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Median Filtering For Power Estimation In Mobile Communication Systems

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Abstract — **In mobile communications, accurate power estimation is important for power control and handoff decisions. Power estimation algorithms aim to filter out the fast variation in the received power due to multipath, and should ideally exhibit very low complexity. For this reason, linear filtering (local mean) is almost exclusively used for power estimation in practice. We derive the maximum likelihood estimator in an idealized setting, and motivated by the statistical properties of the multipath signal, we also propose median filtering for power estimation. Simulations show promising results, indicating gains in RMSE and robustness to increased window size.**

I. INTRODUCTION

In mobile communications, the received signal strength fluctuates as the vehicle travels through the interference patterns caused by multipath, shadowing due to obstructions, and the change in the mobile station (MS) - base station (BS) distance. Estimation of the average signal strength, which is a measure of communication link quality is important for handoff algorithms, power control, and optimal tuning of system parameters to changing channel conditions in adaptive transmission systems. In [7, Fig.7], it is shown that for a variety of power control schemes, even a 1 dB gain in power estimation error can yield up to 5 more users in the system for a fixed outage probability. Similar gains are also possible in handoff performance with better power estimation. Hence, it is of interest to develop accurate power estimators which are simple indicators of link quality.

In this paper, we will investigate how the conventional linear filtering techniques [2] for power estimation compare with the maximum likelihood (ML) estimator, the uniformly minimum variance unbiased (UMVU) estimator of [9], and the median filtering approach proposed herein.

The paper is organized as follows. In Section II we introduce the signal model and state the problem of local power estimation. In Section III, we review existing local power estimators, and derive the ML estimator. In Section IV, we propose median filtering for estimation of the shadow fading process. Section V provides simulation examples comparing the existing techniques with the proposed approach, and Section VI concludes the paper.

II. SIGNAL MODEL AND PROBLEM STATEMENT

A widely accepted model for the received power at the mobile station is in the following product form[2]:

$$p(t) = |h(t)|^2\, s(t) \ , \qquad (1)$$

where $s(t)$ is the slow power fluctuation due to shadowing, also referred to as the local mean, and $h(t)$ is the fast narrowband channel response due to multipath, in complex baseband form [6]. The multipath component is commonly assumed to be statistically independent of the shadow process, and without loss of generality, we will assume that $E[|h(t)|^2] = 1$. When the power is measured through a logarithmic amplifier, its decibel (dB) value has the following additive form:

$$P(t) = H(t) \ + \ S(t) \ , \qquad (2)$$

where $P(t) := 10\log[p(t)]$, $S(t) := 10\log[s(t)]$, $H(t) := 10\log[|h(t)|^2]$, and $\log(\cdot)$ denotes base-10 logarithm.

In many applications, it is of interest to filter out the fast variations in $P(t)$ due to the fast-fading component $H(t)$ (see Fig. 1). Filtering out $H(t)$ to obtain an estimate of $S(t)$ is the topic of this paper. More precisely, given $P[n] := P(nT_s)$, we are interested in estimating $S[n] := S(nT_s)$, where T_s is the sampling period. For this purpose, the statistics of the multipath and the shadow processes are of interest, and are discussed in what follows (for a more comprehensive overview see e.g., [8]).

The multipath signal $h(t)$ is a superposition of many sinusoids with random phases, and random frequencies corresponding to the Doppler induced by the angles of arrival. If the angles of arrival have a continuous uniform distribution (which we will assume throughout), the correlation function of $h(t)$ in the absence of a line of sight component is given by the well-known Clarke-Jakes's model: $E[h(t)h^*(t+\tau)] = J_0(\omega_D\tau)$, where $\omega_D = 2\pi v/\lambda$ is the normalized maximum Doppler spread, λ is the wavelength corresponding to the carrier frequency, and v is the velocity of the mobile [4]. Being the sum of many independent sinusoids, $h(t)$ is commonly assumed to be Gaussian, and hence $|h(t)|^2$ is exponentially distributed. Since we will be working with (2), it is of interest to know the statistical properties of $H(t)$. The probability density function (pdf) of $H(t)$ is given by [9]:

$$f_H(x) = C\exp[Cx - \exp(Cx)] \ , \qquad C := \frac{\ln(10)}{10} \ , \qquad (3)$$

where $\ln(\cdot)$ is base-e logarithm.

The mean of $H(t)$ can be shown to be [2, 9],

$$\overline{H} := E\,[H(t)] = -\frac{\gamma}{C} \ , \qquad (4)$$

where $\gamma = 0.577216\ldots$ is Euler's constant, which makes $\overline{H} \approx 2.5068$. The covariance of $H(t)$ was derived in [1] as:

$$c_H(\tau) := E\,[H(t)\ H(t+\tau)] - \overline{H}^2 = [10/\ln(10)]^2 \sum_{k=1}^{\infty} \frac{q^k(\tau)}{k^2}, \quad (5)$$

where $q(\tau) := J_0^2(\omega_D\tau)$, $c_H(\tau)$ is the normalized covariance, and the variance of $H(t)$ is given by $\sigma_H^2 := c_H(0) = [10/\ln(10)]^2(\pi^2/6)$. We emphasize that the second order statistics of $H(t)$ are a completely known function of ω_D.

The dB value of the shadow fading, $S(t)$, is commonly modeled as a Gaussian process with mean $\mu_S(t)$ and variance σ_S^2. The mean is given by the path loss which decreases monotonically with the MS - BS distance. However, for the purposes of signal strength estimation, we will assume $\mu_S(t) \approx \mu_S$ to be constant during the time window that the received signal $P(t)$ is processed.

A first order autoregressive (AR) model for the shadow process is suggested in [3] based on the measured autocovariance function of $S(t)$ in urban and suburban environments:

$$E\,[S(nT_s)S((n+m)T_s)] = \sigma_S^2 \exp(-vT_s|m|/X_c) \ , \qquad (6)$$

where X_c is the so-called effective correlation distance. Notice that $a := \exp(-vT_s/X_c)$ is the parameter of the autoregression. The pair (σ_S, X_c) determine the second-order statistics of the shadow process we are interested in estimating.

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[2]All the expressions for the received power are given as a function of time and can be equivalently represented in terms of the MS - BS distance d through the MS velocity v by substituting $t = d/v$.

In what follows, we review some existing local power estimators, and propose new ones.

III. Local Power Estimators

The most computationally simple and conceptually straightforward approach for local power estimation can be obtained by averaging the samples $P[n]$, or more generally linear filtering the received power [8]. In [2] an integrate-and-dump (ID) filter (with an impulse response that is constant over its finite support), and an RC filter (with an exponential impulse response) were considered, and similar results were obtained. Hence, in this paper we only consider the ID approach as a representative of the conventional linear filtering technique for local power estimation, which, in discrete-time corresponds to taking the average of the received samples $P[n]$.

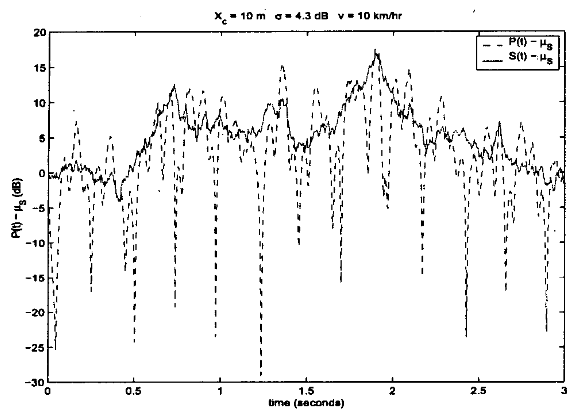

Figure 1: Simulated Received Power

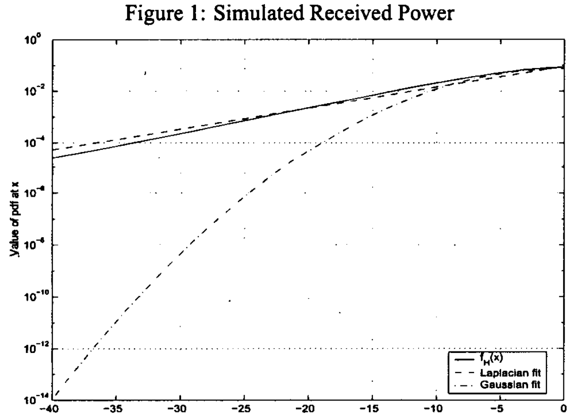

Figure 2: $f_H(x)$ with Gaussian and Laplacian fits

A UMVU estimator was proposed in [9] under two assumptions: (i) $S(t) \approx S$, i.e., the value of the local power is constant over the duration of the available samples, which is possible if σ_S is small or X_c is large; (ii) the samples $P(nT_s)$ are independent, which can be approximated by making T_s large enough. Note that (i) and (ii) are conflicting requirements, since making T_s large conflicts with the constancy of the shadow process for a fixed number of samples. However, this idealization of the problem let [9] derive the UMVU estimator for S, and also will enable us to derive the ML estimator. The UMVU estimator under (i) and (ii) is given by

$$\hat{S}_{\text{UMVU}} := 10 \left[\log\left(T(N)\right) - \frac{\sum_{n=1}^{N-1} \frac{1}{n}}{\ln(10)} \right] - \overline{H} ; \tag{7}$$

where $T(N) := \sum_{n=0}^{N-1} |p(nT_s)|^2$ is a sufficient statistic for S, $p(t)$ is given in (1) and $\overline{H}$ is given in (4) and can be precomputed. The linear sample mean estimator $\hat{S}_{\text{SM}} := N^{-1} \sum_{n=0}^{N-1} P(nT_s) - \overline{H}$ is also unbiased and consistent, but it is not minimum variance like $\hat{S}_{\text{UMVU}}$.

Assuming (i) and (ii), we derived the ML estimator using the pdf in (3). The resulting estimator is given by

$$\hat{S}_{\text{ML}} = \hat{S}_{\text{UMVU}} + \overline{H} + 10 \left[\frac{\sum_{n=1}^{N-1} \frac{1}{n}}{\ln(10)} - \log(N) \right] \tag{8}$$

where the bias term following $\hat{S}_{\text{UMVU}}$ in (8) can be shown to go to zero as $N \to \infty$. Hence the UMVU and ML estimators are asymptotically equivalent, and both depend on the sufficient statistic $T(N)$.

IV. Median Filters for Local Power Estimation

Motivated by the impulsive character of the multipath process $H(t)$ (see Fig. 1), which is also evidenced in the plot of $f_H(x)$ in Fig. 2, we propose a median filtering approach, which is known to be robust to heavy-tailed noise. Median filtering is widely used in signal and image processing, and is known to outperform linear filtering alternatives in the presence of heavy tailed noise. In fact, it can be shown that the maximum-likelihood (ML) estimate of a constant immersed in independent Laplacian noise is obtained by the median of the available samples. Moreover, fast algorithms exist for median filtering of one- or two-dimensional signals [5].

Our estimator for $S[n]$, which relies on the median filter can be expressed as follows:

$$\hat{S}_{\text{MD}}[n] = \text{median}\left(P[n-M], \ldots, P[n+M]\right) - \beta , \tag{9}$$

where the median operator orders the samples within the bracket, and selects the middle value, $N = 2M+1$ is the window length of the running median filter, and $\beta \approx -1.5920$ is a correction term that satisfies $\int_{\beta}^{\infty} f_H(x)\, dx = \int_{-\infty}^{\beta} f_H(x)\, dx = 0.5$, and should be included since $H(t)$, as a random variable, has a nonzero median value given by β.

V. Simulations

In this section, we will compare the methods discussed for local power estimation under the idealized, and practical settings.

In Fig. 3, we compare the four aforementioned local power estimators when the assumption of constancy of $S(t)$ is satisfied over the averaging window. In practice this situation is approximated if σ_S is small, or X_c is very large. In this case, for all of the estimators, using more data points (a larger window size) improves the mean squared error (MSE) performance of each of the estimators. The MSE in all the simulations was calculated by computing the MSE between $S[n]$ and the output of each of the four estimators with a sliding window, over a single record of length 10,000 (corresponding to a 100-second data record). We observe that, as expected, the UMVU has the least mean-squared error, and that the ML estimator is very close to the UMVU as the available data points increase. The surprising conclusion is that, despite the impulsive nature of the multipath process, for the same window size, the linear averaging does better than median filtering. However, in the practical setting where the shadow process $S(t)$ is not constant (i.e., when $X_c < \infty$, and $\sigma_S > 0$) the median filter outperforms the other three alternatives, as we show next.

Consider now Fig. 4 where we have the realistic scenario with the correlation distance $X_c = 100\lambda$ and the shadow standard deviation

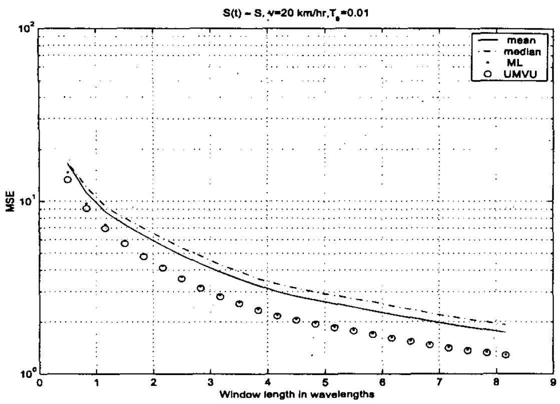

Figure 3: Idealized Case

$\sigma_S = 10$ dB. The first thing we notice is that, unlike the idealized case where the shadow process is constant, increasing the window duration beyond a certain point degrades the MSE performance of each of the estimators. The median filter exhibits the smallest minimum compared to the other three alternatives. Moreover, the degradation in MSE beyond the optimum point is much less severe for the median filter. This is important because the selection of the optimum window size requires knowledge of the vehicle velocity whose estimation is subject to unavoidable modeling and estimation errors [8]. Hence, in the presence of such errors, the loss in MSE performance for the median filter will be less than that of the UMVU, ML, and linear filtering alternatives. It should be noted that the median filter is not causal (see (9)), and the other three filters are causal (i.e., the power estimate does not depend on future values of the received power). This is one of the reasons why the median filter has a more graceful degradation beyond the optimum point in Fig. 4.

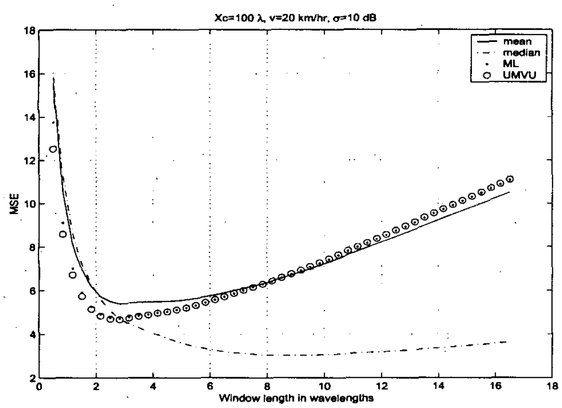

Figure 4: Realistic Case

It is clear that the vehicle velocity is related linearly to the optimum window size (OWS) measured in units of time. A natural question that arises is how the OWS depends on σ_S and X_c. In Fig. 5 we plot the OWS versus X_c and σ_S for the linear filtering and the median filtering alternatives. We observe the intuitive result of the OWS increasing with decreasing σ_S and increasing X_c, both of which have an affect of making $S(t)$ closer to a constant. The knowledge of X_c and σ_S could be known a priori from field measurements, or could be estimated online [8].

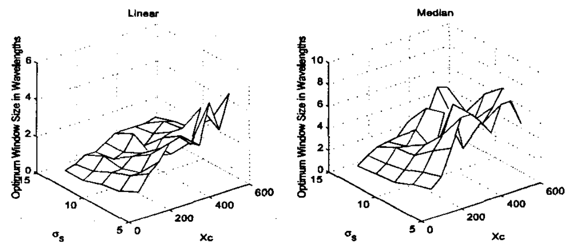

Figure 5: Optimum Window Size vs. σ_S and X_c

VI. Conclusions

We derived the maximum likelihood estimator for the power estimation problem under the assumption that $S(t)$ is constant over the duration of averaging, and that the received power samples are independent, and showed that the ML estimator is the same as the UMVU estimator of [9] except for a bias term that goes to zero with the window size. We also proposed a median filtering approach to estimating $S(t)$, motivated by the statistics of $H(t)$, and showed that the median filtering algorithm outperforms the conventional linear filter, as well as the ML and UMVU alternatives when the correlation distance is finite, and the shadow variance is nonzero (i.e., when $S(t)$ changes with time). The improvement in performance is partly attributed to the non-causality of the median filter.

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