BLIND JOINT ESTIMATION OF CARRIER FREQUENCY OFFSET AND CHANNEL USING NON-REDUNDANT PERIODIC MODULATION PRECODERS

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ABSTRACT

Recent results have shown that periodic modulation precoders allow blind estimation of FIR communication channels from the output second-order cyclic statistics, irrespective of the location of channel zeros, color of additive stationary noise, or channel order overestimation errors. The present paper extends these results to the more general blind problem of joint channel and carrier frequency offset estimation. The performance of a subspace channel identification algorithm is investigated in the presence of carrier frequency offset. Estimation of the frequency offset is simply obtained by exploiting the cyclostationary statistics of received data.

1. INTRODUCTION

It has been shown in [1], [4], [9], [11], that blind identification of FIR communication channels is possible only from the second-order statistics of the received data, without imposing any restriction on the channel zeros, color of additive noise, and channel order over-estimation errors. The underlying channel identification approach relies on the output cyclostationary (CS) statistics, induced by precoding the input symbol stream with a periodic sequence. These algorithms are referred to as Transmitter Induced Cyclostationarity (TIC) approaches. In the present paper, the problem of blind joint estimation of the carrier frequency offset, the frame timing, and the unknown channel is considered within the TIC framework. Correction of the carrier frequency offset and symbol timing is required by most blind channel identification/ equalization approaches. The recovery of the frame timing refers to finding the timing information of the modulating sequence $f(n)$ from the received samples $x(n)$.

So far, only a few blind algorithms have been proposed for joint estimation of the channel and the carrier offset. In [5], [7], the approach consists of two steps: first the channel is equalized by minimizing a certain nonlinear cost function (CMA), and then the carrier phase is tracked from the equalized output. However, in the presence of residual intersymbol interference (ISI) at the equalizer output, performance of these frequency offset estimators may degrade. In the present approach, the influence of residual ISI on the carrier offset estimate is eliminated, because channel equalization is not needed for carrier offset estimation. In addition, a closed-form estimate for the channel is obtained without minimizing a nonlinear cost function such as CMA. Thus, the potential local minima or loss of convergence associated with [5], [7], in the presence of noise, are avoided.

2. PROBLEM FORMULATION

Consider the general communication channel set-up depicted in Fig. 1, where the i.i.d. symbol stream $s(n)$ is modulated by the periodic sequence $f(n)$ ($f(n) = f(n+P), \forall n$). The modulation with the periodic sequence $f(n)$ represents the non-redundant periodic modulation precoder, and its role is to induce CS in the transmitted sequence $w(n) := f(n)s(n)$. The resulting sequence $w(n)$ is converted by the Digital to Analog converter D/A into the continuous waveform $\sum_n w(n)\, \delta(t - nT_s)$, which is then pulse shaped by the baseband transmit filter $\tilde{h}_c^{(tr)}(t)$, is modulated by the carrier exp $(j\ \omega_c\ t)$, and propagates through the unknown *bandpass* channel $h_c^{(ch)}(t)$. Due to local oscillator drifts, the demodulated signal at the receiver includes a carrier frequency offset ω_e. Signal $u_2(t)$ in Fig. 1 is filtered by the receive filter $h_c^{(rec)}(t)$, and the resulting continuous-time waveform $y_c(t)$ is given by

$$y_c(t) = e^{j\omega_e t} \sum_n w(n) \iint h_c^{(rec)}(\rho) h_c^{(ch)}(\tau) \times \tilde{h}_c^{(tr)}(t - nT_s - \rho - \tau) e^{-j\omega_c \tau} e^{-j\omega_e \rho} d\tau d\rho\ . \quad (1)$$

Defining the equivalent *baseband* components $\tilde{h}_c^{(ch)}(\tau) := h_c^{(ch)}(\tau) e^{-j\omega_c \tau}$, $\tilde{h}_c^{(rec)}(\rho) := h_c^{(rec)}(\rho) e^{-j\omega_e \rho}$, $\tilde{h}_c(t) := (\tilde{h}_c^{(rec)} * \tilde{h}_c^{(ch)} * \tilde{h}_c^{(tr)})(t)$, sampling at the symbol rate $1/T_s$ and supposing $h_c(t)$ of support LT_s, it can be shown that

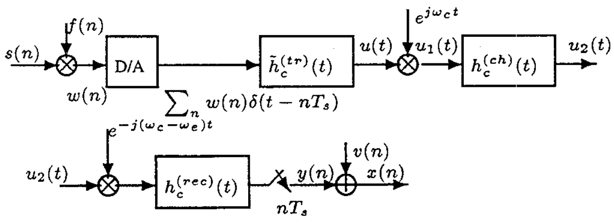

Figure 1: Communication Channel with Frequency Offset

the equivalent discrete-time model of (1) is given by

$$x(n) := y(n)+v(n) = e^{j\theta_e n}\sum_{l=0}^{L} h(l)w(n-l)+v(n), \quad (2)$$

where $x(n) := x_c(nT_s)$, $h(n) := \tilde{h}_c(nT_s)$, $\theta_e := \omega_e T_s$, and $v(n)$ stands for additive stationary noise independently distributed of $s(n)$. Our goal is blind joint estimation of the unknown channel $\mathbf{h} := [h(0) \ \ldots \ h(L)]^T$, the frequency offset θ_e, and the frame timing, from knowledge only of the receive data $x(n)$ and the CS induced in the transmitted sequence $s(n)$ by the non-redundant precoder $f(n)$.

3. BLIND ESTIMATION

We first show that the second-order cyclic spectra of $x(n)$ allow unique identification of the channel modulo an unknown complex scale factor δ and a θ_e-dependent phase rotation, having the expression $\delta h(l)\exp(j\theta_e l)$, $l = 0,\ldots,L$. From (2), the output time-varying correlation is given by $c_{xx}(n;\tau) := Ex^*(n)x(n+\tau) = c_{xx}(n+P;\tau)$, and is periodic. Hence, it admits a Fourier Series (FS) decomposition, whose coefficients are the cyclic correlations. The cyclic correlation at cycle k and lag τ is given by $C_{xx}(k;\tau) := (1/P)\sum_{n=0}^{P-1} c_{xx}(n;\tau)\exp(-j2\pi kn/P) = \sigma_s^2 F_2(k)\exp(j\theta_e\tau)\sum_l h^*(l)\,h(l+\tau)\exp(-j2\pi kl/P) + \sigma_v^2\delta(k)$, where $F_2(k) := (1/P)\sum_{n=0}^{P-1}|f(n)|^2\exp(-j2\pi kn/P)$, $\sigma_s^2 := E\,|s(n)|^2$, and $\sigma_v^2 := E|v\,(n)|^2$. Considering the cyclic spectrum $S_{xx}(k;z) := \sum_\tau C_{xx}(k;\tau)z^{-\tau}$ at a cycle frequency $k \neq 0$, we obtain for $k = 1,\ldots,P-1$

$$S_{xx}(k;z) = \sigma_s^2 F_2(k) H(e^{-j\theta_e}z) H^*(e^{-j2\pi k/P}/(e^{-j\theta_e}z)^*), \quad (3)$$

where $H(z) := \sum_{l=0}^{L} h(l) z^{-L}$. It can be shown that the transfer function $H(e^{-j\theta_e}z)$ can be extracted as the gcd of the family of cyclic spectra $S_{xx}(k;z)$, $k = 1,\ldots,P-1$, provided that the period of $f(n)$ satisfies $P > L+1$ [4, 1]. Henceforth, we choose the period P so that $P > L+1$ is satisfied. Once the frequency offset θ_e is estimated, the channel $\mathbf{h}$ can be recovered by removing the effect of phase rotation. Therefore, we concentrate first on the frequency offset determination problem.

It turns out that for real-valued input constellations (e.g., BPSK, PAM) the *conjugate* cyclic correlation of the output provides enough information for the recovery of θ_e [3, 12]. We justify this for a BPSK input. Consider the conjugate time-varying correlation of the output $\tilde{c}_{xx}(n;\tau) := Ex(n)x(n+\tau) = \sigma_s^2\exp(j\theta_e(2n+\tau))\sum_l h(l)h(l+\tau)\,f^2(n-l) + \tilde{c}_{vv}(\tau)$. Since $\tilde{c}_{xx}(n;\tau)$ is periodically time-varying, the generalized Fourier Series coefficient of $\tilde{c}_{xx}(n;\tau)$, termed the conjugate cyclic correlation, is given by

$$\tilde{C}_{xx}(\alpha;\tau) := \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\tilde{c}_{xx}(n;\tau)e^{-j\alpha n} = \sigma_s^2\tilde{F}_2(\alpha-2\theta_e) \times\sum_l h(l)h(l+\tau)e^{-j(\alpha-2\theta_e)l} + \tilde{c}_{vv}(\tau)\delta(\alpha), \quad (4)$$

where $\tilde{F}_2(\alpha) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1} f^2(n)\exp(-j\alpha n)$. Since $f(n)$ is periodic, $\tilde{F}_2(\alpha)$ consists of Kronecker deltas located at the harmonics $2\pi k/P$, with k being an integer i.e., $\tilde{F}_2(\alpha) = \sum_k \tilde{F}_2(k)\,\delta(\alpha - 2\pi k/P)$. It follows from (4) that $\tilde{C}_{xx}(\alpha;\tau)$ consists also of Kronecker deltas located at the frequencies $2\theta_e + 2\pi k/P$, where k is an integer. Since the location of the Kronecker delta with the smallest positive frequency is equal to $2\theta_e$, a straightforward estimator of the frequency offset θ_e is

$$\hat{\theta}_e := \frac{1}{2}\arg\ \max_{0<\alpha<\frac{2\pi}{P}}|\tilde{C}_{xx}(\alpha;\tau)|\ . \quad (5)$$

The condition $0 \leq 2\theta_e < 2\pi/P$, i.e., $0 \leq \omega_e T_s < \pi/P$, is sufficient to ensure unique estimation of the offset θ_e. A consistent sample estimator of cyclic correlation $\tilde{C}_{xx}(\alpha;\tau)$ is

$$\hat{\tilde{C}}_{xx}(\alpha;\tau) := \frac{1}{N}\sum_{n=0}^{N-\tau-1} x(n)x(n+\tau)e^{-j\alpha n}\ .$$

We choose $\tau = 0$ in (5), for the simplicity of the resulting statistics. Indeed, it follows that the corresponding sample estimator reduces to nothing more than the Discrete Fourier Transform (DFT) of the sequence $x^2(n)$

$$\hat{\theta}_e := \frac{1}{2}\arg\ \max_{0<\alpha<\frac{2\pi}{P}}\left|\frac{1}{N}\sum_{n=0}^{N-1}x^2(n)e^{-j\alpha n}\right|. \quad (6)$$

In practice, (6) is implemented using the Fast Fourier Transform (FFT)

$$\hat{\theta}_e := \frac{\pi}{R}\arg\ \max_{0<k<[R/P]}\left|\frac{1}{N}\sum_{n=0}^{N-1}x^2(n)e^{-j\frac{2\pi kn}{R}}\right|,$$

where the parameter R defines the resolution of the frequency grid, and $[\cdot]$ stands for integer part. In practice, the performance of estimator (6) may degrade if the amplitude of the spectral line at frequency $2\theta_e$ is small. A remedy consists in estimating the offset frequency from the location of highest peak, which leads to the estimator

$$\hat{\alpha} \ := \ \arg\max_{0<\alpha<2\pi}\left|\frac{1}{N}\sum_{n=0}^{N-1}x^2(n)e^{-j\alpha n}\right|,$$

$$\hat{\theta}_e \;:=\; \frac{1}{2}(\hat{\alpha} - \frac{2\pi}{P}\left[\frac{\hat{\alpha}P}{2\pi}\right]) \,. \tag{7}$$

In the full journal version of this paper, we derive the asymptotic unbiasness and consistency of estimators (6), (7). It is shown that the asymptotic variance of frequency offset estimators is inversely proportional to the amplitude of the spectral peak. Thus, (7) may provide a better performance than (6). We also show that var $(\hat{\theta}_e) \approx O_p(N^{-3})$, which implies that the resolution of the frequency grid $(2\pi/R)$ to be sufficiently small. Finally, since the channel taps can be determined in the form $\delta h(l)\exp(j2\pi l\theta_e)$, $l = 0,\ldots,L$, with δ an unknown scaling factor, the magnitude of spectral lines in (6) can be compared in advance.

Next, we consider the more general case of a QAM input constellation for which $Ew^2(n) = 0$, $Ew^3(n) = 0$, and $Ew^4(n) \neq 0$. We note that these conditions are satisfied also by the usual QAM(4) $\div$ QAM(32) constellations. The estimation of the frequency offset is performed by exploiting the information provided by the fourth order cyclic statistics. We define in this sense the 4th-order time-varying correlation $\tilde{c}_{4,xx}(n;\tau) := E\,x^2(n)x^2(n+\tau)$. Using (2), we deduce that $\tilde{c}_{4,xx}(n;\tau) = Es^4(n)\exp(j(4\theta_e n + 2\theta_e\tau))\sum_l h^2(l)\,h^2(l+\tau)\,f^4(n-l) + \tilde{c}_{4,vv}(n;\tau)$. The corresponding generalized Fourier Series coefficient, referred as cyclic correlation, $\tilde{C}_{4,xx}(\alpha;\tau) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1}\tilde{c}_{4,xx}(n;\tau)\exp(-j\alpha n)$ can be expressed as:

$$\tilde{C}_{4,xx}(\alpha;\tau) = Es^4(n)\tilde{F}_4(\alpha - 4\theta_e)\sum_l h^2(l)h^2(l+\tau) \times e^{-j(\alpha-4\theta_e)l}e^{j2\theta_e\tau} + \delta(\alpha)Ev^2(n)v^2(n+\tau), \tag{8}$$

with $\tilde{F}_4(\alpha) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1}f^4(n)\exp(-j\,\alpha\,n)$. It easy to note that $\tilde{F}_4(\alpha)$ consists of a sequence of Kronecker deltas located at frequencies $2\pi k/P$, k being an integer. Thus, $\tilde{C}_{4,xx}(\alpha;\tau)$ consists of a sequence of Kronecker deltas located at $4\theta_e + 2\pi k/P$, $k = 0,\ldots,P-1$. The estimators (6) and (7) can be similarly derived for the present framework. In order to estimate uniquely θ_e the condition $0 \leq 4\theta_e < 2\pi/P$, i.e., $0 \leq \alpha_e\,T_s < \pi/2P$, has to be imposed. The estimator (7) can be written as:

$$\hat{\alpha} \;:=\; \arg\max_{0<\alpha<2\pi}\left|\frac{1}{N}\sum_{n=0}^{N-1}x^4(n)e^{-j\alpha n}\right| \,,$$
$$\hat{\theta}_e \;:=\; \frac{1}{4}(\hat{\alpha} - \frac{2\pi}{P}\left[\frac{\hat{\alpha}P}{2\pi}\right]) \,. \tag{9}$$

We study next the asymptotic performance of the channel identification approach from [9], in the presence of carrier frequency offset. From (3), the following relation can be established for $k, l = 1,\ldots,P-1$, [9]

$$F_2^*(l)H(e^{-j2\pi l/P}/(e^{-j\theta_e}z)^*)S_{xx}^*(k;z) = F_2^*(k)H(e^{-j2\pi k/P}/(e^{-j\theta_e}z)^*)S_{xx}^*(l;z). \tag{10}$$

Associate to the vector of coefficients $\mathbf{f} := [f(0)\ldots f(L_f)]'$ of an arbitrary polynomial $F(z) = \sum_{i=0}^{L_f} f(i)\,z^i$, the $(\tilde{L} + L_f + 1)\times(\tilde{L}+1)$ Toeplitz matrix $\mathcal{T}_F(\tilde{L})$ having as first column the vector $[f(0)\;f(1)\ldots f(L_f)\;0\ldots 0]'$ and as the first row $[f(0)\;0\ldots 0]$ (where prime stands for transpose). Let $\mathcal{T}_{S1}(\tilde{L})$ and $\mathcal{T}_{S2}(\tilde{L})$ denote the $(\tilde{L}+2L+1)\times(\tilde{L}+1)$ Toeplitz matrices associated with the $2L$th-order polynomials $F_2^*(l)\;z^L S_{xx}^*(k;z)$ and $F_2^*(k)\;z^L S_{xx}^*(l;z)$, respectively. Consider also the $(\tilde{L}+1)\times(\tilde{L}+1)$ diagonal matrix $\mathbf{D}_k(\tilde{L}) = \mathrm{diag}\{1, e^{j(2\pi k/P+\theta_e)},\ldots, e^{j2\tilde{L}(2\pi k/P+\theta_e)}\}$, and the $(L+1)\times 1$ vector $\mathbf{h} := [h(0)\;h(1)\;\ldots\;h(L)]'$ of the coefficients of $H(z)$. Note that the vector of coefficients $\mathbf{h}_k$ of the polynomial $H(e^{-j2\pi k/P}/(e^{-j\theta_e}z)^*)$ is given by $\mathbf{h}_k = \mathbf{D}_k(L)\,\mathbf{h}$. Equating coefficients of both sides of (10), we deduce $\mathcal{T}_{S1}(L)\,\mathbf{h}_l = \mathcal{T}_{S2}(L)\mathbf{h}_k$, which can be written as

$$[\mathcal{T}_{S1}(L)\mathbf{D}_l(L) - \mathcal{T}_{S2}(L)\mathbf{D}_k(L)]\mathbf{h} = \mathbf{0}\,. \tag{11}$$

Considering all the eqs. (11) for all 2-tuples (k,l), we obtain $\mathbf{Sh} = \mathbf{0}$. In practice, the channel $\mathbf{h}$ is estimated by solving in the LS-sense $\mathbf{S}(:,2:L)\mathbf{h}_1 = -\mathbf{S}(:,1)$[1], where $\mathbf{h}_1 := \mathbf{h}(2:L)$, and the entries of $\mathbf{S}$ are obtained using consistent estimates for θ_e and cyclic correlation coefficients. It has been established in [9] that the condition $P > L+1$ is sufficient to guarantee identifiability of $\mathbf{h}$ from (11). Elimination of the scaling factor ambiguity in $\mathbf{h}$ is performed by fixing $h(0) = 1$. We re-write (11) in the form $\hat{\mathbf{A}}\mathbf{h}_1 = \hat{\mathbf{b}}$, where $\hat{\mathbf{A}} = \mathbf{S}(:,2:L+1)$ and $\hat{\mathbf{b}} = -\mathbf{S}(:,1)$. The asymptotic performance of the channel estimate follows directly from [8, pp. 96-97]:

Theorem. *The asymptotic estimation error $\sqrt{N}(\hat{\mathbf{h}}_1 - \mathbf{h}_1)$ has a limiting zero-mean complex Gaussian distribution with asymptotic covariance:*

$$\lim_{N\to\infty} NE[(\hat{\mathbf{h}}_1-\mathbf{h}_1)(\hat{\mathbf{h}}_1-\mathbf{h}_1)^*] = (\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\mathbf{D}\boldsymbol{\Sigma}\mathbf{D}^*\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}$$

where $\mathbf{D}$ is the matrix with ith column

$$\mathbf{D}(:,i) = \frac{\partial\mathbf{b}}{\partial\mathbf{c}_i} - \frac{\partial\mathbf{A}}{\partial\mathbf{c}_i}\mathbf{h}_1,$$

and $\boldsymbol{\Sigma}$ is the asymptotic covariance matrix of the vector of cyclic correlations $\mathbf{c}$ present in $\mathbf{A}$ and $\mathbf{b}$, and c_i denotes the ith entry of cyclic vector $\mathbf{c} := [C_{xx}(1;-L)\;\ldots\;C_{xx}(P-1;L)]$.

The last problem we study concerns the frame (symbol) synchronization question. In order to equalize the channel, knowledge of the right timing of the periodic sequence $f(n)$ is necessary. We have shown that from a channel identification viewpoint, an optimal modulating sequence corresponds to the periodic sequence $\{f(n)\}_{n=1}^{P} = \{\beta,\ldots,\beta,\sqrt{\alpha-(P-1)\beta^2}\}$, with $\alpha > P\beta^2$ and $P > L+1$ [2]. For this optimal precoder, the timing of the modulating sequence $f(n)$ can be easily determined by inspection

[1] In writing $\mathbf{S}(:,2:L)$ and $\mathbf{S}(:,1)$, we used the notations from Matlab.

from the plot of $c_{xx}(n;0) := E|x(n)|^2$ versus n. It can be shown that the minimum values of this sequence occur when all the taps of the modulating sequence are equal to β (see also [11]). This follows from the relation $E|x(n)|^2 = \sum_{l=0}^{L} |h(l)|^2 + E|v(n)|^2$, which can be easily obtained from (2).

4. SIMULATION RESULTS

We consider a GSM channel with baseband channel impulse response $\mathbf{h} = [0.53 + 0.07i, -0.24 - 0.23i, -0.54 - 0.32i, 0.11 + 0.44i, -0.036 - 0.099i]^T$. The input $s(n)$ belongs to a BPSK constellation and the modulating sequence is $\{f(n)\}_{n=1}^{P} = [0.76; 0.76; 0.76; 0.76; 0.76; 1.74]$. The additive noise $v(n)$ is normally and independently distributed of $s(n)$. For all simulations, the Signal-to-Noise Ratio (SNR) is defined, at the input of the equalizer, as: SNR $:= 10\log(E|y(n)|^2/E|v(n)|^2)$. In Fig. 2 a, the standard deviation of frequency estimator (6) is illustrated for a varying range of SNR's, assuming the frequency offset $\theta_e = \pi/30$. T= 120 samples are used for each one of the MC= 500 Monte-Carlo runs. We observe that good frequency offset estimates are obtained even with a limited number of data samples. In Fig. 2 b, several periods of $E|x(n)|^2$ are plotted with the scope of revealing the timing information of the modulating sequence. In Fig. 3 a, the average root-mean square error of the subspace channel estimate $\hat{\mathbf{h}}$ is plotted vs. SNR for two different scenarios: with and without frequency offset. It is interesting that no difference in performance is observable between the two scenarios. The two scenarios are also compared from a probability of symbol error. In Fig. 3 b, MC= $1,000$ Monte-Carlo runs have been averaged per SNR point. An MMSE FIR equalizer with 17 taps, and T= $1,000$ data samples per Monte-Carlo run are used. It appears that the existence of a carrier frequency offset does not introduce any penalty in the performance of channel identification and equalization.

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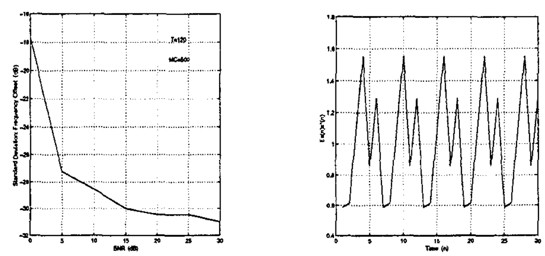

Figure 2: a) Standard Deviation of Frequency Estimator b) Frame Synchronization

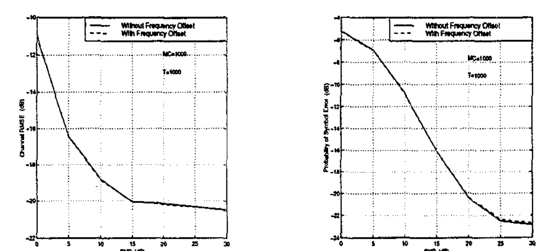

Figure 3: a) Channel RMSE vs. SNR b) Symbol Error Rate vs. SNR