Achievable Rates of Transmitted-Reference Ultra-Wideband Radio With PPM

Xiliang Luo, *Member, IEEE*, and Georgios B. Giannakis, *Fellow, IEEE*

***Abstract*—In this letter, we study the achievable rates of practical ultra-wideband (UWB) systems using pulse position modulation (PPM) and transmitted-reference (TR) transceivers. TR obviates the need for complex channel estimation, which is particularly challenging in the context of UWB communications. Based on an upper bound we derive for the error probability with random coding, we establish that for signal-to-noise ratio values of practical interest, PPM-UWB with TR can achieve rates on the order of $C(\infty) = P/N_0$ (nats/s), where $C(\infty)$ denotes the capacity of an additive white Gaussian noise channel in the UWB regime for average received power P and noise power spectrum density N_0.**

***Index Terms*—Achievable rates, transmitted-reference (TR), ultra-wideband (UWB).**

I. Introduction

IDEAL for providing short-range high-rate wireless connectivity in a personal area network (PAN), ultra-wideband (UWB) technology [a.k.a. impulse radio (IR)] relies on ultra-narrow pulses (at nanosecond scale) to convey information, and has received a lot of attention recently. However, there are still major design challenges to overcome. For instance, timing synchronization with pulse-level accuracy is difficult, due to the fact that the transmitted pulse duration is very small. Meanwhile, the channel typically consists of hundreds of multipath returns, which renders channel estimation prohibitively costly. For this reason, the RAKE receiver, which is typically adopted to collect the multipath energy, is not as efficient in the context of UWB.

To overcome these difficulties, transmitted-reference (TR) transceivers relying on noncoherent detection have received revived interest for UWB systems (see [4] and [11]). TR entails two pulses per symbol period; the first one is unmodulated, while the second one is information-bearing and delayed, relative to the first, by an amount exceeding the channel's delay spread. This way, the first pulse can serve as a template at the receiver side to demodulate the message carried by the second one. As a result, TR bypasses the costly channel estimation required by RAKE reception. Motivated by the work of Souilmi and Knopp [12], where achievable rates of UWB pulse position modulation (PPM) are studied using energy detection at the receiver, we will investigate here the achievable rates of UWB radios using PPM and TR transceivers. Related works on UWB capacity analysis with PPM include [7], [16], and [17]. Specifically, the effect of timing-estimation errors on achievable rates with a correlator receiver was considered in [7]. Unlike [7], TR receivers are relatively robust to timing errors, because they do not rely on a local template to correlate the received waveform.

The rest of this paper is organized as follows. Section II describes the system, while Section III deals with the derivation of pertinent detection-error probabilities and the calculation of achievable rates. In Section IV, numerical results of the achievable rates are provided and compared against the additive white Gaussian noise (AWGN) channel capacity. Finally, conclusions are drawn in Section V.

II. Modeling

The multipath fading channel is modeled as

$$h(t,\tau) = \sum_{l=0}^{L-1} a_l(t)\delta(\tau - \tau_l(t)) \tag{1}$$

where L is the number of paths, $a_l(t)$ is the gain of path l at time t, and $\tau_l(t)$ is the corresponding delay. The model in (1) does not account for pulse distortions that may arise when the transmission bandwidth is extremely wide [10]. But for TR receivers, this does not entail loss of generality, because such distortions are basically identical to both reference and information-bearing pulses, and thus, do not affect detection performance. Each path gain a_l is assumed to be zero-mean, and different path gains are assumed to be uncorrelated, but not necessarily independent; i.e., $E[a_l a_k] = 0$, for $l \neq k$. Without loss of generality, we assume $\tau_0 = 0 < \tau_1 < \cdots < \tau_{L-1}$ with τ_{L-1} denoting the channel delay spread. In this letter, we are interested in channels with coherence period T_c, which is much larger than τ_{L-1}, i.e., $T_c \gg \tau_{L-1}$. In practice, the coherence period of a typical UWB indoor channel is about 20 ms [5], and the delay spread is about 20 ns [6], which clearly satisfies the previous condition. We consider a block fading channel, meaning that $\{a_l(t), \tau_l(t)\}$ remain constant over each T_c-period, but change independently across coherence periods.

In the UWB regime, extremely large bandwidth ($\geq$ 1 GHz) enables the receiver to resolve a large number of paths. If the channel has a high diversity order (the case in dense multipath fading environments), the aggregate channel gain $\sum_{l=0}^{L-1} a_l^2(t)$ varies slowly compared with $a_l(t)$ and $\tau_l(t)$. We can thus assume for all practical purposes that the total channel gain is con-

Paper approved by D. I. Kim, the Editor for Spread Spectrum Transmission and Access of the IEEE Communications Society. Manuscript received June 13, 2005; revised November 14, 2005. This work was supported in part by the NSF-ITR under Grant EIA-0324864, and in part through collaborative participation in the Communications and Networks Consortium sponsored by the U.S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon.

X. Luo was with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA. He is now with Qualcomm, Inc., San Diego, CA 92121 USA (e-mail: xluo@qualcomm.com).

G. B. Giannakis is with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: georgios@ece.umn.edu).

Color versions of Figs. 1–4 are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCOMM.2006.881203

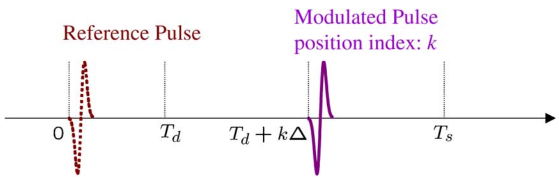

Fig. 1. PPM with transmitted reference.

stant and, without loss of generality, can be normalized to 1; i.e., $\sum_{l=0}^{L-1} a_l^2(t) = 1$ (see also [12]).

A. Transmitter Structure

Flash-signaling is known to enjoy first-order optimality in the sense of achieving capacity in the wideband regime, even when the receiver does not have channel knowledge [15]. As a practical means of implementing flash-signaling, we adopt here baseband m-ary PPM for transmitting information bits. In particular, the transmitted waveform per channel use is $k \in [0, m-1]$, $t \in [0, T_s]$

$$x(t; k) = \sqrt{PT_s}\left[\alpha p(t) + \beta p(t - T_d - k\Delta)\right] \tag{2}$$

where P is the transmission power, $p(t)$ is the normalized monocycle[1] with duration $T_p \approx 1/W$ with W denoting the UWB bandwidth, Δ denotes the modulation index, and α, β are positive scalars satisfying $\alpha^2 + \beta^2 = 1$, which will be optimized later. Delay T_d is selected so that $T_d \geq \tau_{L-1} + T_p$, and the symbol period T_s is chosen such that $T_s = 2T_d + (m-1)\Delta < T_c$, which avoids intersymbol interference (ISI). See also Fig. 1 for reference.

In order to transmit M messages, we generate a random codebook $\mathcal{C} = \{C_1, \ldots, C_M\}$, where each codeword C_w is a length-N sequence $C_w = [C_{w,1}, \ldots, C_{w,N}]$, with $C_{w,n}$ specifying the transmitted waveform during the nth channel use when message w is sent. The entries of each codeword $\{C_{w,n}\}_{n=1,\ldots,N; w=1,\ldots,M}$ are independently generated according to the uniform distribution over $[0, m-1]$. The aggregate transmitted waveform for message w is, thus, $w \in [1, \ldots, M]$, $t \in [0, NT_s]$

$$u_w(t) = \sum_{n=1}^{N} x(t - (n-1)T_s; C_{w,n}). \tag{3}$$

B. Receiver Structure

Assuming that message $w = 1$ has been sent, after propagation through the channel in (1), the received signal is then $r(t) = h(t) \star u_1(t) + z(t)$, where "$\star$" stands for linear convolution, and $z(t)$ denotes AWGN with double-sided power spectrum density $N_0/2$. When the system has bandwidth W, the temporal resolution is approximately $1/W \approx T_p$. Upon selecting T_d to be an integer multiple of T_p, say $T_d = K_d T_p$, we can equivalently express the channel in (1) per coherence period as

$$h(\tau) = \sum_{q=0}^{K_d-1} \tilde{a}_q \delta(\tau - qT_p) \tag{4}$$

where $\tilde{a}_q := \sum_{l=0}^{L-1} a_l \mathbf{1}_{\tau_l \in [qT_p, (q+1)T_p)}$ is the equivalent path gain, and $\mathbf{1}_{\{.\}}$ denotes the indicator function. With the channel as in (4), we obtain for $t \in [0, T_s]$

$$\begin{aligned} r(t + (n-1)T_s) &= x(t; C_{1,n}) \star h(t) + z(t + (n-1)T_s) \\ &= \sqrt{PT_s}\alpha \sum_{q=0}^{K_d-1} \tilde{a}_q^{(n)} p(t - qT_p) \\ &\quad + z(t + (n-1)T_s) \\ &\quad + \sqrt{PT_s}\beta \sum_{q=0}^{K_d-1} \tilde{a}_q^{(n)} \\ &\quad \times p(t - T_d - C_{1,n}\Delta - qT_p) \end{aligned}$$

where $\{\tilde{a}_q^{(n)}\}_{q=0}^{K_d-1}$ are the equivalent path gains during nth channel use. Without loss of generality and for clarity in exposition,[2] we can assume that $\{\tilde{a}_q^{(n)}\}_{q=0}^{K_d-1}$ and $\{\tilde{a}_q^{(v)}\}_{q=0}^{K_d-1}$ are independent for $n \neq v$.

Letting $T_s = K_s T_p$ and $\Delta = K_\Delta T_p$, we can project the received signal onto the set of bases $\{p(t - iT_p)\}_{i=0}^{K_s-1}$ to obtain

$$r_{n,i} := \int p(t - iT_p) r(t + (n-1)T_s) dt, \quad i = 0, 1, \ldots, K_s - 1. \tag{5}$$

Upon defining $\tilde{\boldsymbol{a}}^{(n)} := [\tilde{a}_0^{(n)}, \ldots, \tilde{a}_{K_d-1}^{(n)}]^T$ and $\boldsymbol{r}_n = [r_{n,0}, \ldots, r_{n,K_s-1}]^T$, we find

$$\boldsymbol{r}_n = \left[\sqrt{PT_s}\alpha \tilde{\boldsymbol{a}}^{(n)T}, \overbrace{0, \ldots, 0}^{C_{1,n}K_\Delta}, \sqrt{PT_s}\beta \tilde{\boldsymbol{a}}^{(n)T}, 0, \ldots, 0\right]^T + \boldsymbol{z}_n \tag{6}$$

where $\boldsymbol{z}_n := [z_{n,0}, \ldots, z_{n,K_s-1}]^T$ with $z_{n,i} := \int p(t - iT_p) z(t + (n-1)T_s) dt$ is found to be a zero-mean Gaussian vector with covariance matrix $(N_0/2)\boldsymbol{I}$. In order to detect the transmitted message, the receiver formulates the following decision statistic per message w:

$$\begin{aligned} d(w) &= \frac{1}{N} \sum_{n=1}^{N} d_{w,n} \\ &= \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{r}_n(1 : K_d)^T \boldsymbol{r}_n \\ &\quad \times (K_d + C_{w,n}K_\Delta : 2K_d + C_{w,n}K_\Delta) \end{aligned} \tag{7}$$

[1]We suppose here that the monocycle $p(t)$ already incorporates antenna differentiation effects.

[2]Assuming $T_c = \mathcal{K}T_s$ with $\mathcal{K} \geq 1$, we can divide the time axis to $\mathcal{K}$ groups of time slots $\{\{[kT_s, (k+1)T_s]\}_k + nT_c\}, k \in [0, \mathcal{K} - 1]$. Within each group, i.e., for a fixed k, channels are independent from slot to slot.

where we have used the notation $\boldsymbol{u}(l:q)$ to denote $[u(l),\ldots,u(q)]^T$. When $w=1$, we find

$$\begin{aligned} d_{1,n} =& \boldsymbol{r}_n(1:K_d)^T\boldsymbol{r}_n(K_d+C_{1,n}K_\Delta:2K_d+C_{1,n}K_\Delta)\\ =& \sum_{q=0}^{K_d-1}\left[\sqrt{PT_s}\alpha\tilde{a}_q^{(n)}+z_n(q+1)\right]\\ &\times\left[\sqrt{PT_s}\beta\tilde{a}_q^{(n)}+z_n(K_d+C_{1,n}K_\Delta+q+1)\right]. \end{aligned} \quad (8)$$

As N grows large, $d(1)$ converges to $E[d_{1,n}] = E[\sum_{q=0}^{K_d-1}PT_s\alpha\beta\tilde{a}_q^{(n)2}] = PT_s\alpha\beta$, where we have used the assumption that the total channel gain is constant and has been normalized. When $w\neq 1$, we obtain

$$\begin{aligned} d_{w,n} =& \boldsymbol{r}_n(1:K_d)^T\boldsymbol{r}_n(K_d+C_{w,n}K_\Delta:2K_d+C_{w,n}K_\Delta)\\ =& \sum_{q=0}^{K_d-1}\left[\sqrt{PT_s}\alpha\tilde{a}_q^{(n)}+z_n(q+1)\right]\\ &\times\Big[\sqrt{PT_s}\beta\tilde{a}_{(C_{w,n}-C_{1,n})K_\Delta+q}^{(n)}\\ &+z_n(K_d+C_{w,n}K_\Delta+q+1)\Big]. \end{aligned} \quad (9)$$

Clearly, if $C_{w,n}\neq C_{1,n}$, we have $E[d_{w,n}]=0$, because $\{\tilde{a}_q^{(n)}\}_{q=0}^{K_d-1}$ are uncorrelated to each other and have zero mean.

Based on the decision variables $d(1),\ldots,d(M)$, we assert that message $\hat{w}$ has been sent if $d(\hat{w})\geq\rho$ and $\forall\, w\neq\hat{w}$, $d(w)<\rho$, where $\rho := PT_s\alpha\beta(1-\epsilon)$ is a certain threshold, and $\epsilon>0$ can be made arbitrarily close to zero. Now, let us analyze the decoding error probability $P_e^{(N)}$, for which it is easy to verify the following expression:

$$P_e^{(N)} = \Pr\left(d(1)<\rho\bigcup\cup_{w=2}^{M}d(w)\geq\rho\right). \quad (10)$$

In Section III, we will upper bound $P_e^{(N)}$ and find the rate that is achievable, in the sense that $P_e^{(N)}$ goes to zero as N, the number of channel uses, goes to infinity.

III. Achievable Rates

From the expression of $P_e^{(N)}$ in (10), we can readily obtain the union bound

$$P_e^{(N)}\leq\Pr(d(1)<\rho)+\sum_{w=2}^{M}\Pr(d(w)\geq\rho). \quad (11)$$

Based on the law of large numbers, we know that $\lim_{N\to\infty}d(1)=PT_s\alpha\beta$. Thus, we have

$$\lim_{N\to\infty}\Pr(d(1)<\rho)=\lim_{N\to\infty}\Pr(d(1)<PT_s\alpha\beta(1-\epsilon))=0. \quad (12)$$

To characterize $\Pr(d(w)\geq\rho)$ for $w\neq 1$, we resort to the Chernoff bound

$$\begin{aligned} \Pr(d(w)\geq\rho) =& \Pr\left(\sum_{n=1}^{N}d_{w,n}\geq N\rho\right)\\ \leq& e^{-tN\rho}E\left[e^{t\sum_{n=1}^{N}d_{w,n}}\right]\\ =& e^{-tN\rho}\prod_{n=1}^{N}E\left[e^{td_{w,n}}\right]\quad\forall\, t>0 \end{aligned} \quad (13)$$

where in obtaining the last equality, we have used the fact that $\{d_{w,n}\}_{n=1}^{N}$ are independent.

Now, our task is to find the moment generating function of $d_{w,n}$ in (9). From [13, Ch. 6], we can obtain the following result:

$$E\left[e^{td_{w,n}}\middle|\left\{\tilde{a}_q^{(n)}\right\}_{q=0}^{K_d-1}\right]=\frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\times e^{\Xi} \quad (14)$$

where Ξ is defined as shown in the equation at the bottom of the page. If during the nth channel use, codewords C_1 and C_w collide, i.e., $|(C_{w,n}-C_{1,n})K_\Delta|<K_d$, then we can upper bound (14) as

$$\begin{aligned} E\left[e^{td_{w,n}}\middle|\left\{\tilde{a}_q^{(n)}\right\}_{q=0}^{K_d-1}\right]\leq& \frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\\ &\times\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta 2\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}+2tPT_s\alpha\beta\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\} \end{aligned}$$

where we have used Cauchy's inequality

$$\begin{aligned} &\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)}\tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)}\\ &\leq\sqrt{\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}\cdot\sum_{q=0}^{K_d-1}\tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)2}}\\ &\leq\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}. \end{aligned} \quad (15)$$

$$\Xi := \exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta\sum_{q=0}^{K_d-1}\left(\tilde{a}_q^{(n)2}+\tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)2}\right)}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}\exp\left\{\frac{2tPT_s\alpha\beta\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)}\tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)}}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}$$

Using the assumption that $\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}=1$, we have

$$E\left[e^{td_{w,n}}\middle|\left\{\tilde{a}_q^{(n)}\right\}\right]\leq\frac{1}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}\times\exp\left\{\frac{\frac{N_0}{2}t^2PT_s2\alpha\beta+2tPT_s\alpha\beta}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}:=\phi_{\text{collision}}. \quad (16)$$

If there is no collision between codewords C_1 and C_w over the nth channel use, i.e., $|(C_{w,n}-C_{1,n})K_\Delta|\geq K_d$, then we have

$$E\left[e^{td_{w,n}}\middle|\left\{\tilde{a}_q^{(n)}\right\}\right]=\frac{\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta\sum_{q=0}^{K_d-1}\tilde{a}_q^{(n)2}}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}=\frac{\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2}{4}t^2\right)}\right\}}{\left(1-\frac{N_0^2}{4}t^2\right)^{K_d/2}}:=\phi_{\text{clear}}. \quad (17)$$

With c denoting the number of collisions between codewords C_1 and C_w, we obtain [cf. (13), (16), and (17)]

$$\Pr(d(w)\geq\rho)\leq e^{-tN\rho}\prod_{n=1}^{N}E\left[e^{td_{w,n}}\right]\leq e^{-tN\rho}\phi_{\text{collision}}^{c}\phi_{\text{clear}}^{N-c}. \quad (18)$$

Now, in order to eliminate the effect of a particular codebook generation, we average the probability $\Pr(d(w)\geq\rho)$ over all codebook realizations to arrive at

$$\begin{aligned}&E_{\mathcal{C}}\left[\Pr(d(w)\geq\rho)\right]\\&\quad\leq E_{\mathcal{C}}\left[e^{-tN\rho}\phi_{\text{collision}}^{c}\phi_{\text{clear}}^{N-c}\right]\\&\quad=\sum_{c=0}^{N}\binom{N}{c}\mu^c(1-\mu)^{N-c}e^{-tN\rho}\phi_{\text{collision}}^{c}\phi_{\text{clear}}^{N-c}\\&\quad=e^{-tN\rho}\left[\mu\phi_{\text{collision}}+(1-\mu)\phi_{\text{clear}}\right]^N\quad\forall\, t>0\end{aligned} \quad (19)$$

where $\mu:=\Pr(|C_{w,n}-C_{1,n}|K_\Delta<K_d)$. Substituting (16) and (17) into (19) and recalling (11), we have

$$\begin{aligned}&E_{\mathcal{C}}\left[P_e^{(N)}\right]\\&\quad\leq E_{\mathcal{C}}\left[\Pr(d(1)<\rho)\right]+\sum_{w=2}^{M}E_{\mathcal{C}}\left[\Pr(d(w)\geq\rho)\right]\\&\quad\leq E_{\mathcal{C}}\left[\Pr(d(1)<\rho)\right]+ME_{\mathcal{C}}\left[\Pr(d(w)\geq\rho)\right]\\&\quad\leq E_{\mathcal{C}}\left[\Pr(d(1)<\rho)\right]\\&\qquad+\min_{t>0}\exp\left\{-N\left[-\frac{\ln M}{N}+t\rho+\frac{K_d}{2}\ln\left(1-\frac{N_0^2}{4}t^2\right)\right.\right.\\&\qquad-\ln\left(\mu\exp\left\{\frac{\left[\frac{N_0}{2}t^2+t\right]PT_s2\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right.\\&\qquad\left.\left.\left.+(1-\mu)\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right)\right]\right\}.\end{aligned} \quad (20)$$

One can clearly see from (20) that in order for the error probability to vanish as N goes to infinity, the following condition must be fulfilled:

$$\begin{aligned}\frac{\ln M}{N}<&\max_{t>0}t\rho+\frac{K_d}{2}\ln\left(1-\frac{N_0^2}{4}t^2\right)\\&-\ln\left(\mu\exp\left\{\frac{\left[\frac{N_0}{2}t^2+t\right]PT_s2\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right.\\&\left.+(1-\mu)\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right).\end{aligned} \quad (21)$$

Considering the fact that each channel use lasts $T_s=2T_d+(m-1)\Delta$ seconds, we obtain that the achievable rates (in nats/s) must satisfy

$$\begin{aligned}R:=\frac{\ln M}{NT_s}<R_0:=&\frac{1}{T_s}\max_{t>0}tPT_s\alpha\beta(1-\epsilon)\\&+\frac{K_d}{2}\ln\left(1-\frac{N_0^2}{4}t^2\right)\\&-\ln\left(\mu\exp\left\{\frac{\left[\frac{N_0}{2}t^2+t\right]PT_s2\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right.\\&\left.+(1-\mu)\exp\left\{\frac{\frac{N_0}{2}t^2PT_s\alpha\beta}{2\left(1-\frac{N_0^2t^2}{4}\right)}\right\}\right).\end{aligned} \quad (22)$$

The threshold rate R_0 characterizing the set of achievable rates in (22) will be numerically evaluated and compared with AWGN channel capacity in Section IV for practical UWB system parameters.

IV. Numerical Results

AWGN channel capacity with bandwidth W is given by $C(W)=W\ln(1+P_R/(N_0W))$ with P_R denoting the average received power. As $W\rightarrow\infty$, we have $C(W)\rightarrow C(\infty):=P_R/N_0$. Interestingly, frequency-shift keying (FSK) is capable of achieving $C(\infty)$ even with noncoherent reception in the presence of multipath fading [3], [8], [14]. In this section, we will examine the achievable rates in the practically popular TR-PPM-based UWB system, and compare them with $C(\infty)$.

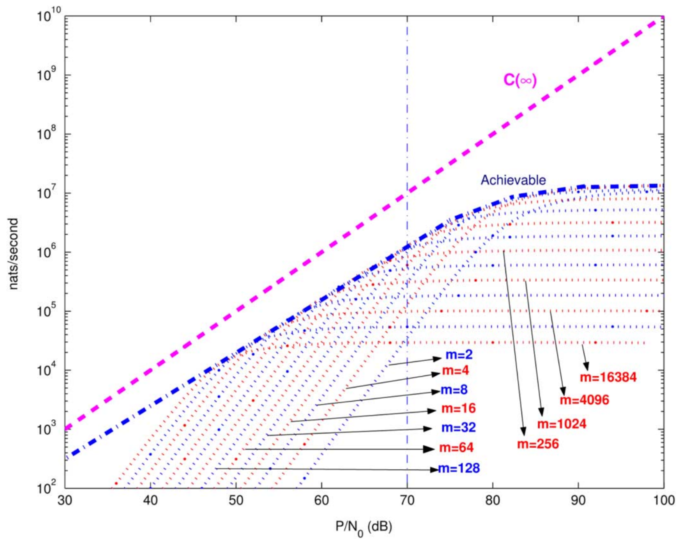

Fig. 2. Achievable rates of a 1-GHz practical UWB system. In (22), we adopt $\alpha = \beta = \sqrt{2}/2, \epsilon = 0, K_d = 20, \mu = 1/m, T_s = (40 + 20(m-1))$ ns.

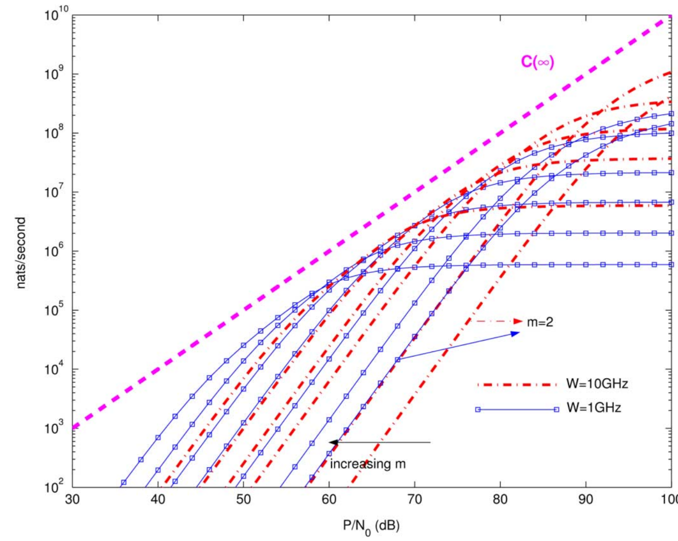

Fig. 3. Effect of bandwidth on achievable rates: free-space propagation. In (22), we adopt $\alpha = \beta = \sqrt{2}/2, \epsilon = 0, K_d = 1, \mu = 1/m, T_s = 2T_p + (m-1)T_p$ where $T_p = 1$ ns when $W = 1$ GHz and $T_p = 0.1$ ns when $W = 10$ GHz.

A. Achievable Rates of a 1-GHz UWB System

In 2002, the Federal Communications Commission (FCC) released a spectral mask for UWB transmissions with 7.5 GHz bandwidth in the range 3.1–10.6 GHz and maximal transmitted power spectral density of -41.3 dbm/MHz [1]. For a 1-GHz UWB system, the maximum transmission power would be $P_T = -12$ dBm. At room temperature, i.e., $T_0 = 300$ K, the noise spectral density is $N_0 = kT_0 \cdot \mathcal{F} \cdot \mathcal{L} =$ -102.83 dBm/MHz, where $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, the noise figure is $\mathcal{F} = 6$ dB, and a link margin $\mathcal{L} = 5$ dB is assumed. Based on experimental measurements [2], an 80-dB path loss is expected at a 10-m Tx–Rx separation, which corresponds to a received-power-to-noise ratio $P_R/N_0 = 70$ dB. Thus, 70 dB can be thought of as a high signal-to-noise-power ratio (SNR) benchmark for practical UWB systems.

The root mean square (RMS) delay spread of a typical UWB channel is on the order of 20 ns for indoor environments [6]. Selecting $\Delta = T_d = 20$ ns in (2), when the modulation size is m, the codeword collision probability μ in (22) is found to be $1/m$. For different values of m, when ϵ is chosen to be 0, rate R_0 in (22) for $W = 1$ GHz is plotted in Fig. 2 (α and β are optimized to be $\sqrt{2}/2$ in (22)), from which we deduce that the achievable rates are indeed on the order $C(\infty)$ within the practical SNR range. When SNR is very low, we should choose very large modulation size m, which renders the transmitted signal extremely "peaky" in time, i.e., exhibiting a very low duty cycle.

B. Bandwidth Scaling

1) Case 1: Free-Space Propagation: When the channel has zero delay spread, which corresponds to free-space propagation, we can choose $T_d = T_p$ and $\Delta = T_p$. For bandwidths of 1 and 10 GHz, the achievable rates are plotted in Fig. 3. It is evident from the figure that larger bandwidths will result in larger achievable rates.

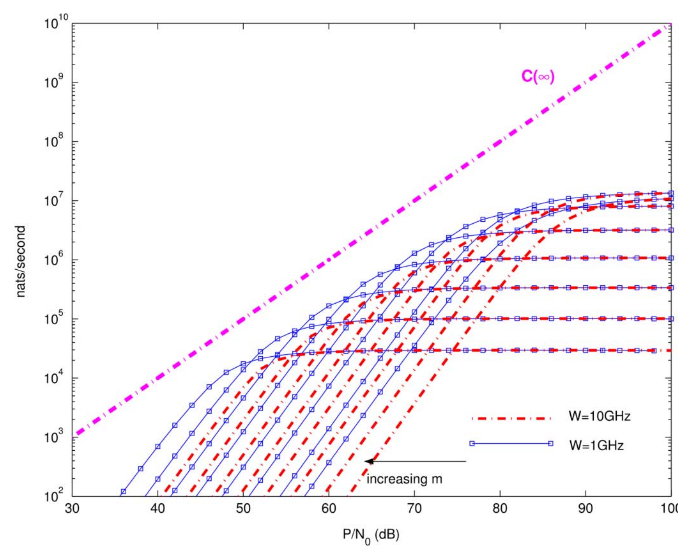

Fig. 4. Effect of bandwidth on achievable rates: multipath fading channel. In (22), we adopt $\alpha = \beta = \sqrt{2}/2, \epsilon = 0, K_d = 20$ when $W = 1$ GHz and $K_d = 200$ when $W = 10$ GHz, $\mu = 1/m, T_s = (40 + 20(m-1))$ ns.

2) Case 2: Multipath Fading Channel: As in Section IV-A, we still choose $T_d = \Delta = 20$ ns. The resulting achievable rates are plotted in Fig. 4, from where we verify that larger bandwidth suffers from rate loss. This is because the noncoherent receiver collects increasingly more noise as bandwidth increases. Interestingly, similar behavior has been observed in [14] and [9], where the noncoherent capacity of spread-spectrum white-noise-like signaling over a multipath fading channel has been shown to approach zero as bandwidth increases.

V. Conclusions and Future Work

In this paper, we investigated the achievable rates of practical UWB systems with PPM and TR. We established that for SNR values of practical interest, PPM-UWB with TR can achieve rates on the order of $C(\infty) = P/N_0$ (nats/s). We further verified that in order to maximize the achievable rates at low SNR, UWB signal transmissions must exhibit an extremely low duty cycle. Future work will explore a tighter bound for the moment generating function of $d_{w,n}$ when codeword collision is present. The Cauchy inequality used in (15) to derive the bound in (16) turns out to be loose when $C_{w,n} \neq C_{1,n}$ and $|C_{w,n} - C_{1,n}|K_\Delta < K_d$, which will happen if we choose $\Delta < T_d$ in (2). In fact, because different path gains are uncorrelated, we have $E[\sum_{q=0}^{K_d-1} \tilde{a}_q^{(n)} \tilde{a}_{q+(C_{w,n}-C_{1,n})K_\Delta}^{(n)}] = 0$.

REFERENCES

[1] Federal Communications Commission, "In the Matter of Revision of Part 15 of the Commissions Rules Regarding Ultra-Wideband Transmission Systems" (2002), FCC Rep. and Order 02-48.

[2] *The Ultra-Wideband Indoor Path Loss Model*, IEEE 802.15-02/278r1-SG3a, 2002, S. S. Gassemzadeh and V. Tarokh.

[3] R. G. Gallager, *Information Theory and Reliable Communication*. New York: Wiley, 1968.

[4] R. Hoctor and H. Tomlinson, "Delay-hopped transmitted-reference RF communications," in *Proc. UWBST Conf.*, Baltimore, MD, May 2002, pp. 265–270.

[5] *Time Variance for UWB Wireless Channels*, IEEE P802.15-02/461r1-SG3a, IEEE P802.15 Working Group for WPANs, 2002.

[6] *Channel Modeling Sub-Committee Report–Final*, IEEE P802.15-02/368r5-SG3a, IEEE P802.15 Working Group for WPANs, 2002.

[7] M. Kamoun, M. de Courville, L. Mazet, and P. Duhamel, "Impact of desynchronization on PPM UWB systems: A capacity-based approach," in *Proc. IEEE Inf. Theory Workshop*, Oct. 2004, pp. 198–203.

[8] R. S. Kennedy, *Fading Dispersive Communication Channels*. New York: Wiley, 1969.

[9] M. Medard and R. G. Gallager, "Bandwidth scaling for fading multipath channels," *IEEE Trans. Inf. Theory*, vol. 48, no. 4, pp. 840–852, Apr. 2002.

[10] R. C. Qiu, "A study of the ultra-wideband wireless propagation channel and optimum UWB receiver design," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 12, pp. 1628–1637, Dec. 2002.

[11] C. K. Rushforth, "Transmitted-reference techniques for random or unknown channels," *IEEE Trans. Inf. Theory*, vol. IT-10, no. 1, pp. 39–42, Jan. 1964.

[12] Y. Souilmi and R. Knopp, "On the achievable rates of ultra-wideband PPM with non-coherent detection in multipath environments," in *Proc. Int. Conf. Commun.*, May 2003, vol. 5, pp. 3530–3534.

[13] M. K. Simon, *Probability Distributions Involving Gaussian Random Variables*. Norwell, MA: Kluwer, 2002.

[14] I. E. Telatar and D. Tse, "Capacity and mutual information of wideband multipath fading channels," *IEEE Trans. Inf. Theory*, vol. 46, no. 7, pp. 1384–1400, Jul. 2000.

[15] S. Verdu, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1319–1343, Jun. 2002.

[16] J. Zhang, R. A. Kennedy, and T. D. Abhayapala, "New results on the capacity of M-ary PPM ultra-wideband systems," in *Proc. Int. Conf. Commun.*, May 2003, vol. 4, pp. 2867–2871.

[17] L. Zhao and A. M. Haimovich, "Capacity of M-ary PPM ultra-wideband communications over AWGN channels," in *Proc. IEEE Veh. Technol. Conf.*, 2001, vol. 2, pp. 1191–1195.