DIRECT BLIND EQUALIZATION FOR TRANSMITTER INDUCED CYCLOSTATIONARITY

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ABSTRACT

Cyclostationarity in a communications signal enables the use of second-order statistics for blind equalization. In this paper, we present a second-order subspace based blind channel estimation technique which introduces cyclostationarity at the transmitter. In contrast with most existing second-order cyclostationary methods which introduce cyclostationarity by fractional-sampling, the proposed method does not require any assumptions on the channel zero locations. Furthermore, the proposed method extends existing transmitter-induced cyclostationary methods to allow for colored random or even deterministic input.

1. INTRODUCTION

In order to maintain reliable performance in wireless communications, Intersymbol Interference (ISI) must be removed at the receiver by equalization. Blind equalizers remove the ISI by exploiting knowledge of the input structure (e.g., whiteness) in conjunction with the outputs. Since they do not require training data, blind equalizers are of both practical and theoretical interest (e.g., [4, Chpt. 10]).

In particular, there has been recent interest in the blind equalization of wireless communication systems where the received signal is cyclostationary. As it turns out, this cyclostationarity allows the use second-order statistics to identify and equalize a certain class of channels [6]. Primarily, the second-order techniques have relied on modifying the receiver either by fractional-sampling (FS) or by using multiple receivers in order to induce cyclostationarity. In contrast, Tsatsanis and Giannakis [7] have proposed modifying the transmitter to repeat blocks of symbols which forces cyclostationarity in the received signal. Surprisingly, this transmitter induced cyclostationarity (TIC) allows for the identification of *any* FIR channel with second-order statistics of the output. Repetition of symbol blocks has the potential penalty of reducing the overall information rate by one half. Recently, Giannakis [1] generalized this TIC system to show that identification of any FIR channel was possible with an arbitrarily small decrease in rate.

In this paper, we describe a blind channel identification method which in contrast to [1], is valid for any color of statistically modeled input or even deterministically modeled input. This method is based on subspace methods for FS receivers (c.f., [3]) but has no restrictions on the channel zeros. And, in contrast with a similar subspace approach in [7], the proposed method is valid for an arbitrarily small information rate change.

2. TIC: BACKGROUND

Consider the general continuous-time communications system shown in Figure 1, where the information symbols $w(n)$ are first transformed by a rate M/P coder (for every M information symbols $w(n)$, P coded symbols $\bar{w}(n)$ are generated). The coded symbols are then converted to the continuous-time signal according to $w_c(t) := \sum_{\ell=-\infty}^{\infty} \bar{w}(\ell)$ $\delta_c(t-\ell T_r)$, where $\delta_c(\cdot)$ is the Dirac delta and T_r is the transmission rate. After transmission through the 'composite' channel $h_c(t)$, the received signal $y_c(t)$ is sampled at a rate $1/T_s$. Mathematically, the discrete time received signal, $y(n) := y_c(t)|_{t=nT_s}$, is $y(n) = \sum_\ell \bar{w}(\ell)\; h_c(nT_s - \ell T_r - d_0)$ $+ v_c(nT_s)$ where d_0 ($d_0 < T_s$) is an arbitrary delay. Unlike the FS receiver which constrains the sample interval $T_s = T_r/P$ where P is an integer, we have selected $T_r = T_s$. With this choice, the output can then be described by:

$$y(n) \;=\; \sum_{\ell} \bar{w}(\ell)\, h(n-\ell) \;+\; v(n) \;\;, \qquad (1)$$

where $h(n) := h_c(nT_s - d_0)$, $v_c(t) := v_c(nT_s)$ are the

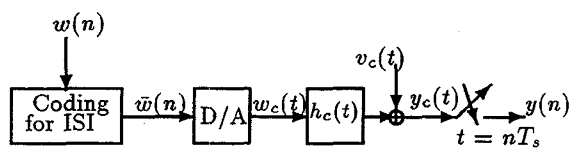

Figure 1. Pre-coding for ISI removal

discrete-time representations of the channel and the additive noise respectively. Note that the discrete-time channel order is $L_h := \lceil\, T_h/T_s \,\rceil$ where T_h is the continuous-time channel span and $\lceil \cdot \rceil$ indicates round up to the nearest integer. For $w(n) = \bar{w}(n)$ (i.e., $M = P$ or no coding), (1) is the standard discrete-time model for a symbol-rate sampled system (c.f., [4, pg. 588]). If, however, we set $\bar{w}(n) = \sum_k$ $w(\ell)\; \delta(n - kP)$ (i.e., the code inserts P - 1 zeros between each symbol), (1) has the same mathematical model as that of FS receivers (c.f., [6]).

In order to introduce cyclostationarity at the transmitter, we will consider codes generated by the system of Figure 2, where the input is grouped into a block of M symbols before coding is used to form the block of P symbols. For this paper, we will focus on the case of $f_i(n) = \delta(n)\ \forall\ i$. In this case, the received output for such a code is:

$$y(n) = \sum_{\ell} \sum_{m=0}^{M-1} w(\ell M + m) h(n - m - \ell P) + v(n)\,. \quad (2)$$

With $*$ denoting complex conjugation, the time-varying input and output correlations are defined as $c_{2\bar{w}}(n;\tau) := \mathrm{E}\{\bar{w}(n)\ \bar{w}^*(n+\tau)\}$ and $c_{2y}(n;\tau) := \mathrm{E}\ \{y(n)\ y^*(n+\tau)\}$ respectively. By assuming the additive noise $v(n)$ is independent of $w(n)$, stationary, and white with variance σ_v^2, the output correlation becomes $c_{2y}(n;\tau) = \sum_{\ell_1} \sum_{\ell_2} h(n-\ell_1)$ $h^*(n+\tau-\ell_2)\ c_{2\bar{w}}(\ell_1;\ell_2-\ell_1) + \sigma_v^2\ \delta(\tau)$. Furthermore, when the information sequence is white with variance σ_w^2, the time-varying output correlation is, [1]

$$\begin{aligned} c_{2y}(n;\tau) &= \sigma_w^2 \sum_{\ell} \sum_{m}^{M-1} h(n - \ell P - m) \\ &\times\ h^*(n+\tau-m-\ell P) + \sigma_v^2\, \delta(\tau)\,. \quad (3) \end{aligned}$$

Eqn. (3) shows that the output sequence $y(n)$ is cyclostationary with period P, i.e., $c_{2y}(n+qP;\tau) = c_{2y}(n;\tau)$ for any integer q. Hence, we have induced cyclostationarity in the received sequence without altering the sample rate. As will be seen, the significance of not altering the sample rate is that the channel order, L_h, remains constant while the period of cyclostationarity, P, is a free design parameter. The significance of this was recognized and exploited in [1] to develop the following theorem:

__Theorem 1:__ *For outputs of a multirate coded system described by Figure 2, the L_h order channel $h(n)$ is identifiable (up to a scale and shift ambiguity) from the output cyclic spectra $\{S_{2y}(k;\omega)\}_{k=0}^{P-1}$ provided $P > L_h$.*

In contrast, there exist channels $h(n) = h_c(t)|_{t=nT_r/P}$ that are never identifiable by FS methods no matter how large P is chosen [6]. Fundamentally, this is a result of the fact that increasing P in a FS system also increases the discrete-time channel order L_h. Hence, unlike a TIC system, it is not possible to choose $P > L_h$. The price paid for this flexibility in choosing P is an increased coding and decoding delay due to the 'block' nature of the coding system in Figure 2.

3. TIC SUBSPACE METHOD

In this section, we present a method for identifying the composite channel $h(n)$ from $y(n)$ only. This method, in contrast with the non-linear correlation matching approach and closed-form methods of [1] is valid for any color input.

Consider writing (2) in matrix form, i.e.,

$$\mathbf{y}(n) = \mathbf{H}_0\, \bar{\mathbf{w}}(n) + \mathbf{H}_1\, \bar{\mathbf{w}}(n-1) + \mathbf{v}(n)\ , \quad (4)$$

where the $P \times 1$ vector $\mathbf{y}(n)$ is $\mathbf{y}(n) := [y(Pn),\ y(Pn+1),\ \ldots,\ y(Pn+P-1)]'$, and $\mathbf{v}(n)$ and $\bar{\mathbf{w}}(n)$ are defined similarly. In addition, the $'$ indicates transpose and the $P \times P$ matrices $\mathbf{H}_0$ and $\mathbf{H}_1$ are:

$$\mathbf{H}_0 := \begin{bmatrix} h(0) & 0 & \ldots & 0 \\ h(1) & h(0) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(P-1) & h(P-2) & \ldots & h(0) \end{bmatrix}, \quad (5)$$

$$\mathbf{H}_1 := \begin{bmatrix} 0 & h(P-1) & \ldots & h(1) \\ 0 & 0 & \ldots & h(2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & h(P-1) \\ 0 & 0 & \ldots & 0 \end{bmatrix}. \quad (6)$$

Similarly, the coded symbols are: $\bar{\mathbf{w}}(n) := [\mathbf{w}'(n),\ \mathbf{0}'_{P-M}]'$, where $\mathbf{0}_{P-M\times 1}$ is the P - $M \times 1$ vector of zeros. Correspondingly, the output becomes: $\mathbf{y}(n) = \mathbf{H}_0(1:P,1:M)\mathbf{w}(n) + \mathbf{H}_1(1:P,1:M)\mathbf{w}(n-1) + \mathbf{v}(n)$, where we have used the MATLAB notation $\mathbf{A}(a:b, c:d)$ to indicate the submatrix of $\mathbf{A}$ formed by rows a through b and columns c through d. Consider choosing P - $M > L_h$. Now, equation (6) implies $\mathbf{H}_1(1:P,1:M) = \mathbf{0}_{P\times M}$ and (4) becomes: $\mathbf{y}(n) = \mathbf{H}_0(1:P,1:M)\mathbf{w}(n) + \mathbf{v}(n)$. We note from (5) that $\mathbf{H}_0(1:M,1:M)$ is lower triangular and hence is always full rank (provided $h(0) \neq 0$). Furthermore, $\mathbf{H}_0(1:P,1:M)$ is at least full column rank.

Defining the input correlation matrix as $\mathbf{C_{2w}} := \mathrm{E}\{\mathbf{w}(n)\ \mathbf{w}^{*'}(n)\}$, the output vector correlation matrix $\mathbf{C_{2y}} := \mathrm{E}\{\mathbf{y}(n)\ \mathbf{y}^{*'}(n)\}$ becomes:

$$\mathbf{C_{2y}} = \mathbf{H}_0(1:P,1:M)\mathbf{C_{2w}}\mathbf{H}_0^{*'}(1:P,1:M) + \sigma_v^2 \mathbf{I}_{P\times P}.$$

The only assumption on the input will be that $\mathbf{C_{2w}}$ is a full-rank $M \times M$ matrix. This assumption together with the fact that $\mathbf{H}_0(1:P,1:M)$ is always full column rank, implies that the ordered eigenvalues of $\mathbf{C_{2y}}$ $\lambda_0 \geq \lambda_1 \geq \ldots \lambda_{P-1}$ can be described by (see e.g., [3]) $\lambda_i > \sigma_v^2$ for $i \leq M-1$ and $\lambda_i = \sigma_v^2$ otherwise. If we denote the eigenvectors corresponding the largest M eigenvalues as $\{\mathbf{s}_0,\ \mathbf{s}_1,\ \ldots,\ \mathbf{s}_{M-1}\}$ and the remaining eigenvectors as $\{\mathbf{z}_0,\ \mathbf{z}_1,\ \ldots,\ \mathbf{z}_{P-M-1}\}$, $\mathbf{C_{2y}}$ is:

$$\mathbf{C_{2y}} = \mathbf{S}\,\mathrm{diag}(\lambda_0,\ \lambda_1,\ \ldots, \lambda_{M-1})\mathbf{S}^{*'} + \sigma_v^2\, \mathbf{Z}\,\mathbf{Z}^{*'}\ , \quad (7)$$

where $\mathbf{S} := [\mathbf{s}_0,\ \mathbf{s}_1,\ \ldots,\ \mathbf{s}_{M-1}]$ and $\mathbf{Z} := [\mathbf{z}_0,\ \mathbf{z}_1,\ \ldots,\ \mathbf{z}_{P-M-1}]$. The signal subspace is the spaced spanned by the columns of $\mathbf{S}$ and is orthogonal to the columns of $\mathbf{Z}$. Since the signal space is also spanned by the columns of $\mathbf{H}_0(1:P,1:M)$, the orthogonality allows us to write

$$\mathbf{Z}^{*'}\ \mathbf{H}_0(1:P,1:M) = \mathbf{0}\ . \quad (8)$$

This type of equation, (8), was first used in [3] to estimate the channel in a FS system. In the FS case though, $\mathbf{H}_0$ is parameterized differently and is not guaranteed to be full column rank for all channels. In the TIC case though, it is guaranteed to be full column rank and therefore (8) can be used to estimate any FIR channel. Equation (8) is also identical to the subspace method presented in [7] although in [7] it is derived only for the case of $P = 2M$ and $f_i(n) = \delta(n) + \delta(n-M)$ (i.e., repetition code). In contrast, the method described in this section is valid for any channel and for any P, M provided P - $M \geq L_h$ and $f_i(n) = \delta(n)$ for all i. This subspace method holds for any colored random input (provided C_{2w} is full-rank). Furthermore, consider the sample or deterministic correlation of the noise-free output:

$$\begin{aligned} \hat{\mathbf{C}}_{\mathbf{y}} &:= \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{y}(n)\, \mathbf{y}^{*'}(n) \quad (9) \\ &= \mathbf{H}_0(1:P,1:M)\, \hat{\mathbf{C}}_{\mathbf{w}}\, \mathbf{H}_0^{*'}(1:P,1:M) \end{aligned}$$

where $\hat{\mathbf{C}}_{\mathbf{W}}$ is defined similarly. Hence, in the absence of noise, the orthogonality relationship (8) holds even for deterministically modeled input. The only condition required on the input is that it be persistently exciting so that $\hat{\mathbf{C}}_{\mathbf{W}}$ is full rank. We note also that while derived under the assumption of identity coding ($f_i(n) = \delta(n)$), the subspace method can be extended to arbitrary codes as will be shown in a future paper.

Remark 1: For $P - M > L_h$, exact zero-forcing equalizers can be found from $\hat{h}(n)$ [1]. Alternatively, for better performance in the presence of additive noise, a minimum mean-square error equalization [2] could be found from the channel estimate. Lastly, optimal performance can be realized by using a maximum likelihood receiver [4].

Remark 2: We note that throughout this paper, we have assumed 'block' synchronization was possible. In other words, to form the correlation matrices correctly, one must know the start and finish time of each code block. This block timing information is available from the cyclostationary nature of the problem and is studied in [7] and [2].

4. SIMULATIONS

In this section we demonstrate the performance of the proposed method in comparison with existing TIC and FS methods. The channel considered has zeros at $\{H(z) = 0: z = 1.5, \pm 0.5j\}$ where $H(z) := \sum_n h(n)\, z^{-n}$. In both experiments, 200 Monte Carlo runs were used to estimate the mean-square channel estimation error which is defined as $\mathrm{MSE} = (1/200) \sum_{i=0}^{199} \sum_{n=0}^{4} |\hat{h}_i(n) - h(n)|^2$, where $\hat{h}_i(n)$ is the estimate of $h(n)$ obtained on the ith Monte Carlo run. Experiment 1 compares the performance of the proposed subspace method with existing TIC non-linear matching [1] and closed-form [1] methods. BPSK symbols ($\pm$ 1) were coded according to Figure 2 with $M = 8$, $P = 12$, and $f_i(n) = \delta(n)$. The output correlations were estimated by using (9) with 336 outputs (i.e., $N = 28$). As seen in Figure 3, the subspace method performance is much greater than existing TIC methods across a large range of SNR's.

Experiment 2 uses the same parameters as Experiment 1 except that the coding rate is allowed to vary. Fig. 4 shows that as the information rate increases (i.e., M/P increases), the MSE increases. This is expected since increasing P implies fewer periods of cyclostationary output are available for a fixed sample size; hence, N in (9) decreases.

Finally, Experiment 3 compares the performance of the proposed subspace method with a FS subspace method [3] and a least-squares deterministic approach described in [8]. We note that the channel considered for all experiments is not identifiable by $P = 2$ FS systems since $H(z) = 0$ at $0.5j$ and $-0.5j$ which are separated in angle by π [6]. Consequently, the performance of the TIC method is superior for this channel.

5. DISCUSSION

This paper extends the ideas of [7] and [1] to derive a method which requires no assumptions on the input statistics and the channel zeros with minimal rate reduction. Relative to an FS system, the penalty paid for the advantages gained through a TIC system are: (1)Coding and decoding delay; (2)stricter 'frame' synchronization; (3)decreased information rate.

The loss in rate is the most severe of the penalties paid by a TIC system. While it is possible to make the rate loss arbitrarily close to one, to do so increases the coding/decoding delay and may, as seen in Figure 4, increase the estimation error. As suggested in [7], one way to overcome the loss information rate is to increase the transmission and receiver sampling rate by the reciprocal factor. For example, if $T_r^{-1} = T_0^{-1}$ is the original transmission rate and a rate P_2, M_2 coder is desired, the new transmission and sampling rate should be $T_r^{-1} = (P/M)\, T_0^{-1}$. By increasing the transmission rate, the effective information rate (number of unique information symbols transmitted in a given time interval) remains constant. However, increasing the transmission rate does not come without penalty. In general, the transmit filter limits the bandwidth of the transmission signal in order to meet design constraints. Increasing the transmission rate though implies either the bandwidth must increase, or, the transmit filter will introduce a known ISI component as in the case of partial response (PR) systems (c.f., [4, Chpt. 9]). Although the ISI introduced by the transmitter for PR systems is known and can be compensated for at the receiver, there is nevertheless a loss in probability of error performance. An alternative is to increase both the transmission rate and the bandwidth of the signal. One scenario where this might be possible (i.e., where bandwidth may not be tightly constrained) is in a spread-spectrum (SS) type system.

Table 1 shows a comparison between HOS, FS, and the three TIC viewpoints (SS, PR, and coding). The first column shows the overall information rate while the second shows the order of the resulting discrete-time equivalent channel when the original continuous-time channel spans T_h seconds. While the discrete-time channel order is simply T_h divided by the sample rate, it demonstrates a subtle but important distinction between the three TIC viewpoints and the FS systems. Namely, a critical property of a TIC system is the ability to pick a P greater than L_h. The value of P determines the period of the cyclostationarity; hence, $P > L_h$ means the period of cyclostationarity is longer than the memory of the channel. This leads to guaranteed identifiability (Theorem 1). In contrast with the three TIC views, the FS system can never have $P > L_h$ unless $T_h/T_0 < 1$. Hence, FS methods have restrictions on channel zeros. Finally we mention that many of the TDMA wireless standards (e.g, IS-54, GSM) typically employ guard-times

Method	Inform Rate	Channel Order L_h	Restrictions for blind ID
HOS	T_0^{-1}	T_h/T_0	Non-Gaussian input
FSE	T_0^{-1}	$P\ (T_h/T_0)$	Non equi-spaced zeros
TIC (Coding)	$(M/P)\ T_0^{-1}$	T_h/T_0	$P > L_h$ $P > M$
TIC (PR)	T_0^{-1}	(P/M) $\times (T_h/T_0)$	$P > M$ $M > (T_h/T_0)$
TIC (SS)	T_0^{-1}	(P/M) $\times (T_h/T_0)$	$P > M$ $M > (T_h/T_0)$

Table 1. Comparison of blind equalization methods

(or zeros) between successive frames. This built in "quiet" time or built in zeros, could potentially be exploited as a multirate identity code system for equalization. In doing so, no additional rate penalty occurs. Also, most TDMA standards apply a periodic power modulation to ease hardware design which could be further exploited since it too induces a periodicity in the output. One final remark is now in order:

Remark 3: Cyclostationarity in $\bar{w}(n)$ is not induced only by the multirate relation (2). Theorem 1 applies even if e.g., $\bar{w}(n) = w(n)p(n)$, where $p(n)$ is a periodic sequence with known period $P > L$ [5]. Such a modulation-driven cyclostationarity can be induced from the filterbank of Figure 2 with a special choice of the $f_m(n)$ filters. In fact, it is shown in [5] that identifiability is guaranteed even from a single cycle, provided that $P > L/2$.

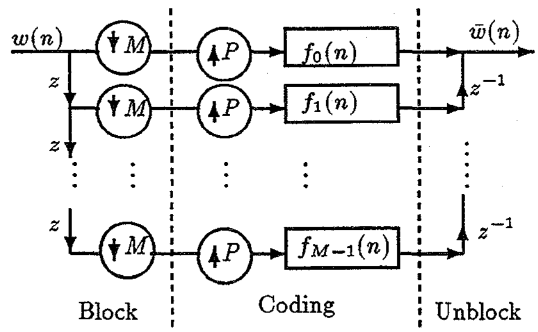

Figure 2. Multirate Coding System for ISI

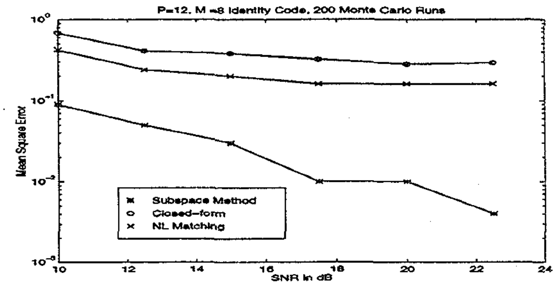

Figure 3. Comparison among TIC methods

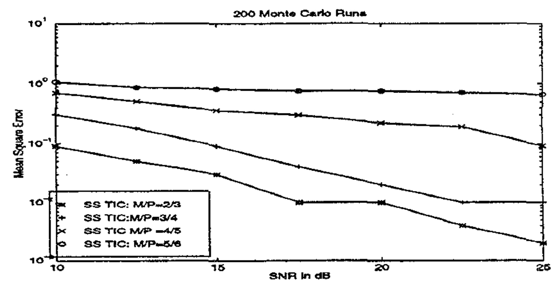

Figure 4. Different Code Rates

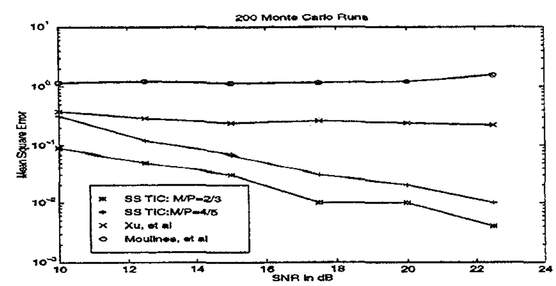

Figure 5. Comparison of FS vs. TIC

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