Filterbanks for Blind Channel Identification and Equalization

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***Abstract*—Multirate precoding using filterbanks induces cyclostationarity at the transmitter and guarantees blind identifiability of frequency selective communication channels with minimal decrease of information rate and without restrictions on zero locations. Finite impulse response (FIR) filterbank decoders are capable of equalizing blindly (and perfectly in the absence of noise) FIR channels without constraints on their zeros.**

I. Introduction

SELF RECOVERING (or blind) equalization of frequency selective channels has gained practical interest recently [5]. Especially with multipoint communications and mobile users inducing changing propagation channels, training sequences interrupt transmission and limit efficient use of the available bandwidth. Blind channel estimation and equalization have traditionally relied on high-order statistics (HOS) of the stationary output, but require relatively long data (e.g., [2]). This fact motivated second-order cyclostationary approaches originated by the work of Tong *et al.* [3], which relies on fractional sampling. The latter exhibit less variance than HOS methods, facilitate synchronization due to oversampling, and allow compensation of finite impulse response (FIR) channels with FIR zero-forcing equalizers. However, inducing cyclostationarity by fractional sampling at the receiver does not come without a price: Longer discrete-time channels arise, and a smaller class of practical channels can be identified (in fact, without sufficient excess bandwidth second-order methods based on fractional sampling may fail [1]).

To obviate these limitations, transmitter-induced cyclostationarity was proposed in [6] by repeating input blocks, but such a scheme reduces the rate by half. In this paper, we derive a precoding (or analysis) filterbank at the transmitter (Section II), and a decoding (or synthesis) filterbank at the receiver (Section III) in order to i) combine the benefits of [6] with an arbitrarily small rate reduction and ii) develop FIR equalizers independent of the channel zero locations. Filterbanks for intersymbol interference (ISI) mitigation can also be found in [7], but the channel is assumed to be known (nonblind scenario).

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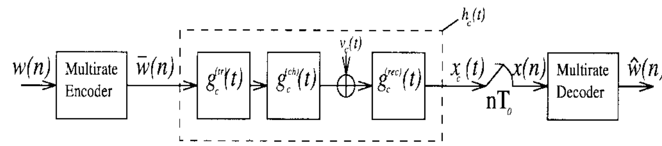

Fig. 1. Multirate transmitter–channel–receiver structure.

II. Channel Identifiability

With reference to Fig. 1, the independent, identically distributed (i.i.d.) information stream $w(n)$ is mapped (or encoded) to $\overline{w}(n)$ using the multirate operations depicted in Fig. 2. For bandwidth control, $\overline{w}(n)$ is pulse-shaped by the transmit filter $g_c^{(tr)}(t)$, propagates through the *unknown* channel $g_c^{(ch)}(t)$, and upon reception, it is filtered by the antialiasing filter $g_c^{(rec)}(t)$ to suppress the stationary additive noise $v_c(t)$. If $h_c(t) := g_c^{(tr)} \star g_c^{(ch)} \star g_c^{(rec)}(t)$, the continuous-time received signal at the baseband is $x_c(t) = \sum_l \overline{w}(l)h_c(t - lT_0 - d) + v_c(t)$, where T_0 denotes the coded-symbol period and $d \in (0, T_0)$ is the propagation delay. Sampling at the symbol rate, and letting $h(n) := h_c(t-d)|_{t=nT_0}$, $v(n) := v_c(t)|_{t=nT_0}$, we arrive at the discrete-time data model $n = 0, 1, \cdots, N-1$,

$$x(n) := x_c(t)|_{t=nT_0} = \sum_l \overline{w}(l)h(n-l) + v(n). \quad (1)$$

The input–output relation of the encoder in Fig. 2 is

$$\overline{w}(n) = \sum_{m=0}^{M-1} \sum_i w(iM+m)f_m(n-m-iP) \quad (2)$$

where $f_m(n)$ are *known* FIR filters. We will assume $P > M$ and define the information rate as the number of uncoded bits per unit time $:= M/(PT_0)$. If $f_m(n) = \delta(n), \forall m$, the multirate encoder simply inserts $P - M$ zero(s) for every M input symbols, thereby decreasing the information rate[1] by a factor M/P. However, as will be shown, even minimal redundancy ($P - M = 1$) will prove useful for blind channel identification and equalization.

Specifically, given $x(n)$ data only, and knowing P, $f_m(n)$, $g_c^{(tr)}(t)$, $g_c^{(rec)}(t)$, we wish to: i) identify the FIR channel h [and thus $g_c^{(ch)}$] using $M = P - 1$ or $M = P - L$; and ii) construct an FIR equalizer (decoder) to recover $w(n)$ blindly using $M = P - L$, where L is the order of h.

[1] If information rate must remain fixed (at $1/T_0$ as in an uncoded transmitter), one should increase the transmission rate from $1/T_0$ used herein, to $1/(T_0') = 1/(MT_0/P)$. In this case, the multirate encoder of Fig. 2 can be interpreted as a structure introducing controlled ISI reminiscent to that in partial response signaling.

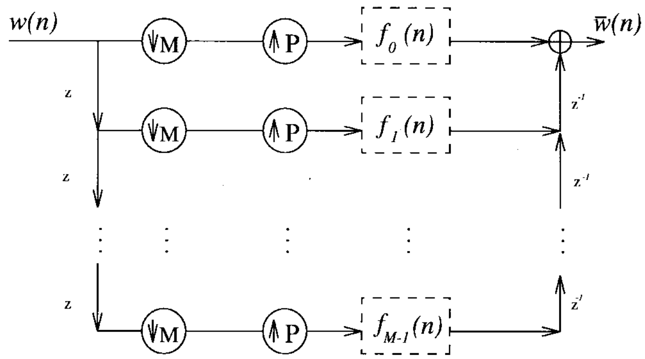

Fig. 2. Multirate encoder (analysis filterbank).

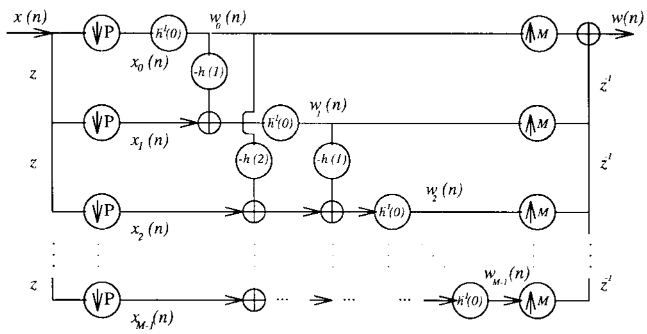

Fig. 3. Multirate decoder (synthesis filterbank).

The key to channel identifiability is that the multirate filterbank induces cyclostationarity in $\overline{w}(n)$, and thus in $x(n)$. Substituting (2) into (1), the correlation $c_{xx}(n;\tau) := E\{x(n)x^*(n+\tau)\}$, is given by

$$c_{xx}(n;\tau) = \sigma_w^2 \sum_{l_1}\sum_{l_2} h(n-l_1)h^*(n+\tau-l_2) \times \sum_{m=0}^{M-1}\sum_i f_m(l_1-m-iP)f_m^*(l_2-m-iP) + \sigma_v^2\delta(\tau) \tag{3}$$

where $*$ stands for conjugation, and σ_w^2 and σ_v^2 denote variances of $w(n)$ and $v(n)$, both of which are assumed zero-mean and mutually independent. It follows from (3) that $c_{xx}(n;\tau)$ is periodic in n with period P (its Fourier series coefficients, $\{C_{xx}(2\pi k/P;\tau)\}_{k=0}^{P-1}$, are called cyclic correlations). Fourier transforming $C_{xx}(2\pi k/P;\tau)$ w.r.t. τ, we arrive at the so-called cyclic spectrum, $k = 0, 1, \cdots, P-1$

$$S_{xx}(k;\omega) = H\left(\frac{2\pi}{P}k-\omega\right)H^*(-\omega)F(k;\omega) + \sigma_v^2\delta(k) \tag{4}$$

where $F(k;\omega) := (\sigma_w^2/P)\sum_{m=0}^{M-1}\sum_{n=0}^{P-1}\sum_l f_m(n-m)f_m^*(n-m-l)\exp[-j(2\pi kn/P)-\omega l]$, and H is the transfer function of h. If $f_m(n) = \delta(n), \forall m$, $F(k;\omega)$ is not a function of ω; and if in addition $M = 1$, (4) reduces to the cyclic spectrum under receiver oversampling by a factor P (e.g., [1]). Suppose $h_c(t)$ has support T_h and extends over $Q \geq 1$ symbols ($T_h = QT_0$). Thus, oversampling by P (sampling period $T_s = T_0/P$) yields a discrete-time channel h of order $L = T_h/T_s = QT_0/(T_0/P) = QP$; i.e., fractional sampling induces cyclostationarity at the receiver end, but leads always to $L > P$.

In contrast, precoding induces cyclostationarity at the transmitter and allows $P > L$ because $T_s = T_0$ and $L = T_h/T_0 = Q$; hence, P can be chosen larger than the channel order L, which allows identification of the L zeros of $H(\omega)$ from the P polynomials $\{S_{xx}(k;\omega)\}_{k=0}^{P-1}$ in (4). Indeed, since : $f_m(n)$ are known, the zeros of $F(k;\omega)$ can be readily recognized among those of $S_{xx}(k;\omega), \forall k$, leaving only ambiguity in deciding whether each of the remaining zeros of $S_{xx}(k;\omega)$ belongs to $H^*(-\omega)$, or, $H(2\pi k/P-\omega)$. The zeros of $H(2\pi k/P-\omega)$ in $S_{xx}(k;\omega)$ rotate by $2\pi/P$ when considering $S_{xx}(k+1;\omega)$, while those of $H^*(-\omega)$ remain fixed. Because $k \in [0, P-1]$, we have P polynomials $S_{xx}(k;\omega)$ to discern rotated zeros of $H(2\pi k/P-\omega)$ from the L zeros of $H^*(-\omega)$; and, since $P > L$, the greatest common divisor of the FIR polynomial family $\{S_{xx}(k;\omega)/F(k;\omega), k = 0, 1, \cdots, P-1\}$ will yield $H^*(-\omega)$. We have proved the following theorem.

Theorem 1: Under the assumptions of (1) and (2), the Lth-order channel $H(\omega)$ is uniquely identifiable[2] from the output cyclic spectra $\{S_{xx}(k;\omega)\}_{k=0}^{P-1}$, provided that $P > L$. Information rate decreases by a factor M/P, and reduction is minimized when $M = P-1$.

Cyclostationarity in $\overline{w}(n)$ is not induced only by the multirate relation (2). Theorem 1 applies even if, e.g., $\overline{w}(n) = w(n)p(n)$, where $p(n)$ is a periodic sequence with known period $P > L$ [note that (4) holds in this case too]. Theorem 1 establishes identifiability in the noise-free case but with $k \neq 0$ and $P > L+1$, the result applies in the presence of (even colored) stationary noise [c.f. (4)].

A simpler identifiability proof leads to a *closed-form* solution, if we restrict ourselves to $f_m(n) = \delta(n), \forall m$, and choose $P \geq L+M$. In this case, (3) reduces to

$$c_{xx}(n;\tau) = \sigma_w^2 \sum_{m=0}^{M-1}\sum_i h(n-m-iP) \times h^*(n+\tau-m-iP) + \sigma_v^2\delta(\tau). \tag{5}$$

Recall that $h(l) \neq 0$ for $l \in [0, L]$, and consider $n \in [0, P-1]$ since $c_{xx}(n;\tau)$ is periodic. Note that $h(n-m-iP) = 0$ for $i > 0$ because $n-m-iP < 0$. Also, if $P \geq L+M$, it follows that $h(n-m-iP) = 0$ for $i < 0$, since $n-m-iP > L$; thus, $i = 0$ in (5) and with $n = 0$, we find $c_{xx}(0;\tau) = \sigma_w^2 h(0)h^*(\tau) + \sigma_v^2\delta(\tau)$. Without loss of generality (see fn. 2), we fix $h(0) = 1$, and estimate the channel for $\tau \neq 0$ using

$$\hat{h}(\tau) = \frac{1}{\sigma_w^2}\hat{c}_{xx}(0;\tau) = \frac{1}{\sigma_w^2 I}\sum_{i=0}^{I-1} x^*(iP)x(iP+\tau) \tag{6}$$

where $I := [N/P]$ and $[y]$ denotes the integer part of y. Exploitation of $\hat{c}_{xx}(n;\tau)$ with $n \neq 0$ is expected to improve estimation accuracy, and such an estimator results if one adopts

[2] Identifiability in all blind methods is understood modulo a complex scale and a shift ambiguity, which are nonidentifiable from output data only.

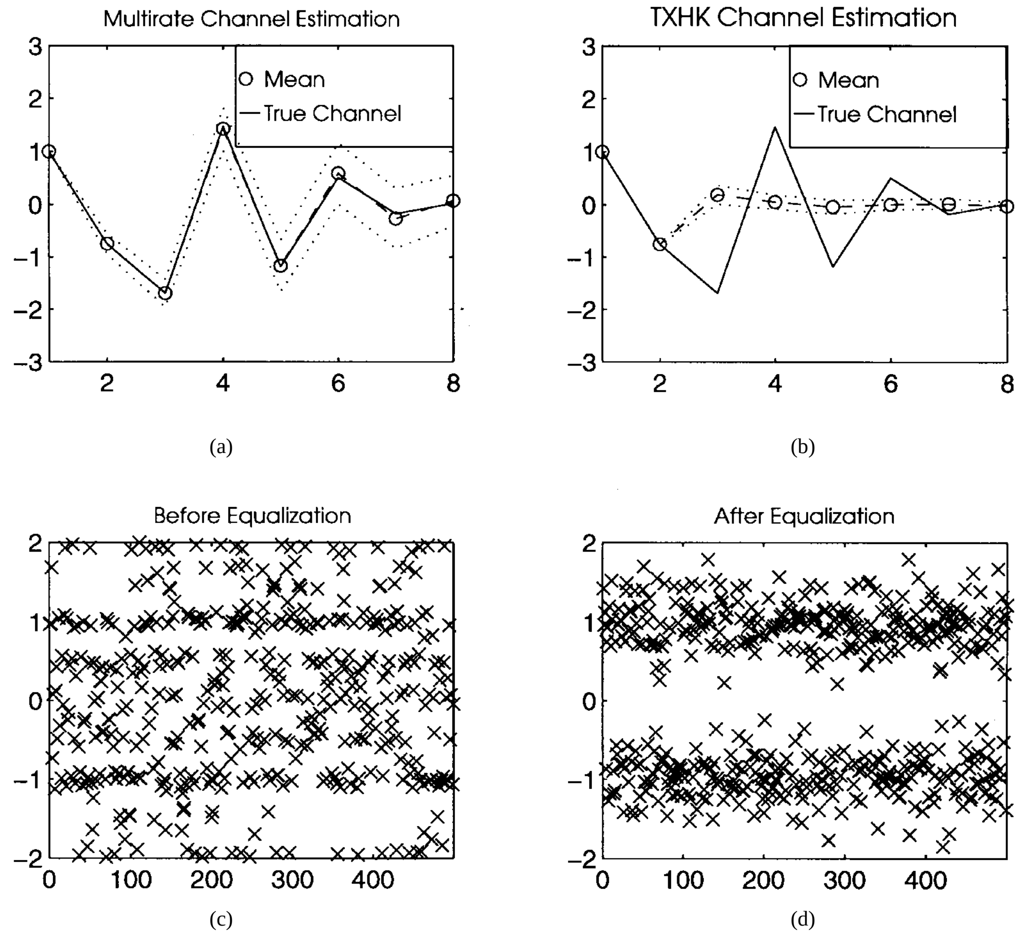

Fig. 4. (a) Channel estimation (mean $\pm$ std) via (6). (b) Channel estimation using the method in [4]. (c) BPSK constellation before equalization and (d) after equalization.

the quadratic correlation matching criterion

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \sum_{n=0}^{P-1} \sum_{\tau=-L}^{L} |c_{xx}(n;\,\tau) - \hat{c}_{xx}(n;\,\tau)|^2. \quad (7)$$

To prevent convergence to local minima, the estimator in (6) can be used as initial condition for the nonlinear search required to estimate the tap vector $\mathbf{h}$ from (7). Interestingly, the estimator in (6) does not require order determination and lends itself naturally to on-line implementation. Subspace versions are also possible along the lines of [6]. On the other hand, (7) allows for $f_m(n) \neq \delta(n)$, and applies even when $P = M+1$, provided that $P > L$.

Selecting as filters $f_m(n)$ the appropriate complex exponentials (Fourier basis), the structure of Fig. 2 with $P = M + L$ reduces to the orthogonal frequency division multiplexer (OFDM) with prefix, which has gained popularity recently for mobile communications and terrestrial television systems. Links of our multirate precoder with OFDM and generalizations to multitone and multicarrier modulation schemes will be discussed elsewhere.

Coding has been traditionally employed for error correction. The repetition approach in [6] was the first to recognize the importance of precoding for blind identification. It is a special case of the multirate transmitter presented here and corresponds to $P = 2M$ with $f_m(n) = \delta(n) + \delta(n-M)\ \forall m$. As noted in [6], the bandwidth does not increase because it depends solely on the pulse shaping filter. However, allowing for $P = M + 1$ implies minimal reduction of information rate. Even when $P = M + L$ [as required by (6)], the rate reduction can be minimized by choosing M sufficiently large so that $M/P = M/(M + L)$ is as close as possible to 1. Having $f_m(n) = \delta(n)$ in Fig. 2 may be more demanding from a hardware point of view because on–off operation is required for the transmitter to insert the $P - M$ zeros in $\overline{w}(n)$. One choice could be to repeat only $P - M$ symbols; i.e., to select $f_m(n) = \delta(n) + \delta(n-M)$ for $m = 0, 1, \cdots, P-M-1$, and $f_m(n) = \delta(n)$ for $m = P - M, \cdots, M - 1$. Results along this direction are beyond the scope of this letter and will be reported elsewhere. Henceforth, we will adopt $P = M + L$ and $f_m(n) = \delta(n)\ \forall m$ for two reasons: i) simplicity of exposition and ii) existence and uniqueness of FIR equalizers without restricting the zeros of $H(\omega)$ as in [3]–[7].

III. Blind Equalization

Adopting $f_m(n) = \delta(n)$ in (2) and substituting in (1) yields, in the absence of noise

$$x(n) = \sum_{m=0}^{M-1} \sum_{l} w(lM + m)h(n - m - lP). \quad (8)$$

In polyphase form, (8) can be interpreted as an M-input/P-output system, $x_i(n) = \sum_{m=0}^{M-1} \sum_l w_m(l) h_{m,i}(n-l)$, where $x_i(n) := x(nP+i)$, $i \in [0, P-1]$, $h_{m,i}(n) := h(nP+i-m)$, and $w_m(l) := w(lM+m)$, $m \in [0, M-1]$. Note that the M inputs, the MP impulse responses, and the P outputs are subsampled versions of w, h, and x, respectively. It follows easily from the definition of $h_{m,i}$ that if $P = M + L$, then $h_{m,i}(n) = h(i-m)\delta(n)$ for $i - m \in [0, L]$ and zero elsewhere; i.e., all $h_{m,i}(n)$ filters are scalars, hence

$$x_i(n) = \sum_{m=0}^{M-1} w_m(n) h(i-m), \qquad i \in [0, P-1]. \tag{9}$$

Because h is causal, the (9) has lower triangular structure and can be solved for $w_0, w_1, \cdots, w_{M-1}$, using back substitution; i.e., $x_0(n) = h(0)w_0(n) \Rightarrow w_0(n) = h^{-1}(0)x_0(n)$; $x_1(n) = h(1)w_0(n) + h(0)w_1(n) \Rightarrow w_1(n) = h^{-1}(0)[x_1(n) - h(1)w_0(n)]$, etc. By construction, we have thus proved the following theorem.

Theorem 2: With $x(n)$ obeying (8) and $P \geq M + L$, blind FIR zero-forcing equalizers of Lth-order FIR channels exist, and are unique irrespective of the channel zero locations.

Algorithmically, given $x(n)$, we first subsample it to obtain $x_i(n)$, and then process it in a triangular fashion (also shown in Fig. 3) using the tap coefficients estimated as in (6). Such a multirate decoding scheme is surprisingly simple, and contrary to [3]–[7], it guarantees zero-forcing equalization (a.k.a., perfect reconstruction) irrespective of the channel zero locations.

The price paid for all these nice features is threefold:

1) Complexity and latency during precoding and decoding increase as P and M increase.
2) Need for synchronization with length P blocks.
3) Possible lack of noise suppression by the FIR equalizers.

All of 1)–3) appear also in [6], where the synchronization problem 2) has been already addressed. The obvious remedy to 3) is to sacrifice zero-forcing and use the channel estimates of Section II to obtain (depending on complexity constraints) mean-square error or Viterbi decoders. Further research is required however, toward noise-resistant zero-forcing equalizers, performance analysis in terms of bit error probability, synchronization error effects, and comparisons.

1) Preliminary Simulation: To test our theoretical findings, $N = 500$ BPSK symbols $w(n)$ were simulated, precoded as in Fig. 2 ($P = 17$, $M = 10$), and passed through a mixed-phase channel of order $L = 7$ (zeros at 0.2, ± 1.5, $\pm j0.5$, $0.2783 \pm j0.3488$). Fig. 4(a) shows the true (solid), mean (dashed), and standard deviation (dotted) of $\hat{h}(n)$ estimated via (6) using 100 Monte Carlo runs (SNR = 20 dB). To check with a method of comparable complexity, the cyclic approach of [4] was also implemented [see Fig. 4(b)] with oversampling factor $P = 2$ ($\Rightarrow$1000 samples). Lower variance is observed in Fig. 4(b) relative to Fig. 4(a). However, the true channel is not estimated correctly, not because [4] performs poorly in the mean, but because the channel chosen contains roots (± 1.5 and $\pm j0.5$), which are nonidentifiable by methods relying on fractional sampling ([3], as well as other subspace methods we have also simulated, failed for the same reason). The constellations before and after the zero-forcing equalization of Fig. 3 are depicted in Fig. 4(c) and (d). Although the eye opens, even at 20 dB, a noise-resistant equalizer could offer further improvement.

Remark: We note that the multirate structure of Fig. 2 is rich in degrees of freedom potentially important also for blind channel estimation and equalization in a multiuser environment. Suppose each of J users precodes bits $w_j(n)$, $j = 1, \cdots, J$, using filterbanks like the one in Fig. 2 with distinct upsamplers P_j and (not necessarily orthogonal) codes $f_{m_j}(n)$. Let P be the least common multiple of $\{P_j\}_{j=1}^J$, and $C_{x_j x_j}(2\pi k/P_j; \tau)$ denote the jth user's cyclic correlation. Because different users' data are mutually independent, and $x(n) = \sum_{j=1}^J x_j(n) + v(n)$, it follows that $C_{xx}(2\pi k/P; \tau) = \sum_{j=1}^J C_{x_j x_j}(2\pi k/P_j; \tau) + c_{vv}(\tau)\delta(k)$. Hence, with distinct P_j's, users can be separated in the cyclic domain since $C_{xx}(2\pi k/P_j; \tau) = C_{x_j x_j}(2\pi k/P_j; \tau)$. Once separated, $c_{x_j x_j}(n; \tau)$ can be obtained from $C_{x_j x_j}(2\pi k/P_j; \tau)$ and the single user approaches can be adopted for blind channel identification. Having identified the channels, demodulation can be achieved using standard decorrelating or Viterbi receivers. Resolution and the role of (non-)orthogonal codes are currently under investigation.

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