Mutual Information Jammer-Relay Games

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***Abstract*—We consider a two-person zero-sum mutual information game between one jammer ($\mathcal{J}$) and one relay ($\mathcal{R}$) in both nonfading and fading scenarios. Assuming that the source ($\mathcal{S}$) and the destination ($\mathcal{D}$) are unaware of the game, we derive optimal pure or mixed strategies for $\mathcal{J}$ and $\mathcal{R}$ depending on the link qualities and whether the players are active during the $\mathcal{S} \rightarrow \mathcal{D}$ channel training. In nonfading scenarios, when both $\mathcal{J}$ and $\mathcal{R}$ have full knowledge of the source signal, linear jamming (LJ) and linear relaying (LR) are shown optimal in the sense of achieving Nash equilibrium. When the $\mathcal{S} \rightarrow \mathcal{J}$ and $\mathcal{S} \rightarrow \mathcal{R}$ links are noisy, LJ strategies (pure or mixed) are still optimal under LR. In this case, instead of always transmitting with full power as when the $\mathcal{S} \rightarrow \mathcal{R}$ link is perfect, $\mathcal{R}$ should adjust the transmit power according to its power constraint and the reliability of the source signal it receives. Furthermore, in fading scenarios, it is optimal for $\mathcal{J}$ to jam only with Gaussian noise if it cannot determine the phase difference between its signal and the source signal. When LR is considered with fading, $\mathcal{R}$ should forward with full power when the $\mathcal{S} \rightarrow \mathcal{R}$ link is better than the jammed $\mathcal{S} \rightarrow \mathcal{D}$ link, and defer forwarding otherwise. Optimal parameters are derived based on exact Nash equilibrium solutions or upper and lower bounds when a closed-form solution cannot be found.**

***Index Terms*—Jammer channel, mutual information, Nash equilibrium (NE), relay channel, two-person zero-sum games.**

I. Introduction

A jammer (or hacker) may be present to inhibit or halt the transmission of signals in a tactical (or commercial) communication system. If a jammer node can fully or partially acquire the signal transmitted by a source, it can disrupt the source-destination link severely by implementing what is referred to as correlated jamming. Because of its severity, the latter has received attention recently from both information-theoretic and game-theoretic perspectives [3], [7], [12], [17]. Optimal source/jammer strategies are reported in [12] for an additive white Gaussian noise (AWGN) channel of a point-to-point link where a source and a jammer participate in a two-person zero-sum game with the mutual information adopted as the objective function. In [7], related strategies are pursued for a single-user multi-input-multi-output (MIMO) fading channel where the jammer has full knowledge of the source signal. This model has been further extended in [3] to consider fading in the channel between the jammer and the receiver. Recently, a noncooperative zero-sum game has been investigated in a setup involving two sources and one-correlated jammer, in both AWGN and fading user channels [17]. For the nonfading two-user channel, the optimal strategy amounts to Gaussian signalling for the sources and linear jamming for the jammer. In fading channels, subgames are defined per channel state and the optimal solution reduces to a set of power allocation strategies for the players.

While a jammer tries to harm, a relay node can facilitate the communication between a source and a destination. Without being necessary to pack multiple antennas per terminal as in MIMO systems, cooperation among distributed single-antenna nodes (source and relays) offers an alternative spatial diversity enabler and brings resilience to shadowing as well as enhanced link coverage [9], [10], [14]–[16]. Different from the jamming channel, the capacity achieving strategy for the relay is generally unknown even for the Gaussian single-relay channel without fading [9]. However, upper and lowerbounds on the capacity of the AWGN relay channel have been developed in various scenarios and many simple relaying strategies have been either proved to be capacity (-bound) achieving under certain conditions [5], or justified in terms of the diversity order [10]. Security issues in relay communications have been investigated in [11] when one of the two relay nodes is adversarial and tries to disrupt communications by sending garbled signals. The objective in [11] is to trace and identify the adversarial relay.

In this paper, we consider a communication system with one jammer ($\mathcal{J}$) and one relay ($\mathcal{R}$) participating in the link between a source ($\mathcal{S}$) and a destination ($\mathcal{D}$), as depicted in Fig. 1. Each node is equipped with a single antenna. Nodes $\mathcal{J}$ and $\mathcal{R}$ have completely antithetical goals regarding the communication over the $\mathcal{S} \rightarrow \mathcal{D}$ link, whose effectiveness is assessed by the mutual information $I(X;Y)$ between the input X and the output Y. The conflicting objectives of $\mathcal{J}$ and $\mathcal{R}$ motivate a two-person zero-sum game formulation well [2], [13] in which the players are $\mathcal{J}$, who try to minimize $I(X;Y)$, and $\mathcal{R}$, who aims to maximize the $I(X;Y)$. Different from existing works where the game is played between $\mathcal{S}$ and $\mathcal{J}$, in our jammer-relay game setup, we assume that $\mathcal{S}$ and $\mathcal{D}$ are unaware of $\mathcal{J}$ and $\mathcal{R}$, while $\mathcal{J}$ ($\mathcal{R}$) can eavesdrop the channel and use the information obtained to perform correlated jamming (relaying). We also differentiate between the availability of perfect and noisy versions of the source signal at $\mathcal{J}$ ($\mathcal{R}$). Under reasonable assumptions on link qualities and the activity of $\mathcal{J}$ and $\mathcal{R}$ during the $\mathcal{S} \rightarrow \mathcal{D}$ channel training, we establish that jammer-relay games reach Nash equilibrium (NE)—a state where no player

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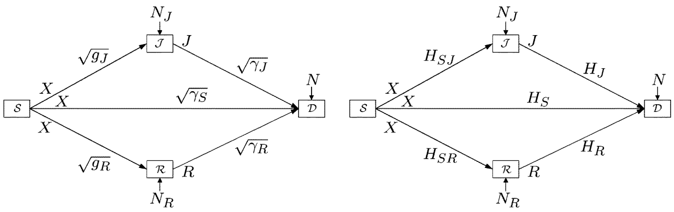

Fig. 1. System model when all the channels are (a) AWGN and (b) fading.

node has anything to gain unilaterally by only changing its own strategy. In scenarios where the single action of a player (a.k.a. pure strategy) cannot reach NE, one resorts to what is known as mixed strategy, which comprises a set of pure strategies with assigned probabilities; in such cases, the payoff is modified to the average mutual information. A strategy (pure or mixed) is claimed to be optimal if it achieves NE.

With nonfading channels, when both $\mathcal{J}$ and $\mathcal{R}$ have perfect information about the source signal, the optimal (pure or mixed) strategies turn out to be linear jamming (LJ) and linear relaying (LR), respectively. If the source signal received at $\mathcal{J}$ and $\mathcal{R}$ is only corrupted by AWGN, the optimal strategy for $\mathcal{R}$ is an open problem for decades even without $\mathcal{J}$. In this case, we prove that corresponding to LR at $\mathcal{R}$, LJ at $\mathcal{J}$ leads to NE. Furthermore, we show that instead of always transmitting with full power as when the $\mathcal{S} \to \mathcal{R}$ link is perfect, $\mathcal{R}$ at NE should adjust its transmit power to a proper value depending on the channel quality of the $\mathcal{S} \to \mathcal{R}$ link. In the presence of fading, when $\mathcal{J}$ cannot determine the phase difference between the jamming signal and the source signal, the best strategy for $\mathcal{J}$ is to jam with Gaussian noise only. When LR is considered with fading, $\mathcal{R}$ should relay to $\mathcal{D}$ with full power when the link $\mathcal{S} \to \mathcal{R}$ has a higher signal-to-noise ratio (SNR) than the jammed $\mathcal{S} \to \mathcal{D}$ link, and stop forwarding otherwise. Optimal parameters are derived based on exact NE solutions or pertinent upper and lower-bounds when a closed-form solution cannot be found.

The rest of this paper is organized as follows. Section II outlines the system model. The jammer-relay game with nonfading channels is dealt with in Sections III and IV, under the assumption of perfect and noisy eavesdropping channels, respectively. Section V generalizes results to fading channel scenarios. Finally, numerical examples and conclusions are given in Sections VI and VII, respectively.

Notation: $\mathcal{CN}(0, \sigma^2)$ denotes a circularly symmetric complex Gaussian distribution with zero mean and variance σ^2; for a real number x, $\operatorname{sgn}(x)$ denotes its sign and $(x)^+ := \max(x, 0)$; for random variables X and Y, $I(X;Y)$ denotes their mutual information; $h(X)$ differential entropy, and $E[X]$ expectation.

II. System Model

We first consider nonfading links with several settings categorized according to the amount of information available at $\mathcal{J}$ and $\mathcal{R}$. In the absence of fading, all channel coefficients are taken without loss of generality (w.l.o.g.) real and positive because they are assumed available both at receiving ends through training and at the transmitting ends through feedback. As illustrated in Fig. 1(a), the received signal at $\mathcal{D}$ is

$$Y = \sqrt{\gamma_S}X + \sqrt{\gamma_J}J + \sqrt{\gamma_R}R + N \tag{1}$$

where X, J, and R denote signals transmitted from the source ($\mathcal{S}$), jammer ($\mathcal{J}$), and relay ($\mathcal{R}$), respectively; $\sqrt{\gamma_S}$, $\sqrt{\gamma_J}$, and $\sqrt{\gamma_R}$ are the gains of the $\mathcal{S} \to \mathcal{D}$, $\mathcal{J} \to \mathcal{D}$ and $\mathcal{R} \to \mathcal{D}$ channels, respectively; and $N \sim \mathcal{CN}(0, \sigma_N^2)$. For the source signal X to be capable of maximizing mutual information, it is assumed zero-mean Gaussian. Power is constrained at $\mathcal{S}$, $\mathcal{J}$, and $\mathcal{R}$ so that $E[X^2] \le P_S$, $E[J^2] \le P_J$, and $E[R^2] \le P_R$, respectively.

We will analyze both cases of perfect and imperfect information about X acquired through eavesdropping, at $\mathcal{J}$ and $\mathcal{R}$. First, we assume that both $\mathcal{J}$ and $\mathcal{R}$ can obtain the exact X. In the second case, we assume that AWGN is present in both $\mathcal{S} \to \mathcal{J}$ and $\mathcal{S} \to \mathcal{R}$ links (i.e., $\mathcal{J}$ and $\mathcal{R}$ observe), respectively

$$Y_J = \sqrt{g_J}X + N_J \tag{2}$$

$$Y_R = \sqrt{g_R}X + N_R \tag{3}$$

where g_J (g_R) denotes the gain of the $\mathcal{S} \to \mathcal{J}$ ($\mathcal{S} \to \mathcal{R}$) channel, $N_J \sim \mathcal{CN}(0, \sigma_{N_J}^2)$, and $N_R \sim \mathcal{CN}(0, \sigma_{N_R}^2)$. Notice that the first case is subsumed by the second after setting $g_J = g_R = 1$ and $N_J = N_R = 0$ in (2) and (3).

When fading is considered in the model, one should also take into account the phases of the channel coefficients. As depicted in Fig. 1(b), the pertinent input–output relationship then becomes

$$Y = H_S X + H_R R + H_J J + N \tag{4}$$

where $H_S \sim \mathcal{CN}(0, \sigma_S^2)$, $H_R \sim \mathcal{CN}(0, \sigma_R^2)$, $H_J \sim \mathcal{CN}(0, \sigma_J^2)$ are the Rayleigh fading channel coefficients from $\mathcal{S}$, $\mathcal{R}$, $\mathcal{J}$ to $\mathcal{D}$, respectively, and the noise term at $\mathcal{D}$ is $N \sim \mathcal{CN}(0, \sigma_N^2)$. The power constraints are the same as before and the received signal Y_R (Y_J) at $\mathcal{R}$ ($\mathcal{J}$) could either coincide with X (perfect eavesdropping) or, in general, obey

$$Y_R = H_{SR}X + N_R \tag{5}$$

$$Y_J = H_{SJ}X + N_J \tag{6}$$

where $H_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ and $H_{SJ} \sim \mathcal{CN}(0, \sigma_{SJ}^2)$ are the Rayleigh fading channel coefficients from $\mathcal{S}$ to $\mathcal{R}$ and $\mathcal{J}$, respectively; and the noise terms at $\mathcal{R}$ and $\mathcal{J}$ are correspondingly $N_R \sim \mathcal{CN}(0, \sigma_{N_R}^2)$ and $N_J \sim \mathcal{CN}(0, \sigma_{N_J}^2)$.

To facilitate the $\mathcal{S} \to \mathcal{D}$ communication, $\mathcal{R}$ aims at maximizing the input–output mutual information $I(X;Y)$, which the adversary $\mathcal{J}$ tries to minimize at the same time. The game ends if $I(X;Y)$ reaches a value for which neither of the player nodes ($\mathcal{R}$ and $\mathcal{J}$) can gain unilaterally by only changing its own strategy. This defines Nash equilibrium (NE), which is the desirable state in game-theoretic problems. When certain links are fading, the same notions carry over except that $I(X;Y)$ needs to be averaged over the channel coefficients first. We will focus on the nonfading scenario in Sections III and IV, and treat the presence of fading effects in Section V.

III. Perfect Source Information at Jammer and Relay

In this section, we will find the best jamming/relaying strategies in the sense of reaching NE in the jammer-relay mutual-information-based game, when both $\mathcal{J}$ and $\mathcal{R}$ know X exactly. Specifically, we will establish that if $\mathcal{R}$ employs linear relaying (LR), through an amplified version of X, the best strategy for $\mathcal{J}$ is linear jamming (LJ), a linear combination of X and Gaussian noise; and vice-versa when $\mathcal{J}$ employs LJ, the best strategy for $\mathcal{R}$ is LR. Subsequently, we will analyze the NE of the mutual-information zero-sum game between $\mathcal{J}$ and $\mathcal{R}$ under various conditions.

A. Linear Jamming and Linear Relaying

Suppose that $\mathcal{J}$ uses LJ (i.e., $J = \rho X + W_J$, where $W_J \sim \mathcal{CN}(0, \sigma_{W_J}^2)$ and ρ, $\sigma_{W_J}^2$ are chosen to satisfy the power constraint $\rho^2 P_S + \sigma_{W_J}^2 \leq P_J$. The relay wishes to find the signalling strategy R which maximizes $I(X;Y)$. To evaluate the latter, introduce the variable $Z := R - XE[XR]/P_S$ which represents the error in linearly estimating R using the source signal X. Using Z, we can rewrite (1) as

$$Y = \left(\sqrt{\gamma_S} + \sqrt{\gamma_J}\rho + \sqrt{\gamma_R}\frac{E[XR]}{P_S}\right)X + \sqrt{\gamma_J}W_J + \sqrt{\gamma_R}Z + N. \tag{7}$$

It is easy to verify that Z is uncorrelated with X and $E[Z^2] + (E[XR])^2/P_S \leq P_R$. We can now upper bound $I(X;Y) := h(Y) - h(Y|X) = h(Y) - h(\sqrt{\gamma_J}W_J + \sqrt{\gamma_R}Z + N|X)$ as

$$\begin{aligned} I(X;Y) &\leq h(Y) - h(\sqrt{\gamma_J}W_J + \sqrt{\gamma_R}Z + N|X, Z) \quad (8)\\ &= h(Y) - h(\sqrt{\gamma_J}W_J + N) \quad (9)\end{aligned}$$

where the equality in (8) holds when $Z = 0$. But for $Z = 0$, Y is Gaussian and the maximum of $h(Y)$ is attained when the upper bound on $E[Y^2]$ is maximized. Using the power constraints and the definition $A := \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho$, it follows readily that:

$$E[Y^2] \leq P_S A^2 + 2A\sqrt{\gamma_R}E[XR] + \gamma_R P_R + \gamma_J \sigma_{W_J}^2 + \sigma_N^2. \tag{10}$$

To maximize $E[Y^2]$ and consequently $I(X;Y)$, it thus suffices to set $Z = 0$, and

$$E[XR] = \begin{cases} \sqrt{P_S P_R}, & \text{if } A \geq 0 \\ -\sqrt{P_S P_R}, & \text{if } A < 0. \end{cases} \tag{11}$$

Recalling that $Z = 0 = R - XE[XR]/P_S$, we arrive at the $I(X;Y)$ maximizing relay strategy

$$R^* = \begin{cases} \sqrt{\frac{P_R}{P_S}}X, & \text{if } A \geq 0 \\ -\sqrt{\frac{P_R}{P_S}}X, & \text{if } A < 0. \end{cases} \tag{12}$$

Equation (12) reveals that if $\mathcal{J}$ relies on LJ, the best strategy for $\mathcal{R}$ is LR. The sign of R depends on the scalar ρ used by $\mathcal{J}$ to restrain reception of the source signal at $\mathcal{D}$.

Assuming now that R is chosen as in (12), we will prove next that the best strategy for $\mathcal{J}$ is LJ. Since $I(X;Y) = h(X) - h(X|Y)$, and $\mathcal{J}$ can only affect $h(X|Y)$, the jammer aims at minimizing $I(X;Y)$ by maximizing $h(X|Y)$, which can be upper bounded as

$$h(X|Y) = h(X - aY|Y) \leq h(X - aY) \leq \frac{1}{2}\log(2\pi e\Lambda) \tag{13}$$

where $\Lambda := E[(X - aY)^2]$. Although the inequalities in (13) hold for any a, we choose $a = E[XY]/E[Y^2]$. We will carry out the rest of the proof in two steps. First, we will prove that if there is an LJ signal giving rise to a certain Λ, it is optimal over all other jamming signals leading to the same Λ. Second, we will prove that any given Λ reachable by a feasible jamming signal can also be reached by a feasible LJ signal. (Feasibility here means adherence to power constraints.)

For the first step, since X is Gaussian and the jamming is linear, J is also Gaussian. Moreover, because $R = R^* = \pm\sqrt{(P_R/P_S)}X$, we infer that Y and $X - (E[XY]/E[Y^2])Y$ are also Gaussian. Now, as $X - (E[XY]/E[Y^2])Y$ is uncorrelated with Y and they are both Gaussian, $X - (E[XY]/E[Y^2])Y$ is independent of Y. Thus, the upper bound in (13) is achieved with equality, i.e.,

$$h(X|Y) = \frac{1}{2}\log\left[2\pi e\left(P_S - \frac{(E[XY])^2}{E[Y^2]}\right)\right] \tag{14}$$

which establishes the optimality of LJ given its existence.

For the second step, substitute (12) into (1) to find $Y = (\sqrt{\gamma_S} \pm \sqrt{\gamma_R}\sqrt{(P_R/P_S)})X + \sqrt{\gamma_J}J + N$, and

$$E[XY] = \left(\sqrt{\gamma_S} \pm \sqrt{\gamma_R}\sqrt{\frac{P_R}{P_S}}\right)P_S + \sqrt{\gamma_J}E[XJ] \tag{15}$$

$$\begin{aligned} E[Y^2] = &\left(\sqrt{\gamma_S} \pm \sqrt{\gamma_R}\sqrt{\frac{P_R}{P_S}}\right)^2 P_S + \gamma_J P_J + \sigma_N^2 \\ &+ 2\sqrt{\gamma_J}\left(\sqrt{\gamma_S} \pm \sqrt{\gamma_R}\sqrt{\frac{P_R}{P_S}}\right)E[XJ]. \quad (16)\end{aligned}$$

Using (15) and (16), it follows that $\Lambda = P_S - (E[XY])^2/E[Y^2]$ is a function of $E[XJ]$. Consider now any J and define $U := J - X(E[XJ]/P_S)$ which is clearly uncorrelated with X. For the jamming signal J to be feasible, we should have $((E[XJ])^2/P_S) \leq P_J$. Now define an LJ signal $J_l := (E[XJ]/P_S)X + W_J$, where $W_J \sim \mathcal{CN}(0, \sigma_{W_J}^2)$ denotes noise uncorrelated with X, having $\sigma_{W_J}^2 = E[U^2]$. Since $E[XJ_l] = E[XJ]$, this J_l results in the same Λ and has the same power as J. Thus, J_l is also feasible. Hence, for any signal in the set of feasible jamming signals, there is an equivalent LJ signal which leads to in the same upper bound (14).

The results so far in this section can be summarized in the following proposition.

Proposition: With nonfading links and perfect source information available at $\mathcal{J}$ and $\mathcal{R}$, the jammer and relay strategies reaching NE in mutual information games under power constraints at $\mathcal{J}$ and $\mathcal{R}$ amounting to LJ and LR, respectively.

Notice that Proposition 1 does not assert that LJ and LR are the only strategies achieving Nash Equilibrium. Having determined the form of $\mathcal{R}$ and $\mathcal{J}$ signals, we proceed to specify them at NE.

B. Nash Equilibria

Since the optimal jamming signal is $J = \rho X + W_J$, to fully describe the NE, we should specify the slope ρ and the noise variance $\sigma_{W_J}^2$ which maximize (14) and, thus, minimize $I(X;Y)$. To this end, note first that since a linear combination of

Gaussian signals is received at $\mathcal{D}$, minimizing $I(X;Y)$ is equivalent to minimizing the output SNR at $\mathcal{D}$. The pertinent minimization problem is thus [cf. (7) with $Z=0$]

$$\min_{\rho,\sigma_{W_J}^2} \frac{\left[\sqrt{\gamma_S}+\sqrt{\gamma_J}\rho+\sqrt{\gamma_R}\frac{E[XR]}{P_S}\right]^2 P_S}{\gamma_J\sigma_{W_J}^2+\sigma_N^2}$$
$$\text{s.t.}\quad \rho^2P_S+\sigma_{W_J}^2\le P_J.$$

The Karush–Kuhn–Tucker (KKT) necessary conditions for optimality are

$$\begin{cases} \frac{\left[\sqrt{\gamma_S}+\sqrt{\gamma_J}\rho+\sqrt{\gamma_R}\frac{E[XR]}{P_S}\right]P_S\sqrt{\gamma_J}}{\gamma_J\sigma_{W_J}^2+\sigma_N^2}+\lambda P_S\rho=0\\ \frac{-\left[\sqrt{\gamma_S}+\sqrt{\gamma_J}\rho+\sqrt{\gamma_R}\frac{E[XR]}{P_S}\right]^2P_S\gamma_J}{\left(\gamma_J\sigma_{W_J}^2+\sigma_N^2\right)^2}+\lambda-\delta=0\\ \lambda\left(\rho^2P_S+\sigma_{W_J}^2-P_J\right)=0, \delta\left(-\sigma_{W_J}^2\right)=0\\ \lambda\ge 0,\delta\ge 0\end{cases}\tag{17}$$

where λ denotes the nonnegative Lagrange multiplier and δ a complementary slackness variable for $\sigma_{W_J}^2$ [4, Ch. 5.5]. Given any LR strategy R, we can solve (17) to obtain the optimal ρ as shown in (18), at the bottom of the page, where the parameter ρ_m in (18) is given by

$$\rho_m=\min\left\{\frac{P_J\gamma_J+\sigma_N^2}{\left|\sqrt{\gamma_S}+\frac{E[XR]\sqrt{\gamma_R}}{P_S}\right|P_S\sqrt{\gamma_J}},\sqrt{\frac{P_J}{P_S}}\right\}\tag{19}$$

and the optimal noise variance is

$$\sigma_{W_J}^{2*}=\left(P_J-\left[\rho^*(R)\right]^2P_S\right)^+.\tag{20}$$

Although it is reasonable to assume that the parameters γ_S, γ_J, γ_R, P_S, P_J, and P_R are available at $\mathcal{J}$ and $\mathcal{R}$, the optimal jamming strategy in (18)–(20) is not fully specified before the relay signalling R is determined at NE.

As we will soon see , determining the J^* and R^* signals at NE depends critically on the activity of $\mathcal{J}$ and $\mathcal{R}$ during the training stage of the $\mathcal{S}\to\mathcal{D}$ link. Specifically, we will differentiate between two operational assumptions as follows.

1) **a1**. $\mathcal{J}$ and $\mathcal{R}$ are inactive during the training of the $\mathcal{S}\to\mathcal{D}$ channel.
2) **a2**. $\mathcal{J}$ and $\mathcal{R}$ are active during the training of the $\mathcal{S}\to\mathcal{D}$ channel.

Under a1, $\mathcal{D}$ acquires the $\mathcal{S}\to\mathcal{D}$ channel phase before the game and relies on it to coherently decode X when the game is played. This rules out the choice corresponding to the negative sign in (12), because $\mathcal{R}$ would then cancel the source signal while $\mathcal{D}$, being unaware of the $\mathcal{J}-\mathcal{R}$ game, will erroneously decode X using the channel phase it acquired during training. A consequence of a1 is that NE is reached by a pair of pure strategies, namely $R^*=\sqrt{(P_R/P_S)}X$ and $J^*=[\rho^*(\sqrt{(P_R/P_S)}X)]X+W_J^*$, where $\rho^*(\sqrt{(P_R/P_S)}X)$ is chosen as in (18) with $R=\sqrt{(P_R/P_S)}X$ and $W_J^*\sim\mathcal{CN}(0,(P_J-[\rho^*(\sqrt{(P_R/P_S)}X)]^2P_S)^+)$.

Under a2, $\mathcal{D}$ acquires the $\mathcal{S}\to\mathcal{D}$ channel phase when the $\mathcal{J}-\mathcal{R}$ game is played. In this case, even if $\mathcal{R}$ cancels X using the negative sign in (12), $\mathcal{D}$ can detect the aggregate ($\mathcal{S}\to\mathcal{D}$ plus $\mathcal{R}\to\mathcal{D}$ plus $\mathcal{J}\to\mathcal{D}$) channel via training and can coherently decode X. Now, either choice in (12) is possible and this provides one more parameter for $\mathcal{J}$ and $\mathcal{R}$ to play with. As a result, the optimal strategies for $\mathcal{J}$ and $\mathcal{R}$ under a2 are not always pure. Whether they are pure or not depends on the following conditions:

- **c1**. $\sqrt{P_J\gamma_J}\le\sqrt{P_S\gamma_S}$ (i.e., $\mathcal{J}$'s power received at $\mathcal{D}$ does not exceed that of $\mathcal{S}$);
- **c2**. $\sqrt{P_S\gamma_S}+\sqrt{P_R\gamma_R}\le\sqrt{P_J\gamma_J}$ (i.e., $\mathcal{J}$'s power at $\mathcal{D}$ exceeds that of $\mathcal{S}$ plus $\mathcal{R}$);
- **c3**. $\sqrt{P_S\gamma_S}<\sqrt{P_J\gamma_J}<\sqrt{P_S\gamma_S}+\sqrt{P_R\gamma_R}$ (i.e., $\mathcal{J}$'s power at $\mathcal{D}$ exceeds that of $\mathcal{S}$ but not that of $\mathcal{S}$ plus $\mathcal{R}$).

Under c1, whether $\mathcal{J}$ chooses or not, it cannot cancel the source signal completely because of its power limitation.[1] From the power constraint at $\mathcal{J}$, it follows that $\rho^2P_S\le P_J$ and, thus, $\rho\ge-\sqrt{P_J/P_S}$. This, in turn, implies $A:=\sqrt{\gamma_S}+\sqrt{\gamma_J}\rho\ge\sqrt{\gamma_S}-\sqrt{\gamma_JP_J/P_S}\ge 0$, where for the last inequality, we used c1. With $A\ge 0$, the best LR signal is [cf. (12)] $R^*=\sqrt{(P_R/P_S)}X$. Correspondingly, the optimal LJ signal has slope $\rho^*=-\rho_m$ [cf. (18) and (19)] and $\sigma_{W_J}^{2*}=(P_J-\rho_m^2P_S)^+$ [cf. (20)]. Notwithstanding, NE is achieved under c1 with a pure strategy.

Under c2, we find that $P_J\gamma_J\ge(\sqrt{P_S\gamma_S}\pm\sqrt{P_R\gamma_R})^2=(\sqrt{P_S\gamma_S}\pm(\sqrt{(P_SP_R}/\sqrt{P_S})\sqrt{\gamma_R})^2=(\sqrt{P_S\gamma_S}+E[XR]\sqrt{\gamma_R}/\sqrt{(P_S)})^2$, where for the last equality, we relied on (11). Now, using (18), this inequality implies that $\rho^*=-(\sqrt{\gamma_S}/\sqrt{\gamma_J})-E[XR]\sqrt{\gamma_R}/(P_S\sqrt{\gamma_J})$. In this case, $\mathcal{J}$ has enough power to cancel signals transmitted by $\mathcal{S}$ and $\mathcal{R}$, regardless of the relaying signal. But since R can either have an opposite or the same sign as X, the players $\mathcal{J}$ and $\mathcal{R}$ cannot arrive at NE with pure strategies. To demonstrate this, let us first check the relationship between $A:=\sqrt{\gamma_S}+\sqrt{\gamma_J}\rho$ and $B:=E[XR]\sqrt{\gamma_R}(/P_S)$. Substituting the optimal $\rho=\rho^*=-(\sqrt{\gamma_S}/\sqrt{\gamma_J})-E[XR]\sqrt{\gamma_R}/(P_S\sqrt{\gamma_J})$ in A and the $E[XR]$ expression from (11) in B, we find

$$A=-\frac{E[XR]\sqrt{\gamma_R}}{P_S}=-B\tag{21}$$

$$B=\begin{cases}\sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{if } A\ge 0\\ -\sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{if } A<0.\end{cases}\tag{22}$$

[1]When $\sqrt{P_J\gamma_J}=\sqrt{P_S\gamma_S}$, either choice in (12) is optimal under a2. When this occurs, we assume for simplicity that $R^*=\sqrt{(P_R/P_S)}X$ and subsume this special case under c1.

$$\rho^*(R)=\begin{cases}-\frac{\sqrt{\gamma_S}}{\sqrt{\gamma_J}}-\frac{E[XR]\sqrt{\gamma_R}}{P_S\sqrt{\gamma_J}}, & \text{if }\left(\sqrt{P_S\gamma_S}+\frac{E[XR]\sqrt{\gamma_R}}{\sqrt{P_S}}\right)^2\le P_J\gamma_J\\ -\rho_m\text{sgn}\left(\sqrt{\gamma_S}+\frac{E[XR]\sqrt{\gamma_R}}{P_S}\right), & \text{if }\left(\sqrt{P_S\gamma_S}+\frac{E[XR]\sqrt{\gamma_R}}{\sqrt{P_S}}\right)^2> P_J\gamma_J\end{cases}\tag{18}$$

Since A (B) depends on the strategy of $\mathcal{J}$ ($\mathcal{R}$), the strategies of $\mathcal{J}$ and $\mathcal{R}$ are clearly coupled. Indeed, $\mathcal{J}$ aims to have a sign opposite of $\mathcal{R}$ while, at the same time, $\mathcal{R}$ wishes to follow the sign of $\mathcal{J}$. This coupling implies that pure individual player actions cannot drive $\mathcal{J}$ and $\mathcal{R}$ to a stable NE.

To reach a stable NE under c2, we consider the following mixed strategies:

$$A = \begin{cases} -\sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{w.p. } p_J \\ \sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{w.p. } 1-p_J \end{cases} \quad (23)$$

$$B = \begin{cases} \sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{w.p. } p_R \\ -\sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{w.p. } 1-p_R \end{cases} \quad (24)$$

where w.p. means with probability. The objective function is now $E[I(X;Y)]$ (i.e., the capacity of the channel linking X to Y); and the players seek the optimal probability assignments (p_J and p_R). For this two-person zero-sum game, optimal strategies can be found through a minimax approach, by which each player node tries to maximize its payoff ($E[I(X;Y)]$ for $\mathcal{R}$ and $-E[I(X;Y)]$ for J) in the worst outcome determined by the opponent's strategy [13, Ch. 3.8].

The relay $\mathcal{R}$ seeks the best probability distribution of pure strategies by solving the following max-min problem:

$$\max_{p_R}\min_{p_J} E\left[\frac{1}{2}\log\left(1+\frac{(A+B)^2P_S}{\gamma_J\sigma_{W_J}^2+\sigma_N^2}\right)\right]$$
$$=\max_{p_R}\min_{p_J}\{(1-p_J)p_R I_- + (1-p_R)p_J I_+\} \quad (25)$$
$$=\max_{p_R}\{\min\{p_R I_-, (1-p_R)I_+\}\} \quad (26)$$

where in deriving (26), we used X and Y that are jointly Gaussian, substituted the binary random variables A and B from (23) and (24), and defined

$$I_- := \frac{1}{2}\log\left[1+\frac{4P_R\gamma_R}{P_J\gamma_J+\sigma_N^2-(\sqrt{P_R\gamma_R}-\sqrt{P_S\gamma_S})^2}\right] \quad (27)$$

$$I_+ := \frac{1}{2}\log\left[1+\frac{4P_R\gamma_R}{P_J\gamma_J+\sigma_N^2-(\sqrt{P_R\gamma_R}+\sqrt{P_S\gamma_S})^2}\right]. \quad (28)$$

It is easy to recognize that the p_R maximizing (26) is $p_R^* = I_+/(I_+ + I_-)$.

Similarly, in selecting its optimum strategy, the jammer node $\mathcal{J}$ solves the min-max problem

$$\min_{p_J}\max_{p_R} E\left[\frac{1}{2}\log\left(1+\frac{(A+B)^2P_S}{\gamma_J\sigma_{W_J}^2+\sigma_N^2}\right)\right]$$
$$=\min_{p_J}\{\max\{(1-p_J)I_-, p_J I_+\}\} \quad (29)$$

to obtain (by symmetry of the corresponding expressions) the optimal $p_J^* = I_-/(I_+ + I_-)$.

By either incorporating p_R^* into (26) or p_J^* into (29), the common payoff at NE is $I_+I_-/(I_+ + I_-)$. This corresponds to a saddle point in two-person zero-sum games by the Minimax Theorem [2, Ch. 2]. Thus, under c2, NE can be achieved with mixed strategies, that is, with $\mathcal{R}$ transmitting

$$R^* = \left\{\left(\sqrt{\frac{P_R}{P_S}}X, -\sqrt{\frac{P_R}{P_S}}X\right), (p_R^*, 1-p_R^*)\right\} \quad (30)$$

where the actions in the first parentheses are taken, in turn, with probabilities in the second parentheses; and similarly with $\mathcal{J}$ transmitting $J^* = \rho^* X + W_J^*$, with $\rho^* = \{(-(\sqrt{\gamma_S}/\sqrt{\gamma_J}) - (\sqrt{P_R\gamma_R}/\sqrt{P_S\gamma_J}), -(\sqrt{\gamma_S}/\sqrt{\gamma_J}) + (\sqrt{P_R\gamma_R}/\sqrt{P_S\gamma_J})), (p_J^*, 1-p_J^*)\}$ and $W_J^* \sim \mathcal{CN}(0, (P_J - \rho^{*2}P_S)^+)$.

Since pure and mixed strategies under c3 follow along the same lines as in c1 and c2, we move the detailed analysis to the Appendix. The counterparts of c1 and c2 under c3 are as follows.

c3-1. $\sqrt{P_S\gamma_S}(\sqrt{P_S\gamma_S}+\sqrt{P_R\gamma_R}) \geq P_J\gamma_J + \sigma_N^2$ and $\sqrt{P_S\gamma_S} < \sqrt{P_J\gamma_J} < \sqrt{P_S\gamma_S}+\sqrt{P_R\gamma_R}$ (i.e., $\mathcal{J}$'s power plus noise power at $\mathcal{D}$ does not exceed the "average" power of $\mathcal{S}$ and $\mathcal{R}$).

c3-2. $\sqrt{P_S\gamma_S}(\sqrt{P_S\gamma_S}+\sqrt{P_R\gamma_R}) < P_J\gamma_J + \sigma_N^2$ and $\sqrt{P_S\gamma_S} < \sqrt{P_J\gamma_J} < \sqrt{P_S\gamma_S}+\sqrt{P_R\gamma_R}$ (i.e., $\mathcal{J}$'s power plus noise power at $\mathcal{D}$ exceeds the "average" power of $\mathcal{S}$ and $\mathcal{R}$).

Under c3-1, we have a high SNR setup at $\mathcal{D}$ when σ_N^2 is relatively small and $\mathcal{J}$ is absent. Having an objective to minimize the SNR at $\mathcal{D}$, it suffices for $\mathcal{J}$ to increase the noise level instead of allocating all of its power to lower the signal level by canceling X. Even though the residual power of $\mathcal{J}$ is not enough to cancel X, $\mathcal{J}$ here has enough power to eliminate X had it not allocated power to generate noise. It turns out that $A > 0$ in this case and, thus, the best pure strategy for $\mathcal{R}$ according to (12) is to use $R^* = \sqrt{(P_R/P_S)}X$, which affects a pure strategy for $\mathcal{J}$ as well.

Under c3-2, $\mathcal{J}$ does not have to considerably increase the noise level because σ_N^2 at $\mathcal{D}$ is high; instead, $\mathcal{J}$ in this case, saves power to cancel X. Meanwhile, under c3, X can be completely cancelled if $\mathcal{J}$ chooses to do so. Parameter A in (10) can be either positive or negative, which allows for either choice in (12) and dictates players to reach NE via mixed strategies.

Characterization of NE under c1–c3 can now be collectively summarized as follows.

Proposition 2: With nonfading links and perfect source information available at $\mathcal{J}$ and $\mathcal{R}$, NE can be achieved with pure strategies under c1 and c3-1 and mixed strategies under c2 and c3-2.

Having characterized NE when the source information is assumed perfect at $\mathcal{J}$ and $\mathcal{R}$, we turn our attention to the more pragmatic situation where this information is imperfect.

IV. Noisy Source Information at Jammer and Relay

Here, we assume that both $\mathcal{S} \rightarrow \mathcal{J}$ and $\mathcal{S} \rightarrow \mathcal{R}$ links are modeled as AWGN channels. The received signals at $\mathcal{J}$ and $\mathcal{R}$ are given by (2) and (3), respectively.

A. Relaying Strategy

We first look to optimize the strategy of $\mathcal{R}$, assuming that $\mathcal{J}$ relies on LJ (i.e., the jamming signal is $J = \rho Y_J + W_J$) and adheres to the power constraint

$$\rho^2 E\left[Y_J^2\right] + \sigma_{W_J}^2 = \rho^2\left(g_J P_S + \sigma_{N_J}^2\right) + \sigma_{W_J}^2 \leq P_J. \quad (31)$$

In this scenario, the overall optimal strategy for $\mathcal{R}$ is difficult to find, primarily because the optimal relaying strategy even for the AWGN relay channel is still unknown [9]. But recent works on cooperative communications have suggested a number of useful relay strategies, including the popular decode-and-forward (DF), amplify-and-forward (AF), and estimate-and-forward (EF) ones [9], [10]. For a Gaussian X, the DF, AF, and EF schemes perform identically as far as mutual information is concerned [6]. Under the power constraint $E[R^2] \leq P_R$, we can thus consider w.l.o.g. the AF relaying strategy

$$R = \frac{\sqrt{P_R}}{\sqrt{E\left[Y_R^2\right]}} Y_R = \frac{\sqrt{P_R}}{\sqrt{g_R P_S + \sigma_{N_R}^2}} Y_R \qquad (32)$$

where Y_R is given by (3). We will later verify that this is not the optimal strategy even if we restrict ourselves to the class of LR functions only.

In general, the optimal R should be a nonlinear function of Y_R. But for analytical tractability, we confine the relaying strategy to linear forwarding (i.e., we assume that $R = \alpha Y_R + Z_R$), where Z_R is a signal independent of Y_R. However, the data-processing inequality asserts that $I(X;Y)$ is maximized if the "interference" Z_R is 0. But then Y is Gaussian and the objective of maximizing $I(X;Y)$ is equivalent to maximizing the output SNR, that is, the relay seeks to

$$\max_{\alpha} \quad \frac{(\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} + \alpha\sqrt{\gamma_R g_R})^2 P_S}{\rho^2 \gamma_J \sigma_{N_J}^2 + \gamma_J \sigma_{W_J}^2 + \sigma_N^2 + \alpha^2 \gamma_R \sigma_{N_R}^2}$$
$$\text{s.t.} \quad \alpha^2 \left(g_R P_S + \sigma_{N_R}^2\right) \leq P_R.$$

The solution of this constrained maximization problem turns out to be

$$\alpha^* = \begin{cases} \operatorname{sgn}(\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J})\alpha_m(\rho), & \text{if } \sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} \neq 0 \\ \pm\sqrt{\frac{P_R}{g_R P_S + \sigma_{N_R}^2}}, & \text{if } \sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} = 0 \end{cases} \qquad (33)$$

where

$$\alpha_m(\rho) = \min\left\{ \frac{\left(\rho^2 \gamma_J \sigma_{N_J}^2 + \gamma_J \sigma_{W_J}^2 + \sigma_N^2\right)\sqrt{g_R}}{|\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J}|\sqrt{\gamma_R}\sigma_{N_R}^2} \sqrt{\frac{P_R}{g_R P_S + \sigma_{N_R}^2}} \right\}. \qquad (34)$$

Interestingly, (34) suggests that $\mathcal{R}$ should not always use its full power to linearly forward the received signal. Fixing all other parameters, mathematically this occurs when $\sigma_{N_R}^2$ is large enough, which implies that the receive SNR at $\mathcal{R}$ is very low. This is reasonable because when Y_R at $\mathcal{R}$ is not reliable, forwarding it to $\mathcal{D}$ with too much power will only downgrade detection performance at $\mathcal{D}$ by decreasing its receive SNR.

B. Jamming Strategy

In this subsection, we establish that when $R = \alpha Y_R = \alpha\sqrt{g_R}X + \alpha N_R$, the best strategy for the jammer is LJ. Since $I(X;Y) = h(X) - h(X|Y)$, and the jammer only affects $h(X|Y)$, in the following steps similar to those in Section III-A, we deduce that if $\mathcal{J}$ relies on LJ (i.e., $J_l = \rho Y_J + W_J$), one can achieve the following upper bound with equality:

$$h(X|Y) = h\left(X - \frac{E[XY]}{E[Y^2]}Y | Y\right) \leq h\left(X - \frac{E[XY]}{E[Y^2]}Y\right) \leq \frac{1}{2}\log(2\pi e \Lambda) \qquad (35)$$

where $\Lambda := E[(X - (E[XY]/E[Y^2])Y)^2]$ depends on $E[XJ]$, as we saw in (15) and (16). These steps establish the optimality of LJ given its existence. But to show the existence of an optimal LJ strategy for any Λ, the method in Section III-A does not carry over because here $J_l = \rho\sqrt{g_J}X + \rho N_J + W_J$ contains the term N_J, whose power cannot be controlled by $\mathcal{J}$. Indeed, ρ can be selected to satisfy $E[XJ] = E[XJ_l]$ as in Section III-A. But due to the presence of N_J, the power of the aggregate noise $W_J' := \rho N_J + W_J$ can never drop below $\rho^2\sigma_{N_J}^2$, making it impossible to guarantee that J_l obeys the power constraint at $\mathcal{J}$.

Since it is impossible to design J_l starting from a feasible J, we will show that the set of all $E[XJ]$ values that are achievable is the same as the set of all $E[XJ_l]$ values achieved by all feasible LJ signals (i.e., LJ achieves the largest possible Λ, which maximizes the upper bound in (35) and, thus, minimizes $I(X;Y)$. First, it is well known that the linear minimum mean-square error (LMMSE) estimate of X based on the noisy source information Y_J is

$$\hat{X}_1 = \frac{E[XY_J]}{E\left[Y_J^2\right]} Y_J = \frac{\sqrt{g_J}E[X^2]}{g_J E[X^2] + \sigma_{N_J}^2} Y_J \qquad (36)$$

and the corresponding MMSE is given by

$$E\left[(\hat{X}_1 - X)^2\right] = E[X^2] - \frac{(E[XY_J])^2}{E\left[Y_J^2\right]} = \frac{\sigma_{N_J}^2 E[X^2]}{g_J E[X^2] + \sigma_{N_J}^2}. \qquad (37)$$

Since Y_J is a linear combination of the Gaussian variables X and N_J, the LMMSE estimate coincides with the MMSE estimate of X [8].

On the other hand, the LMMSE estimate of X based on J is $\hat{X}_2 = (E[XJ]/E[J^2])J$ and the MMSE is given by

$$E\left[(\hat{X}_2 - X)^2\right] = E[X^2] - \frac{(E[XJ])^2}{E[J^2]}. \qquad (38)$$

Since $J = f(Y_J)$, the MMSE of $\hat{X}_1$ must lower bound that of $\hat{X}_2$ since the latter is an LMMSE estimator while the former is an MMSE estimator. Thus, we can write [cf. (37) and (38)]

$$E[X^2] - \frac{(E[XJ])^2}{E[J^2]} \geq \frac{\sigma_{N_J}^2 E[X^2]}{g_J E[X^2] + \sigma_{N_J}^2}. \qquad (39)$$

Rearranging (39), we find that the upper bound of $(E[XJ])^2$ for any jamming strategy J obeys

$$(E[XJ])^2 \leq \frac{g_J\left(E[X^2]\right)^2 E[J^2]}{g_J E[X^2] + \sigma_{N_J}^2} \leq \frac{g_J\left(E[X^2]\right)^2 P_J}{g_J E[X^2] + \sigma_{N_J}^2}. \qquad (40)$$

Meanwhile, for an LJ signal $J_l = \rho Y_J + W_J = \rho(\sqrt{g_J}X + N_J) + W_J$, we have $E[XJ_l] = \rho\sqrt{g_J}E[X^2]$. Due to the power constraint, $E[J_l^2] = \rho^2 E[Y_J^2] + \sigma_{W_J}^2 \leq P_J$, the achievable ρ values must satisfy $\rho^2 \leq (P_J/E[Y_J^2]) = P_J/(g_J E[X^2] + \sigma_{N_J}^2)$. Thus, we can also upper bound $(E[XJ_l])^2$ as

$$(E[XJ_l])^2 \leq \frac{g_J\left(E[X^2]\right)^2 P_J}{g_J E[X^2] + \sigma_{N_J}^2}. \qquad (41)$$

From (40) and (41), we see that $(E[XJ])^2$ and $(E[XJ_l])^2$ have identical upper bounds. Since they are obviously lower bounded by zero, and $E[XJ_l]$ can achieve all of the values between its bounds by modifying ρ, we infer that for any J in the set of feasible jamming signals, there is an equivalent LJ signal which results in the same upper bound (35). Hence, LJ exists which achieves the largest possible $h(X|Y)$ and, therefore, the minimum $I(X;Y)$.

Combining this with the optimality of LJ given its existence, we have established that LJ is the optimum strategy for $\mathcal{J}$ if $\mathcal{R}$ relies on LR, even if both $\mathcal{J}$ and $\mathcal{R}$ know imperfectly the source signal due to the presence of AWGN.

Next, we find the optimal ρ for the jammer. When $R = \alpha Y_R = \alpha\sqrt{g_R}X + \alpha N_R$, we have

$$Y = (\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} + \alpha\sqrt{\gamma_R g_R})X + \sqrt{\gamma_J}W_J + \rho N_J\sqrt{\gamma_J} + \alpha N_R\sqrt{\gamma_R} + N. \quad (42)$$

Opposite of the relay, the jammer wants to select its parameters so that the output SNR at $\mathcal{D}$ is minimized, i.e.,

$$\min_{\rho,\sigma^2_{W_J}} \frac{(\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} + \alpha\sqrt{\gamma_R g_R})^2 P_S}{\rho^2\gamma_J\sigma^2_{N_J} + \gamma_J\sigma^2_{W_J} + \sigma^2_N + \alpha^2\gamma_R\sigma^2_{N_R}} \quad (43)$$

$$\text{s.t.} \quad \rho^2\left(g_J P_S + \sigma^2_{N_J}\right) + \sigma^2_{W_J} \le P_J. \quad (44)$$

After solving the KKT conditions, we find the optimum as shown in (45), at the bottom of the page and $\sigma^{2*}_{W_J} = (P_J - \rho^{*2}(g_J P_S + \sigma^2_{N_J}))^+$ where

$$\rho'_m(\alpha) = \min\left\{\frac{\gamma_J P_J + \sigma^2_N + \alpha^2\gamma_R\sigma^2_{N_R}}{|\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R}|P_S\sqrt{\gamma_J g_J}} \sqrt{\frac{P_J}{g_J P_S + \sigma^2_{N_J}}}\right\}. \quad (46)$$

With the expressions of ρ^* in (45) and α^* in (33), one can pursue characterization of NE under various assumptions and conditions as in Section III-B. Due to a lack of space, we will only consider the optimal jamming and relaying strategies under a1. Specifically, we establish that (see the Appendix for the proof).

Proposition 3: With nonfading links and noisy source information available at $\mathcal{J}$ and $\mathcal{R}$, if $\mathcal{R}$ uses LR, then $\mathcal{J}$ relies on LJ and NE can be achieved with pure strategies under a1.

If $\mathcal{R}$ uses LR, the results so far in this section have readily established the optimality of LJ and LR (i.e., the pair of strategies $(R^* = \sqrt{(P_R/P_S)}X,\ J^* = \rho^* X + W_J^*)$ achieves NE. Arguing as in Section III-B, $\mathcal{R}$ will never transmit with a sign opposite of X under a1; hence, we will always have $\alpha^* \ge 0$ for $\mathcal{R}$ and correspondingly $\rho^* \le 0$ for $\mathcal{J}$ at NE. On the other hand, we know from (34) that $\mathcal{R}$ should not always transmit with full power. For this reason, ρ^* in (45) must be replaced by a more complicated expression depending on the choice of α from (33). Nevertheless, NE can be achieved with pure strategies, while the closed-form expression of the optimal parameters depends on various channel conditions, as detailed in the Appendix. The comparison of $I(X;Y)$ at NE with perfect and noisy source information at $\mathcal{J}$ and $\mathcal{R}$ will be performed using numerical examples in Section VI.

V. Jammer-Relay Games in Fading Channels

Here, we generalize the results of previous sections to fading channels. The generalization will proceed in three stages/setups.

s1) $\mathcal{S}\to\mathcal{D}$, $\mathcal{R}\to\mathcal{D}$, and $\mathcal{J}\to\mathcal{D}$ links are fading but all other links are nonfading (Section V-A).

s2) $\mathcal{S}\to\mathcal{D}$, $\mathcal{S}\to\mathcal{R}$ and $\mathcal{S}\to\mathcal{J}$ links are fading but all others are nonfading (Section V-B).

s3) all links are fading (Section V-C).

Unless otherwise stated, we will assume in each link that the instantaneous channel state information (CSI) is only available at the receiving end. Notice that under s1–s3, channel assumptions are symmetric with respect to $\mathcal{J}$ and $\mathcal{R}$ so that a fair game can be played.

One might also be tempted to consider uncorrelated jamming and relaying when $\mathcal{J}$ and $\mathcal{R}$ do not have any information about the source signals—a setup similar to the uncorrelated jamming considered for single-user MISO fading channels and two-user fading channels in [3] and [17]. But in this setup, basically there is no game between $\mathcal{J}$ and $\mathcal{R}$, or we can say that the obvious solution is to have $\mathcal{R}$ deferring transmission and $\mathcal{J}$ transmitting noise only. The noise power depends on mutual CSI acknowledgements and channel conditions, as discussed in [3] and [17].

A. Fading Links to $\mathcal{D}$

With all channels linked to $\mathcal{D}$ fading under s1, we assume that both $\mathcal{J}$ and $\mathcal{R}$ know the source signals perfectly [i.e., in (5) and (6), we have $H_{SR}\equiv H_{SJ}\equiv 1$ and $N_R\equiv N_J\equiv 0$] and the channel model now is given by (4). We will prove that the optimal strategy for R is LR, while the optimal strategy for J amounts to transmitting Gaussian noise only.

Assuming $J = W_J$, where $W_J\sim\mathcal{CN}(0,\sigma^2_{W_J})$ and $\sigma^2_{W_J}\le P_J$, we will first look for the relay signal R that maximizes $I(X;Y)$. Again, introducing $Z := R - XE[XR]/P_S$ uncorrelated with X, we can write

$$Y = \left(H_S + \frac{E[XR]}{P_S}H_R\right)X + H_R Z + H_J W_J + N \quad (47)$$

where $E[Z^2] + |E[XR]|^2/P_S \le P_R$. Using (47), we can upper bound $I(X;Y|\mathbf{H}_1)$ per realization $\mathbf{H}_1 := (H_S, H_R, H_J)$ as

$$I(X;Y|\mathbf{H}_1) = h(Y|\mathbf{H}_1) - h(Y|X,\mathbf{H}_1) \le h(Y|\mathbf{H}_1) - h(H_J W_J + N) \quad (48)$$

where the upper bound can be achieved with equality when $Z = 0$. If $Z = 0$ and $\mathbf{H}_1$ are given, Y is a linear combination of Gaussian signals and noise terms. Hence, $\mathcal{R}$ should set $Z = 0$

$$\rho^* = \begin{cases} -\frac{\sqrt{\gamma_S}}{\sqrt{\gamma_J g_J}} - \frac{\alpha\sqrt{\gamma_R g_R}}{\sqrt{\gamma_J g_J}}, & \text{if } \left(g_J P_S + \sigma^2_{N_J}\right)(\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R})^2 \le \gamma_J g_J P_J \\ -\rho'_m(\alpha)\text{sgn}(\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R}), & \text{if } \left(g_J P_S + \sigma^2_{N_J}\right)(\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R})^2 > \gamma_J g_J P_J \end{cases} \quad (45)$$

and choose $E[XR]$ to maximize the average mutual information subject to a power constraint, i.e.,

$$\max_{E[XR]} \quad \frac{1}{2}E\left[\log\left(1+\frac{\left|H_S+\frac{E[XR]}{P_S}H_R\right|^2 P_S}{|H_J|^2\sigma_{W_J}^2+\sigma_N^2}\right)\right] \quad (49)$$

$$\text{s.t.} \quad |E[XR]|^2 \le P_S P_R. \quad (50)$$

Since H_S and H_R are jointly Gaussian, $\gamma := |H_S + (E[XR]/P_S)H_R|^2$ is exponentially distributed with mean $\bar{\gamma} = \sigma_S^2 + \sigma_R^2(|E[XR]|^2/P_S^2)$. With $f(\gamma) := \log(1 + C\gamma)$, where $C > 0$ denotes some constant, the average mutual information $E[I(X;Y)]$ in (49) can be written as $(1/2)E_{H_J}[E_\gamma[f(\gamma)]]$. Furthermore, $E_\gamma[f(\gamma)]$ can be expressed in terms of $\bar{\gamma}$ as

$$E_\gamma[f(\gamma)] = \int_0^\infty f(\gamma)\frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)d\gamma = \int_0^\infty \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)\frac{C}{1+C\gamma}d\gamma. \quad (51)$$

From the right-hand side (RHS) of (51), one can easily deduce that $E_\gamma[f(\gamma)]$ is monotonically increasing with $\bar{\gamma}$, and so is $E[I(X;Y)] = (1/2)E_{H_J}[E_\gamma[f(\gamma)]]$. To maximize the average mutual information, it is thus sufficient for $\mathcal{R}$ to maximize $\bar{\gamma} = \sigma_S^2 + \sigma_R^2(|E[XR]|^2/P_S^2)$. Hence, $\mathcal{R}$ should choose $|E[XR]|^2 = P_S P_R$, or $R^* = \sqrt{(P_R/P_S)}X$. Notice that here $\mathcal{R}$ can generate an arbitrary initial phase, if the phase can be detected by the destination during the $\mathcal{S} \to \mathcal{D}$ channel estimation stage, as under a2 in Section III-B. But from now on, we will ignore the phase of R for simplicity.

Next we will prove the converse, namely that if $R = \sqrt{(P_R/P_S)}X$, then the best jamming strategy is to transmit Gaussian noise W_J with power $\sigma_{W_J}^2 = P_J$. Now $Y = (H_S + H_R\sqrt{(P_R/P_S)})X + H_J J + N$, and we have to consider the mutual information conditioned on the realization of the fading channels, i.e.,

$$I(X;Y|\mathbf{H}_1) = h(X|\mathbf{H}_1) - h(X|Y,\mathbf{H}_1) \quad (52)$$
$$= h(X) - h(X|Y,\mathbf{H}_1) \quad (53)$$
$$= h(X) - h(X - A_1Y|Y,\mathbf{H}_1) \quad (54)$$

where A_1 can be any function of $\mathbf{H}_1$ and the second equality in (53) comes from the independence of X and $\mathbf{H}_1$ (X is chosen without any knowledge of $\mathbf{H}_1$). Since the jammer's strategy can only affect the second term of (54), we upper bound it as

$$h(X - A_1Y|Y,\mathbf{H}_1) \le h(X - A_1Y|\mathbf{H}_1) \quad (55)$$
$$\le E\left[\frac{1}{2}\log(2\pi e\Lambda)\right] \quad (56)$$

where $\Lambda := E[|X - A_1Y|^2|\mathbf{H}_1]$. With $\mathbf{H}_1$ given, choosing $A_1 = (E[XY|\mathbf{H}_1]/E[Y^2|\mathbf{H}_1])$, one can render $X - A_1Y$ conditionally uncorrelated with Y. Following arguments similar to those in Section III-A with LR, variables X, J and hence Y are all Gaussian if LJ is in effect. The error $X - (E[XY|\mathbf{H}_1]/E[Y^2|\mathbf{H}_1])Y$ is also Gaussian, independent of Y, and the inequalities in (56) can be achieved with equalities [i.e., assuming LJ, we are ensured that it can achieve the upper bound in (56)].

We next show the existence of an LJ strategy which leads to the maximum Λ and, thus, maximizes the upper bound in (56). Recalling that X and J are independent of $\mathbf{H}_1$, we have

$$E[XY|\mathbf{H}_1] = \left(H_S + H_R\sqrt{\frac{P_R}{P_S}}\right)P_S + H_J E[XJ|\mathbf{H}_1] \quad (57)$$
$$= \left(H_S + H_R\sqrt{\frac{P_R}{P_S}}\right)P_S + H_J E[XJ] \quad (58)$$

since $\mathbf{H}_1$ is unknown to $\mathcal{S}$, $\mathcal{J}$ and $\mathcal{R}$. Similarly, $E[Y^2|\mathbf{H}_1]$ can also be expressed as a function of $E[XJ]$. Since $\Lambda = E[|X - (E[XY|\mathbf{H}_1]/E[Y^2|\mathbf{H}_1])Y|^2|\mathbf{H}_1]$ is a function of $E[XY|\mathbf{H}_1]$ and $E[Y^2|\mathbf{H}_1]$, the upper bound in (56) depends on Λ and, hence, on $E[XJ]$ only. The rest of the arguments mimic those in Section III-A and lead to a systemic design of an LJ signal J_l based on any given feasible J. This proves that LJ can achieve any Λ in (56) and, hence, concludes the proof of optimality of LJ, if LR is in effect.

Having characterized the jamming signal, we proceed to find the optimal parameters in $J = \rho X + W_J$. Based on the input–output relationship

$$Y = \left(H_S + H_R\sqrt{\frac{P_R}{P_S}} + H_J\rho\right)X + H_J W_J + N \quad (59)$$

the jammer aims to minimize the $E[I(X;Y)]$ while adhering to its power constraint, i.e.,

$$\min_{\rho,\sigma_{W_J}^2} \quad \frac{1}{2}E\left[\log\left(1+\frac{\left|H_S+H_R\sqrt{\frac{P_R}{P_S}}+H_J\rho\right|^2 P_S}{|H_J|^2\sigma_{W_J}^2+\sigma_N^2}\right)\right] \quad (60)$$

$$\text{s.t.} \quad \rho^2 P_S + \sigma_{W_J}^2 \le P_J. \quad (61)$$

Fixing H_J, we take the expectation over H_S and H_R first, and rewrite the objective function in (60) as $(1/2)E_{H_J}\{E_{H_S,H_R}[\cdot]\}$, where $[\cdot]$ denotes the term between the square brackets in (60). Since $H_S + H_R\sqrt{(P_R/P_S)}$ is circular complex Gaussian distributed with zero mean, we can use [1, Corollary 3] to prove that

$$\Pr\left\{\left|H_S + H_R\sqrt{\frac{P_R}{P_S}}\right|^2 \le v\right\} \ge \Pr\left\{\left|H_S + H_R\sqrt{\frac{P_R}{P_S}} + H_J\rho\right|^2 \le v\right\}, \quad \forall v \quad (62)$$

where the random variables are H_S and H_R only. Letting $[[\cdot]|\rho = 0]$ denote the term between square brackets in (60) when $\rho = 0$, we find from (62) and (60) that

$$\Pr\{[[\cdot]|\rho = 0] \le v'\} \ge \Pr\{[\cdot] \le v'\}, \ \forall v' \quad (63)$$

and hence

$$E_{H_S,H_R}[[\cdot]|\rho = 0] \le E_{H_S,H_R}[\cdot]. \quad (64)$$

Equation (64) establishes that for each realization of H_J, the jamming strategy minimizing $E[I(X;Y)]$ has $\rho = 0$; and this is also valid after taking average over H_J. Hence, to minimize (60), the jammer should set $\rho = 0$ and transmit only W_J with full power $\sigma_{W_J}^2 = P_J$. This can be intuitively understood because the randomness of $\mathbf{H}_1$ prevents the jammer from cancelling the source signal by using ρ.

In a nutshell, we have proved that under the fading setup s1, NE can be achieved with $R^* = \sqrt{(P_R/P_S)}X$ and $J^* = W_J^* \sim \mathcal{CN}(0, P_J)$.

B. Fading Links to $\mathcal{S}$

Different from the previous subsection where all links connected to $\mathcal{D}$ were assumed fading, here, we consider that all channels linked to $\mathcal{S}$ are fading as per s2, while the channels from $\mathcal{J}$ and $\mathcal{R}$ to $\mathcal{D}$ are nonfading (i.e., $H_J \equiv H_R \equiv 1$). Then, the channels $\mathcal{S} \to \mathcal{R}$ and $\mathcal{S} \to \mathcal{J}$ are modeled as in (5) and (6), respectively, and the received signal at $\mathcal{D}$ is

$$Y = H_S X + J + R + N. \tag{65}$$

Similar to Section IV-A, finding the optimal relaying strategy with imperfect knowledge of X at $\mathcal{R}$ appears analytically intractable. For this reason, we assume *a fortiori* that the relay uses LR. Then $R = \alpha Y_R$ and the channel model becomes

$$\begin{aligned} Y &= H_S X + \alpha Y_R + J + N \\ &= (H_S + \alpha H_{SR})X + \alpha N_R + J + N. \end{aligned} \tag{66}$$

We wish to prove that the optimal jamming strategy is LJ, for each realization of $\mathbf{H}_2 := (H_S, H_{SR}, H_{SJ})$. In this case, the $I(X;Y)$ which $\mathcal{J}$ seeks to minimize is a function of $\mathbf{H}_2$. With A_2 denoting any function of $\mathbf{H}_2$, since $\mathcal{J}$ can only affect the second term in $I(X;Y|\mathbf{H}_2) = h(X) - h(X - A_2 Y|Y, \mathbf{H}_2)$, we upper bound the second term as

$$\begin{aligned} h(X - A_2Y|Y, \mathbf{H}_2) &\le h(X - A_2Y|\mathbf{H}_2) \\ &\le E\left[\frac{1}{2}\log(2\pi e\Lambda)\right] \end{aligned} \tag{67}$$

where $\Lambda := E[|X - A_2Y|^2|\mathbf{H}_2]$. Choosing $A_2 = (E[XY|\mathbf{H}_2]/E[Y^2|\mathbf{H}_2])$ and following arguments similar to those in Section III-A, we infer that if LJ is used, it can achieve the upper bound in (67) with some Λ.

To prove that an optimal LJ exists for any Λ, the method in Section V-A does not work since H_{SJ} is available at $\mathcal{J}$ and, in general, J in (66) can be correlated with $\mathbf{H}_2$. Specifically, Λ here depends on $E[XJ|H_{SJ}]$ instead of $E[XJ]$. Unable to directly design LJ, we generalize the proof in Section IV-B as follows. Given H_{SJ}, the error of the MMSE estimate of X based on Y_J is

$$E\left[|\hat{X}_1 - X|^2|H_{SJ}\right] = \frac{\sigma_{N_J}^2 E[X^2]}{|H_{SJ}|^2 E[X^2] + \sigma_{N_J}^2}. \tag{68}$$

For any feasible $J = f(Y_J)$, the error of the LMMSE estimate of X based on J given H_{SJ} is

$$E\left[|\hat{X}_2 - X|^2|H_{SJ}\right] = E[X^2] - \frac{|E[XJ|H_{SJ}]|^2}{E[J^2|H_{SJ}]} \tag{69}$$

which is lower bounded by (68). Hence, we can further upper bound $|E[XJ|H_{SJ}]|^2$ as

$$|E[XJ|H_{SJ}]|^2 \le \frac{|H_{SJ}|^2 \left|E[X^2]\right|^2 P_J}{|H_{SJ}|^2 E[X^2] + \sigma_{N_J}^2}. \tag{70}$$

Meanwhile, for an LJ signal $J_l = \rho(H_{SJ}X + N_J) + W_J$, we have $E[XJ_l|H_{SJ}] = \rho H_{SJ} E[X^2]$ and $\rho^2 \le (P_J/E[Y_J^2]) = P_J/(|H_{SJ}|^2 E[X^2] + \sigma_{N_J}^2)$ because of the power constraint. Thus, we have

$$|E[XJ_l|H_{SJ}]|^2 \le \frac{|H_{SJ}|^2 \left|E[X^2]\right|^2 P_J}{|H_{SJ}|^2 E[X^2] + \sigma_{N_J}^2}. \tag{71}$$

The bounds in (70) and (71) are identical, which implies that $E[XJ|H_{SJ}]$ and $E[XJ_l|H_{SJ}]$ have the same range of values. Hence, for any Λ achieved by some feasible jamming signal, there is an equivalent LJ signal which yields the same Λ and, thus, the same upper bound in (67). This proves the existence of LJ for any Λ and the fact that LJ is optimal when the relay uses LR.

Having characterized the strategies, we next find the optimal parameters which depend critically on the payoff function

$$E\left[I(X,Y)\right] = \frac{1}{2}E\left[\log\left(1 + \frac{|H_S + \alpha H_{SR} + \rho H_{SJ}|^2 P_S}{\alpha^2\sigma_{N_R}^2 + \rho^2\sigma_{N_J}^2 + \sigma_{W_J}^2 + \sigma_N^2}\right)\right]. \tag{72}$$

For any choice of $\rho = \rho_a$ and $\sigma_{W_J}^2$, this $E[I(X;Y)]$ can be achieved using another jammer with $\sigma_{W_J}^{2\prime} := \sigma_{W_J}^2 + \rho_a^2\sigma_{N_J}^2$, the same jamming power and perfect knowledge of the source signal X ($\sigma_{N_J}^2 = 0$). Based on this observation, it suffices to select jammer parameters minimizing

$$\frac{1}{2}E\left[\log\left(1 + \frac{|H_S + \alpha H_{SR} + \rho H_{SJ}|^2 P_S}{\alpha^2\sigma_{N_R}^2 + \sigma'^2_{W_J} + \sigma_N^2}\right)\right]. \tag{73}$$

Defining $\delta := |H_S + \alpha H_{SR} + \rho H_{SJ}|^2$ and following arguments similar to those in Section V-A, it is easy to show that (73) is monotonically increasing with $\bar{\delta} = \sigma_S^2 + \alpha^2\sigma_{SR}^2 + \rho^2\sigma_{SJ}^2$. As a result, setting $\rho = 0$ and transmitting W_J (or W_J') with power P_J is optimal for the jammer, regardless of α.

Now the relay's optimization problem becomes

$$\max_{\alpha} \quad \frac{1}{2}E\left[\log\left(1 + \frac{|H_S + \alpha H_{SR}|^2 P_S}{\alpha^2\sigma_{N_R}^2 + P_J + \sigma_N^2}\right)\right] \tag{74}$$

$$\text{s.t.} \quad \alpha^2\left(\sigma_{SR}^2 P_S + \sigma_{N_R}^2\right) \le P_R. \tag{75}$$

After taking the expectation, we can rewrite this constrained maximization problem as

$$\begin{aligned} \max_{\alpha} \quad & \frac{1}{2}\exp\left[\frac{\alpha^2\sigma_{N_R}^2 + \sigma_N^2 + P_J}{P_S\left(\alpha^2\sigma_{SR}^2 + \sigma_S^2\right)}\right] \\ & \times \mathrm{E}_1\left[\frac{\alpha^2\sigma_{N_R}^2 + \sigma_N^2 + P_J}{P_S\left(\alpha^2\sigma_{SR}^2 + \sigma_S^2\right)}\right] \end{aligned} \tag{76}$$

$$\text{s.t.} \quad \alpha^2\left(\sigma_{SR}^2 P_S + \sigma_{N_R}^2\right) \le P_R \tag{77}$$

where $\mathrm{E}_1[x]$ is the E_n function with $n = 1$ [18]. Since $\exp(x)\mathrm{E}_1[x] = \int_1^\infty e^{-(t-1)x}/t\, dt$ is monotonically decreasing for positive x, one can readily solve the optimization in (76) and (77) to obtain

$$\alpha^* = \begin{cases} \sqrt{\frac{P_R}{\sigma_{SR}^2 P_S + \sigma_{N_R}^2}}, & \text{if } \sigma_S^2 \sigma_{N_R}^2 < \sigma_{SR}^2 \left(\sigma_N^2 + P_J\right) \\ 0, & \text{if } \sigma_S^2 \sigma_{N_R}^2 \geq \sigma_{SR}^2 \left(\sigma_N^2 + P_J\right). \end{cases} \quad (78)$$

When the $\mathcal{S} \rightarrow \mathcal{R}$ link is not as reliable as the direct link $\mathcal{S} \rightarrow \mathcal{D}$, or say $\sigma_S^2/(\sigma_N^2 + P_J) \geq \sigma_{SR}^2/\sigma_{N_R}^2$, (78) implies that the relay should defer. Otherwise, the relay should transmit with full power. Consequently, under fading setup s2, if $\mathcal{R}$ uses LR, the NE can be reached with $R^* = \alpha^* Y_R$ and $J^* = W_J^* \sim \mathcal{CN}(0, P_J)$, where α^* is given by (78).

Notice that although H_{SJ} is assumed available at $\mathcal{J}$, the LJ slope ρ should still be set to zero because of the randomness of H_S which is not available at $\mathcal{J}$. If we consider a system where H_S is fed back from $\mathcal{D}$ to $\mathcal{S}$ and allow $\mathcal{J}$ to eavesdrop this feedback, then the jammer may choose ρ in (72) to cancel H_S in the numerator and have $H_S + \rho H_{SJ} = 0$. In this case, the source information would be useful when the jammer can determine the phase difference between H_S and H_{SJ}.

C. All Links are Fading

With the results of the previous two subsections on subsets of fading channels, we are ready to consider the most general case, where all of the links are fading as in s3. In lieu of feedback, the jammer would discard the signal from the source totally as in Section V-A, even if it has full information about the source. Hence, when all links are fading, we only need to consider the strategy at the relay, while the optimal jamming signal is still Gaussian noise. As with s2, the relay in s3 is assumed to rely on LR.

Under these considerations, the received signal at $\mathcal{D}$ is given by

$$Y = (H_S + \alpha H_R H_{SR})X + \alpha H_R N_R + H_J W_J + N. \quad (79)$$

With $\sigma_{W_J}^2 = P_J$ denoting the optimal jamming power, the relay seeks to maximize the capacity, i.e.,

$$\max_{\alpha} \ \frac{1}{2} E\left[\log\left(1 + \frac{|H_S + \alpha H_R H_{SR}|^2 P_S}{\alpha^2 |H_R|^2 \sigma_{N_R}^2 + |H_J|^2 P_J + \sigma_N^2}\right)\right] \quad (80)$$

$$\text{s.t. } \alpha^2 \left(\sigma_{SR}^2 P_S + \sigma_{N_R}^2\right) \leq P_R. \quad (81)$$

Taking the expectation over H_S and H_{SR} first, we can rewrite this maximization problem as

$$\max_{\alpha} \ \frac{1}{2} E\left\{\exp\left[\frac{\alpha^2 \sigma_{N_R}^2 |H_R|^2 + \sigma_N^2 + P_J |H_J|^2}{P_S\left(\alpha^2 \sigma_{SR}^2 |H_R|^2 + \sigma_S^2\right)}\right] \times \mathrm{E}_1\left[\frac{\alpha^2 \sigma_{N_R}^2 |H_R|^2 + \sigma_N^2 + P_J |H_J|^2}{P_S\left(\alpha^2 \sigma_{SR}^2 |H_R|^2 + \sigma_S^2\right)}\right]\right\} \quad (82)$$

$$\text{s.t. } \alpha^2 \left(\sigma_{SR}^2 P_S + \sigma_{N_R}^2\right) \leq P_R. \quad (83)$$

Lacking a closed-form solution to this problem, we will look for α's which can provide upper or lower bounds on $E[I(X;Y)]$ at NE. Upon comparing (82) with (76), we can readily derive the optimal α for each realization of H_J and H_R as

$$\alpha^*(H_J, H_R) = \begin{cases} \sqrt{\frac{P_R}{\sigma_{SR}^2 P_S + \sigma_{N_R}^2}}, & \text{if } \sigma_S^2 \sigma_{N_R}^2 < \sigma_{SR}^2 \left(\sigma_N^2 + P_J |H_J|^2\right) \\ 0, & \text{if } \sigma_S^2 \sigma_{N_R}^2 \geq \sigma_{SR}^2 \left(\sigma_N^2 + P_J |H_J|^2\right). \end{cases} \quad (84)$$

If $\mathcal{R}$ can acquire instantaneous knowledge of H_J, then $\alpha^*(H_J, H_R)$ in (84) will maximize (82) per realization H_R; and, thus, it will also maximize $E[I(X;Y)]$ after averaging over H_R. So an upper bound of $E[I(X;Y)]$ at NE can be attained via LR with $\alpha^{ub} := \alpha^*(H_J, H_R)$ in (84).

Before deriving the lower bounds, let us check one situation where the optimal α can be found in closed form. After normalizing the transmit power, if the SNR of the direct $\mathcal{S} \rightarrow \mathcal{D}$ link is lower than the SNR of the $\mathcal{S} \rightarrow \mathcal{R}$ link, i.e., $\sigma_S^2/\sigma_N^2 < \sigma_{SR}^2/\sigma_{N_R}^2$, the second case in (84) will never be in force (since $P_J |H_J|^2 \geq 0$). Thus, for any realization of H_J and H_R, the relay should always transmit will full power. This implies that when all channels are fading, and $\sigma_S^2/\sigma_N^2 < \sigma_{SR}^2/\sigma_{N_R}^2$ holds true, the optimal α for LR is given by

$$\alpha^* = \sqrt{\frac{P_R}{\sigma_{SR}^2 P_S + \sigma_{N_R}^2}}. \quad (85)$$

The optimum α^* in (85) will lead to a lower bound, call it α_1^{lb}, on the $E[I(X;Y)]$ at NE even when $\sigma_S^2/\sigma_N^2 \geq \sigma_{SR}^2/\sigma_{N_R}^2$.

Another simple strategy which yields a lower bound on $E[I(X;Y)]$ at NE, amounts to no transmission. This leads to $\alpha_2^{lb} \equiv 0$ and corresponds to having the direct link $\mathcal{S} \rightarrow \mathcal{D}$ being always better than the link $\mathcal{S} \rightarrow \mathcal{R}$, even in the presence of jammer-induced interference. This could indeed occur if the normalized SNR at the relay $\sigma_{SR}^2/\sigma_{N_R}^2$ is small enough.

To avoid (rather extreme) channel conditions affecting α_1^{lb} and α_2^{lb}, one may want to combine them in a smart way. Without optimality guarantees, α_1^{lb} and α_2^{lb} can be simply combined in (86), shown at the bottom of the page.

All of the upper and lower bounds will be investigated by numerical examples in the next section.

VI. Numerical Examples

Here, we rely on numerical examples to corroborate the analysis in previous sections and test various $\mathcal{J} - \mathcal{R}$ game scenarios. Unless otherwise specified, we will set $P_S = 10$, $\sigma_N^2 = 1$ and measure mutual information in nats.

Figs. 2 and 3 pertain to the nonfading scenario with $\gamma_S = \gamma_J = \gamma_R = g_J = g_R = 1$. Under a1, Fig. 2 depicts

$$\alpha_3^{lb} := \begin{cases} \sqrt{\frac{P_R}{\sigma_{SR}^2 P_S + \sigma_{N_R}^2}}, & \text{if } \Pr\left\{\sigma_S^2 \sigma_{N_R}^2 < \sigma_{SR}^2 \left(\sigma_N^2 + P_J |H_J|^2\right)\right\} > 0.5 \\ 0, & \text{if } \Pr\left\{\sigma_S^2 \sigma_{N_R}^2 < \sigma_{SR}^2 \left(\sigma_N^2 + P_J |H_J|^2\right)\right\} \leq 0.5 \end{cases} \quad (86)$$

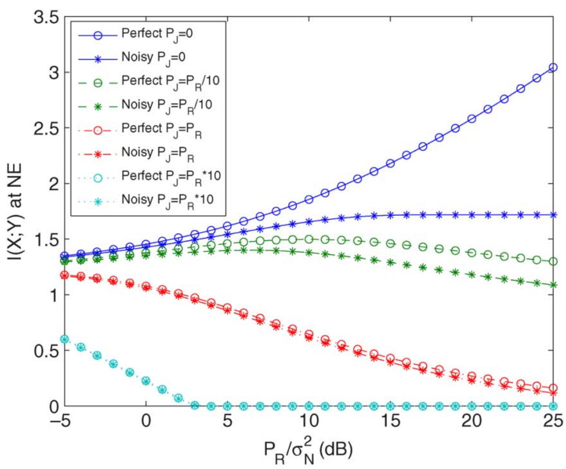

Fig. 2. $I(X;Y)$ at NE under a1 in a nonfading scenario with perfect and noisy source information at $\mathcal{J}$ and $\mathcal{R}$.

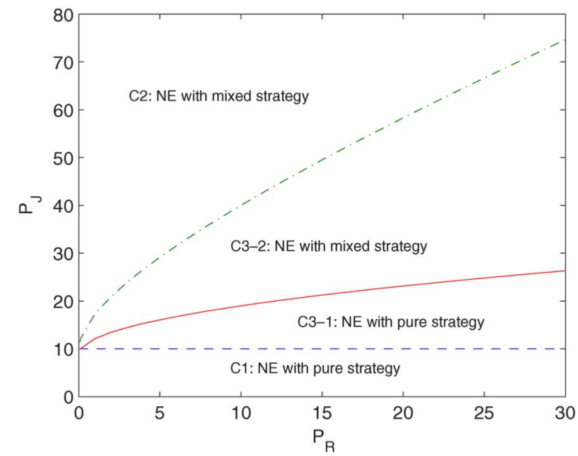

Fig. 3. Decision regions of $\mathcal{J}$ and $\mathcal{R}$ under a2 in a nonfading scenario with perfect source information at $\mathcal{J}$ and $\mathcal{R}$.

$I(X;Y)$ at NE versus P_R/σ_N^2 (in decibels) when perfect or noisy source information is available at $\mathcal{J}$ and $\mathcal{R}$. Recall that in both cases, NE can be achieved with a pair of pure strategies $(R^* = \sqrt{(P_R/P_S)}X,\ J^* = \rho^* X + W_J^*)$. Four curves are shown for each case corresponding to different choices of P_J, namely $P_J = 0$, $P_J = P_R/10$, $P_J = P_R$, and $P_J = 10P_R$. We can see that when the jammer's power is greater than or equal to the power of the relay, $I(X;Y)$ approaches zero when their powers are sufficiently large. The curves of $I(X;Y)$ at NE are marked with "$\circ$" when the $\mathcal{S} \to \mathcal{J}$ and $\mathcal{S} \to \mathcal{R}$ links are perfect, and with "$*$" when the $\mathcal{S} \to \mathcal{J}$ and $\mathcal{S} \to \mathcal{R}$ links are modeled as AWGN channels with $\sigma_{N_J}^2 = \sigma_{N_R}^2 = 0.5$. Since noisy source information "hurts" $\mathcal{R}$ more seriously than $\mathcal{J}$, the $I(X;Y)$ at NE is smaller with noisy source information at $\mathcal{J}$ and $\mathcal{R}$ for all P_J choices. Especially when $P_J = 0$, instead of linearly increasing with perfect X, $I(X;Y)$ at NE with noisy X approaches a constant as P_R grows large. This occurs because when there is noise at $\mathcal{R}$, with $P_J = 0$ and fixed P_S, increasing P_R converts the communication system to an $\mathcal{R} \to \mathcal{D}$ link only while the transmit signal from $\mathcal{R}$ is $X + N_R$. Thus, further increasing P_R will neither bring receive SNR gains nor $I(X;Y)$ gains. In Fig. 3, we plot the decision regions corresponding to the values of P_J and P_R when the source is known perfectly at $\mathcal{J}$ and $\mathcal{R}$ under a2. Depending on channel conditions, $\mathcal{J}$ and $\mathcal{R}$ choose different strategies to maximize their benefit. The plot corroborates that when the jammer's power grows large, NE is reached with mixed strategies.

Figs. 4–7 pertain to fading scenarios with $\sigma_S^2 = \sigma_J^2 = \sigma_R^2 = \sigma_{SR}^2 = \sigma_{SJ}^2 = 1$. In Fig. 4, we plot the average mutual information $E[I(X;Y)]$ at NE in setup s1, against P_R/σ_N^2 in decibels. Different from nonfading scenarios, when $P_J > 0$ and the powers of $\mathcal{J}$ and $\mathcal{R}$ grow large, $E[I(X;Y)]$ at NE approaches a nonzero constant.

In setup s2, since the noise power $\sigma_{N_J}^2$ does not affect the optimal jamming strategy, we only specify the noise at $\mathcal{R}$ to be $\sigma_{N_R}^2 = 0.5$ and $\sigma_{N_R}^2 = 5$ in Fig. 5. The curves without markers correspond to $\sigma_{N_R}^2 = 0.5$, where $\sigma_{N_R}^2$ is so small that the second choice in (78) is ruled out regardless of the value of P_J. Here, the relay always forwards at full power. As $\sigma_{N_R}^2$ grows to 5, the $E[I(X;Y)]$ at NE depicted by the curves with markers decreases relative to the curves with $\sigma_{N_R}^2 = 0.5$ for the same P_J and P_R. Recall that under s2, the optimal jamming signal is just Gaussian noise regardless of whether the source is known. Hence, fading and noisy information of X here impairs

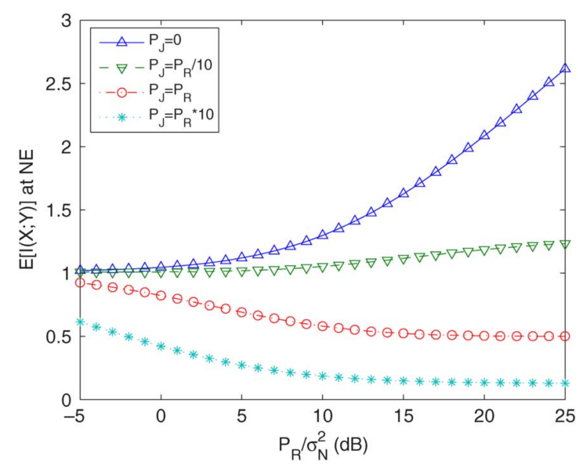

Fig. 4. $E[I(X;Y)]$ at NE in fading setup s1.

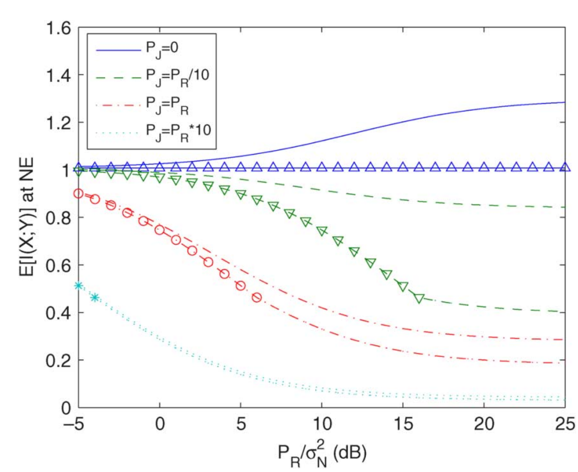

Fig. 5. $E[I(X;Y)]$ at NE in fading setup s2; the curves corresponding to $\sigma_{N_R}^2 = 0.5$ do not have markers while the curves corresponding to $\sigma_{N_R}^2 = 5$ have some markers when $\mathcal{R}$ defers forwarding.

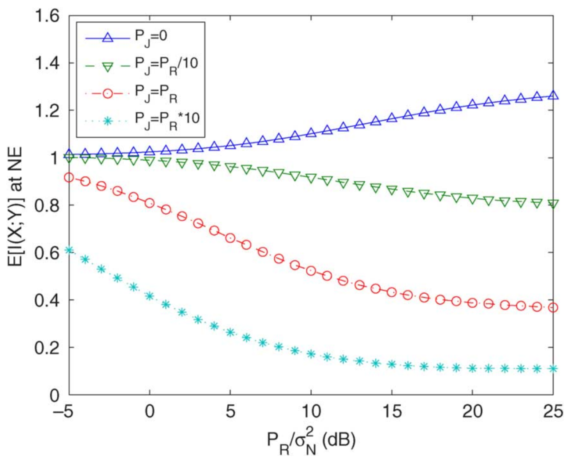

Fig. 6. $E[I(X;Y)]$ at NE in fading setup s3: with $\sigma_{N_R}^2 = 0.5$.

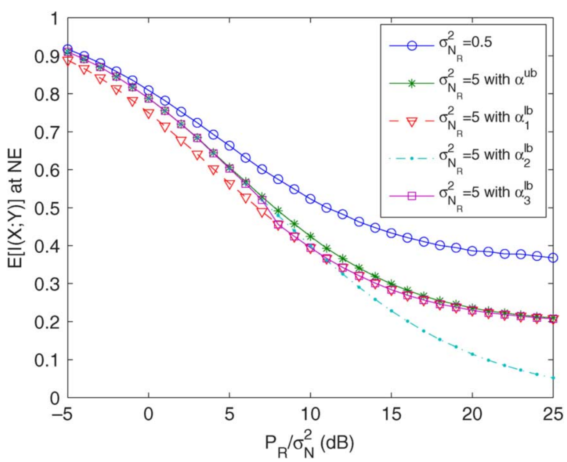

Fig. 7. $E[I(X;Y)]$ at NE in fading setup s3: with $\sigma_{N_R}^2 = 5$ and $P_J = P_R$.

primarily $\mathcal{R}$ and to a lesser extent $\mathcal{J}$. Also, in Fig. 5, $\mathcal{R}$ may stop forwarding when P_J, which is counted in the noise at $\mathcal{D}$ [cf. (78)], is small relative to $\sigma_{N_R}^2$. On the $E[I(X;Y)]$ curves with $\sigma_{N_R}^2 = 5$, markers denote values of P_R/σ_N^2 for which $\mathcal{R}$ defers; while the absence of markers denotes values for which $\mathcal{R}$ transmits at full power. Intuitively speaking, the relay becomes "serious" to play with full power when the jammer's power is sufficiently high.

When all links are fading (setup s3), $E[I(X;Y)]$ is plotted at NE with $\sigma_{N_R}^2 = 0.5$ and $\sigma_{N_R}^2 = 5$ in Figs. 6 and 7, respectively. For $\sigma_{N_R}^2 = 0.5$, we find $\sigma_S^2/\sigma_N^2 < \sigma_{SR}^2/\sigma_{N_R}^2$ and as proved in Section V-C, all α's corresponding to the upper and lower bounds of $E[I(X;Y)]$ in s3 meet at $\alpha^* = \sqrt{P_R/(\sigma_{SR}^2 P_S + \sigma_{N_R}^2)}$. The curves shown in Fig. 6 denote the exact $E[I(X;Y)]$ values at NE. In Fig. 7, we consider $\sigma_{N_R}^2 = 5$ and show all of the upper and lower bounds of $E[I(X;Y)]$ with $P_J = P_R$. By incorporating α_1^{lb} and α_2^{lb}, the relaying strategy with α_3^{lb} provides a decent lower bound on $E[I(X;Y)]$, which is very close to the upper bound obtained via α^{ub}. As a benchmark, we also plot the exact $E[I(X;Y)]$ curve when $\sigma_{N_R}^2 = 0.5$ and $P_J = P_R$.

Finally, in Fig. 8, we compare $I(X;Y)$ and $E[I(X;Y)]$ at NE for different nonfading and fading scenarios, with $P_J = P_R$ and $\sigma_{N_R}^2 = \sigma_{N_J}^2 = 0.5$ (whenever applicable). Besides the obvious result that noisy information at $\mathcal{J}$ and $\mathcal{R}$ decreases the

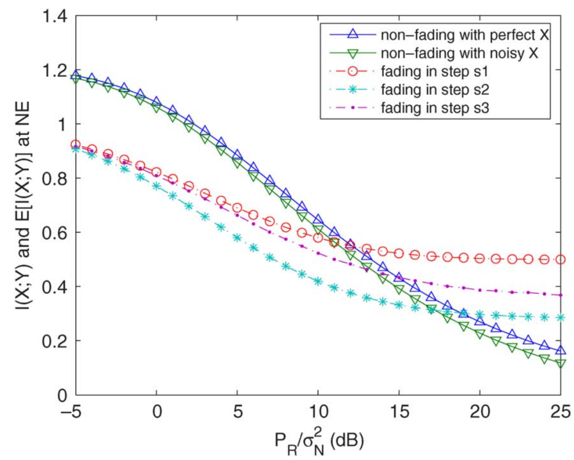

Fig. 8. $I(X;Y)$ and $E[I(X;Y)]$ at NE in nonfading (under a1) and fading scenarios, with $P_J = P_R$.

value of $I(X;Y)$ $(E[I(X;Y)])$ at NE, one can observe two more interesting things. First, $E[I(X;Y)]$ at NE is larger in s3 than in s2 with fading. Since the optimal strategies for s2 and s3 are both $R^* = \sqrt{P_R/(\sigma_{SR}^2 P_S + \sigma_{N_R}^2)}Y_R$ and $J^* = W_J^* \sim \mathcal{CN}(0, P_J)$, upon comparing (82) with (76) the only explanation is that $E[I(X;Y)]$ in (82) is a convex function of $|H_R|^2$ and $|H_J|^2$. Another interesting fact is that at NE, $I(X;Y)$ in the absence of fading becomes smaller than $E[I(X;Y)]$ in the presence of fading, when P_J and P_R are large enough. The reason is that in the nonfading scenario, the jammer relies on the exact phase (sign) difference at $\mathcal{D}$ between the jamming signal and the source signal. So when P_J and P_R are large enough, besides creating noise, the jammer has extra power available to cancel the source signal, especially under a1 which is the case we consider here. While in the fading scenario, the jammer cannot obtain the exact phase difference between its own signal and the source signal because of the insufficient CSI knowledge at the jammer, rendering the optimal strategy to be the generation of noise only. But when P_J grows large, creating Gaussian noise does not only minimize mutual information as efficiently as canceling the source signal. Also, note that we have chosen $\gamma_S = \gamma_J = \gamma_R = g_J = g_R = 1$ in nonfading and $\sigma_S^2 = \sigma_J^2 = \sigma_R^2 = \sigma_{SR}^2 = \sigma_{SJ}^2 = 1$ in fading scenarios. Without considering path loss, what we have here is actually a manifestation of the differences between time-invariant and time-varying channels.

VII. Conclusion

We characterized NE for zero-sum mutual information games between one jammer ($\mathcal{J}$) and one relay ($\mathcal{R}$) in both nonfading and fading scenarios. With source ($\mathcal{S}$) and destination ($\mathcal{D}$) being unaware of the game, the optimal strategies for $\mathcal{J}$ and $\mathcal{R}$ were derived under various assumptions and conditions. In the nonfading scenario, when both $\mathcal{J}$ and $\mathcal{R}$ have full knowledge of the source, the optimal strategies are LJ and LR, respectively. The strategies may be pure or mixed depending on the link qualities and whether $\mathcal{J}$ and $\mathcal{R}$ are active or not during the $\mathcal{S} \to \mathcal{D}$ channel training. If $\mathcal{J}$ and $\mathcal{R}$ receive the source signal in AWGN, and LR is assumed at NE, pure or mixed LJ is still optimal. Instead of always transmitting with full power as when the $\mathcal{S} \to \mathcal{R}$ link is perfect, $\mathcal{R}$ should control transmit power based on its power constraint and the reliability of the received signal at $\mathcal{R}$.

In the fading scenario, when $\mathcal{J}$ cannot determine the phase difference between the jamming and the source signals, the best strategy for $\mathcal{J}$ is to jam with Gaussian noise only. The optimal relaying strategy was either proved or assumed fixed to LR depending on the channel fading setups. When LR is considered, $\mathcal{R}$ should relay with full power if the link $\mathcal{S} \to \mathcal{R}$ is better than the jammed $\mathcal{S} \to \mathcal{D}$ link, and defer forwarding otherwise. Upper and lower bounds were considered when the optimal parameters could not be determined in closed form.[2]

APPENDIX

A. Characterization of NE Under c3

Recall that the pertinent channel conditions are as follows.

c3-1. $\sqrt{P_S\gamma_S}(\sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R}) \geq P_J\gamma_J + \sigma_N^2$ and $\sqrt{P_S\gamma_S} < \sqrt{P_J\gamma_J} < \sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R}$.

c3-2. $\sqrt{P_S\gamma_S}(\sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R}) < P_J\gamma_J + \sigma_N^2$ and $\sqrt{P_S\gamma_S} < \sqrt{P_J\gamma_J} < \sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R}$.

Under c3-1, if $\mathcal{R}$ chooses $R = \sqrt{(P_R/P_S)}X$ and, hence, $E[XR] = \sqrt{P_SP_R}$, from the last inequality in c3-1, we find that $P_J\gamma_J < (\sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R})^2 = (\sqrt{P_S\gamma_S} + (\sqrt{P_SP_R}/\sqrt{P_S})\sqrt{\gamma_R})^2 = (\sqrt{P_S\gamma_S} + (E[XR]\sqrt{\gamma_R}/\sqrt{P_S}))^2$. Now using (18), this inequality implies that $\rho^* = -\rho_m\{\mathrm{sgn}(\sqrt{\gamma_S} + (E[XR]\sqrt{\gamma_R}/P_S)) = -\rho_m$, where ρ_m is defined in (19). Rearranging the first inequality in c3-1, we can easily verify that

$$\sqrt{\gamma_S} \geq \frac{P_J\gamma_J + \sigma_N^2}{\left(\sqrt{\gamma_S} + \sqrt{\frac{P_R\gamma_R}{P_S}}\right)P_S}. \quad (87)$$

Substituting $\sqrt{\gamma_S} < \sqrt{P_J\gamma_J/P_S}$ from c3-1 into (87), we can further determine ρ^* from (19) as

$$\rho^* = -\rho_m = -\frac{P_J\gamma_J + \sigma_N^2}{\left(\sqrt{\gamma_S} + \sqrt{\frac{P_R\gamma_R}{P_S}}\right)P_S\sqrt{\gamma_J}}. \quad (88)$$

Based on (87) and (88), it follows that $A = \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho^*$ in (10) must be nonnegative which, from (12), leads to a pure optimal relay strategy with $R^* = \sqrt{(P_R/P_S)}X$. This justifies our initial choice of $\mathcal{R}$ and proves that NE can be achieved under c3-1 with pure strategies $R^* = \sqrt{(P_R/P_S)}X$ and $J^* = \rho^*X + W_J^*$, where $W_J^* \sim \mathcal{CN}(0, (P_J - \rho^{*2}P_S)^+)$ and ρ^* is chosen as in (88).

Regarding c3-2, following the same rationale as for c3-1, we choose $R = \sqrt{(P_R/P_S)}X$ for $\mathcal{R}$ and arrive at $\rho^* = -\rho_m$. But from the first inequality in c3-2, we infer that

$$\sqrt{\gamma_S} < \frac{P_J\gamma_J + \sigma_N^2}{\left(\sqrt{\gamma_S} + \sqrt{\frac{P_R\gamma_R}{P_S}}\right)P_S}. \quad (89)$$

Now the optimum slope can be found as the maximum of two ratios [cf. (19)]

$$\rho^* = \max\left\{-\frac{P_J\gamma_J + \sigma_N^2}{\left(\sqrt{\gamma_S} + \sqrt{\frac{P_R\gamma_R}{P_S}}\right)P_S\sqrt{\gamma_J}}, -\sqrt{\frac{P_J}{P_S}}\right\}. \quad (90)$$

If ρ^* is equal to the first ratio in (90), we find from (89) that $A = \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho^* < 0$. On the other hand, if $\rho^* = -\sqrt{P_J/P_S}$, substituting $\sqrt{\gamma_S} < \sqrt{P_J\gamma_J/P_S}$ from c3-2 into A we still arrive at $A = \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho^* < 0$. Hence, either choice of ρ^* in (90) ends up with $A < 0$ and leads $\mathcal{R}$ to select $R^* = -\sqrt{(P_R/P_S)}X$ from (12). This contradicts the initial choice of $R = \sqrt{(P_R/P_S)}X$ and proves that pure strategies cannot achieve NE.

Thereby we consider mixed strategies for the same reasons we argued under c2. With B denoting the strategy of $\mathcal{R}$ and $A(B = \pm\sqrt{(P_R\gamma_R/P_S)})$ denoting the strategy of $\mathcal{J}$, when $B = \pm\sqrt{P_R\gamma_R/P_S}$, we assign

$$B = \begin{cases} \sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{w.p. } p_R \\ -\sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{w.p. } 1 - p_R \end{cases} \quad (91)$$

$$A = \begin{cases} A\left(B = \sqrt{\frac{P_R\gamma_R}{P_S}}\right), & \text{w.p. } p_J \\ A\left(B = -\sqrt{\frac{P_R\gamma_R}{P_S}}\right), & \text{w.p. } 1 - p_J. \end{cases} \quad (92)$$

After some derivations, it turns out that the optimum probability assignment for the relay is

$$p_R^* = \begin{cases} 1, & \text{if } p'_R > 1 \\ p'_R, & \text{if } 0 \leq p'_R \leq 1 \\ 0, & \text{if } p'_R < 0 \end{cases} \quad (93)$$

where p'_R is the solution of the equation

$$I\left[B(\sqrt{}), A(\sqrt{})\right]p_R + I\left[B(-\sqrt{}), A(\sqrt{})\right](1 - p_R) = I\left[B(\sqrt{}), A(-\sqrt{})\right]p_R + I\left[B(-\sqrt{}), A(-\sqrt{})\right](1 - p_R)$$

with $I[B(\pm\sqrt{}), A(\pm\sqrt{})]$ denoting $E[(1/2)\log(1 + (A + B)^2P_S/(\gamma_J\sigma_{W_J}^2 + \sigma_N^2))]$ when $B = \pm\sqrt{(P_R\gamma_R/P_S)}$ and $A = A(B = \pm\sqrt{(P_R\gamma_R/P_S)})$. Similarly, the optimal p_J^* for the jammer is the solution of

$$I\left[B(\sqrt{}), A(\sqrt{})\right]p_J + I\left[B(\sqrt{}), A(-\sqrt{})\right](1 - p_J) = I\left[B(-\sqrt{}), A(\sqrt{})\right]p_J + I\left[B(-\sqrt{}), A(-\sqrt{})\right](1 - p_J).$$

The optimal p_J^* satisfies $0 \leq p_J^* \leq 1$ automatically, as the other cases can be ruled out by the given conditions. Thus, we have proved that with nonfading links and perfect knowledge of the source at $\mathcal{J}$ and $\mathcal{R}$, NE can be achieved with pure strategies under c3-1 and mixed strategies under c3-2.

B. Proof of Proposition 3

Here, we derive the optimal jamming and relaying strategies, where we always have $\alpha \geq 0$ for $\mathcal{R}$ and $\rho \leq 0$ for $\mathcal{J}$. Depending on channel conditions, the optimal α^* and ρ^* are given as follows.

[2]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government.

If $\sqrt{\gamma_S} + \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}\sqrt{\gamma_R g_R} \le \sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}\sqrt{\gamma_J g_J}$, then $I(X;Y) = 0$ at NE which is achieved by more than one pair of strategies. A simple but optimal strategy for $\mathcal{J}$ is just to transmit $J^* = -\sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}X$. The relay strategy does not affect the value of $I(X;Y)$ at NE because of the power constraint at $\mathcal{R}$. The NE achieved at $I(X;Y) = 0$ is stable since a small change of each player's values will not change the solution of the game, namely $I(X;Y) = 0$.

If $\sqrt{\gamma_S} \le \sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}\sqrt{\gamma_J g_J} < \sqrt{\gamma_S} + \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}\sqrt{\gamma_R g_R}$, then $\alpha^* = \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}$ and $\rho^* = -\rho'_m(\sqrt{P_R/(g_R P_S + \sigma^2_{N_R})})$, where $\rho'_m(\alpha)$ is defined in (46). For brevity, we omit the detailed derivation here. Basically besides the boundary conditions, one should also check the monotonicity and extreme points of α^* with respect to ρ in (33) and ρ^* with respect to α in (45). This approach is also followed in the next channel condition.

If $\sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}\sqrt{\gamma_J g_J} < \sqrt{\gamma_S}$, then we have to further consider three subcases.

1) If $(\gamma_J P_J + \sigma^2_N)\sqrt{g_R}/(\sqrt{\gamma_S \gamma_R}\sigma^2_{N_R}) > \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}$, then $\alpha^* = \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}$ and $\rho^* = -\rho'_m(\sqrt{P_R/(g_R P_S + \sigma^2_{N_R})})$.
2) If $(\gamma_J P_J + \sigma^2_N)\sqrt{g_R}/\sqrt{(\gamma_S \gamma_R \sigma^2_{N_R})} \le \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}$, but $(\gamma_J P_J + \sigma^2_N)/(\sqrt{\gamma_S} P_S \sqrt{\gamma_J g_J}) > \sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}$, then $\rho^* = -\sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}$ and $\alpha^* = \alpha_m(-\sqrt{P_J/(g_J P_S + \sigma^2_{N_J})})$, where $\alpha_m(\rho)$ is defined in (34).
3) If $(\gamma_J P_J + \sigma^2_N)\sqrt{g_R}/(\sqrt{\gamma_S \gamma_R}\sigma^2_{N_R}) \le \sqrt{P_R/(g_R P_S + \sigma^2_{N_R})}$ and $(\gamma_J P_J + \sigma^2_N)/(\sqrt{\gamma_S} P_S \sqrt{\gamma_J g_J}) \le \sqrt{P_J/(g_J P_S + \sigma^2_{N_J})}$, then $\alpha^* = (\gamma_J P_J + \sigma^2_N)\sqrt{g_R}/(\sqrt{\gamma_S \gamma_R}\sigma^2_{N_R})$ and $\rho^* = -(\gamma_J P_J + \sigma^2_N)/(\sqrt{\gamma_S} P_S \sqrt{\gamma_J g_J})$.

Recall that in all cases, the jammer's noise variance is given by $\sigma^{2*}_{W_J} = (P_J - \rho^{*2}(g_J P_S + \sigma^2_{N_J}))^+$.

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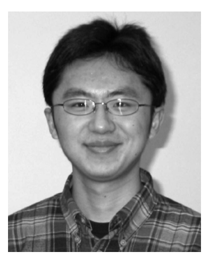

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