GENERALIZED DIFFERENTIAL ENCODING: A NONLINEAR SIGNAL PROCESSING FRAMEWORK

[1] *Fulvio Gini* and [2] *Georgios B. Giannakis*

[1] Dept. "Ingr. dell' Informazione", Univ. of Pisa, via Diotisalvi, 2 I-56126 Pisa, ITALY
Tel: (+39) 50-568550 Fax: (+39) 50-568522 E-mail: gini@iet.unipi.it
[2] Dept. of Electrical Engr., Univ. of Virginia, Charlottesville, VA 22903-2442, U.S.A.
Tel: (804) 924-3659 Fax: (804) 924-8818 E-mail: georgios@virginia.edu

ABSTRACT

Encoding of communication signals compensates for phase distortions introduced by imperfect local oscillators, fading, and multipath effects. Differential encoding is generalized in this paper using a nonlinear transformation called multi-lag High-order Instantaneous Moment (ml-HIM). The ml-HIM decoder is capable of removing not only constant phase ambiguity, but also Doppler frequency, Doppler rate, and even higher-order phase distortions. The multiple lags present in the ml-HIM are exploited to improve system performance. Differential encoding is also generalized to non-constant modulus constellations such as M-ary QAM and AM-PM.

1. INTRODUCTION

In digital communication systems phase ambiguity and frequency errors are likely to be present in the received signal, due to imperfect knowledge of the carrier's phase and frequency, fading, and multipath effects [3, Ch. 6]. Typically, phase distortions are captured in a term $\exp[j\theta(t)]$, which multiplies the received baseband data [see eq. (1)]. When phase variations are induced by the relative motion between transmitter and receiver (such as in mobile and satellite communications), the phase $\theta(t)$ is polynomial in t, and its coefficients are related to the kinematics of the moving station [8, p. 59]. The polynomial coefficients are estimated first and the received data are multiplied next, by $\exp[-j\hat{\theta}(t)]$, to remove phase distortions [4], [5].

Alternatively, phase errors can be pre-compensated by differential encoding at the transmitter and incoherent decoding at the receiver. The latter eliminates the need for carrier phase acquisition and tracking [3, Ch. 5], at the expense of additional signal power required to attain a given probability of error (when compared to the ideal coherent detection). Traditionally, information transmitted with differential phase-shift-keying (DPSK) is encoded in phase differences between two successive symbols. Although tolerant to constant phase errors, differential detection systems are sensitive to carrier frequency variations [9]. To overcome this problem doubly-differential PSK (DDPSK) encoding has been discussed recently in [9], based on less-known results from the Russian literature, [6] (see also [7] and [10]).

Interestingly, differential encoding has been restricted only to MPSK signaling; i.e., constant modulus constellations. In [4] though, blind phase recovery in Quadrature Amplitude Modulated (QAM) signals was proposed based on fourth-order statistics (HOS) of the received data. Unfortunately, unambiguous estimates are guaranteed only if phase errors lie in $[-\pi/4, \pi/4]$, and processing delay is incurred in order to collect the samples required for accurate HOS estimation [4].

In this paper, we revisit the differential encoding idea from a nonlinear signal processing perspective which relies on the multi-lag High-order Instantaneous Moment (ml-HIM) – a transformation originally introduced in [1] and thoroughly studied in [2] (Section 2). Starting from the well-known case of MDPSK modulation [3, Ch. 5], we show how differential encoding can be interpreted in terms of the ml-HIM, and how generalized differential encoding techniques can be derived by means of the multi-lag HIM (Section 3). This result is particularly useful both when the additive noise is colored and when higher- than second-order differential encoding is employed in order to eliminate Doppler effects from the received data. We also show how this approach can be successfully applied to non-constant modulus constellations such as M-ary QAM and Amplitude-Modulation Phase-Modulation (AM-PM) [3, Ch. 5] (Section 4).

2. BACKGROUND

Consider transmission of a linearly modulated signal through a linear time-invariant channel. We assume knowledge of the channel and perfect symbol synchronization. The complex envelope of the received signal after baseband conversion is [3, Ch. 5], [5]:

$$r_c(t) = e^{j\theta(t)} \sum_l w(l)\, g_c^{(tr)}(t - lT_s) + n_c(t) , \qquad (1)$$

where $\theta(t)$ models the phase distortion, $g_c^{(tr)}(t)$ is the cascade of the transmitter's signaling pulse and the channel's impulse response; T_s is the symbol period, and $w(l)$'s are the transmitted complex symbols; noise $n_c(t)$ is assumed complex white Gaussian, with two-sided power spectrum $N_0/2$. After the receiving matched filter, $g_c^{(rec)}(t)$, the signal $x_c(t) = r_c * g_c^{(rec)}(t)$ ($*$ denotes convolution) is sampled at symbol rate $1/T_s$ to obtain: $x(n) = \exp[j\theta(n)]\sum_l w(l)g(n-l) + v(n)$, with $x(n) := x_c(nT_s)$, $v(n) := n_c * g_c^{(rec)}(t)|_{t=nT_s}$, and $g(n) := g_c^{(tr)} * g_c^{(rec)}(t)|_{t=nT_s}$. The model is valid as far as the mismatch between $g_c^{(tr)}(t)$ and $g_c^{(rec)}(t)$ due to $\theta(t)$

0-7803-3944-4/97/$10.00

can be neglected; i.e., the bandwidth of $\exp[j\theta(t)]$ is small as compared to that of $g_c^{(rec)}(t)$.

In this work, we assume $\theta(n) := \theta_0 + 2\pi f_e n + \pi\alpha_e n^2$, with θ_0 denoting the phase offset, while f_e and α_e are the Doppler shift and rate, respectively (higher order terms can be considered as well). If the Nyquist condition is satisfied, sampling every $t_n = nT_s$, we have $g(n) = \delta(n)$ [3, p. 326], where $\delta(n)$ denotes the Kronecker delta function. We thus arrive at the discrete-time model:

$$x(n) = w(n)\, e^{j(\theta_0+2\pi f_e n+\pi\alpha_e n^2)} + v(n) \ . \qquad (2)$$

We wish to encode $w(n)$ to $w_d(n)$ so that $\theta(n)$ in (2) does not affect recovery of $w_d(n)$ from $x(n)$ samples.

Our decoding scheme relies upon the multi-lag Higher-order Instantaneous Moment (ml-HIM) which for on-line implementation is defined recursively as: $x_1(n) := x(n)$, $x_2(n; m_1) := x_1(n)x_1^*(n-m_1), \ldots,$

$$x_k(n; m_1, m_2, \ldots, m_{k-1}) := x_{k-1}(n; m_1, \ldots, m_{k-2})$$
$$\times\ x_{k-1}^*(n - m_{k-1}; m_1,\ \ldots\ , m_{k-2})\ . \qquad (3)$$

To illustrate the role of ml-HIM in differential encoding, let $\theta(n) = \theta_0$ and the digital modulation be M-ary DPSK; i.e., $w(n) := \exp[j\psi(n)]$ is assumed i.i.d., drawn from a discrete M-ary alphabet set $\{\exp[j(2\pi k/M + \psi_0)]\}_{k=0}^{M-1}$, with probability $1/M$ each; w.l.o.g. we assume $\psi_0 = 0$. Let $w_d(n)$ denote the differentially encoded sequence defined as:

$$w_d(n) = e^{j\psi_d(n)} = e^{j[\psi(n)+\psi_d(n-1)]} = w(n)w_d(n-1). \quad (4)$$

From (2), the received discrete-time signal is given by:

$$x(n) = w_d(n)e^{j(\theta_0+2\pi f_e n+\pi\alpha_e n^2)} + v(n)\ . \qquad (5)$$

At the receiver, $w(n)$ can be recovered by "inverting" (4):

$$w(n) = e^{j\psi(n)} = e^{j[\psi_d(n)-\psi_d(n-1)]} = w_d(n)w_d^*(n-1)\ . \ (6)$$

In the noise-free case, if $f_e = 0$ and $\alpha_e = 0$, $w(n)$ is decoded using (6). When noise is present, the decoder output must be quantized; e.g., in the binary case (BDPSK) $w(n) \in \{-1, 1\}$, the transmitted sequence is recovered via:

$$\hat{w}(n) = \mathrm{sgn}\{x(n)x^*(n-1)\}\ , \qquad (7)$$

where $\mathrm{sgn}\{\cdot\}$ denotes the signum function. The encoding-decoding strategy in (5)-(7) is not affected by the phase shift θ_0, and in the noise-free case we have:

$$\hat{w}(n) = x(n)x^*(n-1) = w_d(n)w_d^*(n-1) = w(n)\ . \quad (8)$$

Eq. (8) shows that the decoder implements nothing but the second-order HIM defined in (3) and evaluated for $m_1 = 1$. This observation suggests how to generalize the idea of differential encoding to the case where $\theta(n)$ is not simply a constant θ_0.

3. ML-HIM DIFFERENTIAL DECODING

Consider the ml-HIM (up to the fourth order) of the noise-free signal in (5): $x_1(n) = w_d(n)\exp[j(\theta_0+2\pi f_e n+\pi\alpha_e n^2)]$, $x_2(n; m_1) = w_d(n)w_d^*(n-m_1)\exp[j(2\pi f_e m_1 - \pi\alpha_e m_1^2 + 2\pi\alpha_e m_1 n)]$, and with $k = 3, 4$ we find from (3):

$$x_3(n; m_1, m_2) \;=\; w_d(n)w_d^*(n-m_1)w_d^*(n-m_2)$$
$$\times\ \ w_d(n-m_1-m_2)\, e^{j2\pi\alpha_e m_1 m_2}\ , \ (9)$$

$$x_4(n; m_1, m_2, m_3) = w_d(n)w_d^*(n-m_1)w_d^*(n-m_2)w_d^*(n-m_3)$$
$$\times w_d(n-m_1-m_2)w_d(n-m_1-m_3)w_d(n-m_2-m_3)$$
$$\times\ w_d^*(n-m_1-m_2-m_3)\ . \qquad (10)$$

To remove constant phase, the decoder implements the second-order ml-HIM. The third-order ml-HIM removes both constant phase and Doppler frequency, but not Doppler rate, for which it is necessary to resort to the fourth-order ml-HIM. Generalizing (4), the encoder should correspondingly implement: $w_d(n) = w(n)w_d(n-m_1)$, $w_d(n) = w(n)w_d(n-m_1)w_d(n-m_2)w_d^*(n-m_1-m_2)$,

$$w_d(n) = w(n)w_d(n-m_1)w_d(n-m_2)w_d(n-m_3)$$
$$\times w_d^*(n-m_1-m_2)w_d^*(n-m_1-m_3)w_d^*(n-m_2-m_3)$$
$$\times\ w_d(n-m_1-m_2-m_3)\ . \qquad (11)$$

To assure causality, we select $0 < m_1 \le m_2 \le m_3$. Inductively, to remove a Qth-order polynomial phase distortion $\theta(n)$, we must encode with the kth-order ml-HIM, and select $k = Q + 2$. Under the white noise assumption, x_k is known to be an unbiased and mean-square sense consistent estimator of $w(n)$ [11]. Taking into account that $w(n)$ comes from a finite alphabet, our $\hat{w}(n)$ estimate is obtained as:

$$\hat{w}(n) = \arg\min_{w(n)} |w(n) - x_k(n; m_1, \ldots, m_{k-1})|\ . \qquad (12)$$

The decoding strategy of (9) with $m_1 = m_2 = 1$, was proposed in [7] under the term Double Differential PSK (DDPSK). Probability of error was also derived for symbol by symbol detection [6], [7]. Binary and M-ary PSK and the possibility of multiple symbol detection was discussed in [9]. Cascading n first-order differential encoding blocks and numerically computable probability of error expressions for binary DDPSK, were reported in [10]. Approximately 4 dB of excess SNR is required to attain a given error probability using binary DDPSK relative to binary DPSK without Doppler frequency.

Here, we exploit the degrees of freedom offered by the different lags in order to optimize the system performance. To select the lags, we adopt the deflection criterion. Consider first the case of M-ary PSK and suppose we want to remove a constant phase θ_0 ($f_e = \alpha_e = 0$). The decision at the receiver relies on the noisy second-order HIM:

$$x_2(n, m_1) = w(n) + w_d(n)e^{j\theta_0}v^*(n-m_1) + w_d^*(n-m_1)$$
$$\times\ e^{-j\theta_0}v(n) + v(n)v^*(n-m_1) := w(n) + d(n; m_1)\ , \ (13)$$

where both $w(n)$ and $d(n; m_1)$ are zero mean. We wish to select m_1 that maximizes the deflection:

$$D(m_1) := \frac{\mathrm{var}\{w(n)\}}{\mathrm{var}\{d(n; m_1)\}} = \frac{E\{|w(n)|^2\}}{E\{|d(n; m_1)|^2\}}\ . \qquad (14)$$

The m_1 that maximizes D may not always minimize the probability of error, but we expect it to improve performance relative to $m_1 = 1$. Inserting (13) into (14) and observing that $|w(n)| = |w_d(n)| = 1$, we obtain: $D = 1/(2\sigma_v^2 + \sigma_v^4)$, which does not depend on m_1; hence, our best choice is $m_1 = 1$ because it minimizes the number of "training" symbols required to initialize the differential decoder [3, p. 211]. But what if the noise samples are correlated due to a mismatch between $g_c^{(rec)}(t)$ and $g_c^{(tr)}(t)$? To investigate this aspect, we further assume that $v(n)$ is an MA(1) process; i.e., $v(n) = v_I(n) + bv_I(n-1)$, where $v_I(n)$ is an i.i.d. zero mean Gaussian sequence with variance σ_I^2 such that $\sigma_v^2 = (1+b^2)\sigma_I^2$. The deflection now becomes:

$$D^{-1}(m_1) = 2\sigma_v^2 + \sigma_v^4 + \frac{(1+b^4)\sigma_v^4}{(1+b^2)^2}\delta(m_1) + \frac{b^2\sigma_v^4}{(1+b^2)^2} \times [\delta(m_1 - 1) + \delta(m_1+1)] \ , \quad (15)$$

and depends on m_1. Causality requires $m_1 > 0$, and if deflection is to be maximized with the minimum number of redundant symbols, the best choice is $m_1 = 2$. In general, if L_v denotes the memory of $v(n)$, the minimum m_1 that maximizes deflection is $m_1 = L_v + 1$.

The advantage offered by multiple lags is even more pronounced when Doppler frequency is also present and the decoder implements the third-order ml-HIM. In the presence of noise, the third-order ml-HIM can be written as $x_3(n; m_1, m_2) = w(n) + d(n; m_1, m_2)$. Evaluation of $E\{|d(n; m_1, m_2)|^2\}$ involves 15×15 terms, but most of them are zero mean. Skipping details we find:

$$D^{-1}(m_1, m_2) = 4\sigma_v^2 + 6\sigma_v^4 + 4\sigma_v^6 + \sigma_v^8 + [2\sigma_v^2 + 5\sigma_v^4 + 4\sigma_v^6 + \sigma_v^8]\delta(m_1 - m_2) \ . \quad (16)$$

The deflection in (16) does not depend on the values of θ_0, f_e, and α_e. The choice $m_1 = m_2 = 1$, corresponds to the DDPSK system proposed in [7], [10], and is the worst one in terms of SNR. On the contrary, $m_2 > m_1 = 1$ offers the same performance and in order to minimize the training symbols we select $m_2 = 2$. Fig. 2a shows $D(m_1, m_2)$ in (16) versus SNR with the solid line corresponding to $m_1 = m_2$ and the dashed to $m_1 \neq m_2$.

The deflection of $x_4(n; m_1, m_2, m_3)$ can be evaluated with the same technique, but the calculation involves $(2^8 - 1) \times (2^8 - 1)$ terms. We derived it via Monte Carlo simulations using 5×10^5 symbols for each set of lags. Fig. 2b depicts $D(m_1, m_2, m_3)$ versus SNR for $(m_1, m_2, m_3) = (1,1,1)$, $(1,1,2)$, $(1,2,4)$, $(2,3,4)$, $(1,2,3)$, $(1,4,8)$. Cascading three first-order differential encoding (decoding) cells corresponds to $(m_1, m_2, m_3) = (1,1,1)$, and yields the lowest deflection due to the presence of dependent samples in $x_4(n; m_1, m_2, m_3)$ (note that samples $x(n-1)$ and $x(n-2)$ appear three times each in x_4). When the number of dependent samples in the ml-HIM decreases, the deflection increases. When $(m_1, m_2, m_3) = (1,1,2)$, x_4 contains $x(n)$, $x(n-4)$, and two replicas of $x(n-1)$, $x(n-2)$, and $x(n-3)$. When $(m_1, m_2, m_3) = (1,2,3)$, $x(n-3)$ is present two times; in all other cases the noise samples are independent and the deflection reaches its maximum.

Symbol error rate (SER) curves were obtained by averaging over 8×10^7 decisions for binary PSK with parameters $\theta_0 = \pi/8$, $f_e = 0.05$, $\alpha_e = 0$ (Fig. 3a), and $\alpha_e = 0.0015$ (Fig. 3b). Fig. 3a shows the SER for lags $(m_1, m_2) = (1,1)$ (dashed line) $(1,2)$ (dashdotted), $(1,3)$ (circles), $(1,4)$ (x-marks). The solid lines in Figs. 3a and 3b denote the SER of an ideal (i.e., with $\theta(n) = 0$) binary PSK. As expected, the choice $(m_1, m_2) = (1,1)$ is the worst one also in terms of probability of error performance, while all other choices are equivalent; hence, $(m_1, m_2) = (1,2)$ should be selected because it minimizes the number of training symbols. For a target SER of 10^{-5}, 4.5 dB of additional SNR is required when $m_1 = m_2 = 1$ are used instead of ideal BPSK with $\theta_0 = f_e = 0$. But if we select $(m_1, m_2) = (1,2)$, approximately 2.8 dB of additional SNR is only necessary.

The SER of this system does not depend on the values of θ_0 and f_e, but is affected by the Doppler rate α_e. To remove the Doppler rate effect, the system has to implement the fourth-order ml-HIM at the decoder. Fig. 3b shows the SER of such a system for $\theta_0 = \pi/8$, $f_e = 0.05$, and $\alpha_e = 0.0015$ with lags $(m_1, m_2, m_3) = (1,1,1)$ (upper solid line), $(1,1,2)$ (dashed), $(1,2,4)$ (dotted), $(2,3,4)$ (circles), $(1,2,3)$ (dashdotted). The results show that selecting the lags appropriately improves the performance considerably, relative to the system in [10] which uses $m_1 = m_2 = m_3 = 1$. For example, selecting $(m_1, m_2, m_3) = (1,2,3)$ requires only 4.5 dB of additional SNR relative to the ideal BPSK, while adopting $m_1 = m_2 = m_3 = 1$ incurs greater loss. We note that all sets of lags that correspond to independent noise samples guarantee the same SER, but the optimal choice in terms of SER is $(m_1, m_2, m_3) = (1,2,3)$, whose deflection is slightly lower than that corresponding to the maximum D (see Fig. 2b).

4. NON-CONSTANT MODULUS SIGNALS

Consider 16-QAM modulated symbols $w(n) = a(n) + jb(n) = \rho(n)\exp[j\varphi(n)]$ with i.i.d. $a(n)$ and $b(n)$ drawn from the discrete alphabet set $\{\pm 1, \pm 3\}$. Suppose that only phase and Doppler frequency errors are present in $x(n)$. To remove phase and Doppler frequency errors with the third-order ml-HIM encoder, we define $\bar{w}(n) := w(n)/\rho(n) = \exp[j\varphi(n)]$, and $\bar{w}_d(n) := w_d(n)/\rho(n)$. At the differential encoder output, we have (see Fig. 1): $w_d(n) = \rho(n)\bar{w}_d(n)$ where, according to (9), $\bar{w}_d(n)$ is:

$$\bar{w}_d(n) = \bar{w}(n)\bar{w}_d(n-m_1)\bar{w}_d(n-m_2)\bar{w}_d^*(n-m_1-m_2). \quad (17)$$

At the receiver, we decode by normalizing the ml-HIM:

$$\bar{x}_3(n; m_1, m_2) = \frac{x_3(n; m_1, m_2)}{|x(n-m_1)x(n-m_2)x(n-m_1-m_2)|} \ ,$$

and applying the decision rule in (12) as follows:

$$\hat{w}(n) = \arg\min_{\bar{w}(n)} |w(n) - \bar{x}_3(n; m_1, m_2)| \ . \quad (18)$$

In Fig. 4a we plot the real versus imaginary parts of the received signal (2,000 noise-free 16-QAM symbols) when the uncoded $w(n)$ is transmitted over the channel with phase and frequency errors ($\theta_0 = \pi/8$, $f_e = 0.05$, $\alpha_e = 0$). Fig. 4b depicts real versus imaginary parts of $\bar{x}_3$, when $w_d(n)$ is transmitted with differentially encoded phase according to

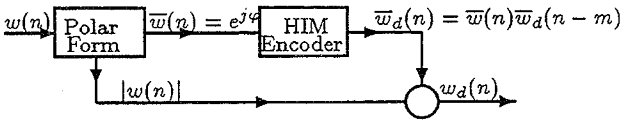

Figure 1. Non-constant modulus signal encoder

(17). It is evident that in the absence of noise the constellation is recovered perfectly. The sames plots with noise at SNR= 25 dB are depicted in Figs. 5a and 5b. Figs. 6a and 6b show the deflection of $\tilde{x}_3$ and the corresponding SER versus SNR for $(m_1, m_2) = (1,1)$, $(1,2)$, $(1,3)$, and $(1,4)$, obtained based on 2×10^5 symbols and with parameters $\theta_0 = \pi/8$, $f_e = 0.05$, and $\alpha_e = 0$. Due to the multilevel constellation (which is less tolerant to noise than BPSK) the loss is greater than in Fig. 3b; however, part of this loss can be recovered if the symbol-by-symbol decision is replaced by multiple symbol detection as suggested in [9].

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Acknowledgements: The work in this paper was supported by ONR Grant No. N00014-93-1-0485.

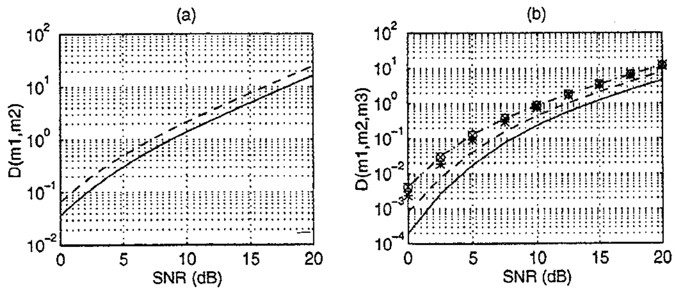

Figure 2. Deflection: (a)3rd-, and (b)4th-order ml-HIM

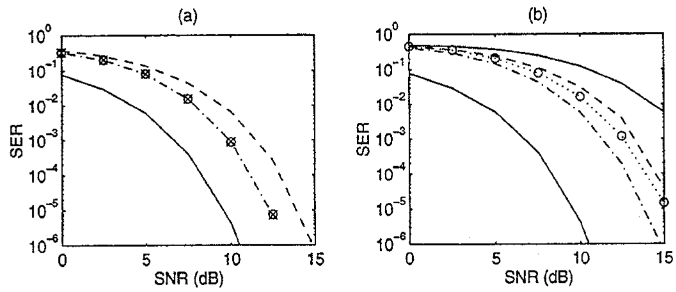

Figure 3. Error Rates: (a)3rd-, and (b)4th-order ml-HIM

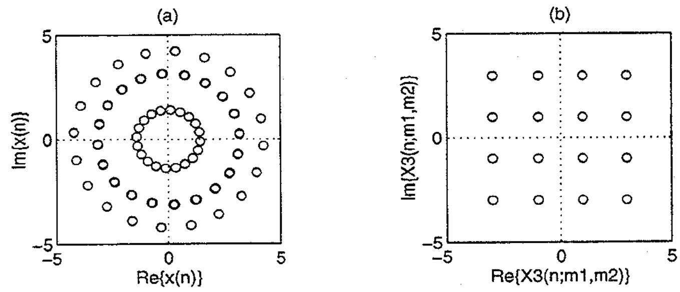

Figure 4. SNR ∞: (a)uncoded, (b)3rd-order HIM

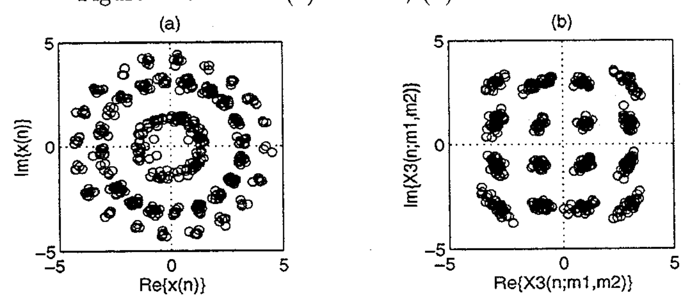

Figure 5. SNR 25dB: (a)uncoded, (b)3rd-order HIM

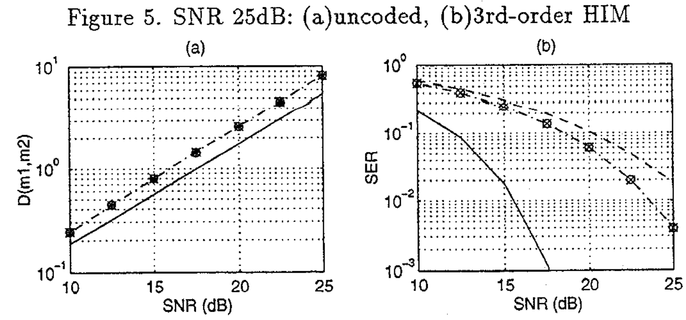

Figure 6. 16-QAM: (a)Deflection, (b)Error Rates