PERFORMANCE ANALYSIS OF REDUNDANT PRECODING SCHEMES WITH BLIND ESTIMATION CAPABILITIES

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ABSTRACT

The aim of this paper is to provide performance analysis of a deterministic method proposed recently for blind identification of linear time-invariant channels, in the presence of additive noise, for block transmission systems using redundant precoding. Robustness of the blind estimation method is also analyzed in the presence of slow channel variations arising due to Doppler shifts or due to phase noise at the transmit/receive oscillators. The theoretical analysis is validated by simulations.

1. INTRODUCTION

Block transmission with appropriate redundancy added per block is a commonly adopted strategy in digital communications [4]. Precoding is introduced to allow for multi-user communications over a shared medium and also to facilitate channel equalization and interference rejection [3]. An example of block transmission with redundancy added in the form of a cyclic prefix is Orthogonal Frequency Division Multiplexing (OFDM), currently adopted for digital audio and video terrestrial broadcasting. In general, given a block of M information symbols $\mathbf{s}(n)$ and using a linear precoder, a vector of coded data $\mathbf{u}(n)$ is generated by multiplying the vector $\mathbf{s}(n)$ by a $P \times M$ precoding matrix $\bar{\mathbf{F}}$, with $P > M$, in order to add redundancy. In OFDM systems for example, the element (k,l) of the precoding matrix $\bar{\mathbf{F}}$ is $\bar{F}_{k,l} = \exp(j2\pi kl/M)$, with $k = 0,\ldots,P-1$ and $l = 0,\ldots,M-1$, where $P = M+L$ and L is the channel order or a known upper bound of it.

Exploiting the redundancy introduced by the precoder, we proposed blind deterministic methods for channel identification and direct equalization in [8], based on the following assumptions:

(a0) Channel $h(l)$ is FIR of order L with $h(0), h(L) \neq 0$.
(a1) For a given L, the pair (P,M) is chosen to satisfy $P > M > L$ and $P = M + L$.
(a2) Precoder matrix $\bar{\mathbf{F}}$ has its L last (trailing) rows all-zero; i.e., $\bar{\mathbf{F}}^T = (\mathbf{F}^T \mathbf{0}^T)$, where $\mathbf{F}$ is a full rank $M \times M$ matrix, i.e., $\text{rank}(\mathbf{F}) = M$; this guarantees one-to-one mapping and thus recovery of $\mathbf{s}(n)$ from the coded symbols $\mathbf{u}(n) = \mathbf{F}\mathbf{s}(n)$.

Under these assumptions, the method of [8] establishes channel identifiability (up to a scalar factor) in the absence of noise. The redundancy introduced by inserting L trailing zeros guarantees on the one hand existence of a zero forcing equalizer *irrespective* of the channel zero locations [7], and on the other hand, similar to a parsed training system, it allows blind channel identification by means of deterministic subspace methods [8].

In this paper, we evaluate performance of the method proposed in [8] in the presence of additive noise and/or channel variations. The validity of our analysis is tested numerically by simulating channel fluctuations due to Doppler shifts and oscillators' phase noise. Specifically, in Section 2 we review briefly the estimation method, and in Section 3 we provide a theoretical performance analysis whose findings are verified numerically with the simulations presented in Section 4.

2. BLIND CHANNEL ESTIMATION

Based on (a0), (a1), (a2) the received block model is [8]:

$$\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{v}(n) = \mathbf{H}\mathbf{F}\mathbf{s}(n) + \mathbf{v}(n), \qquad (1)$$

where $\mathbf{F}$ is the top $M \times M$ part of the precoder matrix, which is the IFFT matrix for an OFDM system, or, the matrix of spreading codes for the downlink of a CDMA system; $P \times M$ channel matrix $\mathbf{H}$ is a Toeplitz matrix (also denoted as $\mathcal{T}(\mathbf{h})$) with first column and row given by

$$\mathbf{c} := (\underbrace{h(0)\ \ldots\ h(L)}_{L+1}\ \underbrace{0\ \ldots\ 0}_{M-1})^T \quad \mathbf{r} := (h(0)\ \underbrace{0\ \ldots\ 0}_{M-1}), \qquad (2)$$

respectively. Furthermore, we assume that:
(a3) There exists an $N \geq P$, such that the $M \times N$ matrix $\mathbf{S}_N := (\mathbf{s}(0)\cdots\mathbf{s}(N-1))$ has full rank M. With white inputs, $\mathbf{S}_N\mathbf{S}_N^H$ tends (as N increases) to the input correlation matrix $\mathbf{R}_{ss}$. But (a3) is satisfied even for colored (e.g., coded) inputs provided that their spectra are non-zero for at least M frequencies (modes).

Collecting N data vectors $\{\mathbf{x}(n)\}_{n=0}^{N-1}$ in a matrix, we arrive at [c.f. (1)]

$$\mathbf{X}_N := (\mathbf{x}(0)\cdots\mathbf{x}(N-1)) = \mathbf{H}\mathbf{F}\mathbf{S}_N, \qquad (3)$$

where $\mathbf{S}_N$ is defined as in (a3). Because of (a0), $h(0) \neq 0$, and thus $\text{rank}(\mathbf{H}) = M$, which along with (a2) and (a3) imply that $\text{rank}(\mathbf{X}_N) = M$. Therefore, the nullity of the matrix $\mathbf{X}_N\mathbf{X}_N^H$ is: $\nu(\mathbf{X}_N\mathbf{X}_N^H) = P - M = L$, and the eigendecomposition

$$\mathbf{X}_N\mathbf{X}_N^H = (\bar{\mathbf{U}}\,\tilde{\mathbf{U}}) \begin{pmatrix} \mathbf{\Sigma}_{M\times M} & \mathbf{0}_{M\times L} \\ \mathbf{0}_{L\times M} & \mathbf{0}_{L\times L} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{U}}^H \\ \tilde{\mathbf{U}}^H \end{pmatrix}, \qquad (4)$$

yields the $P \times L$ matrix $\tilde{\mathbf{U}}$ whose L columns span the nullspace $\mathcal{N}(\mathbf{X}_N)$. Since $\mathbf{F}\mathbf{S}_N$ in (3) is full rank, $\mathcal{R}(\mathbf{X}_N) =$

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$\mathcal{R}(\mathbf{H})$, where $\mathcal{R}$ stands for range space. But since $\mathcal{R}(\mathbf{X}_N)$ is orthogonal to $\mathcal{N}(\mathbf{X}_N)$, it follows that for $l = 1, \ldots, L$

$$\tilde{\mathbf{U}}^H \mathbf{X}_N = \mathbf{0} \Rightarrow \tilde{\mathbf{U}}^H \mathbf{H} = \mathbf{0} \Rightarrow \tilde{\mathbf{u}}_l^H \ \mathcal{T}(\mathbf{h}) = \mathbf{0}^H , \quad (5)$$

where $\tilde{\mathbf{u}}_l$ denotes the lth column of $\tilde{\mathbf{U}}$ and $\mathcal{T}(\mathbf{h}) \equiv \mathbf{H}$ underscores the Toeplitz structure of $\mathbf{H}$. Vector multiplication with a Toeplitz matrix denotes convolution, which is commutative, and thus (5) can be written as:

$$\mathbf{h}^H \ \tilde{\mathbf{U}} \ := \ \mathbf{h}^H \ (\tilde{\mathbf{U}}_1 \cdots \tilde{\mathbf{U}}_L) \ = \ \mathbf{0}^H , \quad (6)$$

where $\tilde{\mathbf{U}}_l$ denotes the $(L+1) \times M$ Hankel matrix formed by $\tilde{\mathbf{u}}_l$. Based on the aforementioned assumptions and (3)-(6), the blind channel identification method is summarized in the following (see [8] for detailed exposition):

Theorem 1 *Let (a0)-(a3) hold true. Starting from the data matrix $\mathbf{X}_N$, we form the $(L+1) \times ML$ matrix $\tilde{\mathbf{U}}$ as in (3)-(6). Channel vector $\mathbf{h}$ can then be obtained as the unique (within a scale) null eigen-vector of $\mathbf{U}$ in (6).*

3. PERFORMANCE EVALUATION

In this section we provide a theoretical performance analysis for the deterministic blind channel estimation method of Theorem 1. The method is analyzed in the presence of slow channel variations modeling Doppler effects and oscillators' phase noise. Theoretical findings are validated by simulations. Although not pursued herein, the results of this section can be adopted with minor changes to analyze performance of all blind methods proposed in [8].

3.1. Perturbation analysis

Ideally, according to the main assumptions underlying the proposed blind channel estimation method, we observe blocks of data of the form $\mathbf{X}_N = \mathbf{H}\,\mathbf{F}\,\mathbf{S}_N$, where the channel matrix $\mathbf{H}$ is Toeplitz. However, in practice our observations are perturbed with respect to the ideal model by the presence of noise and possibly by channel time variations, which render the matrix $\mathbf{H}$ non-Toeplitz. We will evaluate first the effect of a general perturbation $\boldsymbol{\delta}\mathbf{X}_N$ on the blind channel estimation algorithm. We will then specialize our analysis to the two kinds of perturbations specified earlier, namely channel time variations and noise.

In general, a perturbation $\boldsymbol{\delta}\mathbf{X}_N$ induces a perturbation $\boldsymbol{\delta}\tilde{\mathbf{U}}$ on the matrix $\tilde{\mathbf{U}}$ whose columns span the nullspace of $\mathbf{X}_N$. Let us denote by $\delta\tilde{\mathbf{U}}$ and $\delta\mathbf{h}$ the corresponding perturbations on the matrix $\tilde{\mathbf{U}}$ and on the channel vector $\mathbf{h}$ respectively. In the presence of perturbations, Eqn. (6) becomes

$$(\mathbf{h} + \delta\mathbf{h})^H (\tilde{\mathbf{U}} + \delta\tilde{\mathbf{U}}) = \mathbf{0}^H . \quad (7)$$

Hence, at a first order approximation valid for high SNR or small channel variations (small perturbations), we can express the channel estimation error as

$$\delta\mathbf{h}^H \approx -\mathbf{h}^H \delta\tilde{\mathbf{U}} \ \tilde{\mathbf{U}}^\dagger , \quad (8)$$

where $\delta\tilde{\mathbf{U}} := (\delta\tilde{\mathbf{U}}_1, \delta\tilde{\mathbf{U}}_2, \ldots, \delta\tilde{\mathbf{U}}_L)$ and the matrices $\delta\tilde{\mathbf{U}}_l$, $l = 1, \ldots L$ are Hankel matrices built from the vectors $\delta\tilde{\mathbf{u}}_l$, given by the columns of $\delta\tilde{\mathbf{U}}$ († denotes pseudo-inverse). On the other hand, each product $\mathbf{h}^H \delta\tilde{\mathbf{U}}_l$ can be written equivalently as $\delta\tilde{\mathbf{u}}_l^T \mathcal{T}(\mathbf{h})$, where $\mathcal{T}(\mathbf{h})$ is the Toeplitz channel matrix built from $\mathbf{h}$. Therefore, we can write

$$\begin{aligned} \mathbf{h}^H \boldsymbol{\delta}\tilde{\mathbf{U}} &= (\mathbf{h}^H \boldsymbol{\delta}\tilde{\mathbf{U}}_1 \cdots \mathbf{h}^H \boldsymbol{\delta}\tilde{\mathbf{U}}_L) \\ &= (\boldsymbol{\delta}\tilde{\mathbf{u}}_1^H \mathcal{T}(\mathbf{h}) \cdots \boldsymbol{\delta}\tilde{\mathbf{u}}_L^H \mathcal{T}(\mathbf{h})) \\ &= \mathrm{vec}^H(\boldsymbol{\delta}\tilde{\mathbf{U}})(\mathbf{I}_{L\times L} \otimes \mathcal{T}(\mathbf{h})), \quad (9) \end{aligned}$$

where the operator $\mathrm{vec}(\mathbf{A})$ simply stacks all the columns of the matrix $\mathbf{A}$ into one long vector. Hence, substituting (9) into (8), the channel estimation error can be written as

$$\boldsymbol{\delta}\mathbf{h}^H \approx -\mathrm{vec}^H(\boldsymbol{\delta}\tilde{\mathbf{U}})(\mathbf{I}_{L\times L} \otimes \mathcal{T}(\mathbf{h})) \ \tilde{\mathbf{U}}^\dagger . \quad (10)$$

Following steps similar to [6], we consider the SVD

$$\mathbf{X}_N = \left(\bar{\mathbf{U}}\,\tilde{\mathbf{U}}\right) \begin{pmatrix} \boldsymbol{\Sigma}^{\frac{1}{2}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \bar{\mathbf{V}}^H \\ \tilde{\mathbf{V}}^H \end{pmatrix} , \quad (11)$$

neglect perturbation products, and use the fact that $\tilde{\mathbf{U}}^H \mathbf{X}_N = \mathbf{0}$, to arrive at the following approximation:

$$\begin{aligned} &(\tilde{\mathbf{U}} + \boldsymbol{\delta}\tilde{\mathbf{U}})^H (\mathbf{X}_N + \boldsymbol{\delta}\mathbf{X}_N) = \mathbf{0} \\ \Rightarrow \quad &\tilde{\mathbf{U}}^H \boldsymbol{\delta}\mathbf{X}_N + \boldsymbol{\delta}\tilde{\mathbf{U}}^H \mathbf{X}_N \approx \mathbf{0}. \end{aligned}$$

Hence, the perturbation $\boldsymbol{\delta}\tilde{\mathbf{U}}$ can be expressed directly in terms of the perturbation $\boldsymbol{\delta}\mathbf{X}_N$ as

$$\boldsymbol{\delta}\tilde{\mathbf{U}} = -\mathbf{X}_N^{\dagger^H} \boldsymbol{\delta}\mathbf{X}_N^H \ \tilde{\mathbf{U}}. \quad (12)$$

Furthermore, because the vec operator satisfies the property $\mathrm{vec}(\mathbf{ACB}) = (\mathbf{B}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{C})$ [5], we have

$$\mathrm{vec}(\boldsymbol{\delta}\tilde{\mathbf{U}}) = -(\tilde{\mathbf{U}}^T \otimes \mathbf{X}_N^{\dagger^H})\mathrm{vec}(\boldsymbol{\delta}\mathbf{X}_N^H) \quad (13)$$

and thus, using (10), the error on the channel estimate is

$$\begin{aligned} \boldsymbol{\delta}\mathbf{h}^H &= \mathrm{vec}^H(\boldsymbol{\delta}\mathbf{X}_N^H)(\tilde{\mathbf{U}}^T \otimes \mathbf{X}_N^{\dagger^H})^H (\mathbf{I}_{L\times L} \otimes \mathcal{T}(\mathbf{h})) \ \tilde{\mathbf{U}}^\dagger \\ &= \mathrm{vec}^H(\boldsymbol{\delta}\mathbf{X}_N^H)(\tilde{\mathbf{U}}^* \otimes \mathbf{X}_N^\dagger)(\mathbf{I}_{L\times L} \otimes \mathcal{T}(\mathbf{h})) \ \tilde{\mathbf{U}}^\dagger \\ &= \mathrm{vec}^H(\boldsymbol{\delta}\mathbf{X}_N^H)(\tilde{\mathbf{U}}^* \otimes (\mathbf{X}_N^\dagger \mathbf{H}))\tilde{\mathbf{U}}^\dagger , \quad (14) \end{aligned}$$

where * stands for conjugation and for the last equality we used the property: $(\mathbf{A}_1 \otimes \mathbf{A}_2)(\mathbf{B}_1 \otimes \mathbf{B}_2) = (\mathbf{A}_1\mathbf{B}_1 \otimes \mathbf{A}_2\mathbf{B}_2)$. Expression (14) for $\boldsymbol{\delta}\mathbf{h}$ can now be used to evaluate the expected value and the correlation matrix of the channel estimate as follows:

$$E\{\boldsymbol{\delta}\mathbf{h}\} = \tilde{\mathbf{U}}^{\dagger^H}(\tilde{\mathbf{U}}^T \otimes (\mathbf{X}_N^\dagger \mathbf{H})^H) E\{\mathrm{vec}(\boldsymbol{\delta}\mathbf{X}_N^H)\}, \quad (15)$$

and

$$\begin{aligned} \mathbf{R}_{hh} \ := \ & E\{\boldsymbol{\delta}\mathbf{h}\,\boldsymbol{\delta}\mathbf{h}^H\} = \tilde{\mathbf{U}}^{\dagger^H}(\tilde{\mathbf{U}}^T \otimes (\mathbf{X}_N^\dagger \mathbf{H})^H) \quad (16) \\ \cdot \ & E\{\mathrm{vec}(\boldsymbol{\delta}\mathbf{X}_N^H)\mathrm{vec}^H(\boldsymbol{\delta}\mathbf{X}_N^H)\}(\tilde{\mathbf{U}}^* \otimes \mathbf{X}_N^\dagger \mathbf{H})\tilde{\mathbf{U}}^\dagger . \end{aligned}$$

Because (a2) and (a3) guarantee that $\mathbf{FS}_N(\mathbf{FS}_N)^\dagger = \mathbf{I}$, we can write:

$$\mathbf{X}_N^\dagger \mathbf{H} = \mathbf{X}_N^\dagger \mathbf{HFS}_N(\mathbf{FS}_N)^\dagger = \mathbf{X}_N^\dagger \mathbf{X}_N(\mathbf{FS}_N)^\dagger , \quad (17)$$

which, considering that $\mathbf{X}_N^\dagger \mathbf{X}_N \approx \mathbf{I}$ for $M \gg 1$, leads to

$$\mathbf{X}_N^\dagger \mathbf{H} \approx \mathbf{S}_N^\dagger \mathbf{F}^{-1}. \quad (18)$$

Hence, by substituting (18) into (16), we find that the error covariance matrix depends on the channel parameters $\mathbf{h}$ only through the Hankel matrix $\tilde{\mathbf{U}}$, and thus only through the structure of the null space of $\mathbf{H}$.

3.2. Estimation error due to noise

The presence of additive noise gives rise to a perturbation $\boldsymbol{\delta}\mathbf{X}_N := \mathbf{W}_N$, where $\mathbf{W}_N$ is the $P \times N$ noise matrix; i.e., we observe

$$\mathbf{X}_N + \boldsymbol{\delta}\mathbf{X}_N = \mathbf{HFS}_N + \mathbf{W}_N . \quad (19)$$

In the presence of zero mean additive white Gaussian noise (AWGN), with variance σ_w^2, the channel estimation error vector $\boldsymbol{\delta}\mathbf{h}$ is approximated by (14); hence, it is asymptotically (for high SNR) a Gaussian random vector with zero mean and covariance matrix

$$\begin{aligned}\mathbf{C}_{hh} &\approx \mathbf{R}_{hh} = E\{\boldsymbol{\delta}\mathbf{h}\,\boldsymbol{\delta}\mathbf{h}^H\} \\ &= \sigma_w^2 \tilde{\mathbf{U}}^{\dagger^H}(\tilde{\mathbf{U}}^T \otimes \mathbf{H}^H \mathbf{X}_N^{\dagger^H})(\tilde{\mathbf{U}}^* \otimes \mathbf{X}_N^{\dagger}\mathbf{H})\tilde{\mathbf{U}}^{\dagger} \\ &= \sigma_w^2 \tilde{\mathbf{U}}^{\dagger^H}[(\tilde{\mathbf{U}}^T\tilde{\mathbf{U}}^*) \otimes (\mathbf{H}^H \mathbf{X}_N^{\dagger^H}\mathbf{X}_N^{\dagger}\mathbf{H})]\tilde{\mathbf{U}}^{\dagger} \\ &= \sigma_w^2 \tilde{\mathbf{U}}^{\dagger^H}[\mathbf{I}_{L\times L} \otimes (\mathbf{H}^H \mathbf{X}_N^{\dagger^H}\mathbf{X}_N^{\dagger}\mathbf{H})]\tilde{\mathbf{U}}^{\dagger}. \qquad (20)\end{aligned}$$

For a given channel $\mathbf{H}$, (20) offers a closed form expression for evaluating performance of the channel estimates in [8] under AWN perturbations.

3.3. Estimation error due to channel variations

The method proposed in [8] and reviewed in Section 2 requires the channel to be linear time-invariant. However, the method can tolerate some channel fluctuation and still provide reliable estimates. The purpose of this section is precisely to evaluate the method's robustness against variations often encountered with practical systems such as carrier asynchronism, Doppler effects and oscillators' phase noise. Time fluctuations of the channel can be incorporated by letting the channel matrix $\mathbf{H}$ in (1) be a function of the block index n; i.e.,

$$\mathbf{x}(n) + \delta\mathbf{x}(n) = \mathbf{H}(n)\mathbf{F}\mathbf{s}(n), \qquad (21)$$

where $\mathbf{H}(n)$ is given by the sum of a Toeplitz matrix $\mathbf{H} \equiv \mathcal{T}(\mathbf{h})$ plus a (possibly) non-Toeplitz perturbation $\delta\mathbf{H}(n)$:

$$\mathbf{H}(n) = \mathcal{T}(\mathbf{h}) + \delta\mathbf{H}(n) = \mathbf{H} + \delta\mathbf{H}(n). \qquad (22)$$

Correspondingly, the observation $\mathbf{X}_N$ is perturbed by

$$\begin{aligned}\delta\mathbf{X}_N &= [\delta\mathbf{H}(0)\mathbf{F}\mathbf{s}(0), \ldots, \delta\mathbf{H}(N-1)\mathbf{F}\mathbf{s}(N-1)] \\ &= [\delta\mathbf{H}(0), \ldots, \delta\mathbf{H}(N-1)](\mathbf{I}_{N\times N} \otimes \mathbf{F})\mathcal{S}_N \\ &:= \delta\mathbf{H}_N \boldsymbol{\Phi} \mathcal{S}_N\ , \qquad (23)\end{aligned}$$

where we introduced for notational brevity the matrices $\delta\mathbf{H}_N := [\delta\mathbf{H}(0), \ldots, \delta\mathbf{H}(N-1)]$, $\boldsymbol{\Phi} := (\mathbf{I}_{N\times N} \otimes \mathbf{F})$, and

$$\mathcal{S}_N := \begin{pmatrix} \mathbf{s}(0) & \ldots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{s}(N-1) \end{pmatrix}. \qquad (24)$$

Using (23), the perturbation (12) can be expressed as

$$\delta\tilde{\mathbf{U}} = -\mathbf{X}_N^{\dagger^H}(\delta\mathbf{H}_N\boldsymbol{\Phi}\mathcal{S}_N)^H\tilde{\mathbf{U}}. \qquad (25)$$

Substituting (25) into (10), we find the corresponding error on the channel estimate to be given by

$$\begin{aligned}\boldsymbol{\delta}\mathbf{h}^H &\approx \mathrm{vec}^H[\mathbf{X}_N^{\dagger^H}(\delta\mathbf{H}_N\boldsymbol{\Phi}\mathcal{S}_N)^H\tilde{\mathbf{U}}](\mathbf{I}_{L\times L}\otimes\mathbf{H})\ \tilde{\mathbf{U}}^{\dagger} \\ &= \mathrm{vec}^H[(\mathbf{X}_N^{\dagger^H}\mathcal{S}_N^H\boldsymbol{\Phi}^H)\ \delta\mathbf{H}_N^H\ \tilde{\mathbf{U}}](\mathbf{I}_{L\times L}\otimes\mathbf{H})\ \tilde{\mathbf{U}}^{\dagger} \\ &= [\tilde{\mathbf{U}}^T \otimes (\mathbf{X}_N^{\dagger^H}\mathcal{S}_N^H\boldsymbol{\Phi}^H)\mathrm{vec}(\delta\mathbf{H}_N^H)]^H(\mathbf{I}_{L\times L}\otimes\mathbf{H})\ \tilde{\mathbf{U}}^{\dagger} \\ &= \mathrm{vec}^H(\delta\mathbf{H}_N^H)(\tilde{\mathbf{U}}^* \otimes (\boldsymbol{\Phi}\mathcal{S}_N\mathbf{X}_N^{\dagger}))(\mathbf{I}_{L\times L}\otimes\mathbf{H})\ \tilde{\mathbf{U}}^{\dagger} \\ &= \mathrm{vec}^H(\delta\mathbf{H}_N^H)\ \tilde{\mathbf{U}}^* \otimes (\boldsymbol{\Phi}\mathcal{S}_N\mathbf{X}_N^{\dagger}\mathbf{H})\ \tilde{\mathbf{U}}^{\dagger}. \qquad (26)\end{aligned}$$

The error correlation matrix is then found to be:

$$\begin{aligned}\mathbf{R}_{hh} &= \tilde{\mathbf{U}}^{\dagger^H}\tilde{\mathbf{U}}^T \otimes (\boldsymbol{\Phi}\mathcal{S}_N\mathbf{X}_N^{\dagger}\mathbf{H})^H \\ &\quad \cdot\ E\{\mathrm{vec}(\delta\mathbf{H}_N^H)\mathrm{vec}^H(\delta\mathbf{H}_N^H)\}\tilde{\mathbf{U}}^* \otimes (\boldsymbol{\Phi}\mathcal{S}_N\mathbf{X}_N^{\dagger}\mathbf{H})\tilde{\mathbf{U}}^{\dagger}.\end{aligned}$$

3.4. Blind Channel/Carrier-Offset Estimation

Channel perturbation operates on a block by block basis and affects the channel estimation in such a way that it is generally impossible to estimate a (perhaps stationary) channel component $\mathbf{h}$ and its fluctuations separately. However, a stationary channel followed by a non-stationary fluctuation due to carrier frequency asynchronism or Doppler effects, is a notable exception. In fact, denoting by $u(n)$ the transmitted coded sequence (the entries of the coded vector $\mathbf{u}(n) := \mathbf{F}\mathbf{s}(n)$), the channel output sequence $x(n)$, in the presence of a carrier offset f_D, is multiplied by $\exp(j2\pi f_D n)$ and the observed signal is:

$$\begin{aligned}\tilde{x}(n) &:= e^{j2\pi f_D n}x(n) = \sum_{l=0}^{L} e^{j2\pi f_D n(n+l-l)}h(l)u(n-l) \\ &= \sum_{l=0}^{L}\tilde{h}(l)\tilde{u}(n-l)\ , \qquad (27)\end{aligned}$$

where $\tilde{h}(l) := h(l)\exp(j2\pi f_D l)$, and $\tilde{u}(n) := u(n)\exp(j2\pi f_D n)$. If $u(n)$ satisfies (a3), so does $\tilde{u}(n)$. Despite the non-stationarity of the overall channel when a frequency offset is present, the blind method of Section 2 provides in the absence of noise an error-free estimate of $\tilde{h}(l)$ which differs from the LTI channel response $h(l)$ only by a frequency shift. In matrix form, the nth received block in the presence of a frequency offset f_D is

$$\tilde{\mathbf{x}}(n) = \boldsymbol{\Omega}_P(n)\mathbf{H}\mathbf{F}\mathbf{s}(n) = \tilde{\mathbf{H}}\boldsymbol{\Omega}_M(n)\mathbf{F}\mathbf{s}(n), \qquad (28)$$

where $\boldsymbol{\Omega}_K$ is the $K \times K$ diagonal matrix $\boldsymbol{\Omega}_K := \mathrm{diag}(1, \exp(j2\pi f_D)\ldots\exp(j2\pi f_D(K-1)))$ and $\tilde{\mathbf{H}}$ is still a Toeplitz matrix, associated with the channel response $\tilde{h}(l)$. In the absence of noise, our blind method is able to yield an error-free estimate of $\tilde{\mathbf{H}}$. The symbols are recovered as

$$\hat{\mathbf{s}}(n) = \mathbf{F}^{-1}\hat{\boldsymbol{\Omega}}_P^H\tilde{\mathbf{H}}^{\dagger}\tilde{\mathbf{x}}(n), \qquad (29)$$

where $\hat{\boldsymbol{\Omega}}_P$ is the phase rotating matrix built from an estimate of f_D. This estimate can be obtained using, for example, the maximum likelihood method if training sequences are transmitted, or, blindly by exploiting some *a priori* information about the symbol source as in [2]. Specifically, if the symbols $s(n)$ are drawn from a finite alphabet, we can set-up a recursive algorithm that: for every block $\mathbf{x}(n)$ it estimates first $\hat{\mathbf{s}}(n)$ and $\hat{f}_D$; and then updates $\hat{f}_D$ in order to force $\hat{\mathbf{s}}(n)$ to belong to the finite alphabet using the method in [1], or, by constraining the moments of $\hat{\mathbf{s}}(n)$, as in the CMA method of [2].

4. SIMULATIONS

To validate our performance analysis, we compare in this section our theoretical expressions with simulated performance results. In all cases, the precoding matrix $\mathbf{F}$ was chosen to be the OFDM matrix with trailing zeros.

Example 1: (*LTI channel and additive noise*) In Fig. 1 we report the mean square error (MSE) obtained by averaging over 100 independent trials (dashed line) and the corresponding theoretical expression given by the trace of (20) (solid line). The channel has three zeros at 1, 0.9j, and -0.9j. The other parameters are $M = 8$ and $N = P = M+L = 11$.

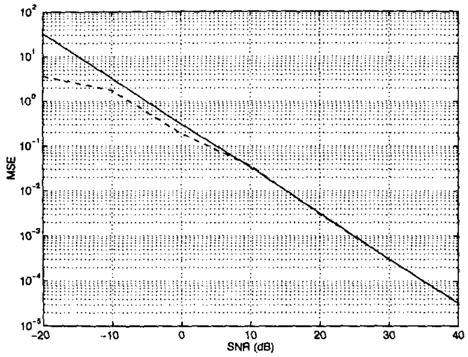

Figure 1. Channel MSE vs. SNR: simulation (dashed); analysis (solid)

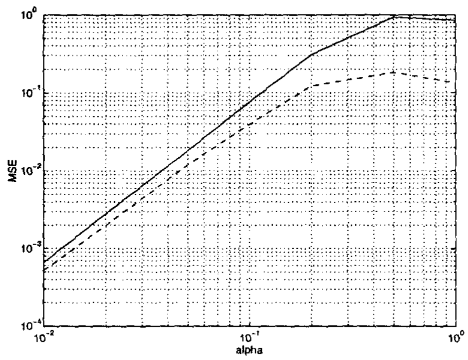

Figure 2. Channel MSE vs. Doppler shifts: simulation (solid); analysis (dashed)

We observe a fairly good agreement between theory and simulations especially when the SNR exceeds 0 dB.

Example 2: (*LTI channel and Doppler shift*) In Fig. 2 we compare the MSE obtained theoretically (dashed line) and by simulation (solid line) on a time-varying multipath channel composed of three rays with amplitudes $[1, 0.9, 0.9]$, delays $[0, T, 2T]$, where $1/T$ is the symbol rate, and Doppler shifts $\alpha[0, 1, 2]/NT$, as a function of the Doppler spread parameter α.

Example 3: (*LTI channel and receiver phase noise*) As a final example, in Fig. 3 we show the performance achieved in the presence of oscillators' phase noise modeled as a multiplicative noise $\exp(j\phi(n))$, where $\phi(n)$ is a Wiener process obtained by integrating a white Gaussian noise with variance $\sigma_n^2 = \gamma/NP$. The curves are obtained by averaging over 40 independent channel and phase noise realizations. The channel was modeled as a multipath Rayleigh fading channel with three independent paths.

5. CONCLUDING REMARKS

In this paper we have derived theoretical performance analysis results for the blind channel identification method proposed in [8]. We have shown that the theory is in close agreement with the simulation results under a variety of conditions that include presence of Doppler shifts and oscillators' phase noise. The theoretical analysis reported in this paper is not only useful to avoid unnecessary simulations, at least under small perturbation conditions, but it is particularly important when it comes to assessing robustness of the method in [8]. The closed form variance expressions are also potentially useful for designing robust precoders against specific channel variations.

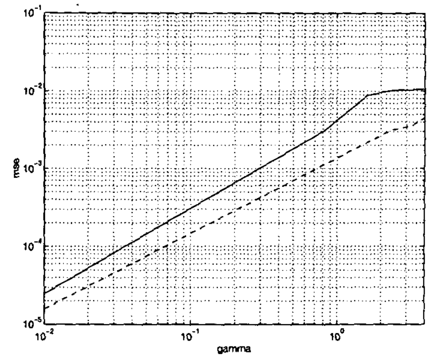

Figure 3. Channel MSE vs. phase noise bandwidth: simulation (solid); analysis (dashed)

REFERENCES

[1] S. Barbarossa, A. Scaglione, "Blind Equalization Using Cost Functions Matched to the Signal Constellation", *Proc. of* 31^{st} *Asilomar Conf. on Signals, Systems & Comp.*, Pacific Grove (CA), pp. 550-554, Nov. 1997.

[2] A. Belouchrani and W. Ren, "Blind Carrier Phase Tracking with Guaranteed Global Convergence," *IEEE Trans. on Signal Processing*, vol. 45, no. 7, pp. 1889-1894, July 1997.

[3] G. D. Forney, Jr., M. V. Eyuboğlu "Combined equalization and coding using precoding", *IEEE Communications Magazine*, pp. 25-34, Dec. 1991.

[4] G. K. Kaleh, "Channel Equalization for Block Transmission Systems," *IEEE Journal on Selected Areas in Communications*, pp. 110-121, Jan. 1995.

[5] P. Lancaster and M. Tismenetsky, *The Theory of Matrices* (second edition), p. 410, Academic Press, Inc., San Diego, 1985.

[6] F. Li and R. J. Vaccaro, "Unified Analysis for DOA Estimation Algorithms in Array Signal Processing", *Signal Process.*, vol. 25, pp. 147-169, Nov. 1991.

[7] A. Scaglione, G.B. Giannakis, S. Barbarossa, "Redundant Filterbank Precoders and Equalizers Part I: Unification and Optimal Designs", *IEEE Trans. on Signal Proc.*, (to appear), 1999.

[8] A. Scaglione, G.B. Giannakis, S. Barbarossa, "Redundant Filterbank Precoders and Equalizers - Part II: Blind Channel Estimation, Synchronization and Direct Equalization", *IEEE Trans. on Signal Proc.*, 1999. See also "Self Recovering Multirate Equalizers Using Redundant Filterbank Precoders," *Proc. of ICASSP '98*, pp. 3501-3504, Seattle (WA), March 1998.