

Linearly Precoded or Coded OFDM against Wireless Channel Fades?

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Abstract — **Orthogonal Frequency Division Multiplexing (OFDM) converts a dispersive channel into parallel subchannels and thus facilitates equalization. But when the channel has nulls close to or on the FFT grid, OFDM faces serious symbol recovery problems. As an alternative to various error-control coding techniques that have been proposed to ameliorate the problem, we linearly precode (LP) symbols before they are multiplexed. We identify the maximum achievable diversity order for i.i.d. or correlated Rayleigh fading channels and also provide design rules for achieving maximum diversity gains with LP-OFDM. Simulated performance comparisons of LP-OFDM with existing block and convolutionally coded OFDM alternatives favor LP-OFDM in a HiperLan2 experiment.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has received a lot of attention recently. It has been included in digital audio/video broadcasting (DAB/DVB) standards in Europe, successfully applied to high-speed digital subscriber line (DSL) modems in the US, and recently proposed for digital cable television systems and local area mobile wireless networks such as the IEEE802.11a and the HiperLAN [1].

By implementing IFFT at the transmitter and FFT at the receiver, OFDM converts an intersymbol interference (ISI)-inducing channel with additive white Gaussian noise (AWGN) into parallel ISI-free subchannels with gains equal to the channel's frequency response values on the FFT grid. Each subchannel can be easily equalized by a single-tap equalizer using scalar division. To avoid inter-block interference (IBI) between successive IFFT processed blocks, a cyclic prefix (CP) of length greater than or equal to the channel order is inserted per block at the transmitter and discarded at the receiver. In addition to suppressing IBI, the CP also converts linear convolution into circular convolution and thus facilitates diagonalization of the associated channel matrix (see e.g., [9]).

Instead of having (superimposed) delayed and scaled replicas of the transmitted symbols as in serial transmission, one penalty of the channel diagonalization in OFDM is the loss of diversity that becomes available from the multipath propagation. Indeed, each OFDM subchannel has its gain being expressed as a linear combination of the dispersive channel taps. When the channel has nulls (deep fades) close to or on the FFT grid, reliable detection of the symbols carried by these faded subcarriers becomes difficult if not impossible.

Error-control codes are usually invoked before the IFFT processing to deal with such a loss in diversity which causes loss in performance. These include convolutional codes, Trellis Coded Modulation (TCM) or coset codes, Turbo-codes, block codes (e.g., Reed-Solomon or BCH), just to mention the common ones.

Such coded OFDM (COFDM) schemes often incur high complexity and/or large decoding delay [2]. Some of them also require Channel State Information (CSI) at the transmitter [5,6], which may be unrealistic or too costly to acquire in wireless applications where the channel changes on a constant basis. Another approach to guaranteeing symbol detectability over ISI channels is to modify the OFDM setup: instead of introducing the CP, each IFFT processed block is zero padded (ZP) by at least as many zeros as the channel order [3,9].

Our way of robustifying OFDM against random frequency-selective fading is to introduce memory into the transmission by *linear precoding* (LP) across the subcarriers (see also [4] where LP is used in conjunction with training). Specifically, instead of sending uncoded symbols (one per subcarrier), our idea is to send linearly combined symbols on the subcarriers. We also design the linear combinations so that maximum diversity gain can be guaranteed without an essential decrease in transmission rate.

Our LP-OFDM system model is presented in Section II. The system performance analysis and optimal design rules are given in Section III. Section IV deals with decoder design and Section V is devoted to comparison of the proposed system with other existing schemes by simulations.

Notations: Bold upper (lower, resp.) letters denote matrices (vectors, resp.); $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose; $[\cdot]_{i,j}$ denotes the (i,j) th entry of a matrix; \mathbf{I}_M denotes identity matrix of size M ; $\text{diag}(\mathbf{x})$ is a diagonal matrix with \mathbf{x} on its diagonal; $\text{E}[\cdot]$ denotes statistical average. We always index matrix and vector entries starting from 0.

II. LINEARLY PRECODED OFDM

Figure 1 represents a LP-OFDM system with N subcarriers, where due to CP-insertion at the transmitter and CP-removal at the receiver, the dispersive channel is represented as an $N \times N$ circulant matrix $\tilde{\mathbf{H}}$, with $[\tilde{\mathbf{H}}]_{i,j} = h((i-j) \bmod N)$, where $h(\cdot)$ denotes the channel impulse response. We assume the channel to be random FIR, consisting of $L+1$ equally spaced zero-mean (possibly correlated) complex Gaussian taps. Let \mathbf{F} denote $N \times N$ FFT matrix with $[\mathbf{F}]_{n,k} = (1/\sqrt{N}) \exp(-j2\pi nk/N)$. Performing IFFT (postmultiplication with the matrix \mathbf{F}^H) at the transmitter and FFT (premultiplication with the matrix \mathbf{F}) at the receiver diagonalizes the circulant matrix $\tilde{\mathbf{H}}$. So, we obtain the parallel ISI-free model for the i th OFDM symbol as (see Figure 1): $\mathbf{x}_i = \mathbf{D}_H \mathbf{u}_i + \boldsymbol{\eta}_i$, where $\mathbf{D}_H := \text{diag}[H(j0), H(j2\pi \frac{1}{N}), \dots, H(j2\pi \frac{N-1}{N})]$, with $H(j\omega)$ being the channel frequency response at ω ; and $\boldsymbol{\eta}_i$ the FFT-processed AWGN.

The difference of our LP-OFDM from the uncoded conventional OFDM is that at the transmitter, instead of transmitting N uncoded symbols (one-per-subcarrier), we first linearly precode $K \leq N$ symbols in the i th block, \mathbf{s}_i , by an $N \times K$ matrix $\boldsymbol{\Theta} \in \mathbb{C}^{N \times K}$ and then multiplex the precoded symbols $\mathbf{u}_i = \boldsymbol{\Theta} \mathbf{s}_i$. Note that here the precoder $\boldsymbol{\Theta}$ does not depend on the OFDM symbol index i . Time-varying precoders may be useful for certain purposes (e.g., power loading), but they will not be pursued here. Combining the precoder with the diagonalized channel model, the i th received block after CP removal

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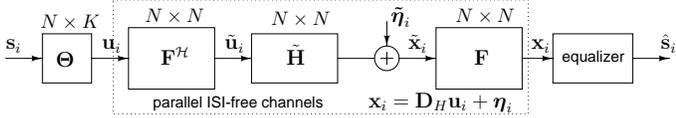


Figure 1: OFDM with Linear Precoding (LP-OFDM)

and FFT processing can be written as:

$$\mathbf{x}_i = \mathbf{F}\tilde{\mathbf{x}}_i = \mathbf{F}(\tilde{\mathbf{H}}\mathbf{F}^H\Theta\mathbf{s}_i + \tilde{\boldsymbol{\eta}}_i) = \mathbf{D}_H\Theta\mathbf{s}_i + \boldsymbol{\eta}_i. \quad (1)$$

Before exploring optimal designs of Θ , let us first look at some special cases of the LP-OFDM system.

C1: (Uncoded OFDM): By setting $K = N$ and $\Theta = \mathbf{I}_N$, we obtain the conventional uncoded OFDM model. In such a case, the one-tap linear equalizer matrix $\mathbf{\Gamma} = \mathbf{D}_H^{-1}$ yields $\hat{\mathbf{s}}_i = \mathbf{\Gamma}\mathbf{x}_i = \mathbf{s}_i + \mathbf{D}_H^{-1}\boldsymbol{\eta}_i$. Under the AWGN assumption, such an equalizer followed by a minimum distance quantizer is optimum in the maximum-likelihood (ML) sense for a given channel with CSI available at receiver. But when the channel has nulls on (or close to) the FFT grid $2\pi n/N$, $n = 0, \dots, N-1$, the matrix \mathbf{D}_H will be ill-conditioned and serious noise-amplification will emerge. Although events of channel nulls being close to FFT grid have relatively low probability, their occurrence has a major impact on the average system performance. Improving the performance of an uncoded transmission thus relies on robustifying the system against the occurrence of such rare but catastrophic events. In this paper, we achieve such a goal through judicious choice of redundant ($N > K$) linear precoders.

C2: (CP-only Transmission): If we choose $K = N$ and $\Theta = \mathbf{F}$, then the IFFT \mathbf{F}^H annihilates precoding and the resulting system is a single-carrier block transmission with CP insertion (c.f. Figure 1): $\tilde{\mathbf{x}}_i = \tilde{\mathbf{H}}\mathbf{s}_i + \tilde{\boldsymbol{\eta}}_i$. The FFT at the receiver is no longer necessary [9].

C3: (ZP-only Transmission): Let $K = N - L$. We choose Θ to be a $N \times K$ truncated FFT matrix (the first K columns of \mathbf{F}); i.e., $[\Theta]_{n,k} = (1/\sqrt{N})\exp(-j2\pi nk/N)$. It can be easily verified that $\mathbf{F}^H\Theta = [\mathbf{I}_K, \mathbf{0}_{K \times L}]^T := \mathbf{T}_{z_p}$, where $\mathbf{0}_{K \times L}$ denotes a $K \times L$ all-zero matrix, and the subscript “z_p” stands for zero-padding (ZP). The matrix \mathbf{T}_{z_p} simply pads zeros at \mathbf{s}_i and the zero-padded block $\tilde{\mathbf{u}}_i = \mathbf{T}_{z_p}\mathbf{s}_i$ is transmitted. Notice that $\mathbf{H} := \tilde{\mathbf{H}}\mathbf{F}^H\Theta = \tilde{\mathbf{H}}\mathbf{T}_{z_p}$ is an $N \times K$ Toeplitz convolution matrix (the first K columns of $\tilde{\mathbf{H}}$), which is always full rank. The symbols \mathbf{s}_i can thus always be recovered from the received signal $\tilde{\mathbf{x}}_i = \mathbf{H}\mathbf{s}_i + \tilde{\boldsymbol{\eta}}_i$ (perfectly in the absence of noise) and no catastrophic channels exist in this case [9]. Also, the CP is not necessary here because the ZP already serves the purpose of separating successive blocks in this single-carrier block transmission.

III. DESIGN CRITERIA AND OPTIMIZATION

To design Θ , we will resort to the pair-wise error probability (PEP) analysis technique that in a different context has been also used in e.g., [7].

For simplicity, we will drop the OFDM symbol index i , and we will assume that the channel $\mathbf{h} := [h(0), h(1), \dots, h(L)]^T$ has independent and identically distributed (i.i.d.) zero-mean complex Gaussian taps (Rayleigh fading), and unit energy. The corresponding correlation matrix of \mathbf{h} is thus $\mathbf{R}_h = \alpha_L \mathbf{I}_{L+1}$, where the constant $\alpha_L := 1/\sqrt{L+1}$.

We suppose ML detection with perfect CSI at the receiver and consider the probability $P(\mathbf{s} \rightarrow \mathbf{s}' | \mathbf{h})$ that a vector \mathbf{s} is transmitted but is erroneously decoded as $\mathbf{s}' \neq \mathbf{s}$.

The PEP can be approximated using the Chernoff bound as:

$$P(\mathbf{s} \rightarrow \mathbf{s}' | \mathbf{h}) \leq \exp(-d^2(\mathbf{y}, \mathbf{y}')/4N_0), \quad (2)$$

where $N_0/2$ is the noise variance per dimension, $\mathbf{y} := \mathbf{D}_H\Theta\mathbf{s}$, $\mathbf{y}' := \mathbf{D}_H\Theta\mathbf{s}'$, and $d^2(\mathbf{y}, \mathbf{y}') = |\mathbf{y} - \mathbf{y}'|^2$ is the Euclidean distance between \mathbf{y} and \mathbf{y}' .

Let us consider the $N \times (L+1)$ matrix \mathbf{V} with entries $[\mathbf{V}]_{n,l} = \exp(-j2\pi nl/N)$, and use it to perform the N -point Fourier transform $\mathbf{V}\mathbf{h}$ of \mathbf{h} . Note that $\mathbf{D}_H = \text{diag}(\mathbf{V}\mathbf{h})$. Using the definitions $\mathbf{e} := \mathbf{s} - \mathbf{s}'$, $\mathbf{u}_e := \Theta\mathbf{s} - \Theta\mathbf{s}'$, and $\mathbf{D}_e := \text{diag}(\mathbf{u}_e)$, we can write $\mathbf{y} - \mathbf{y}' = \mathbf{D}_H\mathbf{u}_e = \text{diag}(\mathbf{V}\mathbf{h})\mathbf{u}_e$. Furthermore, we can express the distance $d^2(\mathbf{y}, \mathbf{y}') = |\mathbf{D}_H\mathbf{u}_e|^2 = |\mathbf{D}_e\mathbf{V}\mathbf{h}|^2$ as

$$d^2(\mathbf{y}, \mathbf{y}') = \mathbf{h}^H \mathbf{V}^H \mathbf{D}_e^H \mathbf{D}_e \mathbf{V} \mathbf{h} := \mathbf{h}^H \mathbf{A}_e \mathbf{h}. \quad (3)$$

Matrix $\mathbf{A}_e = \mathbf{V}^H \mathbf{D}_e^H \mathbf{D}_e \mathbf{V}$ in (3) is symmetric and non-negative definite; therefore, there exists a unitary matrix \mathbf{U}_e such that $\mathbf{U}_e^H \mathbf{A}_e \mathbf{U}_e = \boldsymbol{\Lambda}_e$, where $\boldsymbol{\Lambda}_e$ is a diagonal matrix with non-increasing diagonal entries $\lambda_{e,0}, \lambda_{e,1}, \dots, \lambda_{e,L}$. Define $\bar{\mathbf{h}} := \mathbf{U}_e \mathbf{h}$, with correlation matrix $\mathbf{R}_{\bar{\mathbf{h}}} = \mathbf{U}_e^H \mathbf{R}_h \mathbf{U}_e = \alpha_L \mathbf{U}_e^H \mathbf{U}_e = \alpha_L \mathbf{I}_{L+1}$. Therefore, $\bar{\mathbf{h}}$ is also a zero-mean complex Gaussian vector with i.i.d. entries. With $\bar{\mathbf{h}}$, we can rewrite (3) as

$$d^2(\mathbf{y}, \mathbf{y}') = \mathbf{h}^H \mathbf{U}_e^H \boldsymbol{\Lambda}_e \mathbf{U}_e \mathbf{h} = \bar{\mathbf{h}}^H \boldsymbol{\Lambda} \bar{\mathbf{h}} = \sum_{l=0}^L \lambda_{e,l} |\bar{h}_l|^2. \quad (4)$$

Using (4), we average (2) with respect to the i.i.d. Rayleigh random variables $|\bar{h}_l|$ and obtain the following upper bound on the average PEP:

$$P(\mathbf{s} \rightarrow \mathbf{s}') \leq \prod_{l=0}^L \frac{1}{1 + \lambda_{e,l}/(4N_0)}. \quad (5)$$

If r_e is the rank of \mathbf{A}_e , then $\lambda_{e,l} \neq 0$ if and only if $l \in [0, r_e - 1]$. It thus follows from (5) that

$$P(\mathbf{s} \rightarrow \mathbf{s}') \leq (1/N_0)^{-r_e} \left(\prod_{l=0}^{r_e-1} \lambda_{e,l} \right)^{-1}. \quad (6)$$

As in [7], we call r_e the diversity advantage $G_{d,e}$ and $(\prod_{l=0}^{r_e-1} \lambda_{e,l})^{1/r_e}$ the coding advantage $G_{c,e}$ of the system for the symbol error vector \mathbf{e} . The diversity advantage $G_{d,e}$ determines the slope of the averaged (w.r.t. the random channel) PEP (between \mathbf{s} and \mathbf{s}') as a function of the signal-to-noise ratio (SNR) at high SNR ($N_0 \rightarrow 0$). Correspondingly, $G_{c,e}$ determines the shift of this PEP curve in SNR relative to a benchmark error rate curve of $(1/N_0)^{-r_e}$.

Since both $G_{d,e}$ and $G_{c,e}$ depend on the choice of \mathbf{e} (thus on \mathbf{s} and \mathbf{s}'), we define the diversity and coding advantages for our LP-OFDM system respectively as:

$$G_d := \min_{\mathbf{e} \neq \mathbf{0}} G_{d,e}, \quad \text{and} \quad G_c := \min_{\mathbf{e} \neq \mathbf{0}} G_{c,e}. \quad (7)$$

Since the diversity advantage G_d determines how fast the symbol error probability drops as SNR increases, G_d is to be optimized first. Because \mathbf{A}_e is of size $(L+1) \times (L+1)$, we have that $r_e \leq L+1, \forall \mathbf{e}$. Thus, we obtain the following theorem.

Theorem 1 (Maximum Diversity Advantage) *The Maximum Diversity Advantage (MDA) of LP-OFDM transmissions over i.i.d. FIR Rayleigh fading channels of order L is $L+1$.*

Theorem 1 is intuitively reasonable because the FIR Rayleigh fading channel offers us $L+1$ independent fading taps, which is the maximum possible number of independent replicas of the transmitted signal in the serial transmission mode.

Remark 1: The MDA property of LP-OFDM can also be inherited by conventional OFDM systems provided that channel coding or interleaving is applied and that no memory is introduced across successive OFDM symbols. To see this, it suffices to view the precoded error vector \mathbf{u}_e as the error vector between two coded blocks.

To achieve MDA, we need \mathbf{A}_e to be full rank and thus positive definite for any $\mathbf{e} = \mathbf{s} - \mathbf{s}'$ when $\mathbf{s} \neq \mathbf{s}'$. This is true if and only if $\mathbf{h}^H \mathbf{A}_e \mathbf{h} > 0$ for any $\mathbf{h} \neq \mathbf{0} \in \mathbb{C}^{L+1}$. Equation (3) shows that this is equivalent to $d^2(\mathbf{y}, \mathbf{y}') = |\mathbf{D}_H \Theta \mathbf{e}|^2 \neq 0$ for any $\mathbf{e} \neq \mathbf{0}$ and for any channel \mathbf{h} . The latter means that any two different transmitted vectors should result in different received vectors in the absence of noise, irrespective of the channel. The conditions for achieving MDA and channel-irrespective symbol detectability are summarized in the following theorem:

Theorem 2 (Symbol Detectability \Leftrightarrow MDA) *Under the channel conditions of Theorem 1, the maximum diversity advantage is achieved if and only if symbol detectability is achieved; i.e., $|\mathbf{D}_H \Theta \mathbf{e}|^2 \neq 0, \forall \mathbf{e} = \mathbf{s} - \mathbf{s}' \neq \mathbf{0}$ and $\forall \mathbf{h} \neq \mathbf{0}$.*

We define the Hamming distance between two vectors \mathbf{u} and \mathbf{u}' as the number of non-zero entries in the vector $\mathbf{u} - \mathbf{u}'$. Viewing the LP matrix Θ as a block (K, N) code and following coding theory terminology, we call the precoded symbols *Maximum Distance Separable (MDS)* if the minimum Hamming distance between two precoded vectors is $d_{min} = N - K + 1$. Such a precoder achieves the maximum possible Hamming distance $N - K + 1$ (Singleton Bound) if we allow \mathbf{s} to be any vector in \mathbb{C}^K ; e.g., when we do not require the entries of \mathbf{s} to be drawn from a finite alphabet.

Because \mathbf{D}_H has on its diagonal entries frequency response values (evaluated on the FFT grid) of the order- L channel, there can be at most L zero entries on the diagonal of \mathbf{D}_H . To satisfy the condition in Theorem 2 when $N = K + L$, we need $\mathbf{u}_e = \Theta \mathbf{e}$ to have Hamming distance no less than $L + 1 = N - K + 1$. In other words, we need the precoded symbols to be MDS.

Remark 2: Such an MDS property is also achieved by error control codes such as the (extended) Reed-Solomon codes, repetition codes, and single-parity-check codes. But all these codes require that the coded symbols to come from a given fixed Galois Field. Although Reed-Solomon codes are the least restrictive among them in terms of the number of elements in the field, they are constrained on the code length and the alphabet size. Our linear precoders Θ , on the other hand, operate over the complex field with no restriction on the input symbol alphabet or the precoded symbol alphabet¹.

Theoretically, the symbol detectability condition in Theorem 2 should be checked against all pairs \mathbf{s} and \mathbf{s}' , which is usually not an easy task, especially when the underlying constellations are large and/or when the size K of \mathbf{s} is large. But it is possible to identify sufficient conditions that are relatively easy to check. One such condition is given in the following theorem that is stated without proof due to space limitations.

Theorem 3 (Sufficient Condition for MDA) *Under the channel conditions of Theorem 1, MDA is achieved when $\text{rank}(\mathbf{D}_H \Theta) = K, \forall \mathbf{h} \neq \mathbf{0}$, which is equivalent to the following condition: Any $N - L$ rows of Θ span the $\mathbb{C}^{1 \times K}$ space. The latter in turn implies that $N - L \geq K$.*

To satisfy Theorem 3 and transmit as many symbols in one block as possible, we should have $K = N - L$ and any K rows of Θ be linearly independent.

¹The fact that precoded symbols do not necessarily belong to a given constellation should not be viewed as undesirable in OFDM, simply because the IFFT will be applied to yield a transmitted signal with values over the complex field, anyways.

Notice that the conditions given in Theorem 3 have also been recognized in e.g., [9], for channel-irrespective symbol recovery. Based on the equivalence established in Theorem 2, it should not be surprising that they appear here again.

The following theorem constructively shows two classes of precoders that satisfy Theorem 3 and thus achieve MDA. We omit its proof here due to lack of space.

Theorem 4 (MDA-achieving precoders)

i) **Vandermonde Precoders:** *Choose N points $\rho_n \in \mathbb{C}, n = 0, 1, \dots, N - 1$, such that $\rho_m \neq \rho_n, \forall m \neq n$. Let $\boldsymbol{\rho} := [\rho_0, \rho_1, \dots, \rho_{N-1}]^T$. Then the Vandermonde precoder $\Theta(\boldsymbol{\rho}) \in \mathbb{C}^{N \times K}$ defined by $[\Theta(\boldsymbol{\rho})]_{n,k} = \rho_n^k$ satisfies Theorem 3 and thus achieves MDA.*

ii) **Cosine Precoders:** *Choose N points $\phi_0, \phi_1, \dots, \phi_{N-1} \in \mathbb{R}$, such that $\phi_m \pm \phi_n \neq 2k\pi, \forall m \neq n$ and $\forall k \in \mathbb{Z}$. Let $\boldsymbol{\phi} := [\phi_0, \phi_1, \dots, \phi_{N-1}]^T$. Then the real cosine precoder $\Theta(\boldsymbol{\phi}) \in \mathbb{R}^{N \times K}$ defined by $[\Theta(\boldsymbol{\phi})]_{n,k} = \cos(k + \frac{1}{2})\phi_n$ satisfies Theorem 3 and thus achieves MDA.*

Special cases of the construction in Theorem 4 include selecting $\rho_n = \exp(-j2\pi n/N)$ which will result in a precoding matrix that is a truncated FFT matrix as in the ZP-only transmission case C3. This from a different perspective explains the robustness of ZP-only transmissions because *ZP-only transmissions can indeed achieve MDA*. A special case of the general cosine precoder in ii) is a truncated DCT (Discrete Cosine Transform) matrix defined by choosing $\phi_n = n\pi/N$. Real cosine precoders lead to lower-complexity decoders and have the potential to improve performance in terms of the coding advantage G_c .

Notice that up to now we have been assuming that the channel consists of i.i.d. zero-mean complex Gaussian taps. Such a model is well suited for studying average system performance in wireless fading channels, but is rather restrictive since in general the taps may be correlated. We state without proof (due to space limitations) the following theorem.

Theorem 5 (MDA of Correlated Rayleigh Channels) *Let the channel \mathbf{h} be zero-mean complex Gaussian with correlation matrix \mathbf{R}_h . The maximum achievable diversity advantage equals the rank of \mathbf{R}_h and is achieved by any precoder that achieves MDA with i.i.d. Rayleigh channels.*

Theorem 5 asserts that $\text{rank}(\mathbf{R}_h)$ is the MDA for LP-OFDM systems as well as for COFDM systems that do not code or interleave across OFDM symbols. Also, MDA precoded transmissions through i.i.d. channels can achieve MDA for correlated channels as well.

Coding advantage G_c is another parameter that needs to be optimized among the MDA-achieving precoders. Pertinent optimization criteria include Minimum Mean Squared Error (MMSE) and maximum information rate. These will not be treated here due to space limitations. Some related aspects of redundant precoding are treated in [4].

IV. DECODING SCHEMES

To achieve MDA, LP-OFDM requires ML decoding. ML decoding of LP transmissions belongs to a general class of lattice decoding problems, as the matrix product $\mathbf{D}_H \Theta$ in (1) gives rise to a discrete subgroup of the \mathbb{C}^N space. An exhaustive ML search, in general, has complexity that is exponential in K .

A relatively faster ML search is possible with the sphere decoding (SD) algorithm, which only searches for precoded vectors that are within a sphere centered at the received symbol [8]. The theoretical complexity of SD is polynomial in K (e.g., $\mathcal{O}(K^6)$), which is better than exponential but still too high for practical purposes.

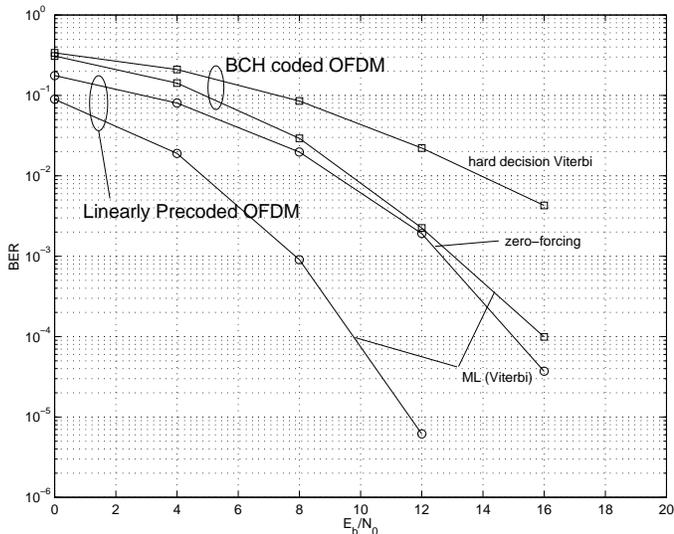


Figure 2: LP-OFDM vs. COFDM with BCH codes

Zero-forcing (ZF) and MMSE equalizers certainly offer low-complexity alternatives. They require $\mathcal{O}(N \times K)$ operations per K symbols (the operations needed to compute the equalizer are not counted). So per symbol, they require only $\mathcal{O}(N)$ operations. When the precoder is in the special form of a truncated FFT (or DCT) matrix as in the ZP-only case, FFT (or fast DCT) can be used to bring the linear equalization complexity down to $\mathcal{O}(\log(N))$ per symbol. Also, in ZP-only transmissions, the FIR channel model $\tilde{\mathbf{x}}_i = \mathbf{H}\mathbf{s}_i + \tilde{\mathbf{\eta}}_i$ accepts Viterbi decoding at complexity of $\mathcal{O}(Q^L)$, where Q is the constellation size of the symbols in \mathbf{s} .

As an example, we list in the following table the approximate number of flops needed for different decoding schemes when $K = 16$, $N = 20$, and BPSK modulation is employed.

Decoding Scheme	Flops/symbol
Exhaustive ML	$> 2^K = 2^{16} = 65,536$
Sphere Decoding	$\approx 1,000$ (empirical)
ZF/MMSE	$\approx N = 20$
ZF/MMSE w/ FFT	$\log_2(N) \approx 5$
Viterbi for ZP-only	$2^L = 2^4 = 16$

Remark 3: When the number of carriers N is very large (e.g., 1,024), it is desirable to keep the decoding complexity manageable. To achieve this we can split the precoder into several smaller precoders. Specifically, we can choose $\Theta = \mathbf{P}\Theta'$ where \mathbf{P} is a permutation matrix that interleaves the subcarriers, and Θ' is a block diagonal matrix. Decoding $\Theta\mathbf{s}$ is now equivalent to decoding a few precoded sub-vectors of smaller sizes. Such a decomposition is particularly important when a high complexity decoder such as the sphere decoder is to be employed.

V. SIMULATED COMPARISONS

Test Case 1: For demonstration and verification purposes, we first compare LP-OFDM with COFDM that uses Galois Field coding. The channel is modeled as FIR with 5 i.i.d. Rayleigh distributed equally spaced taps. In Figure 2, we illustrate Bit Error Rate (BER) performance of LP-OFDM with Vandermonde precoding and that of COFDM with BCH coding. The system parameters are $K = 26$, $N = 31$. Since the (26,31) BCH code can correct only one error, it has minimum Hamming distance 3, and possesses a diversity advantage of order 3 only, which is only half of the maximum possible ($L + 1 = 6$) that LP-OFDM achieves with the same spectral efficiency. We can

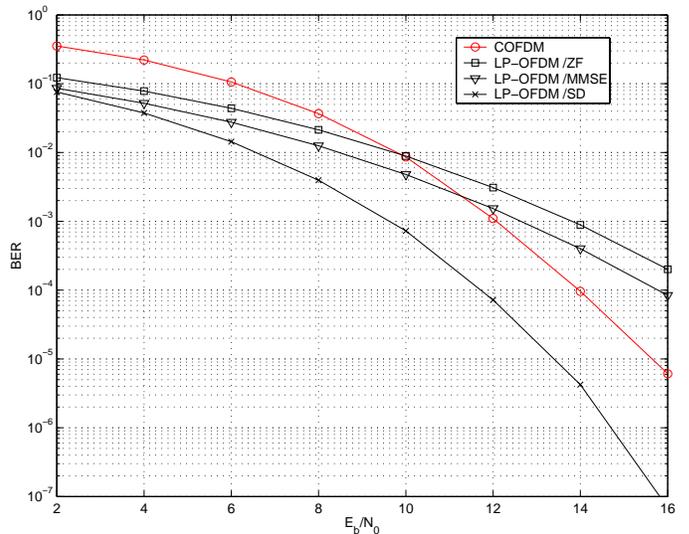


Figure 3: LP-OFDM vs. COFDM in HyperLAN

see that when the optimum ML decoder is employed by both receivers, LP-OFDM outperforms COFDM with BCH coding considerably. The slopes of the corresponding BER curves also confirm the theoretical observations.

Test Case 2: We have also compared (see Figure 3) our LP-OFDM system with truncated DCT precoder against convolutionally coded OFDM (with a rate 1/2 code punctured to rate 3/4 followed by interleaving) that is employed by the HyperLAN standard [1] over the channels used in Test Case 1. With ML decoding, LP-OFDM performs about 2dB better than COFDM. Surprisingly, even with linear MMSE equalization, the performance of LP-OFDM is better than COFDM for SNR values less than 12 dB.

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