

# Space–Time Coding With Maximum Diversity Gains Over Frequency-Selective Fading Channels

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**Abstract**—In this letter, we study space–time (ST) block coding for single carrier zero-padded (ZP) block transmissions through finite impulse response (FIR) multipath fading channels of order  $L$ . We prove that maximum diversity of order  $2(L + 1)$  is achieved with two transmit and one receive antennae. Simulations demonstrate that joint exploitation of multi-antenna diversity and multipath diversity leads to significantly enhanced performance.

**Index Terms**—Frequency selective multipath channels, space–time coding, zero-padded (ZP) block transmissions.

## I. INTRODUCTION

SPACE–TIME (ST) coding offers an attractive means of achieving high data rate wireless transmissions with diversity and coding gains. However, existing ST codes are mainly designed for frequency flat channels. In order to maintain decoding simplicity, most works on ST coding for frequency selective channels employ orthogonal frequency division multiplexing (OFDM) to convert frequency selective channels to a set of flat fading subchannels, see e.g., [4], [9]. ST-OFDM uses ST codes designed for flat fading channels but does not exploit the embedded multipath diversity. The latter can be recovered with e.g., carefully designed trellis codes across subcarriers [2], [5]. However, OFDMs nonconstant modulus multicarrier transmission reduces power efficiency of all ST-OFDM based schemes. This motivates ST codes for single carrier transmissions over frequency selective channels that have been reported recently in [3] for serial transmissions, and in [8] for block transmissions.

In this letter, we investigate single carrier zero-padded (ZP) block transmissions. Different from [8], which focuses on mitigating rapidly time varying channels with suboptimum receivers, we deal with block quasi static channels and prove that a maximum diversity of order  $2(L + 1)$  is achievable in rich scattering environments with two transmit and one receive antennae.

**Notation:** Bold upper (lower) letters denote matrices (column vectors).

$(\cdot)^*$	conjugate;
$(\cdot)^T$	transpose;
$(\cdot)^{\mathcal{H}}$	Hermitian transpose;

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$\mathbf{E}\{\cdot\}$	expectation;
$\mathbf{I}_K$	identity matrix of size $K$ ;
$\mathbf{0}_{M \times N}$	all-zero matrix of size $M \times N$ ;
$\mathbf{F}_N$	$N \times N$ FFT matrix with $(p, q)$ th entry $(1/\sqrt{N})e^{-j((2\pi)/N)(p-1)(q-1)}$ , $\forall p, q \in [1, N]$ ;
$\text{diag}(\mathbf{x})$	diagonal matrix with $\mathbf{x}$ on its diagonal;
$[\cdot]_p$	$p$ th entry of a vector;
$[\cdot]_{p,q}$	$(p, q)$ th entry of a matrix.

We define a set of  $J \times J$  permutation matrices  $\{\mathbf{P}_J^{(n)}\}_{n=0}^{J-1}$  that performs a reversed cyclic shift, i.e., for a  $J \times 1$  vector  $\mathbf{a} = [a(0), a(1), \dots, a(J-1)]^T$ , and we have  $[\mathbf{P}_J^{(n)} \mathbf{a}]_p = a((J-p+n) \bmod J)$ .

## II. SINGLE CARRIER ZP BLOCK TRANSMISSIONS

Fig. 1 depicts the discrete-time baseband equivalent model of a communication system with  $N_t = 2$  transmit antennas and  $N_r = 1$  receive antennas, where the channel between the  $\mu$ th transmit antenna and the receive antenna is modeled as an  $L$ th-order finite impulse response (FIR) filter with coefficients  $\mathbf{h}_\mu = [h_\mu(0), \dots, h_\mu(L)]^T$ ,  $\mu = 1, 2$ . At the transmitter, the information symbols  $s(n)$  are first parsed to form  $K \times 1$  blocks  $\mathbf{s}(i) = [s(iK), s(iK+1), \dots, s(iK+K-1)]^T$ . The ST encoder takes two consecutive blocks  $\mathbf{s}(2i)$  and  $\mathbf{s}(2i+1)$  to output the following  $2K \times 2$  ST-coded matrix:

$$\begin{bmatrix} \bar{\mathbf{s}}_1(2i) & \bar{\mathbf{s}}_1(2i+1) \\ \bar{\mathbf{s}}_2(2i) & \bar{\mathbf{s}}_2(2i+1) \end{bmatrix} = \begin{bmatrix} \mathbf{s}(2i) & -\mathbf{P}_K^{(0)} \mathbf{s}^*(2i+1) \\ \mathbf{s}(2i+1) & \mathbf{P}_K^{(0)} \mathbf{s}^*(2i) \end{bmatrix} \quad (1)$$

which reduces to the well-known Alamouti's ST block code [1] if no symbol blocking (i.e.,  $K = 1$ ) is invoked. Although the output  $\mathbf{P}_K^{(0)} \mathbf{s}(i) = [s(iK+K-1), \dots, s(iK)]^T$  is a time-reversed version of  $\mathbf{s}(i)$ , as in the serial transmission of [3] and the block transmission of [8], our receiver processing and maximum diversity proofs are novel and have not been established in [3] and [8].

At each block transmission time interval  $i$ , the blocks  $\bar{\mathbf{s}}_1(i)$  and  $\bar{\mathbf{s}}_2(i)$  are forwarded to the first and the second antenna, respectively. At each antenna  $\mu \in [1, 2]$ , a tall  $P \times K$  zero padding transmit-matrix  $\mathbf{T}_{zp} = [\mathbf{I}_K^T, \mathbf{0}_{K \times L}^T]^T$  is applied on  $\bar{\mathbf{s}}_\mu(i)$  to obtain  $P \times 1$  blocks  $\mathbf{u}_\mu(i) = \mathbf{T}_{zp} \bar{\mathbf{s}}_\mu(i)$ , which are transmitted through the channels. ZP guards adjacent blocks from inter block interference. Let  $\mathbf{H}_\mu$  be the  $P \times P$  lower triangular Toeplitz channel matrix with  $[\mathbf{H}_\mu]_{p,q} = h_\mu(p-q)$ , and  $\tilde{\mathbf{H}}_\mu$  the  $P \times P$  circulant matrix with  $[\tilde{\mathbf{H}}_\mu]_{p,q} = h_\mu(p-q \bmod P)$ . The

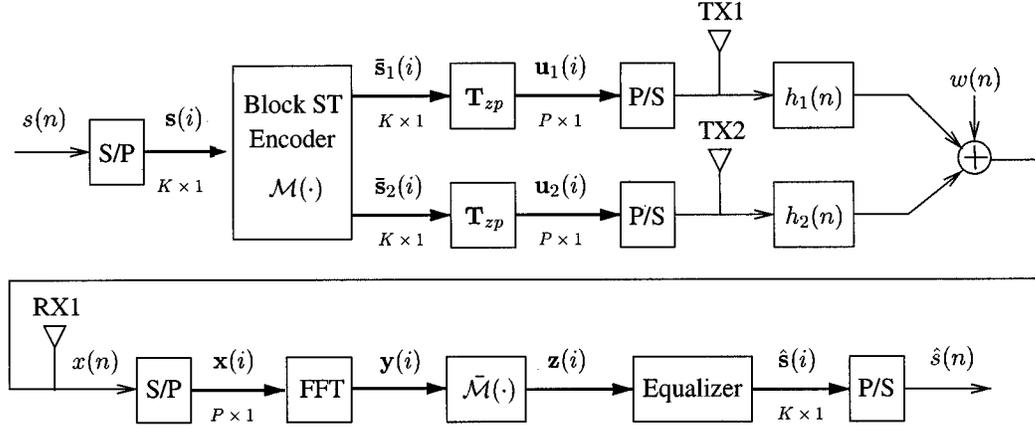


Fig. 1. ST-coded single carrier block ZP transceiver model.

block input-output relationship can then be described by (see [9] for a detailed derivation with single antenna ZP transmissions)

$$\begin{aligned} \mathbf{x}(i) &= \sum_{\mu=1}^2 \mathbf{H}_{\mu} \mathbf{T}_{zp} \bar{\mathbf{s}}_{\mu}(i) + \mathbf{w}(i) \\ &= \sum_{\mu=1}^2 \tilde{\mathbf{H}}_{\mu} \mathbf{T}_{zp} \bar{\mathbf{s}}_{\mu}(i) + \mathbf{w}(i) \end{aligned} \quad (2)$$

where  $\mathbf{w}(i)$  is additive white Gaussian noise, and we have used that  $\mathbf{H}_{\mu} \mathbf{T}_{zp} = \tilde{\mathbf{H}}_{\mu} \mathbf{T}_{zp}$  when deriving the second equality. The reason for replacing the Toeplitz matrix  $\mathbf{H}_{\mu}$  by the circulant  $\tilde{\mathbf{H}}_{\mu}$  is that circulant matrices enjoy two nice properties.

- 1) Pre- and postmultiplying circulant matrices by (I)FFT matrices diagonalizes them

$$\tilde{\mathbf{H}}_{\mu} = \mathbf{F}_P^H \mathbf{D}(\tilde{\mathbf{h}}_{\mu}) \mathbf{F}_P \quad \text{and} \quad \tilde{\mathbf{H}}_{\mu}^H = \mathbf{F}_P^T \mathbf{D}(\tilde{\mathbf{h}}_{\mu}^*) \mathbf{F}_P \quad (3)$$

where  $\tilde{\mathbf{h}}_{\mu} := [H_{\mu}(e^{j0}), \dots, H_{\mu}(e^{j((2\pi)/P)(P-1)})]^T$  with  $[\tilde{\mathbf{h}}_{\mu}]_p$  being the channel frequency response  $H_{\mu}(z) := \sum_{l=0}^{L-1} h_{\mu}(l)z^{-l}$  evaluated at  $z = e^{j((2\pi)/P)(p-1)}$ , and  $\mathbf{D}(\tilde{\mathbf{h}}_{\mu}) := \text{diag}(\tilde{\mathbf{h}}_{\mu})$ .

- 2) Pre- and postmultiplying  $\tilde{\mathbf{H}}_{\mu}$  by  $\mathbf{P}_P^{(n)}$  yields  $\tilde{\mathbf{H}}_{\mu}^T$ :

$$\mathbf{P}_P^{(n)} \tilde{\mathbf{H}}_{\mu} \mathbf{P}_P^{(n)} = \tilde{\mathbf{H}}_{\mu}^T \quad \text{and} \quad \mathbf{P}_P^{(n)} \tilde{\mathbf{H}}_{\mu}^* \mathbf{P}_P^{(n)} = \tilde{\mathbf{H}}_{\mu}^H. \quad (4)$$

To verify (4), notice that

$$\left[ \mathbf{P}_P^{(n)} \tilde{\mathbf{H}}_{\mu} \mathbf{P}_P^{(n)} \right]_{p,q} = h_{\mu}(q - p \bmod P) = [\tilde{\mathbf{H}}_{\mu}]_{q,p}. \quad (5)$$

Defining  $\tilde{\mathbf{P}} := \mathbf{P}_P^{(K)}$ , it follows that  $\tilde{\mathbf{P}} \mathbf{T}_{zp} = \mathbf{T}_{zp} \mathbf{P}_P^{(0)}$ , and using (1), we can establish the following key relationships:

$$\begin{aligned} \mathbf{T}_{zp} \bar{\mathbf{s}}_1(2i+1) &= -\tilde{\mathbf{P}} \mathbf{T}_{zp} \bar{\mathbf{s}}_2^*(2i), \\ \mathbf{T}_{zp} \bar{\mathbf{s}}_2(2i+1) &= \tilde{\mathbf{P}} \mathbf{T}_{zp} \bar{\mathbf{s}}_1^*(2i). \end{aligned} \quad (6)$$

Now we can write two consecutive received blocks as

$$\mathbf{x}(2i) = \tilde{\mathbf{H}}_1 \mathbf{T}_{zp} \bar{\mathbf{s}}_1(2i) + \tilde{\mathbf{H}}_2 \mathbf{T}_{zp} \bar{\mathbf{s}}_2(2i) + \mathbf{w}(i) \quad (7)$$

$$\begin{aligned} \mathbf{x}(2i+1) &= -\tilde{\mathbf{H}}_1 \tilde{\mathbf{P}} \mathbf{T}_{zp} \bar{\mathbf{s}}_2^*(2i) + \tilde{\mathbf{H}}_2 \tilde{\mathbf{P}} \mathbf{T}_{zp} \bar{\mathbf{s}}_1^*(2i) \\ &\quad + \mathbf{w}(2i+1). \end{aligned} \quad (8)$$

Conjugating  $\tilde{\mathbf{P}} \mathbf{x}(2i+1)$  and using (4), we arrive at

$$\begin{aligned} \tilde{\mathbf{P}} \mathbf{x}^*(2i+1) &= -\tilde{\mathbf{H}}_1^H \mathbf{T}_{zp} \bar{\mathbf{s}}_2(2i) + \tilde{\mathbf{H}}_2^H \mathbf{T}_{zp} \bar{\mathbf{s}}_1(2i) \\ &\quad + \tilde{\mathbf{P}} \mathbf{w}^*(2i+1). \end{aligned} \quad (9)$$

We will next pursue frequency domain processing by forming  $\mathbf{y}(2i) := \mathbf{F}_P \mathbf{x}(2i)$ ,  $\mathbf{y}^*(2i+1) := \mathbf{F}_P \tilde{\mathbf{P}} \mathbf{x}^*(2i+1)$ , and  $\tilde{\mathbf{y}}(i) := [\mathbf{y}^T(2i), \mathbf{y}^H(2i+1)]^T$ . Likewise, we define  $\tilde{\eta}(2i) := \mathbf{F}_P \mathbf{w}(2i)$  and  $\tilde{\eta}^*(2i+1) := \mathbf{F}_P \tilde{\mathbf{P}} \mathbf{w}^*(2i+1)$ . For notational simplicity, we also define  $\mathcal{D}_1 := \mathbf{D}(\tilde{\mathbf{h}}_1)$  and  $\mathcal{D}_2 := \mathbf{D}(\tilde{\mathbf{h}}_2)$ . Using (3) in (7) and (9) and after straightforward manipulations, we obtain

$$\tilde{\mathbf{y}}(i) = \underbrace{\begin{bmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2^* & -\mathcal{D}_1^* \end{bmatrix}}_{:=\mathcal{D}} \begin{bmatrix} \mathbf{F}_P \mathbf{T}_{zp} \mathbf{s}(2i) \\ \mathbf{F}_P \mathbf{T}_{zp} \mathbf{s}(2i+1) \end{bmatrix} + \begin{bmatrix} \tilde{\eta}(2i) \\ \tilde{\eta}^*(2i+1) \end{bmatrix} \quad (10)$$

where the identities  $\bar{\mathbf{s}}_1(2i) = \mathbf{s}(2i)$  and  $\bar{\mathbf{s}}_2(2i) = \mathbf{s}(2i+1)$  have been used following the design in (1).

Defining the diagonal matrix  $\tilde{\mathcal{D}}_{12} := [\mathcal{D}_1^* \mathcal{D}_1 + \mathcal{D}_2^* \mathcal{D}_2]^{1/2}$ , and based on the fact that  $\mathcal{D}^H \mathcal{D} = \mathbf{I}_2 \otimes \tilde{\mathcal{D}}_{12}^2$ , where  $\otimes$  stands for Kronecker product, we can construct the unitary matrix  $\mathbf{U} := \mathcal{D}(\mathbf{I}_2 \otimes \tilde{\mathcal{D}}_{12}^{-1})$  which satisfies<sup>1</sup>  $\mathbf{U}^H \mathbf{U} = \mathbf{I}_{2P}$  and  $\mathbf{U}^H \mathcal{D} = \mathbf{I}_2 \otimes \tilde{\mathcal{D}}_{12}$ . Multiplying  $\tilde{\mathbf{y}}(i)$  by  $\mathbf{U}^H$  does not incur any loss of decoding optimality. Thus, forming  $\tilde{\mathbf{z}}(i) := [\mathbf{z}^T(2i), \mathbf{z}^T(2i+1)]^T$  and defining  $\Theta := \mathbf{F}_P \mathbf{T}_{zp}$ , we obtain  $\tilde{\mathbf{z}}(i) = \mathbf{U}^H \tilde{\mathbf{y}}(i)$  as

$$\tilde{\mathbf{z}}(i) = \begin{bmatrix} \tilde{\mathcal{D}}_{12} \Theta \mathbf{s}(2i) \\ \tilde{\mathcal{D}}_{12} \Theta \mathbf{s}(2i+1) \end{bmatrix} + \mathbf{U}^H \begin{bmatrix} \tilde{\eta}(2i) \\ \tilde{\eta}^*(2i+1) \end{bmatrix} \quad (11)$$

where the noise  $\tilde{\eta}(i) := [\eta^T(2i), \eta^T(2i+1)]^T$  is still white.

We deduce from (11) that the blocks  $\mathbf{s}(2i)$  and  $\mathbf{s}(2i+1)$  can be demodulated separately. Equivalently, we need to demodulate  $\mathbf{s}(i)$  from the following blocks:

$$\mathbf{z}(i) = \tilde{\mathcal{D}}_{12} \Theta \mathbf{s}(i) + \eta(i). \quad (12)$$

Although we can solve (12) for  $\mathbf{s}(i)$  in the zero-forcing (ZF) or minimum mean-square error (MMSE) sense, we will next focus

<sup>1</sup>We assume here that  $\mathbf{h}_1$  and  $\mathbf{h}_2$  do not share common roots, i.e.,  $\tilde{\mathcal{D}}_{12}$  is invertible. However, this assumption can be removed [10].

on maximum likelihood (ML) decoding in order to find the best achievable performance.

### III. PROOF OF MAXIMUM DIVERSITY GAINS

Assume that  $h_\mu(t)$ 's are independently and identically distributed (i.i.d.) zero mean Gaussian with variance  $1/(L+1)$ , i.e.,  $\mathbf{R}_{h_\mu} = E\{\mathbf{h}_\mu \mathbf{h}_\mu^H\} = \mathbf{I}_{L+1}/(L+1)$ , which is satisfied when scattering is rich (see [10] for non-i.i.d. channels).

Dropping the block index  $i$  for brevity, we will henceforth denote, e.g.,  $\mathbf{s}(i)$  by  $\mathbf{s}$ . With channel state information (CSI) at the receiver, we consider the pairwise error probability (PEP)  $P(\mathbf{s} \rightarrow \mathbf{s}' | \mathbf{h}_1, \mathbf{h}_2)$  that the symbol block  $\mathbf{s}$  is transmitted but is erroneously decoded as  $\mathbf{s}' \neq \mathbf{s}$ . The PEP can be approximated using the Chernoff bound as

$$P(\mathbf{s} \rightarrow \mathbf{s}' | \mathbf{h}_1, \mathbf{h}_2) \leq \exp(-d^2(\mathbf{z}, \mathbf{z}')/4N_0) \quad (13)$$

where  $d(\mathbf{z}, \mathbf{z}')$  is the Euclidean distance between  $\mathbf{z}$  and  $\mathbf{z}'$ . Defining  $\mathbf{e} := \mathbf{s} - \mathbf{s}'$  and starting with (12), we have

$$\begin{aligned} d^2(\mathbf{z}, \mathbf{z}') &= |\bar{\mathbf{D}}_{12} \Theta \mathbf{e}|^2 = \mathbf{e}^H \Theta^H \bar{\mathbf{D}}_{12}^2 \Theta \mathbf{e} \\ &= \mathbf{e}^H \Theta^H (\mathcal{D}_1^* \mathcal{D} + \mathcal{D}_2^* \mathcal{D}_2) \Theta \mathbf{e} \\ &= |\mathcal{D}_1 \Theta \mathbf{e}|^2 + |\mathcal{D}_2 \Theta \mathbf{e}|^2 \\ &= |\mathbf{D}_e \check{\mathbf{h}}_1|^2 + |\mathbf{D}_e \check{\mathbf{h}}_2|^2 \\ &= |\mathbf{D}_e \mathbf{V} \mathbf{h}_1|^2 + |\mathbf{D}_e \mathbf{V} \mathbf{h}_2|^2 \end{aligned} \quad (14)$$

where  $\mathbf{D}_e := \text{diag}(\Theta \mathbf{e})$  and  $[\mathbf{V}]_{p,q} = e^{j(2\pi/P)(p-1)(q-1)}$  such that  $\check{\mathbf{h}}_\mu = \mathbf{V} \mathbf{h}_\mu$ . Defining  $\mathbf{A}_e := \mathbf{D}_e \mathbf{V}$ , we note that  $\mathbf{A}_e^H \mathbf{A}_e$  is Hermitian symmetric and nonnegative definite. Hence, there exists a unitary matrix  $\mathbf{U}_e$  such that  $\mathbf{U}_e^H \mathbf{A}_e^H \mathbf{A}_e \mathbf{U}_e = (L+1) \mathbf{\Lambda}_e$ , where  $\mathbf{\Lambda}_e$  is a diagonal matrix with nonincreasing diagonal entries collected in the vector  $\lambda_e := [\lambda_e(0), \lambda_e(1), \dots, \lambda_e(L)]^T$ .

Let  $\mathbf{h}'_\mu := \sqrt{L+1} \mathbf{U}_e^H \mathbf{h}_\mu$  have correlation matrix  $\mathbf{R}_{h'_\mu} = (L+1) \mathbf{U}_e^H \mathbf{R}_{h_\mu} \mathbf{U}_e = \mathbf{I}_{L+1}$ . Hence,  $\mathbf{h}'_\mu$  is a zero mean complex Gaussian vector with unit variance i.i.d. entries. We can thus rewrite (14) as

$$\begin{aligned} d^2(\mathbf{z}, \mathbf{z}') &= \sum_{\mu=1}^2 (\mathbf{h}'_\mu)^H \mathbf{U}_e^H \mathbf{A}_e^H \mathbf{A}_e \mathbf{U}_e \mathbf{h}'_\mu / (L+1) \\ &= \sum_{l=0}^L \lambda_e(l) |h'_1(l)|^2 + \sum_{l=0}^L \lambda_e(l) |h'_2(l)|^2. \end{aligned} \quad (15)$$

Using (15), we average (13) with respect to the i.i.d. Rayleigh random variables  $|h'_1(l)|, |h'_2(l)|$  to obtain the following upper bound on the average PEP:

$$P(\mathbf{s} \rightarrow \mathbf{s}') \leq \prod_{l=0}^L \frac{1}{(1 + \lambda_e(l)/(4N_0))^2}. \quad (16)$$

If  $r_e$  is the rank of  $\mathbf{A}_e$  (and of  $\mathbf{A}_e^H \mathbf{A}_e$ ), then  $\lambda_e(l) \neq 0$ , if and only if,  $l \in [0, r_e - 1]$ . It follows from (16) that:

$$P(\mathbf{s} \rightarrow \mathbf{s}') \leq (1/N_0)^{-2r_e} \left( \prod_{l=0}^{r_e-1} \lambda_e(l)/4 \right)^{-2}. \quad (17)$$

As in [6], we call  $2r_e$  the diversity advantage  $G_{d,e}$  for a given symbol error vector  $\mathbf{e}$ . At high SNR, the diversity advantage

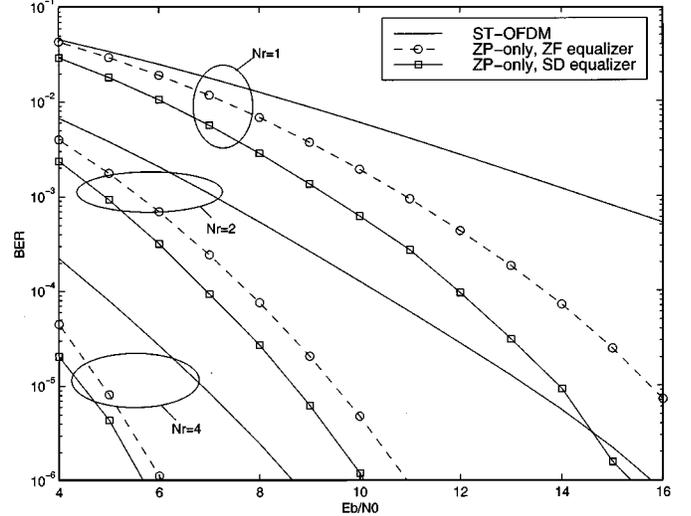


Fig. 2. Comparisons between ZP-only and ST-OFDM,  $N_r = 1, 2, 4$ .

$G_{d,e}$  determines the slope of the averaged (with regard to the random channel) PEP (between  $\mathbf{s}$  and  $\mathbf{s}'$ ) as a function of the SNR. Different from [6], which relied on PEP to design ST codes for flat fading channels, we here invoke PEP bounds to prove diversity properties of ZP block transmissions (termed as ZP-only in [9]) over frequency selective channels.

Since  $G_{d,e}$  depends on the choice of  $\mathbf{e}$  (thus on  $\mathbf{s}$  and  $\mathbf{s}'$ ), we define the overall diversity advantage for our system as  $G_d := \min_{\mathbf{e} \neq \mathbf{0}} G_{d,e}$ . Using that  $\mathbf{A}_e^H \mathbf{A}_e$  is  $(L+1) \times (L+1)$ , our first result states that the *maximum achievable* diversity for ZP-only is  $G_d = 2(L+1)$ .

We will prove next that ZP-only achieves this maximum diversity gain. Since  $\Theta$  is Vandermonde and any of its  $K$  rows are linearly independent,  $\Theta \mathbf{e}$  has at least  $(L+1)$  nonzero entries for any  $\mathbf{e}$ . Indeed, if  $\Theta \mathbf{e}$  has only  $L$  nonzero entries for some  $\mathbf{e}$ , then it has  $K$  zero entries. Picking the corresponding  $K$  rows of  $\Theta$  to form the truncated matrix  $\check{\Theta}$ , we have  $\check{\Theta} \mathbf{e} = \mathbf{0}$ . The latter shows that these  $K$  rows are linearly dependent, which is impossible. With  $\mathbf{D}_e = \text{diag}(\Theta \mathbf{e})$  having at least  $L+1$  nonzero diagonal entries, we have that  $\mathbf{A}_e = \mathbf{D}_e \mathbf{V}$  has full rank because any  $L+1$  rows of  $\mathbf{V}$  are linearly independent. Thus, ZP only achieves the maximum achievable diversity gain of  $G_d = 2(L+1)$ , which is much larger than the diversity order two of ST-OFDM that uses ST block codes on each subcarrier<sup>2</sup> and thus only exploits multiantenna but not multipath diversity [4]. More generally, we prove in [10] that  $G_d = N_t N_r (L+1)$  when  $N_t$  transmit- and  $N_r$  receive-antennas are used, which reveals that frequency selectivity can boost system diversity gain.

ML decoding is required to collect full diversity gains, which is certainly prohibitive when the constellation size and/or the block length increases. A relatively faster ML search is possible with the sphere decoding (SD) algorithm [7]. Linear ZF and MMSE equalizers certainly offer low complexity alternatives and are able to collect almost full diversity gains as  $N_t N_r$  increases [10].

In a test case, we set  $L = 2$  (three-ray channels) and the block size  $K = 8$ . We use QPSK constellations and let  $E_b/N_0$  denote

<sup>2</sup>Unlike [2] and [5], channel coding across subcarriers is not considered here.

the average received bit energy to noise ratio on each receive antenna. With  $N_t = 2$  and  $N_r = 1$ , we infer from Fig. 2 that uncoded ZP-only outperforms uncoded ST-OFDM significantly (it will be interesting to compare ZP-only with OFDM when both systems invoke channel coding with identical rates and/or comparable complexity). In Fig. 2, we also increase our multi-antenna diversity by deploying  $N_r = 2, 4$  receive antennas and employ coherent combining. An interesting observation is that the difference between linear ZF and near-optimal SD equalization becomes smaller as multi-antenna diversity increases. With  $N_r = 4$ , the difference between ZF and SD equalizers for ZP-only is only within several tenths of a dB, which is very encouraging from an overall performance-complexity perspective.

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