mid \mathbf{0} \leq \bar{\boldsymbol{r}} \leq \bar{\boldsymbol{r}}(\boldsymbol{\alpha}, \boldsymbol{p})\}. \quad (4)$$

Proposition 1: The ergodic capacity $c_{j,k}(\alpha, p)$ in (2) is a jointly concave function of α and p; and thus the ergodic region $\mathcal{C}$ in (4) is a convex set of $\bar{\boldsymbol{r}}$ vectors.

Proof: See Appendix A. ■

Since $\mathcal{C}$ is convex, each boundary point of $\mathcal{C}$ can be attained by maximizing a weighted sum of average rates [8]; i.e.,

$$\bar{\boldsymbol{r}}^*(\boldsymbol{w}) = \arg \max_{\bar{\boldsymbol{r}} \in \mathcal{C}} \boldsymbol{w}^T \bar{\boldsymbol{r}} \quad (5)$$

where the weight vector $\boldsymbol{w} := [w_1, \ldots, w_J]^T \geq \mathbf{0}$. Varying the weights allows one to reach all the boundary points and thus determine $\mathcal{C}$. Clearly, at a boundary point associated with a weight vector $\boldsymbol{w}$, one finds the optimal rate vector $\bar{\boldsymbol{r}}^* = \bar{\boldsymbol{r}}(\boldsymbol{\alpha}^*, \boldsymbol{p}^*)$ for a certain $(\boldsymbol{\alpha}^*, \boldsymbol{p}^*) \in \mathcal{F}$. This optimal allocation will be pursued next using a Lagrange dual-based approach.

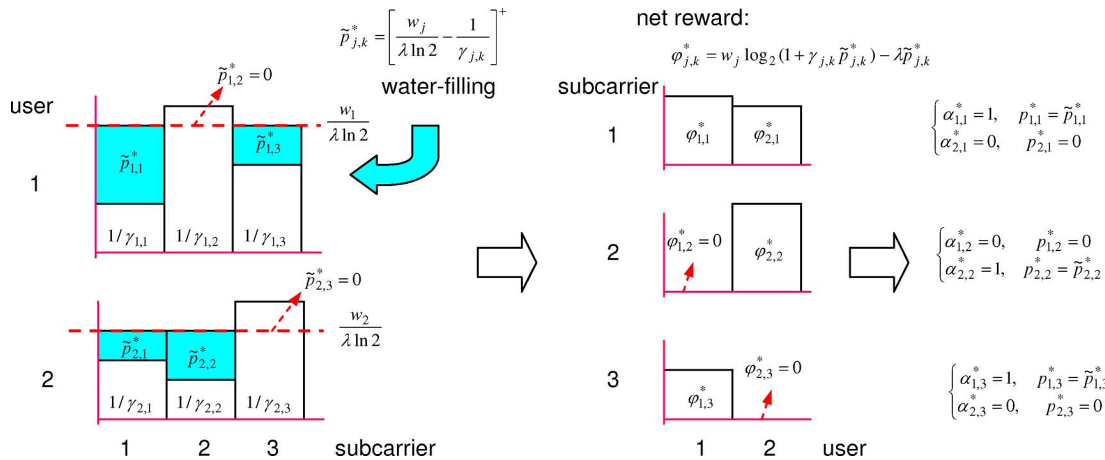

Fig. 1. Greedy water-filling approach.

B. Optimal Subcarrier and Power Allocation

After substituting (3) and (4) into (5), finding the optimal $(\boldsymbol{\alpha}^*, \boldsymbol{p}^*)$ requires solving the optimization problem

$$\max_{(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} \quad \sum_{j=1}^{J} w_j \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))\right]$$
$$\text{s. to} \quad \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1}^{J}\sum_{k=1}^{K} p_{j,k}(\boldsymbol{\gamma})\right] \le \check{P}. \tag{6}$$

Since $c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))$ is a concave function of $(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))$, it follows that (6) is a convex optimization problem, which can be solved using a Lagrange dual approach [8], [16]. With λ denoting the Lagrange multiplier associated with the constraint, the Lagrangian function of (6) is

$$L(\lambda, \boldsymbol{\alpha}, \boldsymbol{p}) = \sum_{j=1}^{J} w_j \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))\right] + \lambda\left(\check{P} - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1}^{J}\sum_{k=1}^{K} p_{j,k}(\boldsymbol{\gamma})\right]\right)$$
$$= \lambda\check{P} + \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1}^{J}\sum_{k=1}^{K} \varphi_{j,k}(\lambda, \alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))\right]$$

where the functional $\varphi_{j,k}$ is defined as

$$\varphi_{j,k}(\lambda, \alpha_{j,k}, p_{j,k}) := w_j c_{j,k}(\alpha_{j,k}, p_{j,k}) - \lambda p_{j,k}. \tag{7}$$

The dual function is then given by

$$D(\lambda) = \max_{(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} L(\lambda, \boldsymbol{\alpha}, \boldsymbol{p}) \tag{8}$$

and the dual problem of (6) is

$$\min_{\lambda\ge 0} D(\lambda). \tag{9}$$

Due to the convexity of (6), there is no duality gap [8]. Therefore, the solution of (6) can be obtained by solving (9). Specifically, if λ^* denotes the minimizer of (9), then the optimal $\boldsymbol{\alpha}^*(\lambda^*)$ and $\boldsymbol{p}^*(\lambda^*)$ that maximize $L(\lambda^*, \boldsymbol{\alpha}, \boldsymbol{p})$ in (8), are the optimal variables for (6) provided that the complementary slackness condition is satisfied [8].

To obtain $D(\lambda)$ for a given λ, consider what will be later interpreted as link quality indicator

$$\varphi^*_{j,k}(\lambda, \boldsymbol{\gamma}) = \begin{cases} \frac{w_j}{\ln 2}\ln\left(\frac{w_j\gamma_{j,k}}{\lambda\ln 2}\right) - \frac{w_j}{\ln 2} + \frac{\lambda}{\gamma_{j,k}}, & \gamma_{j,k} > \frac{\lambda\ln 2}{w_j} \\ 0, \quad \text{otherwise.} \end{cases} \tag{10}$$

Lemma 1: For ergodic fading channels with continuous cdf, the almost surely unique solution of (8) yields the optimal time fractions and powers per k and $\boldsymbol{\gamma}$ as

$$\begin{cases} \alpha^*_{j^*_k,k}(\lambda, \boldsymbol{\gamma}) = 1, \quad p^*_{j^*_k,k}(\lambda, \boldsymbol{\gamma}) = \left[\frac{w_{j^*_k}}{\lambda\ln 2} - \frac{1}{\gamma_{j^*_k,k}}\right]^+ \\ \alpha^*_{j,k}(\lambda, \boldsymbol{\gamma}) = p^*_{j,k}(\lambda, \boldsymbol{\gamma}) = 0, \quad \forall j \ne j^*_k(\lambda, \boldsymbol{\gamma}) \end{cases} \tag{11}$$

where $j^*_k(\lambda, \boldsymbol{\gamma}) = \arg\max_{j=1,\ldots,J} \varphi^*_{j,k}(\lambda, \boldsymbol{\gamma})$.

Proof: See Appendix B. ∎

Basically, Lemma 1 asserts that a "winner-takes-all" assignment per subcarrier along with a water-filling power allocation across $\boldsymbol{\gamma}$ realizations constitutes with probability one (w.p. 1) the optimal solution of (6), provided that the distribution function of the random fading channel is continuous. Regarding w_j as a rate reward weight and λ as power price, $\varphi_{j,k}(\lambda, \alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))$ in (7) is indeed a a*net reward* (rate reward minus power cost) for user j over subcarrier k per slot. The optimal resource allocation should maximize the total net reward across users and subcarriers per $\boldsymbol{\gamma}$. As illustrated in Fig. 1, this amounts to a *greedy water-filling* solution, where power and subcarrier allocations are decoupled.

In the first step, transmit-power $\tilde{p}_{j,k} := p_{j,k}/\alpha_{j,k}$ during the active time fraction $\alpha_{j,k} > 0$, is allocated per user across subcarriers following a water-filling principle, i.e.,

$$\tilde{p}^*_{j,k}(\lambda, \boldsymbol{\gamma}) = \left[\frac{w_j}{\lambda\ln 2} - \frac{1}{\gamma_{j,k}}\right]^+. \tag{12}$$

The link quality indicators $\varphi^*_{j,k}(\lambda, \boldsymbol{\gamma})$ in (10), obtained based on such power values

$$\varphi^*_{j,k}(\lambda, \boldsymbol{\gamma}) = w_j\log_2(1 + \gamma_{j,k}\tilde{p}^*_{j,k}(\lambda, \boldsymbol{\gamma})) - \lambda\tilde{p}^*_{j,k}(\lambda, \boldsymbol{\gamma}) \tag{13}$$

represent the highest potential of net reward when allocating subcarrier k to terminal j for the fading realization $\boldsymbol{\gamma}$. In the second step, the entire subcarrier k is then greedily assigned to the "winner"-user $j_k^*(\lambda,\boldsymbol{\gamma}) = \arg\max_{j=1,\ldots,J}\ \varphi_{j,k}^*(\lambda,\boldsymbol{\gamma})$ for maximum net reward per subcarrier.

With the optimal allocation of Lemma 1, only the optimal multiplier λ^* is left to specify in order to obtain the optimal solution for (6). With the dual-optimal power allocation $p_{j,k}^*(\lambda,\boldsymbol{\gamma})$ in (11), consider the resultant average sum-power which is a well-defined function of λ since the associated expected value is unique due to the almost surely unique $p_{j,k}^*(\lambda,\boldsymbol{\gamma}), \forall\boldsymbol{\gamma}$. Function $\bar{P}(\lambda)$ is monotonic as asserted by the following lemma.

Lemma 2: The instantaneous power $p^*_{j_k^*(\lambda,\boldsymbol{\gamma}),k}(\lambda,\boldsymbol{\gamma})$ specified in (11) as well as the corresponding average sum-power $\bar{P}(\lambda)$ are both nonincreasing functions of λ.

Proof: See Appendix C. ■

As dictated by the complementary slackness condition, the optimal λ^* should satisfy: either i) $\lambda^* = 0$ and $\bar{P}(\lambda^*) < \check{P}$ or ii) $\lambda^* > 0$ and $\bar{P}(\lambda^*) = \check{P}$. However, here we should have $\lambda^* > 0$, because otherwise $p^*_{j_k^*,k}(\lambda^*,\boldsymbol{\gamma})$ must be infinity [cf. (11)], which clearly violates the sum-power constraint. Therefore, we must have $\bar{P}(\lambda^*) = \check{P}$. Thanks to the monotonicity established in Lemma 2, 1-D search algorithms (such as bi-section) employed to numerically solve $\bar{P}(\lambda^*) = \check{P}$ are ensured to converge to λ^* geometrically fast.

Summarizing, we have established the following result.

Theorem 1: For ergodic fading channels with continuous cdf, the almost surely optimal subcarrier assignment and power allocation for (6) is given by $\alpha_{j,k}^*(\lambda^*,\boldsymbol{\gamma})$ and $p_{j,k}^*(\lambda^*,\boldsymbol{\gamma}), \forall k, \forall\boldsymbol{\gamma}$, in (11), where the optimal λ^* is chosen (via bisection iterations) such that $\bar{P}(\lambda^*) = \check{P}$.

Interestingly, although each subcarrier was allowed to be time shared at the outset, the almost surely optimal solution in Theorem 1 dictates no sharing. In fact, this "winner-takes-all" policy per subcarrier k subsumes three cases: (i) no transmission when all user subchannels experience deep fades; i.e., $\gamma_{j,k} \le (\lambda^* \ln 2)/w_j, \forall j$; (ii) allocation to a single winner if $\varphi_{j,k}^*(\lambda^*,\boldsymbol{\gamma}), \forall j$ admits a unique maximizer; and (iii) allocation to a randomly chosen winner if $\varphi_{j,k}^*(\lambda^*,\boldsymbol{\gamma})\ \forall j$ has multiple maxima. Continuity of the channel cdf ensures that having multiple "winners," i.e., case (iii), is an event of Lebesgue measure zero. Thus, the pair $(\boldsymbol{\alpha}^*(\lambda^*,\boldsymbol{\gamma}), \boldsymbol{p}^*(\lambda^*,\boldsymbol{\gamma}))$ is almost surely unique; and hence, it is almost surely optimal for all wireless channel models including Rayleigh, Rice, and Nakagami that indeed have continuous cdfs [17]. Note that this optimality w.p. 1 does not apply when the channel fading distribution is discrete (as is the case with static channel fading [9], [20]). In this case, the "winner-takes-all" policy may not be optimal and optimal time-sharing among users needs to be determined (even when one considers maximization of the *average* rates); see also [23].

The optimal resource allocation in Theorem 1 can be used to determine boundary points of the ergodic rate region $\mathcal{C}$. Such a greedy approach "water-fills" the available power resources across subcarriers and $\boldsymbol{\gamma}$ realizations, with higher power (and rate) assigned to higher quality $(\gamma_{j,k})$ links. This achieves optimal utilization of the rich spectral and temporal modes of diversity that become available with random OFDM channels. In addition, the water-filling power allocation together with the "winner-takes-all" subcarrier assignment specified in (11) captures the multiuser diversity [22], by intelligently scheduling the user terminal with the "best" channel as quantified by the highest net reward per subcarrier. Note that the term "winner-takes-all" should not be misunderstood. Although one winner is chosen per subcarrier per $\boldsymbol{\gamma}$, the optimally scheduled winners (as well as their assigned powers) vary across subcarriers and fading realizations. In this spirit, all the types of diversity available by fading are exploited by the proposed scheme.

From a Shannontheoretic perspective, the ergodic rate region $\mathcal{C}$ in (4) does not yield a maximum achievable rate (i.e., capacity) region. Shannon's capacity of general OFDM broadcast channels can be approached by allowing superposition coding and successive decoding per subcarrier. In fact, the capacity-achieving schemes for OFDM downlink should not be difficult to derive by combining the proposed approach with those in [14], [25]. This direction is not pursued because superposition coding and successive decoding per subcarrier are generally considered too complex for practical OFDM systems. For this reason, most existing works on resource allocation for OFDM systems do not advocate superposition coding [9], [27], [20]. In fact, many existing works assume *a fortiori* that each subcarrier is assigned to a single user (i.e., time-sharing is not allowed), and formulate the resultant optimization problem as a nonconvex 0-1 integer program (which is NP-hard). Bearing these considerations in mind, we allow time-sharing among users per subcarrier at the outset. This prevents the NP-hard integer program, and facilitates a convex optimization formulation. On the other hand, we prove that the optimal resource allocation policy for our formulation admits a "winner-takes-all" strategy, that is a single user assignment per subcarrier, almost surely, provided that the channel fading has a continuous distribution function—a condition met by all fading models used in practice. Hence, we provide a provably optimal solution for OFDM systems with low-complexity transceiver design (since each subcarrier is assigned to a single user). Notwithstanding, this optimality cannot be claimed by existing sub-optimum schemes [9], [27], [20]. Furthermore, the proposed optimal scheme incurs low computational complexity. With the optimal power price λ^*, optimal allocation of subcarriers and power entails just evaluating and comparing J

$$\bar{P}(\lambda) = \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1}^{J}\sum_{k=1}^{K} p_{j,k}^*(\lambda,\boldsymbol{\gamma})\right] = \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} p^*_{j_k^*(\lambda,\boldsymbol{\gamma}),k}(\lambda,\boldsymbol{\gamma})\right]$$

link quality indicators $\varphi^*_{j,k}(\lambda^*, \boldsymbol{\gamma})$ per subcarrier $k = 1, \ldots, K$. Hence, the almost surely "winner-takes-all" optimal resource allocation over *random* OFDM channels exhibits a linear complexity $\mathcal{O}(JK)$. This is remarkable if one takes into account that with deterministic multiuser OFDM channels, optimal "winner-takes-all" subcarrier allocation amounts to an integer programming problem, which is known to be exponentially complex; and even the sub-optimal algorithms in [9] incur "reduced complexity" in the order of $\mathcal{O}(JK^2)$. Interestingly, wireless fading with infinitely many states is actually a "blessing" rather than a "curse" as far as optimal OFDM resource allocation is concerned.

We remark that a Lagrange dual approach to "optimal" downlink OFDM resource allocation was independently developed in [27][1] This approach is actually a variant of our "greedy water-filling" approach. But assuming each subcarrier is assigned to a single user a fortiori, the optimality of the proposed scheme in [27] cannot be shown; instead, a so-called "99.9999% practical optimality" is claimed through extensive simulations using a specific distribution function for the fading process, namely white, zero-mean, circular-symmetric complex Gaussian. Differently, we analytically establish that a "winner-takes-all" policy such as the scheme in [27] is almost surely optimal for any fading process with continuous distribution function. This almost sure optimality holds for any continuous fading case, in contrast to the "99.9999% practical optimality" for a specific distribution.

The Lagrange dual method was also used to solve a *static* optimization for resource allocation per slot (i.e., per fading realization) in OFDM downlink [10]. Differently, here we maximize the ergodic rates for time-varying fading channels; thus, our scheme can also capitalize on the temporal fading diversity to achieve better performance. A time-sharing argument was also employed in [10] to obtain a convex optimization formulation[2]. The optimal resource allocation strategy derived in [10] has different form when compared to the "greedy water-filling" strategy of this paper. The proposed "greedy water-filling" method with decoupled power and time allocation provide useful insights on the structure of the optimal policy.

It is worth mentioning that an attractive aspect of the proposed greedy water-filling approach with regard to those in [27], [10] is its generality. Theorem 1 for OFDM downlink is derived assuming that both transmit-rates and fading channel cdfs are continuous. The result carries over readily to the uplink by taking into account the individual (instead of sum) power constraints; our approach can also account for discontinuous channel cdfs as well as discrete rate adaptation, and can be extended to general orthogonally channelized systems, along the lines of our recent work in [23] and [24].

[1]We exchanged conference papers with the authors of [27], and they cited/acknowledged our work. In fact their first conference paper reporting this result appeared one month after ours in ICASSP, April 16–20, 2007; whereas we first reported our result in CISS, March 14–16, 2007.

[2]In fact, the approach in [10] is not on safe grounds because $\alpha \log(1+\gamma p/\alpha)$ is used as the rate function for a time fraction α, and power allocation p; but this function is undefined at $\alpha = 0$, especially $\alpha = p = 0$, and thus concavity cannot be claimed over the entire feasible set $\alpha \geq 0$ and $p \geq 0$. This is actually neither trivial nor irrelevant since the optimal policy indeed chooses one of such extreme points. (Recall that at the optimal "winner-takes-all" allocation, it holds that $\alpha^*_j = p^*_j = 0$, for all but a single user.)

IV. Utility-Based Scheduling With Average Rate Guarantees

The schemes of Section III provide proper benchmarks for resource allocation and scheduling over wireless OFDM connections. However, they are insufficient to address user fairness and QoS guarantees, as well as to cope with the uncertainty associated with wireless fading propagation. The approach of "opportunistic scheduling" in [13], [21], [15], [5], [28] holds promise to address these challenges in the OFDM context. To devise fair and efficient schedulers for communication networks, these works rely on utility functions—a notion originally studied in economics, to quantify user fairness and benefits of a resource. Following such an approach and using the derived ergodic rate region as benchmark, this section deals with utility based multiuser scheduling for OFDM downlink systems, where continuous rate adaptation and continuous fading distributions are assumed for specificity; but results of this section carry over readily to uplink, and to other (discontinuous cdf, discrete rate adaptation) setups.

For J user connections requesting QoS guarantees in terms of minimum prescribed rates $\check{\boldsymbol{r}} := [\check{r}_1, \ldots, \check{r}_J]^T \geq \mathbf{0}$, a general utility-based scheduler seeks the solution of

$$\max_{\bar{\boldsymbol{r}}} U(\bar{\boldsymbol{r}}), \quad \text{s. to} \quad \bar{\boldsymbol{r}} \geq \check{\boldsymbol{r}}, \quad \bar{\boldsymbol{r}} \in \mathcal{C} \tag{14}$$

where $U(\bar{\boldsymbol{r}})$ is a chosen utility function of the average rate vector $\bar{\boldsymbol{r}}$, and the constraint $\bar{\boldsymbol{r}} \in \mathcal{C}$ bounds average rate limits that the physical channels can handle. Similar rate (utility) maximization problem with minimum rate requirements were also formulated in different contexts to address user fairness and/or QoS guarantees [5], [16], [21], [25].

To ensure that the global optimum of (14) can be attained in principle, it is assumed that:

(A1) Function $U(\bar{\boldsymbol{r}})$ is selected to be concave, increasing and uniformly bounded, $\forall \bar{\boldsymbol{r}} \in \mathcal{C}$; and the prescribed minimum rates reside within the interior of $\mathcal{C}$, i.e., $\check{\boldsymbol{r}} \in \mathbf{int}(\mathcal{C})$.

Choosing U to be increasing is consistent with the fact that the benefit of the jth user should increase as $\bar{r}_j$ increases, whereas a concave U can balance the tradeoff between rate efficiency and user fairness [23]. Other conditions in (A1) are imposed to assure that (14) is a strictly feasible convex optimization problem, provided that $\mathcal{C}$ is a closed convex set of $\bar{\boldsymbol{r}}$.

A. Scheduling With Known Channel CDF

To bypass the requirement for differentiable U, one can resort to the Lagrange dual approach. To this end, consider re-formulating (14) to [cf. the definition of $\mathcal{C}$]

$$\begin{aligned} &\max_{\bar{\boldsymbol{r}} \geq \check{\boldsymbol{r}}, (\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{A}} U(\bar{\boldsymbol{r}}) \\ &\text{s. to} \quad \bar{r}_j \leq \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{k=1}^{K} c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma})) \right], \quad \forall j \\ &\qquad \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{j=1}^{J} \sum_{k=1}^{K} p_{j,k}(\boldsymbol{\gamma}) \right] \leq \check{P} \end{aligned} \tag{15}$$

where the constraint $\bar{\boldsymbol{r}} \geq \check{\boldsymbol{r}}$ is put under the $\max$ operator for notational brevity. Due to the concavity of functions U and $c_{j,k}$, the primal problem in (15) is convex.

With λ denoting the Lagrange multiplier associated with the average sum-power constraint and $\boldsymbol{\mu} := [\mu_1, \ldots, \mu_J]^T$ collecting the Lagrange multipliers for $\bar{r}_j \leq \mathbb{E}_{\boldsymbol{\gamma}}[\sum_k c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma}))]$, $\forall j$, the Lagrangian for (15) is then as shown in (16), at the bottom of the page, where the net-reward per user-subcarrier is defined as

$$\varphi_{j,k}(\lambda, \boldsymbol{\mu}, \alpha_{j,k}, p_{j,k}) := \mu_j c_{j,k}(\alpha_{j,k}, p_{j,k}) - \lambda p_{j,k}. \quad (17)$$

The corresponding Lagrange dual function can be found as

$$D(\lambda, \boldsymbol{\mu}) = \max_{\bar{\boldsymbol{r}} \geq \check{\boldsymbol{r}}, (\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{A}} L(\lambda, \boldsymbol{\mu}, \bar{\boldsymbol{r}}, \boldsymbol{\alpha}, \boldsymbol{p}) \quad (18)$$

and the dual problem of (15) is $\min_{\lambda > 0,\, \boldsymbol{\mu} \geq \boldsymbol{0}} D(\lambda, \boldsymbol{\mu})$.

Since (15) is a strictly feasible convex optimization problem, it follows that there is no duality gap between the primal (15) and its dual. Hence, the solution of (15) can be obtained via solving its dual problem.

To this end, it is necessary to first solve (18), which amounts to solving two decoupled subproblems (across $\bar{\boldsymbol{r}}$ and $(\boldsymbol{\alpha}, \boldsymbol{p})$). The first one involves only $\bar{\boldsymbol{r}}$ [cf. (16)]

$$\max_{\bar{\boldsymbol{r}} \geq \check{\boldsymbol{r}}} U(\bar{\boldsymbol{r}}) - \boldsymbol{\mu}^T \bar{\boldsymbol{r}}. \quad (19)$$

For any (possibly nondifferentiable) concave U, (19) is a simple convex optimization problem, for which efficient algorithms are available to obtain the optimal $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu})$. If $U(\cdot)$ is differentiable and its gradient ∇U has a well-defined inverse ∇U^{-1}, then (19) can be solved in closed form

$$\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}) = \max(\check{\boldsymbol{r}},\ \nabla U^{-1}(\boldsymbol{\mu})). \quad (20)$$

The second subproblem associated with $(\boldsymbol{\alpha}, \boldsymbol{p})$ is [cf. (16)] as shown in

$$\max_{(\boldsymbol{\alpha}, \boldsymbol{p}) \in \mathcal{A}} \lambda \check{P} + \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{j=1}^{J} \sum_{k=1}^{K} \varphi_{j,k}(\lambda, \boldsymbol{\mu}, \alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma})) \right]. \quad (21)$$

But problem (21) is identical to the one in (8) with $\boldsymbol{w} \equiv \boldsymbol{\mu}$; hence, its solution $\{\alpha^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}), \alpha^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}), \forall j, k\}$ is provided by Lemma 1.

Relying on $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu})$ and $\{\alpha^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}), \alpha^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}), \forall j, k\}$, the optimal multipliers λ^* and $\boldsymbol{\mu}^*$ can be obtained by solving the dual problem using gradient projection iterations [7], as shown in (22), at the bottom of the page, where a shorthand notation is used for the jth user's transmit-rate over subcarrier k, namely

$$r^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}) := c_{j,k}(\alpha^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma}), p^*_{j,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma})). \quad (23)$$

The iterations in (22) are guaranteed to converge to the optimal λ^* and $\boldsymbol{\mu}^*$ from any initial $\lambda[0] \geq 0$ and $\boldsymbol{\mu}[0] \geq \boldsymbol{0}$ [7, p. 641].

Strong duality between the primal (15) and its dual ensures that replacing λ and $\boldsymbol{\mu}$ with λ^* and $\boldsymbol{\mu}^*$ provides the (almost surely) optimal resource schedule $(\boldsymbol{\alpha}^*(\lambda^*, \boldsymbol{\mu}^*), \boldsymbol{p}^*(\lambda^*, \boldsymbol{\mu}^*))$, and the resultant optimal rate vector $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*)$ for (15), or equivalently, for (14). Notice that since U is an increasing function, it must hold that

$$\bar{r}^*_j(\boldsymbol{\mu}^*) = \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{k=1}^{K} c_{j,k}(\alpha^*_{j,k}(\lambda^*, \boldsymbol{\mu}^*, \boldsymbol{\gamma}), p^*_{j,k}(\lambda^*, \boldsymbol{\mu}^*, \boldsymbol{\gamma})) \right], \quad \forall j.$$

In other words, the optimal $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*)$ resides on a boundary point of $\mathcal{C}$. Furthermore, reasoning as in Theorem 1, it should also hold that

$$\mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{k=1}^{K} p^*_{j^*_k,k}(\lambda^*, \boldsymbol{\mu}^*, \boldsymbol{\gamma}) \right] = \check{P}.$$

$$\begin{aligned} L(\lambda, \boldsymbol{\mu}, \bar{\boldsymbol{r}}, \boldsymbol{\alpha}, \boldsymbol{p}) &= U(\bar{\boldsymbol{r}}) + \sum_{j=1}^{J} \mu_j \left(\mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{k=1}^{K} c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma})) \right] - \bar{r}_j \right) + \lambda \left(\check{P} - \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{j=1}^{J} \sum_{k=1}^{K} p_{j,k}(\boldsymbol{\gamma}) \right] \right) \\ &= U(\bar{\boldsymbol{r}}) - \boldsymbol{\mu}^T \bar{\boldsymbol{r}} + \lambda \check{P} + \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{j,k} \varphi_{j,k}(\lambda, \boldsymbol{\mu}, \alpha_{j,k}(\boldsymbol{\gamma}), p_{j,k}(\boldsymbol{\gamma})) \right] \end{aligned} \quad (16)$$

$$\begin{aligned} \lambda[i+1] &= \left[\lambda[i] + \beta \left(\mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{k=1}^{K} p^*_{j^*_k,k}(\lambda[i], \boldsymbol{\mu}[i], \boldsymbol{\gamma}) \right] - \check{P} \right) \right]^+ \\ \mu_j[i+1] &= \left[\mu_j[i] + \beta \left(\bar{r}^*_j(\boldsymbol{\mu}[i]) - \mathbb{E}_{\boldsymbol{\gamma}} \left[\sum_{k=1}^{K} r^*_{j,k}(\lambda[i], \boldsymbol{\mu}[i], \boldsymbol{\gamma}) \right] \right) \right]^+ \end{aligned} \quad (22)$$

Therefore, all the inequality constraints associated with λ and $\boldsymbol{\mu}$ are active (and the optimal multipliers $\lambda^* > 0$, $\boldsymbol{\mu}^* > \mathbf{0}$). This implies that the projection operators $[\cdot]^+$ in (22) are not in effect. One can then rely on the duality theorem [8, p. 225] to show that the dual iterations without $[\cdot]^+$; that is

$$\lambda[i+1] = \lambda[i] + \beta\left(\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} p^*_{j_k^*,k}(\lambda[i],\boldsymbol{\mu}[i],\boldsymbol{\gamma})\right] - \check{P}\right)$$
$$\mu_j[i+1] = \mu_j[i] + \beta\left(\bar{r}^*_j(\boldsymbol{\mu}[i]) - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} r^*_{j,k}(\lambda[i],\boldsymbol{\mu}[i],\boldsymbol{\gamma})\right]\right) \tag{24}$$

can replace (22) to find λ^* and $\boldsymbol{\mu}^*$.

Summarizing, we have the following proposition.

Proposition 2: Under (A1), the iterates in (24) converge to λ^* and $\boldsymbol{\mu}^*$ from any initial $\lambda[0]$ and $\boldsymbol{\mu}[0]$, and the optimal solution of (14) is given by $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*)$, while the corresponding optimal subcarrier and power allocation are specified by $\boldsymbol{\alpha}^*(\lambda^*,\boldsymbol{\mu}^*)$ and $\boldsymbol{p}^*(\lambda^*,\boldsymbol{\mu}^*)$ in Lemma 1.

It is clear from Proposition 2 that the globally optimal allocation maximizing the selected utility coincides with the optimal one for (5) with $\boldsymbol{w} \equiv \boldsymbol{\mu}^*$; that is, the optimal $\bar{\boldsymbol{r}}^*$ for (14) resides on a boundary point of the ergodic rate region when the rate reward vector is $\boldsymbol{\mu}^*$, which is, of course, unknown prior to convergence of the gradient iterations. In fact, for a differentiable U, it holds that: i) if all minimum rate constraints are inactive, i.e., $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*) > \check{\boldsymbol{r}}$, then $\boldsymbol{\mu}^* = \nabla U(\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*))$; and ii) if some of the rate constraints are active, then $\boldsymbol{\mu}^*$ satisfies $\bar{\boldsymbol{r}}^*(\boldsymbol{\mu}^*) = \max(\check{\boldsymbol{r}},\ \nabla U^{-1}(\boldsymbol{\mu}^*))$ [cf. (20)].

Relying on Proposition 2, a utility-based scheduling algorithm when the channel cdf is known can be devised as:

Algorithm 1 Dual-gradient iterations:

1) **initialize** with any $\{\lambda[0],\boldsymbol{\mu}[0]\}$ at the AP, and run off-line (24) until convergence to find the optimal $\{\lambda^*,\boldsymbol{\mu}^*\}$.
2) **repeat on-line:** for channel realization $\boldsymbol{\gamma}[n]$ per slot n, AP schedules in accordance with the globally optimal $\{\boldsymbol{\alpha}^*(\lambda^*,\boldsymbol{\mu}^*,\boldsymbol{\gamma}[n]),\boldsymbol{p}^*(\lambda^*,\boldsymbol{\mu}^*,\boldsymbol{\gamma}[n])\}$.

B. On-Line Scheduling Without Channel CDF

Knowledge of the channel cdf is required to evaluate the expectations involved in (24) and thus implement the scheduling Algorithm 1. If this knowledge is not available a priori, the channel distribution can be only accurately estimated by a sufficiently large number of measurements. Actually even when the channel cdf is available, it is rarely possible to find $\mathbb{E}_{\boldsymbol{\gamma}}$ in closed form. These considerations motivate "stochastic" schedulers based on a single or a limited number of observations that can learn the required cdf on-the-fly in order to approach the optimal strategy. In fact, such stochastic schemes also make more sense in practical mobile applications. The optimization based on *a priori* knowledge of the channel cdf is not robust in the sense that it simply fails if the underlying channel distribution changes due to e.g., user mobility or topology changes. On the other hand, the stochastic optimization based on a single channel observation is capable of "learning" the channel statistics on-the-fly, and thus it could even track changes in the distribution function. These considerations guide the scheduling scheme of this subsection in which the ensemble iterations (24) are replaced with stochastic approximation iterations.

Stochastic approximation iterations are typically adopted by adaptive signal processing algorithms; e.g., the well-known least-mean-square (LMS) algorithm is a result of dropping the expectation operator from the steepest descent iterations [19, p.77]. In the same spirit, consider dropping $\mathbb{E}_{\boldsymbol{\gamma}}$ from (24), and replacing the iteration index i with the slot index n. The resultant on-line iterations based on the per slot fading realization $\boldsymbol{\gamma}[n]$ are

$$\hat{\lambda}[n+1] = \hat{\lambda}[n] + \beta\left(\sum_{k=1}^{K} p^*_{j_k^*,k}(\hat{\lambda}[n],\hat{\boldsymbol{\mu}}[n],\boldsymbol{\gamma}[n]) - \check{P}\right)$$
$$\hat{\mu}_j[n+1] = \hat{\mu}_j[n] + \beta\left(\bar{r}^*_j(\hat{\boldsymbol{\mu}}[n]) - \sum_{k=1}^{K} r^*_{j,k}(\hat{\lambda}[n],\hat{\boldsymbol{\mu}}[n],\boldsymbol{\gamma}[n])\right) \tag{25}$$

where hats are to stress that these iterations involve stochastic estimates of their counterparts in (22), based on *instantaneous* (instead of average) power and rates. The optimal subcarrier and power allocation for the given $\hat{\lambda}[n]$, $\hat{\boldsymbol{\mu}}[n]$ and $\boldsymbol{\gamma}[n]$ per slot n is provided by Lemma 1, which is then used to determine $p^*_{j_k^*,k}(\hat{\lambda}[n],\hat{\boldsymbol{\mu}}[n],\boldsymbol{\gamma}[n])$ and $r^*_{j,k}(\hat{\lambda}[n],\hat{\boldsymbol{\mu}}[n],\boldsymbol{\gamma}[n])$ in (25).

Based on (25), the LMS-alike on-line scheduler can be summarized as follows.

Algorithm 2 Stochastic dual-gradient iterations:

1) **initialize** with any $\hat{\lambda}[0]$ and $\hat{\boldsymbol{\mu}}[0]$; and
2) **repeat on-line:** with $\boldsymbol{\gamma}[n]$, $\hat{\lambda}[n]$ and $\hat{\boldsymbol{\mu}}[n]$ available per slot n, the AP schedules according to the allocation $\boldsymbol{\alpha}^*(\hat{\lambda}[n],\hat{\boldsymbol{\mu}}[n],\boldsymbol{\gamma}[n])$ and $\boldsymbol{p}^*(\hat{\lambda}[n],\hat{\boldsymbol{\mu}}[n],\boldsymbol{\gamma}[n])$, and subsequently updates $\hat{\lambda}[n+1]$ and $\hat{\boldsymbol{\mu}}[n+1]$ using (25).

Different from Algorithm 1, the multipliers $\hat{\lambda}[n]$, $\hat{\boldsymbol{\mu}}[n]$ are updated on-line without knowing the channel cdf. The interesting feature of Algorithm 2 is that multiplier iterates converge to the optimal λ^* and $\boldsymbol{\mu}^*$; thus, the subcarrier and power allocation converges also to the globally optimal one.

To rigorously establish this claim, start by defining $\hat{\boldsymbol{z}}[n] := [\hat{\lambda}[n],\hat{\mu}_1[n],\ldots,\hat{\mu}_J[n]]^T$, $g_\lambda(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]) := \sum_{k=1}^{K} p^*_{j_k^*,k}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]) - \check{P}$, $g_{\mu_j}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]) := \bar{r}^*_j(\hat{\boldsymbol{\mu}}[n]) - \sum_{k=1}^{K} r^*_{j,k}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n])$, and $\boldsymbol{g}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]) := [g_\lambda(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]), g_{\mu_1}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]),\ldots,g_{\mu_J}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n])]^T$. Using these definitions, rewrite (25) as

$$\hat{\boldsymbol{z}}[n+1] = \hat{\boldsymbol{z}}[n] + \beta\boldsymbol{g}(\hat{\boldsymbol{z}}[n],\boldsymbol{\gamma}[n]). \tag{26}$$

Following standard practice in adaptive systems, to establish convergence of the *primary* iteration in (26), we link it with its *averaged* system (and thus time-invariant) iteration

$$\boldsymbol{z}[n+1] = \boldsymbol{z}[n] + \beta\bar{\boldsymbol{g}}(\boldsymbol{z}[n]), \quad \text{with } \bar{\boldsymbol{g}}(\boldsymbol{z}) := \mathbb{E}_{\boldsymbol{\gamma}}[\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\gamma})] \tag{27}$$

where $\boldsymbol{z}[n] := [\lambda[n], \mu_1[n], \ldots, \mu_J[n]]^T$. Putting $\mathbb{E}_{\boldsymbol{\gamma}}$ back into (25), it clearly follows that (27) is (within iteration re-indexing) equivalent to (24).

Having clarified that (25) and (24) is a pair of primary and averaged systems, it is possible to employ the stochastic locking theorem in [19] to prove the following.

Lemma 3: For ergodic fading channels with continuous cdf, if the primary system (26) and its averaged system (27) are both initialized with $\hat{\boldsymbol{z}}[0] \equiv \boldsymbol{z}[0]$, then in any time interval T it holds that

$$\max_{1 \leq n \leq T/\beta} \|\hat{\boldsymbol{z}}[n] - \boldsymbol{z}[n]\| \leq c_T(\beta) \quad \text{w.p. } 1 \tag{28}$$

where $c_T(\beta) \rightarrow 0$ as $\beta \rightarrow 0$.

Proof: See Appendix D. ■

The stochastic locking theorem in [19] holds true only when some regularity conditions (primarily stochastic Lipschitz conditions for system perturbations) are satisfied. The contribution of Lemma 3 is to confirm that these regularity conditions are satisfied for the primary and averaged systems of the wireless OFDM setup, provided that the random fading channel has continuous cdf. The continuous fading distribution renders the sequential difference $\bar{\boldsymbol{g}}(\boldsymbol{z})$ Lipschitz continuous in the averaged system (27). This together with the monotonicity of $p^*_{j^*_k,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma})$ and $\bar{P}(\lambda, \boldsymbol{\mu}) := \sum_{k=1}^{K} p^*_{j^*_k,k}(\lambda, \boldsymbol{\mu}, \boldsymbol{\gamma})$ in λ (as well as $\boldsymbol{\mu}$) proved by Lemma 2, then implies that both $\boldsymbol{g}(\boldsymbol{z}, \boldsymbol{\gamma})$ in the primary system (26) and the total deviations of the two systems are stochastic Lipschitz, so that the stochastic locking theorem is in effect.

Trajectory locking is key to the asymptotic optimality of the stochastic iterations (25). Here, Lemma 3 rigorously establishes that for sufficiently small stepsize β, the trajectories of the primary and averaged systems corresponding to (25) and (24) remain close over any time interval T in the sense that the distance of the two trajectories is bounded in probability by a constant $c_T(\beta)$, which vanishes as $\beta \rightarrow 0$. Since Proposition 2 asserts that the averaged system (24) converges to λ^* and $\boldsymbol{\mu}^*$ (geometrically fast from arbitrary initialization), the stochastic hovering theorem in [19] further implies the following result.

Theorem 2: Under (A1), the iterates in (25) converge to $\boldsymbol{z}^* := [\lambda^*, \boldsymbol{\mu}^{*T}]^T$ in probability, i.e., $\sup_{n \to \infty} \Pr\{\|\hat{\boldsymbol{z}}[n] - \boldsymbol{z}^*\| > \epsilon\} \rightarrow 0$, from any initial $\hat{\boldsymbol{z}}[0]$ as $\beta \rightarrow 0$; and thus the corresponding subcarrier and power allocation converges to the globally optimal one for (14).

Theorem 2 establishes both convergence as well as stability of the stochastic iterations in Algorithm 2 in the following sense. Given any $\epsilon > 0$, there exist (controllably small) constants $\delta(\epsilon)$ and $\beta(\epsilon, \delta)$ such that the probability of the stochastic iterates $\hat{\boldsymbol{z}}[n]$ in Algorithm 2 to escape from a ball of radius ϵ around the optimal $\boldsymbol{z}^*$ is less than $\delta(\epsilon)$ when using a (sufficiently small but constant) stepsize $\beta \leq \beta(\epsilon, \delta)$. It is clear that this convergence does not rely on finite-state Markovianity of the random fading channels, and only ergodicity of the fading process suffices.

Theorem 2 exemplifies also a tradeoff between convergence speed and optimality. This is a well-known tradeoff especially in the adaptive signals and systems literature [21], [19], [23]. As in any stochastic approximation scheme, the Lagrange multipliers in (25) only converge to or hover within a small neighborhood with size proportional to the stepsize β around optimal $\boldsymbol{z}^*$; hence, one needs a small β to come "closer" to optimality, but the smaller β is chosen, the slower convergence speed is experienced.

The novel dual-based scheduling approach overcomes limitations of schedulers developed in [6], [4], [21], [13], [5], [15] for time-division systems and their extensions to OFDM networks, since: i) it remains operational for both differentiable and nondifferentiable utility functions and ii) its convergence to the globally optimal schedule is established for typical wireless channels with continuous fading. A similar on-line dual iteration was also developed for resource allocation in multi-antenna broadcasting [28], but no proof was provided for its convergence.

In the foregoing derivation, the "ensemble" scheduler implemented by Algorithm 1 seems as a middle step to the stochastic scheme. However, it is worth mentioning that both schemes have complementary strengths. Algorithms 1 and 2 are developed based on ensemble and stochastic gradient iterations, respectively. Compared with Algorithm 2, Algorithm 1 can take advantage of the channel cdf when known, to obtain the optimal Lagrange multipliers off-line; and uses them in the on-line phase to implement the optimal resource allocation and scheduling per slot with fairness and average rate guarantees. On the other hand, Algorithm 2 is capable of learning the channel statistics on-the-fly and Theorem 2 ensures that this algorithm converges asymptotically to the optimal strategy regardless of initialization. The latter implies that Algorithm 2 can also track even nonstationary channels, and thus provides robustness to e.g., network dynamics and user mobility. With the same algorithmic structure, Algorithms 1 and 2 can be also seamlessly integrated to take advantage of the available cdf information as well as remain robust to network dynamics, by playing roles similar to "estimation" and "tracking" in adaptive signal processing.

V. Numerical Tests

In this section, we provide numerical examples to verify the proposed schemes. We consider two-user OFDM downlink or uplink transmissions over frequency-selective wireless channels. The total bandwidth is 320 KHz, and there are 32 sub-carriers, each with sub-bandwidth 10 KHz. For each user's wireless link, a profile of 20 μs exponentially and independently decaying tap gains is assumed changing independently across slots of 500 μs.

Supposing average normalized SNRs $\bar{\gamma}_1 = \bar{\gamma}_2 = 8$ dBW, Fig. 2 shows the ergodic rate regions found by the proposed schemes for the OFDM downlink and uplink channels with: i) sum-power budget $\check{P} = 1$ Watt and individual power budgets; ii) $\check{P}_1 = \check{P}_2 = 0.5$ Watt; iii) $\check{P}_1 = 0.2$ Watt, $\check{P}_2 = 0.8$ Watt; and iv) $\check{P}_1 = 0.8$ Watt, $\check{P}_2 = 0.2$ Watt. (The receive SNR is $\bar{\gamma}_j$ dBW multiplied by the transmit-power measured in Watts.) Clearly, the individual average power constraints can be seen as realizations of the average sum-power constraint, i.e., $\check{P}_1 + \check{P}_2 = \check{P}$. Therefore, the downlink region contains the uplink regions,

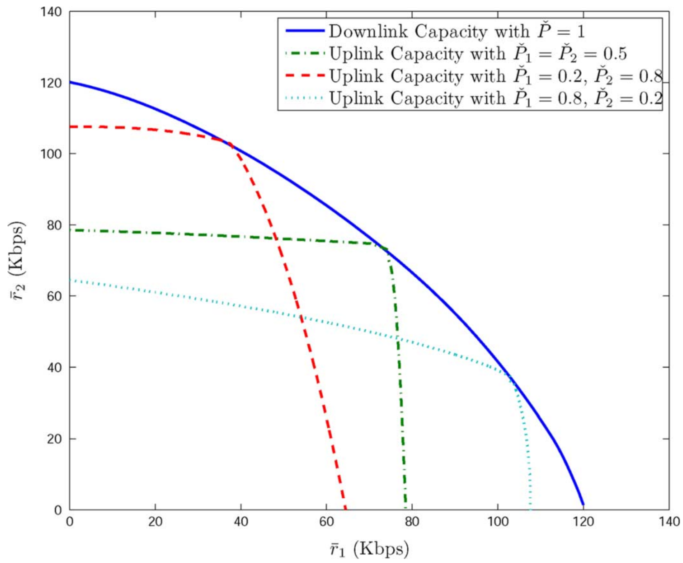

Fig. 2. Ergodic rate regions for 2-user OFDM systems when average SNRs for both users are 8 dB.

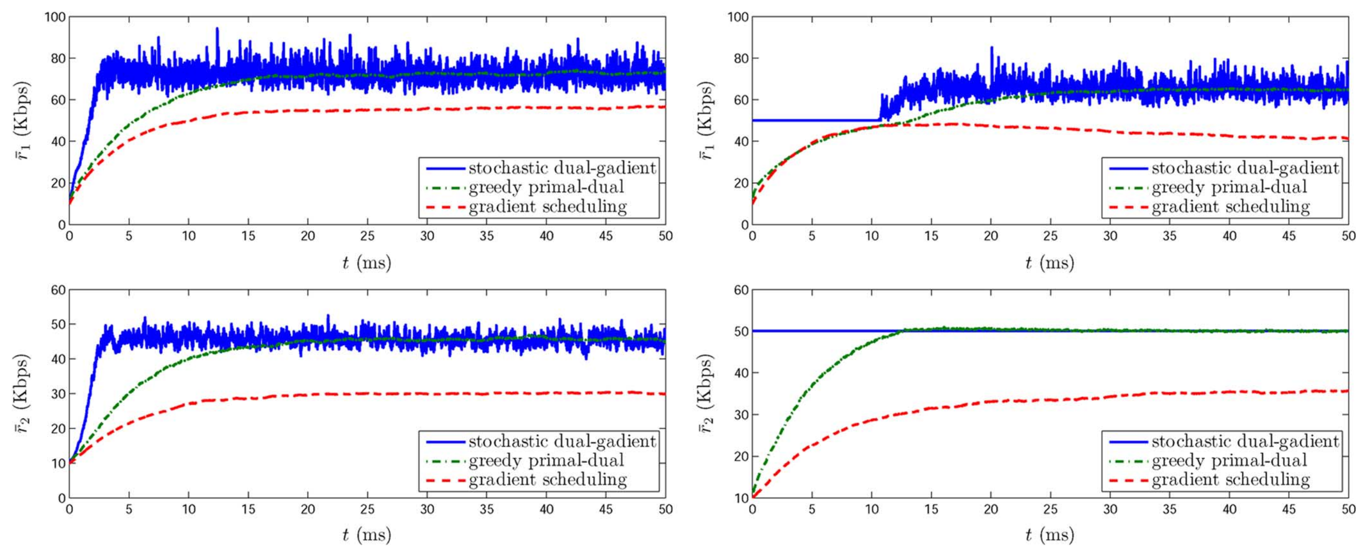

Fig. 3. Learning curves of Algorithm 2, GPD and GS schemes for BE traffic (*left*), and nRT traffic (*right*).

and each uplink region touches the downlink region at one point.

We next test the utility-based scheduling Algorithm 2 and compare it with existing alternatives in an OFDM downlink. The two baseline schemes chosen to compare with are the greedy primal-dual (GPD) scheme in [21] which indeed exhibits best performance among the existing opportunistic scheduling schemes, and the gradient scheduling (GS) algorithm of [6]. The GPD and GS schemes were introduced for time-division slotted networks. With these schemes modified appropriately for the OFDM context, we test them together with Algorithm 2 in an OFDM downlink where average normalized SNRs for users 1 and 2 are 8 dBW and 5 dBW, respectively. The utility function is selected as $U(\bar{\boldsymbol{r}}) = \ln(\bar{r}_1) + \ln(\bar{r}_2)$. We first consider that the two users support BE services (without rate requirements). Fig. 3 (*left*) compares the evolutions of average user rates for the three schemes. It is evident that the proposed stochastic dual-gradient and the GPD schemes converge to the same optimal $\bar{\boldsymbol{r}}^*$. Since the GS algorithm does not perform adaptive power allocation, it converges to a suboptimal rate vector strictly less than $\bar{\boldsymbol{r}}^*$. It is also seen that the stochastic dual-gradient algorithm converges faster but exhibits larger variation after convergence which also depends on the utility function adopted. This is because the trajectory of $\hat{\boldsymbol{\mu}}[n]$ is locked to $\boldsymbol{\mu}^*$ but the final average rate is obtained as $\nabla U^{-1}(\boldsymbol{\mu}^*)$.

We consider next that the two users require nRT services with minimum average rate constraints $\check{r}_1 = \check{r}_2 = 50$ Kbps. Fig. 3 (*right*) compares the average user rates for the stochastic dual-gradient, the GPD and the GS schemes in this case. Again, the stochastic dual-gradient and the GPD schemes converge to the optimal $\bar{\boldsymbol{r}}^*$ with both minimum rate requirements satisfied. The stochastic dual-gradient algorithm converges faster but exhibits larger variation after convergence than the GPD. Notice that due to the $\max$ operator in (20), the average user rate estimated by $\bar{r}_2^*(\hat{\boldsymbol{\mu}}[n])$ in Algorithm 2 is equal to 50 kbps from the beginning and stays there afterwards since this is the optimal value upon convergence. Following [5], the GS algorithm is also

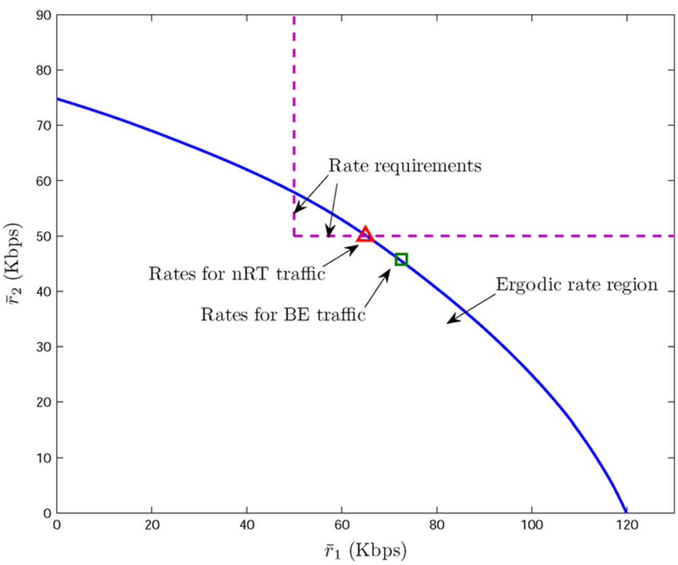

Fig. 4. Ergodic rate region (benchmark) and achieved rates with stochastic dual-gradient scheme for BE and nRT traffic.

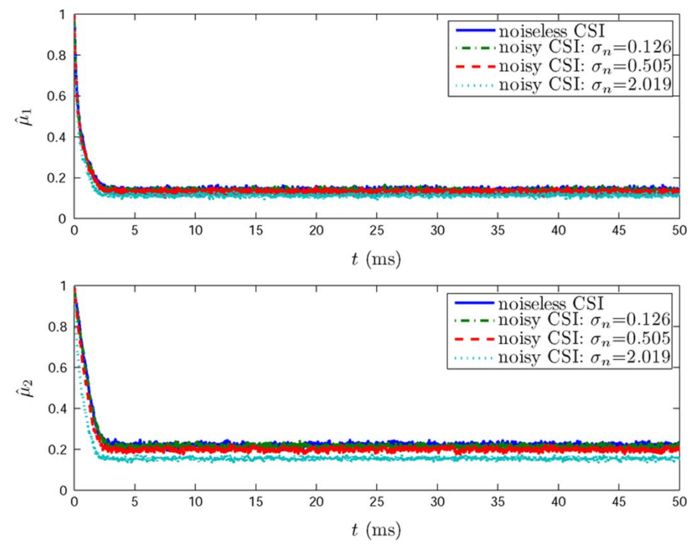

Fig. 5. Evolutions of Lagrange multipliers with imperfect CSI.

possible to guarantee the minimum average rate requirements, if feasible, using a token-based approach. Although such a scheme is employed in the simulations, Fig. 3 (*right*) demonstrates that $\check{\mathbf{r}}$ cannot be met by the GS. This is because without performing adaptive power allocation in the GS, the rate requirements become infeasible for the given average SNRs since spectral and temporal diversity have not been fully exploited.

Additional evidence on the merits of the stochastic dual-gradient scheduling is provided by Fig. 4, which depicts the ergodic rate region for the investigated OFDM downlink as well as the resultant average user rates by the stochastic dual-gradient scheme at the end of the simulations for the aforementioned BE and nRT traffic. The optimality of the stochastic dual-gradient algorithm is confirmed by the fact that the achieved rate vectors settle at boundary points of the ergodic rate region. The minimum rates are not enforced, and are thus not guaranteed in the resultant user rate vector for BE services. Notice however, that except for the minimum rate constraints, the stochastic dual-gradient algorithms for BE or nRT traffic solve the same utility maximization problem [cf. (14)]. Therefore, taking into account the minimum rate constraints, the nRT solution should be the projection of the BE solution onto the feasible set dictated by the area between the boundaries of the ergodic rate region and the two dashed lines $\check{r}_1 = 50$ Kbps and $\check{r}_2 = 50$ Kbps. This is clearly seen in Fig. 4, which further corroborates the optimality of the stochastic dual-gradient scheduling.

Lastly, we study the impact of the imperfect CSI on the proposed algorithm. To account for the estimation/feedback errors, suppose that the CSI per user acquired by the AP is corrupted by a circularly symmetric Gaussian noise with variance σ_n. Fig. 5 shows the evolutions of Lagrange multipliers $\hat{\mu}_j$, $j = 1, 2$, with the Algorithm 2 for the case considered by Fig. 3 (*left*). Adding a noise term to the CSI changes the distribution of the fading coefficients. The stochastic iteration (22) then simply learns the so-adjusted cdf of $\boldsymbol{\gamma}$ and converges to a different "optimal" point. It is seen from Fig. 5 that the impact of noisy CSI on the proposed algorithm is noticeable only when the noise variance is large; i.e., $\sigma_n = 2.019$ in which case the noisy CSI has a SNR of 4.95 dB and of 1.95 dB for users 1 and 2, respectively. This demonstrates the robustness of the proposed stochastic dual-gradient algorithm to the imperfect CSI.

VI. Conclusion

Relying on convex optimization and stochastic approximation tools, a novel approach was introduced for solving the joint subcarrier, power and rate allocation for multiuser OFDM systems. Based on this approach, the ergodic rate region of wireless multiuser OFDM channels was characterized, and corresponding optimal resource allocation schemes were developed in closed form. In addition, a unifying framework was presented for designing and analyzing utility-based stochastic scheduling schemes for OFDM networks with provable optimality and prescribed rate guarantees. The merits of the proposed stochastic schemes were corroborated by simulations. Robustness of the proposed schemes to noisy CSI was also illustrated using simulated tests. In future work, it will be interesting to further study the impact of delayed CSI encountered with mobile settings as in the IEEE 801.16 and 3GPP networks.

Appendix
Proof of Propositions and Lemmas

A. *Proof of Proposition 1*

To prove the concavity of the function $c_{j,k}$, it suffices to show for a convex combination $(\alpha_c, p_c) = \theta(\alpha_a, p_a) + (1-\theta)(\alpha_b, p_b)$ with $\theta \in (0,1)$, that $c_{j,k}(\alpha_c, p_c) \geq \theta c_{j,k}(\alpha_a, p_a) + (1-\theta)c_{j,k}(\alpha_b, p_b)$. To this end, consider the following three cases.

1) $\alpha_a > 0$ and $\alpha_b > 0$: In this case, $\alpha_c > 0$ and $c_{j,k}(\alpha, p) = \alpha \log_2(1+\gamma_{j,k}p/\alpha)$. Because $\log_2(1+\gamma_{j,k}p)$ is a concave function of p, it follows that its *perspective* $\alpha \log_2(1+\gamma_{j,k}p/\alpha)$ is jointly concave in α and p when

$\alpha > 0$ [8, p. 89]. Using the latter, it follows readily that for $\alpha_a,\ \alpha_b,\ \alpha_c > 0$

$$\alpha_c \log_2(1+\gamma_{j,k}p_c/\alpha_c) \geq \theta\alpha_a \log_2(1+\gamma_{j,k}p_a/\alpha_a) + (1-\theta)\alpha_b \log_2(1+\gamma_{j,k}p_b/\alpha_b)$$

and thus $c_{j,k}(\alpha_c,p_c) \geq \theta c_{j,k}(\alpha_a,p_a)+(1-\theta)c_{j,k}(\alpha_b,p_b)$.

2) $\alpha_a = 0$ and $\alpha_b > 0$: Since $c_{j,k}(\alpha_a,p_a)=0$, we have $\theta c_{j,k}(\alpha_a,p_a)+(1-\theta)c_{j,k}(\alpha_b,p_b) = (1-\theta)\alpha_b\log_2(1+\gamma_{j,k}p_b/\alpha_b)$. On the other hand, with $\alpha_c=(1-\theta)\alpha_b>0$

$$\begin{aligned} c_{j,k}(\alpha_c,p_c) &= \alpha_c\log_2(1+\gamma_{j,k}p_c/\alpha_c)\\ &= (1-\theta)\alpha_b\log_2\left(1+\gamma_{j,k}\frac{\theta p_a+(1-\theta)p_b}{(1-\theta)\alpha_b}\right)\\ &= (1-\theta)\alpha_b\log_2\left(1+\frac{\gamma_{j,k}\theta p_a}{(1-\theta)\alpha_b}+\frac{\gamma_{j,k}p_b}{\alpha_b}\right)\\ &\geq (1-\theta)\alpha_b\log_2(1+\gamma_{j,k}p_b/\alpha_b). \end{aligned}$$

It then follows that $c_{j,k}(\alpha_c,p_c) \geq \theta c_{j,k}(\alpha_a,p_a)+(1-\theta)c_{j,k}(\alpha_b,p_b)$.

3) $\alpha_a = \alpha_b = 0$: It is clear that $c_{j,k}(\alpha_c,p_c) = \theta c_{j,k}(\alpha_a,p_a)+(1-\theta)c_{j,k}(\alpha_b,p_b)=0$.

From 1)–3), it always holds that $c_{j,k}(\alpha_c,p_c) \geq \theta c_{j,k}(\alpha_a,p_a)+(1-\theta)c_{j,k}(\alpha_b,p_b)$; and thus, $c_{j,k}(\alpha,p)$ is a jointly concave function in α and p.

Based on the concavity of $c_{j,k}(\alpha,p)$, we can now proceed to prove the convexity of $\mathcal{C}$. For any two $\bar{\boldsymbol{r}}_a\in\mathcal{C}$ and $\bar{\boldsymbol{r}}_b\in\mathcal{C}$, there exist two allocation policies $(\boldsymbol{\alpha}_a,\boldsymbol{p}_a)\in\mathcal{F}$, $(\boldsymbol{\alpha}_b,\boldsymbol{p}_b)\in\mathcal{F}$ such that $\bar{\boldsymbol{r}}_a\leq\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_a,\boldsymbol{p}_a)$ and $\bar{\boldsymbol{r}}_b\leq\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_b,\boldsymbol{p}_b)$. The convex combination $(\boldsymbol{\alpha}_c,\boldsymbol{p}_c):=\theta(\boldsymbol{\alpha}_a,\boldsymbol{p}_a)+(1-\theta)(\boldsymbol{\alpha}_b,\boldsymbol{p}_b)$, with $\theta\in(0,1)$, satisfies $(\boldsymbol{\alpha}_c,\boldsymbol{p}_c)\in\mathcal{F}$ since it belongs to $\mathcal{A}$ and obeys the average sum-power constraint. Furthermore, since $c_{j,k}(\alpha_{j,k},p_{j,k})$ is a concave function of $(\alpha_{j,k},p_{j,k})$, it clearly holds that $\bar{r}_j(\boldsymbol{\alpha},\boldsymbol{p})=\mathbb{E}_{\boldsymbol{\gamma}}[\sum_{k=1}^{K}c_{j,k}(\alpha_{j,k}(\boldsymbol{\gamma}),p_{j,k}(\boldsymbol{\gamma}))]$ is a concave function of $(\boldsymbol{\alpha},\boldsymbol{p})$. This implies that $\theta\bar{r}_j(\boldsymbol{\alpha}_a,\boldsymbol{p}_a)+(1-\theta)\bar{r}_j(\boldsymbol{\alpha}_b,\boldsymbol{p}_b)\leq\bar{r}_j(\theta\boldsymbol{\alpha}_a+(1-\theta)\boldsymbol{\alpha}_b,\theta\boldsymbol{p}_a+(1-\theta)\boldsymbol{p}_b):=\bar{r}_j(\boldsymbol{\alpha}_c,\boldsymbol{p}_c)$, and thus $\theta\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_a,\boldsymbol{p}_a)+(1-\theta)\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_b,\boldsymbol{p}_b)\leq\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_c,\boldsymbol{p}_c)$. We then have $\theta\bar{\boldsymbol{r}}_a+(1-\theta)\bar{\boldsymbol{r}}_b\leq\theta\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_a,\boldsymbol{p}_a)+(1-\theta)\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_b,\boldsymbol{p}_b)\leq\bar{\boldsymbol{r}}(\boldsymbol{\alpha}_c,\boldsymbol{p}_c)$. Since $(\boldsymbol{\alpha}_c,\boldsymbol{p}_c)\in\mathcal{F}$, any convex combination $\theta\bar{\boldsymbol{r}}_a+(1-\theta)\bar{\boldsymbol{r}}_b$ of two vectors $\bar{\boldsymbol{r}}_a,\bar{\boldsymbol{r}}_b\in\mathcal{C}$ must also belong to $\mathcal{C}$. The convexity of $\mathcal{C}$ thus follows readily.

B. Proof of Lemma 1

To prove the lemma, we need the following two properties.

Property B.1: For any $\lambda\geq 0$, it holds that $\alpha\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\geq\varphi_{j,k}(\lambda,\alpha,p)$, $\forall\alpha\geq 0$ and $\forall p\geq 0$.

Proof: Consider the following two cases:

1) If $\alpha>0$, substituting (2) into (7) yields

$$\varphi_{j,k}(\lambda,\alpha,p)=w_j\alpha\log_2\left(1+\gamma_{j,k}\frac{p}{\alpha}\right)-\lambda p.$$

Upon defining $\tilde{p}:=p/\alpha$, the latter can be rewritten as

$$\varphi_{j,k}(\lambda,\alpha,p)=\alpha[w_j\log_2(1+\gamma_{j,k}\tilde{p})-\lambda\tilde{p}]\ :=\alpha\tilde{\varphi}_{j,k}(\lambda,\tilde{p})$$

where $\tilde{\varphi}_{j,k}(\lambda,\tilde{p}) := w_j\log_2(1+\gamma_{j,k}\tilde{p})-\lambda\tilde{p}$. Since $\tilde{\varphi}_{j,k}(\lambda,\tilde{p})$ is a concave function of $\tilde{p}\geq 0$, the optimal $\tilde{p}^*_{j,k}(\boldsymbol{\gamma})$ maximizing $\tilde{\varphi}_{j,k}(\lambda,\tilde{p})$ is given by the water-filling formula

$$\tilde{p}^*_{j,k}(\lambda,\boldsymbol{\gamma})=\left[\frac{w_j}{\lambda\ln 2}-\frac{1}{\gamma_{j,k}}\right]^+,\quad\forall j,k. \tag{29}$$

Substituting the latter into $\tilde{\varphi}_{j,k}(\lambda,\tilde{p})$ yields the link quality indicator in (10); i.e.,

$$\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})=\tilde{\varphi}_{j,k}(\lambda,\tilde{p}^*_{j,k}(\lambda,\boldsymbol{\gamma})). \tag{30}$$

It thus holds that $\alpha\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\geq\alpha\tilde{\varphi}_{j,k}(\lambda,\tilde{p})=\varphi_{j,k}(\lambda,\alpha,p)$, $\forall p\geq 0$.

2) If $\alpha=0$, it clearly holds that $\alpha\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})=0$. On the other hand, (7) and (2) imply that $\varphi_{j,k}(\lambda,\alpha,p)=-\lambda p\leq 0$, $\forall p\geq 0$. Therefore, $\alpha\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\geq\varphi_{j,k}(\lambda,\alpha,p)$, $\forall p\geq 0$.

Cases 1) and 2) together imply that $\alpha\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\geq\varphi_{j,k}(\lambda,\alpha,p)$, $\forall\alpha\geq 0,\forall p\geq 0$. □

Property B.2: For any λ, it holds that $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\geq 0$, $\forall\boldsymbol{\gamma}$, and $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})>0$ if $\gamma_{j,k}>\lambda\ln 2/w_j$.

Proof: Differentiating (10) yields

$$\frac{\partial\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})}{\partial\gamma_{j,k}}=\begin{cases}\frac{1}{\gamma_{j,k}}\left(\frac{w_j}{\ln 2}-\frac{\lambda}{\gamma_{j,k}}\right), & \text{if } \gamma_{j,k}>\frac{\lambda\ln 2}{w_j}\\ 0, & \text{if } \gamma_{j,k}\leq\frac{\lambda\ln 2}{w_j}\end{cases} \tag{31}$$

and thus $\frac{\partial\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})}{\partial\gamma_{j,k}}>0$, $\forall\gamma_{j,k}>\lambda\ln 2/w_j$. Since $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})$ is a continuous function of $\gamma_{j,k}$ and $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})=0$, $\forall\gamma_{j,k}\leq\lambda\ln 2/w_j$, the property follows readily. □

We are now ready to prove Lemma 1 based on Properties B.1 and B.2. With winner user index $j^*_k(\lambda,\boldsymbol{\gamma})$ defined in (11), it holds for each fading state $\boldsymbol{\gamma}$ that

$$\begin{aligned}\sum_{j=1}^{J}\sum_{k=1}^{K}\varphi_{j,k}(\lambda,\alpha_{j,k}(\boldsymbol{\gamma}),p_{j,k}(\boldsymbol{\gamma}))&\leq\sum_{j=1}^{J}\sum_{k=1}^{K}\alpha_{j,k}(\boldsymbol{\gamma})\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\\ &\leq\sum_{k=1}^{K}\left(\varphi^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})\sum_{j=1}^{J}\alpha_{j,k}(\boldsymbol{\gamma})\right)\leq\sum_{k=1}^{K}\varphi^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})\end{aligned}$$

where the first inequality is due to Property B.1; the second inequality is due to the definition of $j^*_k(\lambda,\boldsymbol{\gamma}) := \arg\max_j\ \varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})$ in (11); and the third one is due to the facts that $\varphi^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})\geq 0$ from Property B.2 and $\sum_{j=1}^{J}\alpha_{j,k}(\boldsymbol{\gamma})\leq 1$ $\forall k$. Furthermore, the equality can be achieved using the allocation $(\boldsymbol{\alpha}^*(\lambda,\boldsymbol{\gamma}),\boldsymbol{p}^*(\lambda,\boldsymbol{\gamma}))$ specified in (11), which is thus optimal for (8).

To show the almost sure uniqueness of $(\boldsymbol{\alpha}^*(\lambda,\boldsymbol{\gamma}),\boldsymbol{p}^*(\lambda,\boldsymbol{\gamma}))$, consider this optimal allocation at subcarrier k for the following three cases (recall that $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})\geq 0,\forall j,k$).

1) If $\max_j\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})=0$, we must have $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})=0$ $\forall j$, and all users are "winners". This is the case when all users' channels experience deep fading over subcarrier k such that $\gamma_{j,k}\leq\lambda\ln 2/w_j$, $\forall j$ [cf. (10)]. Upon such a deep fading state, in fact any user j, if scheduled, will be allocated with transmit-power $\tilde{p}^*_{j,k}(\lambda,\boldsymbol{\gamma})=0$ [cf. (29)].

Therefore, the *unique* optimal strategy for AP is to defer its transmission at subcarrier k, which can be represented by the policy in (11) where the subcarrier is assigned to a randomly chosen "winner" but zero transmit-power is allocated.

2) If $\max_j \varphi^*_{j,k}(\lambda,\boldsymbol{\gamma}) > 0$ and it is attained by a single winner, then the optimal allocation given by (11) is clearly unique.

3) If $\max_j \varphi^*_{j,k}(\lambda,\boldsymbol{\gamma}) > 0$ and it is attained by multiple users, then assigning the entire subcarrier k to a randomly chosen winner or allowing (arbitrary) time-sharing among multiple winners is optimal for maximizing the dual function in (8). However, since $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma})$ is an increasing function of $\gamma_{j,k}$ when it is greater than zero [cf. (31)], the event that two winners k and $k' \neq k$ have identical but nonzero net reward; i.e., event $\{\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma}) - \varphi^*_{j,k'}(\lambda,\boldsymbol{\gamma}) \equiv 0\}$ must have Lebesgue-measure zero when the fading process has a continuous cdf. Likewise, having more than two "winners" tie is also a measure zero event. Hence, the nonuniqueness of $(\boldsymbol{\alpha}^*(\lambda,\boldsymbol{\gamma}),\boldsymbol{p}^*(\lambda,\boldsymbol{\gamma}))$ upon such events has "measure-zero" effect.

Combining 1)–3), we readily deduce the almost sure uniqueness of $(\boldsymbol{\alpha}^*(\lambda,\boldsymbol{\gamma}),\boldsymbol{p}^*(\lambda,\boldsymbol{\gamma}))$.

C. Proof of Lemma 2

We first prove that $p^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})$ is nonincreasing in λ, $\forall k$ and $\forall\boldsymbol{\gamma}$, by considering two cases.

1) If $j^*_k(\lambda_1,\boldsymbol{\gamma}) = j^*_k(\lambda_2,\boldsymbol{\gamma}) = j$; i.e., we have the same winner at subcarrier k for λ_1 and λ_2, then $p^*_{j^*_k,k}(\lambda_1,\boldsymbol{\gamma}) = [\frac{w_j}{\lambda_1 \ln 2} - \frac{1}{\gamma_{j,k}}]^+$ and $p^*_{j^*_k,k}(\lambda_2,\boldsymbol{\gamma}) = [\frac{w_j}{\lambda_2 \ln 2} - \frac{1}{\gamma_{j,k}}]^+$. In this case, it is clear that $p^*_{j^*_k,k}(\lambda_1,\boldsymbol{\gamma}) \geq p^*_{j^*_k,k}(\lambda_2,\boldsymbol{\gamma})$ for $\lambda_1 < \lambda_2$.

2) If $j^*_k(\lambda_1,\boldsymbol{\gamma}) = j$ and $j^*_k(\lambda_2,\boldsymbol{\gamma}) = j' \neq j$, the winner selection rule in (11) implies that

$$\begin{aligned}\tilde{\varphi}_{j,k}(\lambda_1,\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})) &= \varphi^*_{j,k}(\lambda_1,\boldsymbol{\gamma})\\ &\geq \varphi^*_{j',k}(\lambda_1,\boldsymbol{\gamma}) = \tilde{\varphi}_{j',k}(\lambda_1,\tilde{p}^*_{j',k}(\lambda_1,\boldsymbol{\gamma})) \qquad (32)\\ \tilde{\varphi}_{j,k}(\lambda_2,\tilde{p}^*_{j,k}(\lambda_2,\boldsymbol{\gamma})) &= \varphi^*_{j,k}(\lambda_2,\boldsymbol{\gamma})\\ &\leq \varphi^*_{j',k}(\lambda_2,\boldsymbol{\gamma}) = \tilde{\varphi}_{j',k}(\lambda_2,\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})) \qquad (33)\end{aligned}$$

where $\varphi^*_{j,k}(\lambda,\boldsymbol{\gamma}) = \tilde{\varphi}_{j,k}(\lambda,\tilde{p}^*_{j,k}(\lambda,\boldsymbol{\gamma}))$ from (30). Now recognizing that $\tilde{p}^*_{j',k}(\lambda_1,\boldsymbol{\gamma})$ and $\tilde{p}^*_{j,k}(\lambda_2,\boldsymbol{\gamma})$ maximize $\tilde{\varphi}_{j',k}(\lambda_1,\tilde{p})$ and $\tilde{\varphi}_{j,k}(\lambda_2,\tilde{p})$, respectively, $\forall p \geq 0$, we further deduce from (32) and (33) that

$$\begin{aligned}\tilde{\varphi}_{j,k}(\lambda_1,\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})) &\geq \tilde{\varphi}_{j',k}(\lambda_1,\tilde{p}^*_{j',k}(\lambda_1,\boldsymbol{\gamma}))\\ &\geq \tilde{\varphi}_{j',k}(\lambda_1,\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})) \qquad (34)\\ \tilde{\varphi}_{j,k}(\lambda_2,\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})) &\leq \tilde{\varphi}_{j,k}(\lambda_2,\tilde{p}^*_{j,k}(\lambda_2,\boldsymbol{\gamma}))\\ &\leq \tilde{\varphi}_{j',k}(\lambda_2,\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})). \qquad (35)\end{aligned}$$

Substituting $\tilde{\varphi}_{j,k}(\lambda,\tilde{p}) = w_j \log_2(1+\gamma_{j,k}\tilde{p}) - \lambda\tilde{p}$ into the leftmost and rightmost terms in (34) and (35), yields

$$\begin{aligned}&w_j \log_2(1+\gamma_{j,k}\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})) - \lambda_1\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})\\ &\quad\geq w_{j'} \log_2(1+\gamma_{j',k}\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})) - \lambda_1\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma}),\\ &w_j \log_2(1+\gamma_{j,k}\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})) - \lambda_2\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})\\ &\quad\leq w_{j'} \log_2(1+\gamma_{j',k}\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})) - \lambda_2\tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma}).\end{aligned}$$

Subtracting the latter two inequalities implies that $(\lambda_1 - \lambda_2)(\tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma}) - \tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})) \leq 0$. But since $j^*_k(\lambda_1,\boldsymbol{\gamma}) = j$, $j^*_k(\lambda_2,\boldsymbol{\gamma}) = j'$ imply $p^*_{j^*_k,k}(\lambda_1,\boldsymbol{\gamma}) = \tilde{p}^*_{j,k}(\lambda_1,\boldsymbol{\gamma})$, $p^*_{j^*_k,k}(\lambda_2,\boldsymbol{\gamma}) = \tilde{p}^*_{j',k}(\lambda_2,\boldsymbol{\gamma})$, we clearly have $p^*_{j^*_k,k}(\lambda_1,\boldsymbol{\gamma}) \geq p^*_{j^*_k,k}(\lambda_2,\boldsymbol{\gamma})$ for $\lambda_1 < \lambda_2$.

Cases 1) and 2) prove the wanted nonincreasing property of $p^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})$. The nonincreasing property of $\bar{P}(\lambda)$ then follows from that of $p^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})$ upon taking expectation. In fact, taking into account the strict concavity of the $\log_2$ function, we readily see that $p^*_{j^*_k,k}(\lambda,\boldsymbol{\gamma})$ and $\bar{P}(\lambda)$ are strictly decreasing in λ when they are greater than zero.

D. Proof of Lemma 3

To prove the wanted locking between primary and averaged trajectories, it suffices to verify that the five conditions (9.2A1)–(9.2A5) in [19, Theorem 9.1] are satisfied. These conditions with our notational conventions are:

(C1) In the primary system (26), $\mathbb{E}_{\boldsymbol{\gamma}[n]}[\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\gamma}[n])]$, is time-invariant.

(C2) For $\|\boldsymbol{z}\| \leq B_z$ in the averaged system (27), it holds that $\|\bar{\boldsymbol{g}}(\boldsymbol{z})\| \leq B_g$.

(C3) Initialization $\boldsymbol{z}[0]$ is small enough so that iterates of the averaged system remain bounded; i.e., $\|\boldsymbol{z}[n]\| \leq B_z$ for $n \in [1, T/\beta]$.

(C4) Function $\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\gamma}[n])$ obeys a stochastic Lipschitz condition; i.e.,

$$\|\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\gamma}[n]) - \boldsymbol{g}(\boldsymbol{z}',\boldsymbol{\gamma}[n])\| \leq L_g[n]\|\boldsymbol{z}-\boldsymbol{z}'\|, \quad \forall\, \|\boldsymbol{z}\|, \|\boldsymbol{z}'\| \qquad (36)$$

where $L_g[n]$ is a random sequence obeying $N^{-1}\sum_{n=1}^{N} L_g[n] \to L_g$ w.p. 1, as $N \to \infty$.

(C5) The total deviation $\Delta(N,\boldsymbol{z}) := \sum_{n=1}^{N}(\boldsymbol{g}(\boldsymbol{z},\boldsymbol{\gamma}[n]) - \bar{\boldsymbol{g}}(\boldsymbol{z}))$, is also stochastic Lipschitz; i.e.,

(a) $\|\Delta(N,\boldsymbol{z})\| \leq B_\Delta[N]$; and (b) $\|\Delta(N,\boldsymbol{z}) - \Delta(N,\boldsymbol{z}')\| \leq L_\Delta[N]\|\boldsymbol{z}-\boldsymbol{z}'\|$, $\forall\, \|\boldsymbol{z}\|, \|\boldsymbol{z}'\|$, where as $N \to \infty$, $B_\Delta[N]/N \to 0$ and $L_\Delta[N]/N \to 0$ w.p. 1.

Condition (C1) clearly holds since the fading process is assumed stationary and ergodic.

To check (C2), notice that all the average transmit-rates and powers in $\bar{\boldsymbol{g}}(\boldsymbol{z})$ are bounded if $\lambda \neq 0$. But since $\lambda^* > 0$ from Theorem 1, we can select a small $\epsilon > 0$, and restrict the iterates of $\lambda[n]$ within the positive interval $[\epsilon, \infty)$. Then (C.2) is satisfied for a sufficiently large B_g.

To establish (C3), one needs to show first that a finite optimal Lagrange multiplier vector $\boldsymbol{z}^*$ always exists. For notational brevity, let $\boldsymbol{X}$ collect all the primal optimization variables. Writing also the constraints in (15) in a compact form $\boldsymbol{h}(\boldsymbol{X}) \geq 0$, there must also exist under (A1) a strictly feasible $\boldsymbol{X}_0$ such that $\boldsymbol{h}(\boldsymbol{X}_0) \geq C$ for a constant $C > 0$. By definition, the dual function in (18) is $D(\boldsymbol{z}) = \max_{\boldsymbol{X}}[U(\boldsymbol{X}) + \boldsymbol{z}^T h(\boldsymbol{X})]$. Therefore, it holds that

$$D(\boldsymbol{z}) \geq U(\boldsymbol{X}_0) + \boldsymbol{z}^T h(\boldsymbol{X}_0) \geq U(\boldsymbol{X}_0) + C\boldsymbol{z}^T\mathbf{1}$$

which readily implies that $\boldsymbol{z} \leq \frac{1}{C}(D(\boldsymbol{z}) - U(\boldsymbol{X}_0))$ for any $\boldsymbol{z}$; thus, $\boldsymbol{z}^* \leq \frac{1}{C}(D(\boldsymbol{z}^*) - U(\boldsymbol{X}_0))$. The boundedness of $\boldsymbol{z}^*$ now follows immediately from the finite value of $D(\boldsymbol{z}^*)$, which is equal to $U(\bar{\boldsymbol{r}}^*)$ because the duality gap is zero. Now since $\boldsymbol{z}[n]$ converges to $\boldsymbol{z}^*$ under (A1), there exists a finite $N_0(\beta,\delta)$ for a stepsize β, and any constant δ, such that

$$\|\boldsymbol{z}[n]-\boldsymbol{z}^*\|\le\delta,\quad \forall n\ge N_0(\beta,\delta).$$

This implies that $\|\boldsymbol{z}[n]\|\le\|\boldsymbol{z}^*\|+\delta\ \forall n\ge N_0(\beta,\delta)$. Since $\boldsymbol{z}[n]=\boldsymbol{z}[0]+\beta\sum_{\nu=1}^{n-1}\bar{\boldsymbol{g}}(\boldsymbol{z}[\nu])$ and $\|\bar{\boldsymbol{g}}(\boldsymbol{z})\|\le B_g$, it is also easy to see that $\|\boldsymbol{z}[n]\|\le\|\boldsymbol{z}^*\|+\delta+\beta N_0(\beta,\delta)B_g, \forall n<N_0(\beta,\delta)$. Overall, we must have $\|\boldsymbol{z}[n]\|\le\|\boldsymbol{z}^*\|+\delta+\beta N_0(\beta,\delta)B_g, \forall n$. By selecting a δ to minimize $\delta+\beta N_0(\beta,\delta)B_g$ for a given β, the tightest bound is obtained. Letting B_z denote this bound, it follows that $\|\boldsymbol{z}[n]\|\le B_z$ for any $n\in[1,T/\beta]$. This holds for any (arbitrarily large) T.

Checking the validity of (C4) and (C5) is nontrivial. We next only detail those for the iterations related to the power price λ, whereas the rest can be verified using similar arguments. To this end, we first establish the ensuing property:

Property D.1: Function $\bar{g}_\lambda(\boldsymbol{z})$ is Lipschitz; i.e., $|\bar{g}_\lambda(\lambda)-\bar{g}_\lambda(\lambda')|\le L_{\lambda,g}|\lambda-\lambda'|$ for a constant $L_{\lambda,g}$.

Proof: By definition, we have

$$\bar{g}_\lambda(\boldsymbol{z})=\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_k p^*_{j_k^*,k}(\boldsymbol{z},\boldsymbol{\gamma})\right]-\check{P}=\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j,k} p^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})\right]-\check{P}$$

where [cf. Lemma 1]

$$p^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})=\begin{cases}\left[\frac{\mu_j}{\lambda\ln 2}-\frac{1}{\gamma_{j,k}}\right]^+, & \text{if } \varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})\ge\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}),\ \forall j'\\ 0, & \text{otherwise.}\end{cases}\quad (37)$$

Recall that when $\gamma_{j,k}<\frac{\lambda\ln 2}{\mu_j}$, it holds that $p^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})=0$, by the definition of the $[\cdot]^+$ operator. Define $\forall j,k$ the set

$$\Gamma_{j,k}:=\{\boldsymbol{\gamma}\ :\ \gamma_{j,k}\ge\lambda\ln 2/\mu_j\}$$

. Using the latter and (37), we can express $\bar{g}_\lambda(\boldsymbol{z})$ as a product of unit step functions $\mathcal{U}(\cdot)$

$$\begin{aligned}\bar{g}_\lambda(\boldsymbol{z})&=\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j,k}\left\{\left(\prod_{j'\ne j}\mathcal{U}(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))\right)\times\left[\frac{\mu_j}{\lambda\ln 2}-\frac{1}{\gamma_{j,k}}\right]^+\right\}\right]-\check{P}\\&=\sum_{j,k}\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma_{j,k}}\left[\left(\prod_{j'\ne j}\mathcal{U}(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))\right)\times\left[\frac{\mu_j}{\lambda\ln 2}-\frac{1}{\gamma_{j,k}}\right]\right]-\check{P}\end{aligned}\quad (38)$$

where taking the expectation over $\boldsymbol{\gamma}\in\Gamma_{j,k}$ allows removing the $[\cdot]^+$ operator. Since the derivative of the step function $\mathcal{U}$ is the Dirac delta function δ, and all other terms in (38) are continuously differentiable, Leibniz's rule implies that the first-order derivative of $\bar{g}_\lambda(\boldsymbol{z})$ w.r.t. λ is

$$\begin{aligned}\dot{\bar{g}}_\lambda(\lambda)=&\sum_{j,k}\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma_{j,k}}\left[\left(\prod_{j'\ne j}\mathcal{U}(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))\right)\frac{\mu_j}{\lambda^2\ln 2}\right]\\&+\sum_{j,k}\mathbb{E}_{\boldsymbol{\gamma}\in\Gamma_{j,k}}\left[\sum_{j''\ne j}\left\{\delta(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))\right.\right.\\&\times\left(\frac{\partial\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})}{\partial\lambda}-\frac{\partial\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma})}{\partial\lambda}\right)\\&\left.\left.\times\left[\prod_{j'\ne j,j''}\mathcal{U}(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))\right]\left[\frac{\mu_j}{\lambda\ln 2}-\frac{1}{\gamma_{j,k}}\right]\right\}\right].\end{aligned}$$

Clearly, all terms except the delta functions are bounded (for $\lambda\ge\epsilon$ specified in proving (C2)), and the sums over j,k and j' are finite. Therefore, $\dot{\bar{g}}_\lambda(\lambda)$ is uniformly bounded if we can show that $\mathbb{E}_{\boldsymbol{\gamma}}[\delta(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))]$ is bounded. If we introduce a continuous random variable $d\equiv\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma})$, it can be shown that d has continuous distribution since $\boldsymbol{\gamma}$ has a continuous cdf and function $\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})$ is monotonic w.r.t. $\gamma_{j,k}$ (from Property B.2). This then implies that $\mathbb{E}_{\boldsymbol{\gamma}}[\delta(\varphi^*_{j,k}(\boldsymbol{z},\boldsymbol{\gamma})-\varphi^*_{j',k}(\boldsymbol{z},\boldsymbol{\gamma}))]$ is given by the bounded density function of d at 0. This shows the uniform boundedness of $\dot{\bar{g}}_\lambda(\lambda)$, and in turn the Lipschitz continuity of $\bar{g}_\lambda(\boldsymbol{z})$. □

We are now ready to check (C4). Recall that $g_\lambda(\boldsymbol{z},\boldsymbol{\gamma}[n])=\sum_k p^*_{j_k^*,k}(\boldsymbol{z},\boldsymbol{\gamma}[n])-\check{P}$. Assuming w.l.o.g. that $\lambda\le\lambda'$, non-increasing of $p^*_{j_k^*,k}(\boldsymbol{z},\boldsymbol{\gamma})$ in λ from Lemma 2 implies that $|g_\lambda(\lambda,\boldsymbol{\gamma}[n])-g_\lambda(\lambda',\boldsymbol{\gamma}[n])|=g_\lambda(\lambda,\boldsymbol{\gamma}[n])-g_\lambda(\lambda',\boldsymbol{\gamma}[n])$. Therefore, we have

$$\begin{aligned}N^{-1}\sum_{n=1}^N|g_\lambda(\lambda,\boldsymbol{\gamma}[n])-g_\lambda(\lambda',\boldsymbol{\gamma}[n])|=\\N^{-1}\sum_{n=1}^N[g_\lambda(\lambda,\boldsymbol{\gamma}[n])-g_\lambda(\lambda',\boldsymbol{\gamma}[n])]=\bar{g}_\lambda(\lambda)-\bar{g}_\lambda(\lambda')\end{aligned}$$

as $N\to\infty$ since the fading process is stationary and ergodic. Because $\bar{g}_\lambda(\lambda)$ is Lipschitz continuous from Property D.1, it readily follows that $|\bar{g}_\lambda(\lambda)-\bar{g}_\lambda(\lambda')|=\bar{g}_\lambda(\lambda)-\bar{g}_\lambda(\lambda')\le L_{\lambda,g}|\lambda-\lambda'|$. And there must exist a random sequence $L_{\lambda,g}[n]$ satisfying (36), and $N^{-1}\sum_{n=1}^N L_{\lambda,g}[n]\to L_{\lambda,g}$ w.p. 1, as $N\to\infty$. In fact, from the mean-value theorem, such $L_{\lambda,g}[n]$ can be given by the first derivative (w.r.t. λ) $\dot{g}_\lambda(\lambda_0,\boldsymbol{\gamma}[n])$ for a certain $\lambda_0\in[\lambda,\lambda']$.

To verify (C5a), simply select

$$B_\Delta[N]:=\sup_{\{\boldsymbol{\gamma}[n],\ n=1,\ldots,N\}}|\Delta(N,\lambda)|$$

which clearly satisfies $|\Delta(N,\lambda_p)|\le B_\Delta[N]$; and also $B_\Delta[N]/N\to 0$ since

$$\lim_{N\to\infty}N^{-1}\Delta(N,\lambda)=\lim_{N\to\infty}N^{-1}\sum_{n=1}^N g_\lambda(\lambda,\boldsymbol{\gamma}[n])-\bar{g}_\lambda(\lambda)=0$$

w.p. 1, when the fading process is stationary and ergodic.

To confirm (C5b), start with the definition of total deviation and apply the mean-value theorem to obtain $\Delta(N,\lambda) - \Delta(N,\lambda') = L_\Delta[N](\lambda - \lambda')$, where $L_\Delta[N] := -N\dot{\bar{g}}_\lambda(\lambda_0) + \sum_{n=1}^{N} \dot{g}_\lambda(\lambda_0, \boldsymbol{\gamma}[n])$. But as $N \to \infty$, sample averages converge to ensemble ones, and therefore

$$\lim_{N\to\infty} N^{-1}L_\Delta[N] = -\dot{\bar{g}}_\lambda(\lambda_0) + \mathbb{E}_{\boldsymbol{\gamma}}[\dot{g}_\lambda(\lambda_0,\boldsymbol{\gamma})] = -\dot{\bar{g}}_\lambda(\lambda_0) + \dot{\bar{g}}_\lambda(\lambda_0) = 0, \quad \text{w.p. } 1$$

where the second equality holds since $\bar{g}_\lambda(\lambda)$ has uniformly bounded derivative (cf. Property D.1). Therefore, there exists a random sequence $L_\Delta[N]$ satisfying the requirement of (C5b).

Having verified (C1)–(C5), the lemma follows from [19, Theorem 9.1].

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