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Universal Redundant Precoders for Guaranteed Recovery of Block Transmissions through Unknown FIR Channels

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Abstract — **Without channel state information available, universal designs of wireless OFDM transmitters and receivers offer performance that is robust to the underlying channel fades and the adopted signal constellation. We show that judiciously designed precoders with redundancy twice as long as the channel order are necessary and sufficient for guaranteed constellation- and channel-irrespective symbol recovery as well as for blind channel identifiability. Simulated comparisons of the novel precoders with zero-padded and cyclic-prefixed block transmissions delineate their relative merits.**

I. Introduction

Block transmissions relying on linear redundant filterbank precoding have gained increasing interest recently for mitigating frequency-selective multipath effects (see e.g., [1, 5, 7, 8] and references therein). To enable low-complexity block-by-block processing at the receiver, inter block interference (IBI) due to the frequency-selective channel should be eliminated. There are at least two ways to remove IBI [8]: one pads guard zeros in each transmission block and thus forms a zero-padded (ZP) redundant block transmission, while the other allows for nonzero redundant symbols and because it discards the first few components of the received vector it is referred to as the leading receiver zeros (LRZ) approach.

The LRZ approach includes the important class of cyclic prefixed (CP) block transmissions that have found widespread application in the context of orthogonal frequency-division multiplexing (OFDM) - the basic multicarrier modulation adopted by many standards (IEEE802.11a, HIPERLAN 2, DAB and DVB). Although channel-irrespective (even blind) channel identifiability and symbol recovery are guaranteed for ZP block transmissions [3, 7, 8], such attractive features have not been established for general standardized redundant block transmissions with CP that require LRZ receivers.

In this paper, we bridge this gap by offering precoding matrices that assure constellation- and channel-irrespective (and thus universal) symbol recovery, and also enable blind channel identification for the practically important case of LRZ reception. After developing our unifying block modeling framework (Section II), we present a necessary and sufficient condition on the number of redundant symbols for the existence of precoders that assure constellation- and channel-irrespective symbol recovery (Section III). Then, we propose a class of precoders that enable blind channel identification (Section IV). Simulation results are finally presented to compare the novel precoders against ZP precoders (Section V).

II. Block Modeling and Preliminaries

We consider a discrete-time baseband equivalent model for block transmissions that are linearly precoded using redundant filterbanks and propagate through frequency-selective communication channels [8]. At the transmitter, the $M \times 1$ information block $\boldsymbol{s}(i)$ is precoded with an $(M+K) \times M$ precoding matrix $\boldsymbol{T}$ of full column rank to yield

$$\boldsymbol{u}(i) = \boldsymbol{T}\boldsymbol{s}(i), \tag{1}$$

which is transmitted through the channel. Note that the redundancy offered with $K > 0$ subsumes but is neither limited to the cyclic prefix (CP) employed by conventional OFDM [2], nor to the zero-padding (ZP) proposed recently for OFDM [3, 7, 8]. We correspondingly form the received block by stacking $M+K$ samples of the receive-filter output taken at the symbol rate $1/T_s$. We express the finite impulse response (FIR) of the underlying discrete-time baseband equivalent channel as $\{h(n)\}$ and set as an upper bound on its order L, the ratio $L := \lceil \tau_{max,d}/T_s \rceil$, where $\tau_{max,d}$ denotes the maximum multipath delay-spread and $\lceil\ \rceil$ stands for integer-ceiling.

We ignore for the moment additive noise effects. To remove inter block interference (IBI), we utilize the so-called leading receiver zeros (LRZ) receiver, i.e., we discard the first L components of the received vector to obtain [8]

$$\boldsymbol{x}(i) = \boldsymbol{H}\boldsymbol{T}\boldsymbol{s}(i), \tag{2}$$

where $\boldsymbol{H}$ denotes the $(M+K-L) \times (M+K)$ Toeplitz channel matrix with first row $[h(L), h(L-1), \ldots, h(0), 0, \ldots, 0]$. The LRZ approach is applied to the orthogonal frequency devision multiplexing (OFDM), which has low complexity and is widely used for, e.g., mobile wireless networks.

Our system's operating conditions are the following:

- **C1** *the FIR channel has at least one non-zero coefficient and its order is not greater than L.*
- **C2** *the number K of redundant symbols is selected greater than or equal to L,* i.e., $K \geq L$.
- **C3** *the transmitter precoding matrix $\boldsymbol{T}$ is chosen to be full column rank.*

Conditions C2 and C3 can be easily satisfied by the transmitter design, while C1 requires only an upper bound on the channel order which is readily acquired by estimating the maximum delay-spread and dividing it by the symbol period T_s. Note also that no assumption is imposed on the symbol constellation which is an important difference between the linear block equalizers dealt with in this paper and existing nonlinear schemes such as constant-modulus, decision-feedback, or, maximum-likelihood alternatives that either incur convergence and error propagation effects, or, they have high computational complexity.

It is known that equalization for conventional uncoded OFDM with L redundant symbols is only possible when the channel has no zero on the FFT grid [7, 8]. Although channel-irrespective (even blind) channel identifiability and symbol recovery are guaranteed for zero-padded (ZP) block transmission, such attractive features have not been established for general standardized redundant block transmissions with CP that require LRZ receivers. In this paper, we will offer precoding matrices that assure constellation- and channel-irrespective symbol recovery, and also enable blind channel identification for the LRZ reception.

III. Symbol recovery for the LRZ approach

The constellation- and channel-irrespective (and thus universal) symbol recovery from (2) is assured if and only if $\text{rank}\{\boldsymbol{HT}\} = M$ for *any* FIR channel of order L. First, we state the following basic (albeit negative) result on symbol recovery for a general block transmission with insufficient redundancy and LRZ reception:

Theorem 1 *For $K < 2L$ and for every $(M+K) \times M$ full column rank matrix $\boldsymbol{T}$, universal symbol recovery from (2) is impossible under C1-C3; i.e., there exits at least one $(M+K-L) \times (M+K)$ channel matrix $\boldsymbol{H}$ such that*

$$\text{rank}\{\boldsymbol{HT}\} < M. \tag{3}$$

Proof: See Appendix A. ■

Theorem 1 implies that if $K = L$ (as it is with e.g., CP-OFDM), then there is no universal redundant precoder; i.e., there is no $\boldsymbol{T}$ that assures symbol recovery for all FIR channels of order L. Theorem 1 corroborates and can be considered as a generalization of [7, Thm. 1.1.2 in Part I], which deals with CP transmissions and asserts that with L cyclic prefixed redundant symbols one can not guarantee symbol recovery if the channel has nulls located at $\zeta_i = \rho\exp(j2\pi i/M)$ for some ρ and an integer i. Note however, that the insufficient redundancy $K < 2L$ preventing universal precoders in Theorem 1 does not have to be in the form of CP. From Theorem 1, we can deduce:

Corollary 1 *A necessary condition for the existence of precoding matrices enabling constellation-irrespective symbol recovery with the LRZ receiver for any FIR channel of order L is that $K \geq 2L$, where K is the number of redundant symbols in each transmitted block $\boldsymbol{u}(i)$.*

The following theorem characterizes and constructively designs universal precoders for LRZ receivers:

Theorem 2 *Let $\boldsymbol{V}$ be any $(M+K)\times(M+K-L)$ Vandermonde matrix of full column rank and $\boldsymbol{\Phi}$ any $(M+K-L) \times M$ full column rank matrix such that any M rows of $\boldsymbol{\Phi}$ are linearly independent. If for $K \geq 2L$, the redundant precoder is chosen as*

$$\boldsymbol{T} = \boldsymbol{V\Phi}, \tag{4}$$

then symbol recovery for any FIR channel of order L is assured regardless of the symbol constellation under C1-C3.

Proof: Let $\boldsymbol{v}_m := [1, v_m, \ldots, v_m^{M+K-1}]^T$ be the mth column of $\boldsymbol{V}$ for $m \in [0, M+K-L-1]$ and the channel transfer function be $H(z) := \sum_{l=0}^{L} h(l)z^{-l}$. Then, since $\boldsymbol{H}$ is Toeplitz and $\boldsymbol{V}$ is Vandermonde, we have

$$\boldsymbol{HT} = \boldsymbol{HV\Phi} = \tilde{\boldsymbol{V}}\boldsymbol{D}_H\boldsymbol{\Phi}, \tag{5}$$

where $\tilde{\boldsymbol{V}}$ is constructed from the last $M+K-L$ rows of $\boldsymbol{V}$, and $\boldsymbol{D}_H$ is a diagonal matrix defined as

$$\boldsymbol{D}_H := \text{diag}[H(v_0), H(v_1), \ldots, H(v_{M+K-L-1})]. \tag{6}$$

For $H(z)$ of order L, at most L diagonal elements of $\boldsymbol{D}_H$ are zero. Since any M rows of $\boldsymbol{\Phi}$ are linearly independent, so are any M rows of $\boldsymbol{D}_H\boldsymbol{\Phi}$, i.e., $\text{rank}\{\boldsymbol{D}_H\boldsymbol{\Phi}\} = M$. It follows from the non-singularity of $\tilde{\boldsymbol{V}}$ that $\text{rank}\{\boldsymbol{HT}\} = M$ for any channel matrix $\boldsymbol{H}$, which completes the proof. ■

The requirement on $\boldsymbol{\Phi}$ is fulfilled deterministically by any Vandermonde (e.g., FFT) matrix with $M+K-L$ distinct generators, and also with probability one by any pseudo-random (PN) matrix with ± 1 entries. We denote the $(M+K)\times(M+K-L)$ CP-inducing matrix $\boldsymbol{T}_{cp}$ as $\boldsymbol{T}_{cp} := [\boldsymbol{I}_{cp}^T, \boldsymbol{I}_{M+K-L}^T]^T$, where $\boldsymbol{I}_{cp} := [\boldsymbol{0}_{L\times(M+K-2L)}, \boldsymbol{I}_L]$; $\boldsymbol{I}_L$ denotes the identity matrix of size L; and $\boldsymbol{0}_{L\times(M+K-2L)}$ stands for the $L \times (M+K-2L)$ zero matrix. If we set $\boldsymbol{V}$ to be $\boldsymbol{T}_{cp}\boldsymbol{F}_{M+K-L}^{\mathcal{H}}$, where $\boldsymbol{F}_{M+K-L}$ is the $(M+K-L)\times(M+K-L)$ FFT matrix and $^{\mathcal{H}}$ denotes conjugated transposition, the precoding scheme results in what we term precoded (P)-OFDM [6].

Remark: Theorem 2 is important because it proves that with redundancy twice the channel length, P-OFDM with CP does not suffer from channel fades (i.e., it is universal).

IV. Blind identification for LRZ approach

Collecting N data vectors $\boldsymbol{x}(i)$ to form the matrix

$$\boldsymbol{X}_N := [\boldsymbol{x}(0), \boldsymbol{x}(1), \ldots, \boldsymbol{x}(N-1)], \tag{7}$$

we obtain from (2):

$$\boldsymbol{X}_N = \boldsymbol{HTS}_N, \tag{8}$$

where $\boldsymbol{S}_N := [\boldsymbol{s}(0), \boldsymbol{x}(1), \ldots, \boldsymbol{s}(N-1)]$. We further assume that the input symbol sequence satisfies a persistence of excitation condition for identification:

(A1) *There exists an $N \geq \underline{N}$ such that the $M \times N$ matrix $\boldsymbol{S}_N$ is full row rank.*

In the following, we consider transmitter precoding matrices as per in Theorem 2. We will show that $K \geq 2L$ is sufficient to assure blind identifiability for any FIR channel of order L by using the subspace approach of [7].

Let the left null vectors of $\boldsymbol{HT}$ be $\boldsymbol{u}_l := [u_l(0), \ldots, u_l(M+K-L-1)]^T$ for $l \in [0, K-L-1]$, i.e., $\boldsymbol{u}_l^{\mathcal{H}}\boldsymbol{HT} = \boldsymbol{0}^T$ for $l \in [0, K-L-1]$, or equivalently,

$$\boldsymbol{U}^{\mathcal{H}}\boldsymbol{HT} = \boldsymbol{0}, \tag{9}$$

where $\boldsymbol{U} := [\boldsymbol{u}_0, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_{K-L-1}]$. For given $\boldsymbol{U}$ and $\boldsymbol{T}$, if the non-trivial solution of (9) is unique within a scale factor, then blind identification is possible by the so-called subspace method as follows: Vector multiplication with a Toeplitz matrix denotes convolution which is commutative and hence $\boldsymbol{u}_l^{\mathcal{H}}\boldsymbol{HT} = \boldsymbol{0}^T$ can be expressed as $\boldsymbol{h}^T\boldsymbol{\mathcal{U}}_l\boldsymbol{T} = \boldsymbol{0}^T$, where $\boldsymbol{h} := [h(0), \ldots, h(L)]^T$; $\boldsymbol{\mathcal{U}}_l$ is an $(L+1)\times(M+K)$ Hankel matrix with first column $[0, \ldots, 0, u_l^*(0)]^T$ and last row $[\boldsymbol{u}_l^{\mathcal{H}}, 0, \ldots, 0]$. Stacking $\boldsymbol{h}^T\boldsymbol{\mathcal{U}}_l\boldsymbol{T} = \boldsymbol{0}^T$ for $l = 0, \ldots, K-L-1$, we obtain

$$\boldsymbol{h}^T[\boldsymbol{\mathcal{U}}_0\boldsymbol{T}, \boldsymbol{\mathcal{U}}_1\boldsymbol{T}, \ldots, \boldsymbol{\mathcal{U}}_{K-L-1}\boldsymbol{T}] = \boldsymbol{0}^T. \tag{10}$$

If the non-trivial solution of (9) is unique within a scale factor, so is the non-trivial solution of (10) and hence blind identification is achieved by computing the non-trivial left null vector of $[\boldsymbol{\mathcal{U}}_0\boldsymbol{T}, \ldots, \boldsymbol{\mathcal{U}}_{K-L-1}\boldsymbol{T}]$. We show in Appendix B that for $K \geq 2L$, the non-trivial solution of (9) (or equivalently (10)) is unique within a scale factor and establish the following theorem:

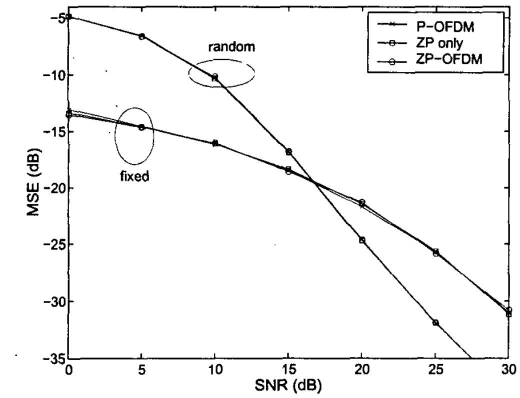

Figure 1: Channel MSE for fixed and random channels

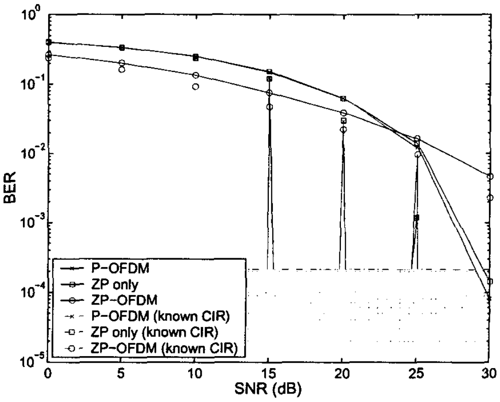

Figure 2: BER (a fixed channel)

Theorem 3 *Under C1-C3 and A1, for $K \geq 2L$, the precoding matrices of Theorem 2 enable blind identification of any FIR channel of order L regardless of the underlying symbol constellation.*

Theorems 2 and 3 imply that $2L$ redundant symbols are necessary and sufficient for universal symbol recovery and blind identification of FIR channels up to order L, with LRZ reception. For $K = 2L$, the bandwidth efficiency (that depends on the relative redundancy K/M) is given by $\mathcal{E}_{LRZ}(M, L) := M/(M + 2L)$. On the other hand, ZP block transmission requires L redundant (zero) symbols for universal symbol recovery and blind identification, and its bandwidth efficiency is $\mathcal{E}_{ZP}(M, L) := M/(M + L)$. For universal symbol recovery and blind identification with fixed M and L, ZP block transmissions have a slightly better bandwidth efficiency than redundant transmissions with LRZ reception.

V. Simulated Performance

We conducted simulations to compare LRZ and ZP block transmissions. For LRZ, we set $K = 2L$ and adopted P-OFDM with $\boldsymbol{V} = \boldsymbol{T}_{cp}\boldsymbol{F}^{\mathcal{H}}_{M+L}$ and $\boldsymbol{\Phi}$ having the first M columns of $\boldsymbol{F}^{\mathcal{H}}_{M+L}$. We tested ZP only transmission with precoding matrix $[\boldsymbol{I}_M, \boldsymbol{0}_{M\times L}]^T$ and ZP-OFDM with $[\boldsymbol{F}_M, \boldsymbol{0}_{M\times L}]^{\mathcal{H}}$, where $\boldsymbol{F}_M$ is the FFT matrix of size M. We dealt with two cases: a fixed FIR channel of 4th order with zeros at 1, -1, $3\exp(-j9\pi/20)$, and 0.8; and Rayleigh channels of 4th order with i.i.d. complex zero-mean Gaussian taps. For every system, we set the guard interval length to be $L = 4$ and blocks of information bearing symbols to have size $M = 32$. The information symbols were drawn from a BPSK constellation. We used the received signal-to-noise ratio (SNR) defined as $E\{||\boldsymbol{HTs}(i)||^2\}/E\{||\boldsymbol{w}(i)||^2\}$, where $\boldsymbol{w}(i)$ denotes additive i.i.d. zero-mean circular Gaussian noise. For the fixed channel, we conducted 10^2 Monte-Carlo simulations, while for random channels we averaged results over 10^3 channel realizations.

For blind identification of P-OFDM, we utilized the following procedure:

1. *Construct $\boldsymbol{X}_N$ in (7) from $N = 50$ received blocks.*
2. *Compute the eigenvectors of $\boldsymbol{X}_N\boldsymbol{X}_N^{\mathcal{H}}$ corresponding to the L smallest eigenvalues.*
3. *Compute the eigenvector of $\sum_{l=0}^{L-1} \boldsymbol{U}_0\boldsymbol{T}\boldsymbol{T}^{\mathcal{H}}\boldsymbol{U}_0^{\mathcal{H}}$ corresponding to the smallest eigenvalue.*
4. *Set the channel estimate $\hat{\boldsymbol{h}}$ to be the complex conjugate of the eigenvector obtained above.*

For blind channel identification with ZP transmissions, we used the algorithm developed in [7]. As a performance measure of channel estimation quality, we computed the (normalized) channel mean squared error (MSE) defined as $(1/N_t)\sum_{i=1}^{N_t} \min_{c_i} ||\boldsymbol{h} - c_i\hat{\boldsymbol{h}}_i||^2/||\boldsymbol{h}||^2$, where N_t is the number of trials; $\hat{\boldsymbol{h}}_i$ is the ith estimate in the Monte-Carlo experiments; and c_i is to remove the constant ambiguity, which is also utilized to construct zero-forcing (ZF) equalizers for symbol recovery.

Fig. 1 compares channel MSE with P-OFDM, ZP only and ZP-OFDM transmissions. There is no significant difference among them. It is confirmed that P-OFDM enables blind identification for the channel with nulls on the FFT grid.

Fig. 2 illustrates BER performance for the fixed channel. The dashed dotted BER curves are obtained from ZF equalizers constructed from exact channel impulse responses (CIR). The BER curves of P-OFDM basically coincide with those of the ZP only transmission. This is because $\boldsymbol{HT}$ in P-OFDM is just a permuted version of the transfer matrix from $\boldsymbol{s}(i)$ to the received block in the ZP only transmission. At moderate and low SNR, below about 25dB for estimated channels (20dB for exact CIR), ZP-OFDM exhibits the best performance, while at high SNR P-OFDM has better performance than ZP-OFDM. Similar observations are also evident from Fig. 3, which depicts BER performance for Rayleigh random channels.

A. Proof of Theorem 1

Suppose that for a fixed $\boldsymbol{H}$, an $(M + K) \times M$ full column rank matrix $\boldsymbol{T}$ satisfies that $\text{rank}\{\boldsymbol{HT}\} = M$. The necessary and sufficient condition for $\text{rank}\{\boldsymbol{HT}\} = M$ is

$$\dim\{\mathcal{N}(\boldsymbol{H}) \cap \mathcal{R}(\boldsymbol{T})\} = 0, \tag{11}$$

where $\mathcal{N}(\boldsymbol{X})$ and $\mathcal{R}(\boldsymbol{X})$ is the range and the null space of $\boldsymbol{X}$ respectively, and $\dim(\mathcal{A})$ denotes the dimension of a space $\mathcal{A}$.

Since $\boldsymbol{H}$ has full row rank, $\dim(\mathcal{N}(\boldsymbol{H})) = L$ and $\dim(\mathcal{R}(\boldsymbol{H}^{\mathcal{H}})) = M + K - L$. Similarly, we obtain that

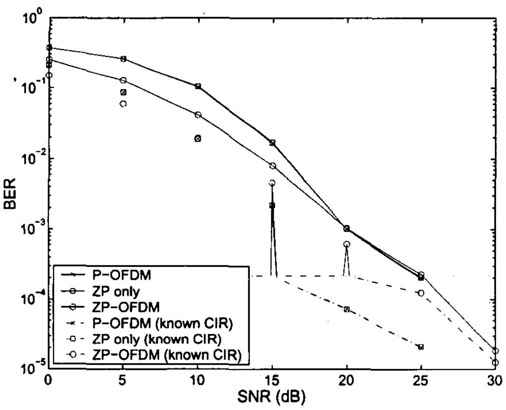

Figure 3: BER (random channels)

$\dim(\mathcal{R}(\boldsymbol{T})) = M$ and $\dim(\mathcal{N}(\boldsymbol{T}^{\mathcal{H}})) = K$. Noting that $\mathcal{R}(\boldsymbol{T})$ is the orthogonal complement of $\mathcal{N}(\boldsymbol{T}^{\mathcal{H}})$, and $\mathcal{N}(\boldsymbol{H})$ is the orthogonal complement of $\mathcal{R}(\boldsymbol{H}^{\mathcal{H}})$, we find that (11) is satisfied if and only if $\mathcal{N}(\boldsymbol{T}^{\mathcal{H}}) \supseteq \mathcal{N}(\boldsymbol{H})$.

The null space of $\boldsymbol{H}$ is spanned by the (generalized) Vandermonde vectors associated with the zeros $\{\gamma_l\}_{l\in[0,L-1]}$ of the channel transfer function $H(z) := \sum_{l=0}^{L} h(l)z^{-l} = \Pi_{l=0}^{L-1}(1 - \gamma_l z^{-1})$. Then, from $\mathcal{N}(\boldsymbol{T}^{\mathcal{H}}) \supseteq \mathcal{N}(\boldsymbol{H})$, the null space of $\boldsymbol{T}^{\mathcal{H}}$ is given by

$$\mathcal{N}(\boldsymbol{T}^{\mathcal{H}}) = \mathrm{span}\{\boldsymbol{\gamma}_0, \dots, \boldsymbol{\gamma}_{L-1}, \boldsymbol{r}_0, \dots, \boldsymbol{r}_{K-L-1}\}, \qquad (12)$$

where $\boldsymbol{\gamma}_l$ is the Vandermonde vector defined as $\boldsymbol{\gamma}_l := [1, \gamma_l, \dots, \gamma_l^{M+K-L-1}]^T$ and the $\boldsymbol{r}_l$'s are linearly independent of each other and of the $\boldsymbol{\gamma}_l$'s.

Consider $\boldsymbol{H}'$ with transfer function $H'(z) = \sum_{l=0}^{L} h'(l)z^{-l} = \Pi_{l=0}^{L-1}(1 - \gamma'_l z^{-1})$ such that $\gamma'_i \neq \gamma_j$ for all $i, j \in [0, L-1]$. The null space of $\boldsymbol{H}'$ can be expressed as $\mathcal{N}(\boldsymbol{H}') = \mathrm{span}\{\boldsymbol{\gamma}'_0, \dots, \boldsymbol{\gamma}'_{L-1}\}$, where $\boldsymbol{\gamma}'_i$ is the Vandermonde vector associated with γ'_i. Since $\gamma'_i \neq \gamma_j$ for all i, j, the $\boldsymbol{\gamma}'_i$'s are linearly independent of $\boldsymbol{\gamma}_i$'s. Thus, for $(K-L) < L$, i.e., $K < 2L$, it is impossible that $\mathcal{N}(\boldsymbol{T}^{\mathcal{H}}) = \mathrm{span}\{\boldsymbol{\gamma}_0, \dots, \boldsymbol{\gamma}_{L-1}, \boldsymbol{r}_0, \dots, \boldsymbol{r}_{K-L-1}\} \supseteq \mathrm{span}\{\boldsymbol{\gamma}'_0, \dots, \boldsymbol{\gamma}'_{L-1}\} = \mathcal{N}(\boldsymbol{H}')$. This means that $\mathrm{rank}\{\boldsymbol{H}'\boldsymbol{T}\} < M$. Therefore, for $K < 2L$, there is no universal precoder, which proves the theorem.

B. Proof of uniqueness for the solution in (9)

We can prove the uniqueness along the lines in [4, Appen. II]. But here we provide an alternative proof.

For $K \geq 2L$, w.l.o.g., we can assume that $H(v_m) \neq 0$ for $m \in [0, M-1]$. We denote the $M \times M$ matrix having the first M rows of, $\boldsymbol{\Phi}$ in $\boldsymbol{T} = \boldsymbol{V}\boldsymbol{\Phi}$, as $\boldsymbol{\Phi}_1$. Since any M rows of $\boldsymbol{\Phi}$ are linearly independent, $\boldsymbol{\Phi}_1$ is nonsingular. Thus, we can re-express $\boldsymbol{\Phi}$ as

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \boldsymbol{\Phi}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_M \\ \boldsymbol{\Phi}_2\boldsymbol{\Phi}_1^{-1} \end{bmatrix} \boldsymbol{\Phi}_1. \qquad (13)$$

Since $\boldsymbol{\Phi}_1$ is nonsingular, $\boldsymbol{U}^{\mathcal{H}}\boldsymbol{H}\boldsymbol{T} = \boldsymbol{0}$ is equivalent to $\boldsymbol{U}^{\mathcal{H}}\boldsymbol{H}\boldsymbol{V}\boldsymbol{R} = \boldsymbol{0}$, where $\boldsymbol{R} := [\boldsymbol{I}_M, \boldsymbol{\Theta}^{\mathcal{H}}]^{\mathcal{H}}$ and $\boldsymbol{\Theta} := \boldsymbol{\Phi}_2\boldsymbol{\Phi}_1^{-1}$. Then, it follows from (5) that

$$\boldsymbol{U}^{\mathcal{H}}\tilde{\boldsymbol{V}}\boldsymbol{D}_H\boldsymbol{R} = \boldsymbol{0}. \qquad (14)$$

Suppose that there exist two non-trivial solutions of (14) that are denoted by $\boldsymbol{h} := [h(0), \dots, h(L)]^T$ and $\boldsymbol{h}' := [h'(0), \dots, h'(L)]^T$, respectively. Let us define $\boldsymbol{D}_{H'} := \mathrm{diag}[H'(v_0), \dots, H'(v_{M+K-L-1})]$, where $H'(z) = \sum_{l=0}^{L} h'(l)z^{-l}$. Since $\mathrm{rank}\{\boldsymbol{U}^{\mathcal{H}}\tilde{\boldsymbol{V}}\} = K - L$ and $\mathrm{rank}\{\boldsymbol{D}_H\boldsymbol{R}\} = M$, it follows from (14) and $\boldsymbol{U}^{\mathcal{H}}\tilde{\boldsymbol{V}}\boldsymbol{D}_{H'}\boldsymbol{R} = \boldsymbol{0}$ that $\boldsymbol{D}_{H'}\boldsymbol{R} = \boldsymbol{D}_H\boldsymbol{R}\boldsymbol{\Psi}$, where $\boldsymbol{\Psi}$ is an $M \times M$ nonsingular matrix.

Let us partition $\boldsymbol{D}_H$ to obtain $\boldsymbol{D}_H = \mathrm{diag}(\boldsymbol{D}_{H,1}, \boldsymbol{D}_{H,2})$, where $\boldsymbol{D}_{H,1}$ and $\boldsymbol{D}_{H,2}$ is an $M \times M$ and an $(K-L) \times (K-L)$ submatrix of $\boldsymbol{D}_H$. Similarly, we partition $\boldsymbol{D}_{H'}$. Then, $\boldsymbol{D}_{H'}\boldsymbol{R} = \boldsymbol{D}_H\boldsymbol{R}\boldsymbol{\Psi}$ is re-expressed as

$$\boldsymbol{D}_{H',1} = \boldsymbol{D}_{H,1}\boldsymbol{\Psi} \qquad (15)$$

$$\boldsymbol{D}_{H',2}\boldsymbol{\Theta} = \boldsymbol{D}_{H,2}\boldsymbol{\Theta}\boldsymbol{\Psi}. \qquad (16)$$

Since every diagonal entry of $\boldsymbol{D}_{H,1}$ is non-zero and $\boldsymbol{\Psi}$ is nonsingular, from (15), $\boldsymbol{\Psi}$ has to be diagonal with non-zero diagonal entries. We substitute $\boldsymbol{\Psi} := \mathrm{diag}(\psi_0, \psi_1, \dots, \psi_{M-1})$ into (16) to obtain

$$H'(v_{M+l})\boldsymbol{\theta}_l^T = H(v_{M+l})\boldsymbol{\theta}_l^T \,\mathrm{diag}(\psi_0, \psi_1, \dots, \psi_{M-1}), \qquad (17)$$

for $l \in [0, K-L-1]$, where $\boldsymbol{\theta}_l^T$ is the lth row of $\boldsymbol{\Theta}$.

It is easy to see that a necessary condition for $\boldsymbol{\Phi}$ to have the property that any M rows are linearly independent is that all entries of $\boldsymbol{\Theta}$ are non-zero. Suppose that $H(v_{M+l}) \neq 0$ for some $l \in [0, K-L-1]$. Then, from (17) and $\boldsymbol{\theta}_l \neq \boldsymbol{0}$, all ψ_i should be the same, which means from (15) that $\boldsymbol{D}_{H',1} = c\boldsymbol{D}_{H,2}$ and hence $\boldsymbol{h}' = c\boldsymbol{h}$ for a non-zero constant c. If $H(v_{M+l}) = 0$ for all $l \in [0, K-L-1]$, which is possible only if $K = 2L$, then it follows from (17) that $H'(v_{M+l}) = 0$ for all $l \in [0, L-1]$, i.e., $\boldsymbol{h}' = c\boldsymbol{h}$. Thus, we can conclude that for $K \geq 2L$, $\boldsymbol{h}' = c\boldsymbol{h}$, which completes the proof.

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