SELF-RECOVERING TRANSCEIVERS FOR BLOCK TRANSMISSIONS: FILTERBANK PRECODERS AND DECISION-FEEDBACK EQUALIZERS

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ABSTRACT

Transmitter induced redundancy using FIR filterbanks can be used to equalize FIR channels irrespective of channel zeros and facilitate blind channel estimation. At the receiver end, linear or non-linear (DFE) FIR filterbanks can be applied to recover the transmitted data. Closed form expressions are derived for the FIR linear or DFE filterbank receivers. By applying blind channel estimation methods, the resulting DFE-framework becomes self-recovering. Extensive simulations illustrate the merits of our designs.

1. INTRODUCTION

Block transmission systems have been proposed as a means of combating fading effects in frequency selective channels [2]. Among the ways to model block transmission of data is the unifying framework of [4], where it is shown that most of the currently used block transmission systems (e.g., orthogonal frequency-division multiplexing, OFDM) can be realized using pairs of filterbank transmitters and receivers (transceivers). It is also shown that by introducing redundancy at the transmitter, it is possible to perform blind channel estimation and block synchronization [5]. The redundancy, which is in the form of trailing zeros ("guard interval"), offers degrees of freedom which can be exploited in the design of transceivers under bit error rate (BER) and information rate (throughput) constraints.

On the other hand, decision feedback equalizers (DFE) have also been proposed as a way to approach the performance of a maximum-likelihood receiver (e.g., [1]). Results on finite-length fractionally-sampled DFE have established that it is possible with FIR transmitter and receiver filters to achieve low bit error rates [1]. Moreover, with adaptive DFE techniques (e.g., [3]), the DFE receiver structure lends itself naturally to decision-directed channel estimation. Blind DFE channel estimation methods have been also proposed (see [6] and references therein).

In this paper, we generalize the linear filterbank framework of [4] and [5] to include non-linear (DFE) filterbank receivers, which offer improved BER performance. Though results on zero-forcing (ZF) or minimum-mean-square-error (MMSE) DFE receivers have appeared [2], in this work we develop closed-form FIR DFE filterbank receivers which take into account the transmitter precoder, the (perhaps unknown) channel response, the autocorrelation of the finite-alphabet input data, and the additive noise. In addition, contrary to [2] that requires channel status information, we develop self-recovering DF filterbanks relying upon the blind channel estimation of [5].

Hence, the resulting framework combines the strengths of the two aforementioned transceiver structures of [2] and [4, 5]. It improves upon [2] by showing that transmitter redundancy (in the form of known or zero symbols between the blocks) can be used for blind channel estimation; it improves upon [4, 5] by showing that a DFE receiver, which exploits the finite alphabet property of the input symbols, can improve the system performance.

2. MODEL DESCRIPTION

Fig. 1 depicts the discrete-time model of a baseband block transmission communication system. The transmitted data are parsed into blocks using advance elements and downsamplers. The transmit filters $\{f_m(n)\})_{m=0}^{M-1}$ are FIR of maximum order $P-1$ (Fig. 2). The FIR channel $\{h(n)\}$ includes multipath effects and transmit/receive filters. The input to the upsampler of the mth branch is $s_m(n){:=}s(nM+m)$, which represents the m-th symbol in the n-th block of M symbols. With the insertion of $P-1$ zeros, the corresponding upsampler's output is: $\sum_i s_m(i)\delta(n-iP)$, where $\delta(n)$ denotes Kronecker's delta. The transmitted sequence is: $u(n)=\sum_{m=0}^{M-1} u_m(n)=\sum_i\sum_{m=0}^{M-1} s_m(i)f_m(n-iP)$. The received samples, $y(n)=x(n)+v(n)$, consist of the noise-free data $x(n)=\sum_l h(l)u(n-l)$ plus additive zero-mean stationary noise, with covariance matrix $\mathbf{R}_{vv}$ ($=\sigma_{vv}^2\mathbf{I}_P$ when the noise is white). We will assume that:

(a0) Channel $h(l)$ is Lth order FIR with $h(0),h(L)\neq 0$.

(a1) (P,M,L) are chosen such that the triplet (P,M,L) satisfies: $P=M+L$, and $M>L$.

(a2) The last L samples of the filters $\{f_m(n)\}_{m=0}^{M-1}$ are zero,

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and the $P\times M$ matrix $\mathbf{F}$ (with entries $\mathbf{F}_{mn}:=f_m(n)$) has full column rank M.

Under (a0)-(a2) and thanks to the FIR nature of $h(n)$ and $f_m(n)$, the p-th polyphase component (delayed and down-sampled version) of $x(n)$ is:

$$x_p(n):=x(nP+p)=\sum_{m=0}^{M-1} s_m(n)\sum_{l=0}^{L} h(l)f_m(p-l), \quad (1)$$

which shows that no inter-block interference arises due to ISI. Let us define the $M\times 1$ vectors $\mathbf{s}(n):=(s_0(n)\ s_1(n)\ \ldots s_{M-1}(n))^T$, and $\mathbf{u}(n):=(u(nP)u(nP+1)\ldots u(nP+M-1))^T$; the $P\times 1$ vector $\mathbf{x}(n):=(x(nP)x(nP+1)\ldots x(nP+P-1))^T$, and the $P\times P$ Toeplitz lower triangular matrix $\mathbf{H}$ with first column $(h(0)\cdots h(L)0\cdots 0)^T$. Based on these definitions, we can cast (1) in matrix form:

$$\mathbf{x}(n)=\mathbf{H}\mathbf{u}(n)=\mathbf{H}\mathbf{F}\mathbf{s}(n). \quad (2)$$

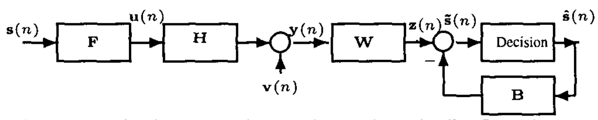

Figure 1: Block Transmitter, Channel, and DFE Receiver

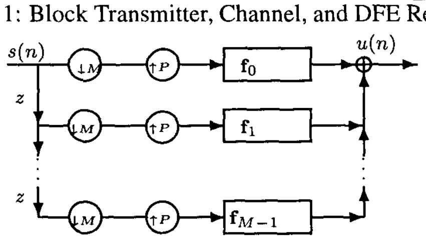

Figure 2: Transmitter Structure

The decision feedback equalizer consists of the feedforward filterbank represented by the $M\times P$ matrix $\mathbf{W}$, the decision making device and the feedback filterbank represented by the $M\times M$ matrix $\mathbf{B}$. By defining the $M\times 1$ vectors: $\mathbf{z}(n):=(z(nM)\ z(nM+1)\ \ldots z(nM+M-1))^T$, $\mathbf{t}(n):=(t(nM)\ t(nM+1)\ \ldots t(nM+M-1))^T$, $\hat{\mathbf{s}}(n):=(\hat{s}(nM)\ \hat{s}(nM+1)\ \ldots\hat{s}(nM+M-1))^T$, we can write in matrix form: $\mathbf{z}(n)=\mathbf{W}\mathbf{y}(n)=\mathbf{W}\mathbf{H}\mathbf{F}\mathbf{s}(n)+\mathbf{W}\mathbf{v}(n)$, $\mathbf{t}(n)=\mathbf{z}(n)-\mathbf{B}\hat{\mathbf{s}}(n)$, $\hat{\mathbf{s}}(n)=Q(\mathbf{t}(n))$, where $Q(\cdot)$ is the quantizer used by the decision making device. Note that $\mathbf{B}$ is chosen to be upper triangular which makes successive cancellation possible. By successive cancellation we mean that for every block indexed by n, first the $(M-1)$th symbol is recovered; then the estimate $\hat{s}(nM+M-1)$ is weighted by the last column of $\mathbf{B}$ and is removed from $\mathbf{z}(n)$ so that the remaining symbols can be recovered. When this is done, the $(M-2)$nd symbol is recovered, and the estimate $\hat{s}(nM+M-2)$ is removed from $\mathbf{z}(n)$. This procedure is carried out until all the symbols of the current block n have been recovered.

3. FILTERBANK RECEIVERS

In this section we design linear and non-linear (DFE) receivers under ZF and MMSE criteria. We assume that the channel matrix $\mathbf{H}$, the precoder $\mathbf{F}$, and the correlation matrices $\mathbf{R}_{ss}$[1], $\mathbf{R}_{vv}$ are known.

3.1. Linear and Non-Linear ZF Receivers

Linear ZF Receiver: It is proved in [4] that for a given full-rank precoding matrix $\mathbf{F}$ and channel matrix $\mathbf{H}$ (which is also full-rank by construction), there exists a ZF equalizer filterbank $\mathbf{G}_{zf}$ so that $\mathbf{G}_{zf}\mathbf{x}(n)=\mathbf{s}(n)$. The minimum norm ZF filterbank is unique and it is given by: $\mathbf{G}_{zf}=(\mathbf{H}\mathbf{F})^{\dagger}$. The equalized blocks are given by: $\hat{\mathbf{s}}(n)=\mathbf{G}_{zf}\mathbf{H}_0\mathbf{F}_0\mathbf{s}(n)+\mathbf{G}_{zf}\mathbf{v}(n)$, where $\hat{\mathbf{s}}(n):=(\hat{s}(nM)\ \hat{s}(nM+1)\ \ldots\hat{s}(nM+M-1))^T$. The structure of the linear filterbank receiver is illustrated in Fig. 3.

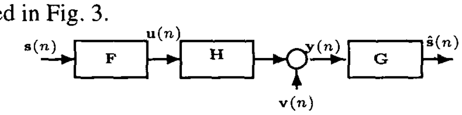

Figure 3: Linear Filterbank Transceiver

ZF-DFE Receiver: A ZF-DFE receiver improves the BER performance (especially at low SNR) in two ways: (a) by exploiting the finite alphabet of the input and taking into account decisions about the symbols in the same block, and (b) by whitening the noise at the input of the decision device. Three are the requirements that a ZF-DFE receiver should meet: zero-forcing, noise whitening, and allowance for "successive cancellation". By zero-forcing we mean that in the absence of noise and under the assumption of correct past decisions, the decision statistic should be equal to the transmitted data: $\bar{\mathbf{s}}(n)=\mathbf{s}(n)=\hat{\mathbf{s}}(n)$, which translates the ZF requirement to: $\mathbf{W}\mathbf{H}\mathbf{F}=\mathbf{B}+\mathbf{I}_M$. To render symbol-by-symbol detection optimal, it is necessary that the noise in the input of the decision device is white. Noise-whitening is achieved by selecting $\mathbf{W}$ such that: $\mathbf{W}\mathbf{W}^{\mathcal{H}}=\mathbf{I}_M$.

To derive our filterbank block ZF-DFE, we borrow the "whitened matched filter" concept (see e.g., [7, pg. 59]). Because $(\mathbf{H}\mathbf{F})^{\mathcal{H}}(\mathbf{H}\mathbf{F})$ is positive semi-definite, its Cholesky decomposition is given by $(\mathbf{H}\mathbf{F})^{\mathcal{H}}(\mathbf{H}\mathbf{F})=\mathbf{U}^{\mathcal{H}}\mathbf{U}$, where $\mathbf{U}$ is upper triangular. Then the ZF ($\mathbf{W}$,$\mathbf{B}$) receiver pair is given by: $\mathbf{W}=\mathbf{U}^{-\mathcal{H}}(\mathbf{H}\mathbf{F})^{\mathcal{H}}$, and $\mathbf{B}=\mathbf{U}-\mathbf{I}_M$ (which is upper triangular). We can verify by direct substitution that: $\mathbf{W}\mathbf{H}\mathbf{F}=\mathbf{U}$, $\mathbf{W}\mathbf{W}^{\mathcal{H}}=\mathbf{I}_M$.

3.2. Linear and Non-linear MMSE Receivers

Linear MMSE Receiver: It is shown in [4] that the linear receiver which minimizes the mean square error (MSE) is:

$$\mathbf{G}_{\text{mmse}}=\mathbf{R}_{ss}\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}(\mathbf{R}_{vv}+\mathbf{H}\mathbf{F}\mathbf{R}_{ss}\mathbf{F}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}})^{-1}.$$

MMSE-DFE: A performance measure of a DFE receiver is the error $e(n):=\tilde{s}(n)-s(n)$ at the input of the decision device. In vector form, by defining $\mathbf{e}(n):=(e(nM)e(nM+1)\ldots e(nM+M-1))^T$, we have $\mathbf{e}(n)=\bar{\mathbf{s}}(n)-\mathbf{s}(n)$. Using the standard assumption of correct past decisions, we infer that: $\mathbf{e}(n)=\mathbf{W}\mathbf{y}(n)-(\mathbf{B}+\mathbf{I}_M)\mathbf{s}(n)$.

[1] $R_{ss}:=E\{\mathbf{s}(n)\mathbf{s}^{\mathcal{H}}(n)\}$; all other correlation matrices are defined accordingly.

The MMSE-DFE $(\mathbf{W},\mathbf{B})$ receiver pair should be chosen so that the MSE $E\{|\mathbf{e}(n)|^2\}=\text{tr}\{\mathbf{R}_{ee}\}$ is minimized, under the constraint that $\mathbf{B}$ is upper triangular with unit diagonal. First, we assume that $\mathbf{B}$ is fixed and we obtain the matrix $\mathbf{W}$ which minimizes $E\{|\mathbf{e}(n)|^2\}$. Using the orthogonality principle, we find that $\mathbf{e}(n)$ should be orthogonal to $\mathbf{y}(n)$, which yields: $\mathbf{W}\mathbf{R}_{yy}=(\mathbf{B}+\mathbf{I}_M)\mathbf{R}_{sy}$. Using (2), we obtain: $\mathbf{R}_{xx}=(\mathbf{HF})\mathbf{R}_{ss}(\mathbf{HF})^{\mathcal{H}}$, and $\mathbf{R}_{sy}=\mathbf{R}_{ss}(\mathbf{HF})^{\mathcal{H}}=\mathbf{R}_{ys}^{\mathcal{H}}$. As the additive noise is independent of the transmitted data, we obtain: $\mathbf{R}_{yy}=\mathbf{R}_{xx}+\mathbf{R}_{vv}$. Hence, we relate the feedforward and feedback filterbanks via:

$$\mathbf{W}=(\mathbf{B}+\mathbf{I}_M)\mathbf{R}_{sy}\mathbf{R}_{yy}^{-1}.$$

As a result, the error $\mathbf{e}(n)$ can be written as:

$$\mathbf{e}(n)=(\mathbf{B}+\mathbf{I}_M)\boldsymbol{\epsilon}(n), \text{ with } \boldsymbol{\epsilon}(n):=\mathbf{R}_{sy}\mathbf{R}_{yy}^{-1}\mathbf{y}(n)-\mathbf{s}(n).$$

After some algebra we find that:

$$\mathbf{R}_{\epsilon\epsilon}=\mathbf{R}_{ss}-\mathbf{R}_{sy}((\mathbf{HF})\mathbf{R}_{ss}(\mathbf{HF})^{\mathcal{H}}+\mathbf{R}_{vv})^{-1}\mathbf{R}_{ys}.$$

Using the matrix inversion lemma[2], we have:

$$\mathbf{R}_{\epsilon\epsilon}=\left(\mathbf{R}_{ss}^{-1}+(\mathbf{HF})^{\mathcal{H}}\mathbf{R}_{vv}^{-1}(\mathbf{HF})\right)^{-1},$$

which yields:

$$\mathbf{R}_{ee}=(\mathbf{B}+\mathbf{I}_M)\left(\mathbf{R}_{ss}^{-1}+(\mathbf{HF})^{\mathcal{H}}\mathbf{R}_{vv}^{-1}(\mathbf{HF})\right)^{-1}(\mathbf{B}+\mathbf{I}_M)^{\mathcal{H}}.$$

The minimization of MSE amounts to minimizing the $\text{tr}\{\mathbf{R}_{ee}\}$. Consider now the Cholesky factorization of $\mathbf{R}_{\epsilon\epsilon}$, namely,

$$\mathbf{R}_{ss}+(\mathbf{HF})^{\mathcal{H}}\mathbf{R}_{vv}^{-1}(\mathbf{HF})=\mathbf{U}^{\mathcal{H}}\mathbf{D}\mathbf{U},$$

where $\mathbf{U}$ is upper triangular with unit diagonal. By setting $\mathbf{B}=\mathbf{U}-\mathbf{I}_M$, we obtain: $\mathbf{R}_{ee}=\mathbf{D}^{-1}$. As $\mathbf{D}$ is diagonal, this has the implication that the noise at the input of the decision device is *white*, which makes symbol-by-symbol detection optimal.

4. SELF-RECOVERING DFE FILTERBANKS

Blind channel estimation dispenses with transmission of training sequences, which results in bandwidth savings. In this section we explore how the trailing-zeros redundancy can be used for channel estimation. We utilize results from [5] and apply them to our DFE filterbank at the receiver end. The basic idea is that N received data blocks are collected and from them we obtain the estimated channel vector $\hat{\mathbf{h}}$. This vector is used to construct the estimated channel matrix $\hat{\mathbf{H}}$, which is used to define along with our precoder $\mathbf{F}$ the matrices $\mathbf{W}$ and $\mathbf{B}$ under the ZF- or MMSE-DFE criteria. In [5], it is proved that if $P=M+L$, channel estimation can be carried out blindly and deterministically (i.e., input need not be white). What is required is an upper bound on the order of the FIR channel $h(n)$. The channel estimation method can be summarized as follows:

- collect $N\geq P$ blocks of data $\mathbf{y}(0),\ldots,\mathbf{y}(N-1)$ and form the matrix $\mathbf{Y}:=(\mathbf{y}(0)\ldots\mathbf{y}(N-1))$
- determine the L eigenvectors $\mathbf{v}_l$, $l=1\ldots L$, which correspond to the L smallest eigenvalues of $\mathbf{Y}\mathbf{Y}^{\mathcal{H}}$
- for $l=1\ldots L$, form the Hankel matrices $\mathbf{V}_l$ defined as:

$$\mathbf{V}_l:=\begin{pmatrix} v_l(0) & v_l(1) & \ldots & v_l(P-L-1) \\ v_l(1) & v_l(2) & \ldots & v_l(P-L) \\ \vdots & \vdots & \vdots & \vdots \\ v_l(L) & v_l(L+1) & \ldots & v_l(P-1) \end{pmatrix}_{(L+1)\times M}$$

- estimate the channel vector $\mathbf{h}$ as the non-trivial solution of the system: $\mathbf{h}^{\mathcal{H}}(\mathbf{V}_1\ldots\mathbf{V}_L)_{(L+1)\times ML}=\mathbf{0}_{1\times ML}$.

It is also shown in [5] that the channel estimation method does not pose any restrictions on the FIR channel zeros and it is robust even when the channel order is overestimated. These merits make the aforementioned channel estimation method suitable not only for the blind linear filterbanks of [5] but also for our self-recovering DFE filterbanks.

5. SIMULATION EXAMPLES

In this section we present simulation results to illustrate the characteristics of our filterbank transceivers. In all examples the figure of merit is BER as a function of E_b/N_o. The BER is calculated using Monte Carlo simulations assuming BPSK modulation.

Example 1 *MMSE-DFE achieves lowest BER*: We consider an OFDM precoder with $P=M+L$. Specifically, $M=32$, $P=36$ for an FIR channel of order $L=4$ with zeros at 1, $0.9\exp(j9\pi/20)$, $1.1\exp(-j9\pi/20)$, -0.8. Fig. 4 depicts the BER performance as a function of E_b/N_o, where $E_b=E_s=\frac{1}{M}\text{tr}\{\mathbf{F}\mathbf{F}^{\mathcal{H}}\}$. The input correlation matrix is taken to be $\mathbf{R}_{ss}=\mathbf{I}_M$, and the additive noise is simulated as white with autocorrelation matrix $\mathbf{R}_{vv}=\sigma_v^2\mathbf{I}_P$, $\sigma_v^2=N_o$. The precoder matrix is $\mathbf{F}=[\mathbf{F}]_{m,n}$ with trailing zeros; i.e.,

$$\mathbf{F}_{m,n}=\begin{cases} e^{j\frac{2\pi}{M}mn} : 0\leq m\leq M-1, 0\leq n\leq M-1 \\ 0 \quad : M\leq m\leq P-1, 0\leq n\leq M-1 \end{cases}.$$

From Fig. 4 we observe that the DFE receivers outperform the linear receivers in both cases. As expected, as the signal-to-noise ratio E_b/N_o increases, the performance improvement becomes more evident.

Example 2 *The precoder matrix does make a difference*: To test whether the selection of a precoder matrix has an impact on the BER we have simulated the system of Example 1 using three different precoders: the OFDM precoder of Example 1, a Hadamard precoder $\mathbf{F}_{\text{HAD}}$, and the optimal ZF-precoder $\mathbf{F}_{\text{opt}}$ of [4]. The precoder $\mathbf{F}_{\text{HAD}}$ is given by the MATLAB function `hadamard(M)` concatenated with trailing zeros. The M first rows of $\mathbf{F}_{\text{opt}}$ are given by $\mathbf{V}_{\text{opt}}\boldsymbol{\Lambda}_{\text{opt}}^{-1/2}$, where $\mathbf{V}_{\text{opt}}$, $\boldsymbol{\Lambda}_{\text{opt}}$ are given by the eigen-decomposition $\mathbf{H}^{\mathcal{H}}\mathbf{R}_{vv}^{-1}\mathbf{H}=\mathbf{V}_{\text{opt}}\boldsymbol{\Lambda}_{\text{opt}}\mathbf{V}_{\text{opt}}^{\mathcal{H}}$. Fig. 5 depicts the BER performance of the MMSE-DFE receiver. As it can be observed from Fig. 5, $\mathbf{F}_{\text{opt}}$ results in the best BER performance.

Example 3 *Blind Channel Estimation*: To study the performance of our blind DFE algorithm, we use the settings of Example 1, but now the channel matrix $\mathbf{H}$ is not known at the receiver end. The receiver estimates the channel matrix $\mathbf{H}$ and using the estimate $\hat{\mathbf{H}}$ defines the receive filter-

[2] For A,B,C,D matrices of compatible dimensions it holds that: $(A-CB^{-1}D)^{-1}=A^{-1}+A^{-1}C(B-DA^{-1}C)^{-1}DA^{-1}$

banks (of course the receiver knows the transmit precoder). Fig. 6 depicts the performance of the four receivers. Even in the blind scenario, the DFE filterbanks exhibit better performance than their linear counterparts; the MMSE-DFE receiver shows the best BER performance. Moreover, the simulations indicate that thanks to our block approach for blind equalization, channel estimation errors do not result in catastrophic errors in the DFE receivers. Contrary to serial DFE schemes, error propagation in block DFE's is "limited" within a block. As expected, the BER performance of all receivers is worse than that of the receivers in Example 1.

6. CONCLUSIONS

Transmitter induced redundancy using FIR filterbanks provides us with degrees of freedom which can be exploited to achieve blind channel estimation and DF equalization with FIR filterbanks. The receiver structure and the corresponding BER performance depend critically on the selection of the transmitter precoder. We have derived closed form solutions for the FIR linear and non-linear decision-feedback receivers. Using block channel estimation methods relying on redundant filterbank precoders enables a self-recovering framework where the DFE filterbank acquires channel status information blindly and adjusts its FIR filters accordingly. As the data inside in a block could belong to different users, multiuser transmissions (e.g., [7]) are intrinsically related to block transmissions. Linking our block transmission results to multiuser transmissions is among our future research directions.

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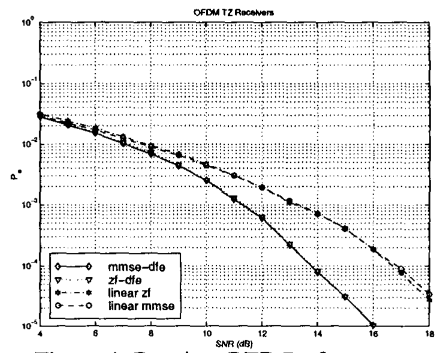

Figure 4: Receiver BER Performance

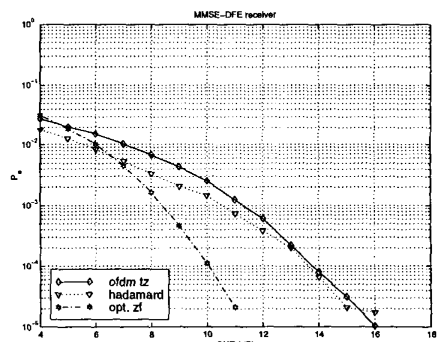

Figure 5: MMSE-DFE Performance for Three Precoders

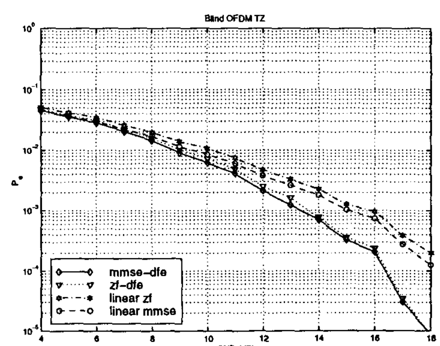

Figure 6: Blind Channel Estimation