Transmitter Induced Cyclostationarity for Blind Channel Equalization

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***Abstract*—Fractional sampling has received considerable interest recently as a means of developing blind equalization techniques without resorting to higher order statistics. Instead, cyclostationarity introduced at the receiver by fractional sampling is exploited. In this paper, we show that simpler solutions are possible if cyclostationarity is introduced at the transmitter instead of the receiver. We propose specific coding and interleaving strategies at the transmitter that induce cyclostationarity and facilitate the equalization task. Novel batch and adaptive equalization algorithms are derived that make no assumptions on the channel zeros locations. Subspace methods are also proposed and, in the absence of noise, guarantee perfect estimation from finite data. Synchronization issues and bandwidth considerations are briefly discussed, and simulation examples are presented.**

I. Introduction

BLIND or "self-recovering" channel equalization techniques simplify the design and management of multipoint networks, as they require no individual training of each network node [14], [16]. Moreover, in the case of fading communication links, like the ones encountered in TDMA mobile radio channels, blind techniques can potentially eliminate the need for periodic retraining of the receiver, and thereby increase the data throughput.

Blind equalizers can generally be divided in symbol spaced and fractionally spaced ones, depending on the receiver's sampling rate (e.g., [24]). Unfortunately, nonminimum phase channels cannot be successfully estimated or equalized in a symbol-spaced framework unless the receiver relies (explicitly or implicitly) on higher-than-second-order statistics of the received signal [1], [25], [23]. This fact limits the applicability of symbol spaced schemes since long data records, which are required for accurate estimation of higher order statistics (HOS), are either not available in a rapidly fading environment or violate the time invariance assumption for the underlying channel.

The fractional sampling framework, on the other hand, presents the possibility of estimating a certain class of channels using only second-order statistics [29]. For this reason, it has received a growing interest recently [7], [8], [17], [20], [26], [29], [34]. However, not all channels fall in that class [32], and HOS methods are also popular in the FSE setup (most notably the constant modulus algorithm [19]).

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In essence, the FSE approach consists of altering the hardware structure of the receiver (sampling rate) in order to facilitate the removal of intersymbol interference (ISI). Following the same reasoning, one may consider altering the structure of the transmitter for the same purpose. In this work, we propose a specific coding and interleaving of the input symbols, prior to their transmission, so that the ISI removal procedure at the receiver is greatly simplified. The proposed method can guarantee identifiability of all FIR channels regardless of their zero locations. Thus, it overcomes the limitations of FSE schemes without resorting to HOS. Moreover, it allows for a variety of batch and adaptive algorithms, some of which turn out to be surprisingly simple.

Traditionally, channel coding has been performed with the sole objective of error correction in mind and with little concern about channel dispersion problems. On the other hand, channel equalization methods typically assume i.i.d. inputs, ignoring any possible coding at the transmitter. In this paper, we introduce a novel viewpoint where coding information can be exploited to facilitate the receiver's equalization task.

The price paid for these advantages is the introduction of a small decoding delay, which is equal to a few symbol periods, due to coding and interleaving in the transmitter. In addition, a moderate increase in the transmitter complexity is introduced.

The details of the problem statement are presented in the next session, whereas the novel coding and interleaving approach is analyzed in Section III. A globally convergent adaptive channel estimation algorithm that illustrates the feasibility of our approach is developed. More accurate batch methods are also developed based on subspace approaches. Synchronization issues, as well as power and bandwidth considerations, are briefly discussed in Section VI, and some experimental comparisons are presented in Section VII.

II. Problem Statement

Let us consider a linear modulation system, where the received continuous time signal can be expressed as

$$y_c(t) = \sum_{k=-\infty}^{\infty} w(k) h_c(t-kT) + v_c(t) \tag{1}$$

where

- $_c$ continuous time signals;
- $w(k)$ i.i.d., zero mean information symbol stream,
- T symbol period;
- $v_c(t)$ additive, zero mean noise;
- $h_c(t)$ combined impulse response of the channel along with the transmitter and receiver's filters.

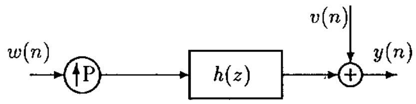

Fig. 1. FS multirate channel model.

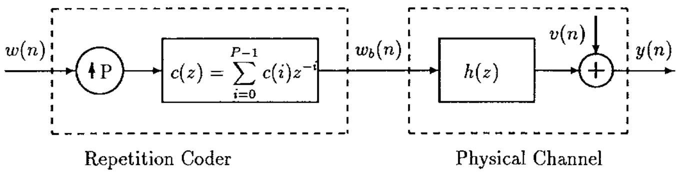

Fig. 2. Repetition coding model.

If fractional sampling is performed at the receiver and P samples are collected per symbol period, then the discrete-time received signal is

$$y(n) := y_c(nT/P) = \sum_{k=-\infty}^{\infty} w(k) h_c((n-kP)T/P) + v_c(nT/P). \quad (2)$$

The discrete time model of (2), shown in Fig. 1, indicates that $y(n)$ is cyclostationary with period P (e.g., [10]).

A number of blind methods have been developed to identify the channel by exploiting either the cyclostationarity of $y(n)$ in Fig. 1 [7], [10] or an equivalent multichannel formulation [22], [26], [34].

Since the role of cyclostationarity is important in the FSE approach, it is natural to examine if other forms of cyclostationarity in the transmitted signal can also be beneficial for equalization. In this work, we will consider the case where the transmitter repeats each symbol P times

$$w_b(k) = \sum_{l} w(l) c(k - lP) \quad (3)$$

where $w_b(k)$ is the repeated or blocked transmitted signal for some constants $c(k), k = 0, \cdots, P-1$ (exact repetition corresponds to $c(k) = 1, k = 0, \cdots, P-1$). If the symbol period is decreased by a factor of P so that the information rate does not change, then the received signal is

$$y(n) := y_c(nT/P) = \sum_{k=-\infty}^{\infty} w_b(k) h_c((n-k)T/P) + v_c(nT/P) \quad (4)$$

where $w_b(k)$ is given by (3). Fig. 2 shows the equivalent discrete time model for this symbol repetition framework.

No motivation has been given up to this point for introducing this new architecture. Although the benefits will become clear later on, it is instructive to notice here the similarities between Figs. 1 and 2 [or (1), (3), or (4)]. Note that with $c(k) = \delta(k)$, the two setups can become identical.

A number of different interpretations can be given to this repetition framework. If the pulse duration remains unchanged so that $h_c(t)$ is the same in (1) and (4), then some controlled ISI is introduced at the transmitter (there is more overlap between successive pulses due to the increased data rate). In this respect, the scheme is similar to partial response signaling, e.g., [24, p. 548], where controlled ISI is introduced to simplify the pulse design. The induced ISI is expected to have a negative effect in performance, but in partial response channels, this effect has been observed to be minimal [24]. One might be tempted to discard all repetition-based techniques by arguing that it is preferable to insert training symbols in the place of repeated symbols and perform trained equalization. This argument ignores the fact that in this case, half of the transmitter's power would be devoted to training (for $P = 2$), resulting in a 3-dB penalty even under perfect ISI removal.

On the other hand, if the transmitter's spectral pulse bandwidth increases by a factor of P to avoid inducing ISI, then the scheme resembles a repetition coding setup. However, due to the poor error correcting performance of repetition codes, we will not pursue this direction any further. Note that our main goal in this paper is concerned with combating ISI rather than achieving coding gain.

One exception where the increased bandwidth could be tolerated is in spread spectrum and CDMA applications. In this case, $c(k)$ should represent the spreading code [30]. Although this might present an interesting future research topic, we will focus on the narrowband case in the sequel.

Before proceeding, however, a note is due on the impact of more general block or convolutional codes on the statistics of the transmitted signal. Unfortunately, most common codes produce an i.i.d. output when applied to an i.i.d. equiprobable input. More specifically, let $\boldsymbol{w}$ be a vector of i.i.d., equiprobable, binary random variables, and $\boldsymbol{w}_b = \boldsymbol{C}\boldsymbol{w}$, where $\boldsymbol{C}$ is a binary matrix. If the operations are considered in the GF(2), then $\boldsymbol{w}_b$ contains i.i.d. random variables unless $\boldsymbol{C}$ has at least one zero row or at least two identical rows [2]. Therefore, the only way to introduce dependence in the input samples using a linear code is to either periodically transmit fixed symbols (null rows) or to repeat some of the (information or parity) symbols (identical rows).

Other nonlinear or spectral shaping codes could be of interest [5], [21], with applications to magnetic and optical recording [18]. Additionally, if the constraint of GF(2) is removed (as, for example, in partial response signaling), then more opportunities arise. Extension of the current analysis to those cases, however, is outside the scope of this paper.

With this in mind, the choice of the repetition framework seems less arbitrary. It is still not clear, however, why this framework is advantageous when compared with the FSE structure. It will soon become evident that the current approach is considerably more flexible as the symbols $w_b(n)$ can be interleaved to give rise to different equivalent channels, as will be explained next.

A. Coding and Interleaving

Let us consider the interleaving procedure of Fig. 3, where the input signal is partitioned in successive blocks of length $M, \boldsymbol{w}_M(l) := [w(Ml) \cdots w(Ml + M - 1)]^T$ with ^{T} denoting

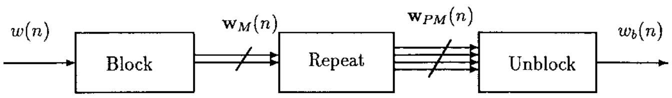

Fig. 3. Repetition coding with interleaving.

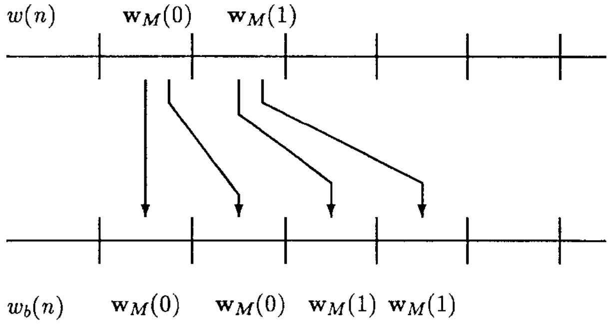

Fig. 4. Block coding/interleaving (each block repeated twice, $P = 2$).

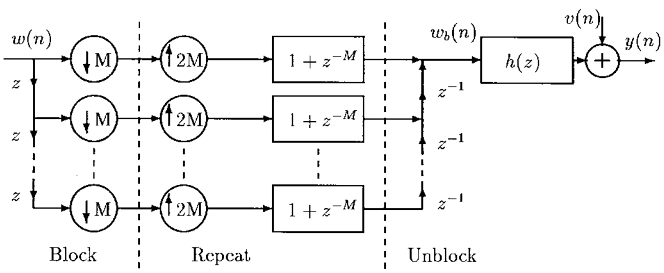

Fig. 5. Multirate structure for repetition coding.

transpose. Each block is transmitted P times, i.e.,

$$\boldsymbol{w}_b := [\underbrace{c(0)\boldsymbol{w}_M^T(0), \cdots, c(P-1)\boldsymbol{w}_M^T(0)}_{P\text{ times}}, \underbrace{c(0)\boldsymbol{w}_M^T(1), \cdots, c(P-1)\boldsymbol{w}_M^T(1)}_{P\text{ times}}, \cdots]^T$$

multiplied perhaps by different constants $c(k)$ (see also Fig. 4). More formally, the transmitted signal is

$$w_b(PMl+k) = \begin{cases} c(0)w(Ml+k) & 0 \le k < M \\ c(1)w(Ml+k-M) & M \le k < 2M \\ \vdots & \\ c(P-1)w(Ml+k-(P-1)M) & (P-1)M \le k < PM \end{cases} \tag{5}$$

whereas the received discrete-time signal is

$$y(n) = \sum_{k=0}^{q} h(k)w_b(n-k) + v(n). \tag{6}$$

Notice that fractional sampling in (6) is also possible, but it would further increase the data rate and is not considered here. In addition, for reasons of notational simplicity, only the case $P = 2$ with $c(k) = 1, k = 0, \cdots, P-1$ will be presented in the sequel. Generalizations for $P > 2$ can be derived following similar steps.

The operations carried out in Fig. 3 can be more accurately described using multirate blocks, as shown in Fig. 5 (where $P = 2$, and $c(k) = 1, 0 \le k < P$). It is easy to verify from this figure that the maximum rate change is $2M$ and that $w_b(n)$ [and hence $y(n)$] has periodically time-varying statistics with period $2M$ (see also Appendix A). Therefore, in order to stationarize the (repetition induced), cyclostationary problem, we will consider a polyphase vector representation of order $2M$. The polyphase components of $h(n)$ are defined as $h_k(n) := h(2Mn + k), k = 0, 1, \cdots, 2M-1$ and represent different decimated versions of the original impulse response $h(n)$. Using vector notation, we define $\boldsymbol{h}_{2M}(n) := [h_0(n), \cdots, h_{2M-1}(n)]^T$ and its z-transform $\boldsymbol{h}_{2M}(z) := [h_0(z), \cdots, h_{2M-1}(z)]^T$. Vector sequences $\boldsymbol{w}_b(n), \boldsymbol{v}(n)$ and $\boldsymbol{y}(n)$ are similarly defined.

III. A Simple Multirate Algorithm

We will show in the sequel that under some conditions, appropriately chosen correlation lags coincide with a scaled version of the impulse response. To this end, it would be useful to express the channel input/output relationship in a polyphase form. It can be shown using multirate theory [33, p. 431] that a time-invariant channel is described in the (polyphase) z domain by

$$\boldsymbol{y}_{2M}(z) = \boldsymbol{H}_{2M}(z)\boldsymbol{w}_{b,2M}(z) + \boldsymbol{v}_{2M}(z) \tag{7}$$

where $\boldsymbol{H}_{2M}(z)$ is a pseudo-circulant matrix of the polyphase vector $\boldsymbol{h}_{2M}(z)$

$$\boldsymbol{H}_{2M}(z) = \begin{bmatrix} h_0(z) & z^{-1}h_{2M-1}(z) & \cdots & z^{-1}h_1(z) \\ h_1(z) & h_0(z) & \ddots & \vdots \\ \vdots & \vdots & \ddots & z^{-1}h_{2M-1}(z) \\ h_{2M-1}(z) & h_{2M-2}(z) & \cdots & h_0(z) \end{bmatrix}. \tag{8}$$

Notice that due to the repetition present in the sequence $w_b(n)$, its polyphase components obey the symmetry

$$w_{b,k}(n) = w_{b,M+k}(n), \qquad k = 0, \cdots, M-1$$

or equivalently

$$\boldsymbol{w}_{b,2M}(z) = [\boldsymbol{w}_M^T(z) \quad \boldsymbol{w}_M^T(z)]^T.$$

Using this fact in (7) and (8), we obtain

$$\begin{aligned} \boldsymbol{y}_{2M}(z) &= \begin{bmatrix} \boldsymbol{H}_{11}(z) & \boldsymbol{H}_{12}(z) \\ \boldsymbol{H}_{21}(z) & \boldsymbol{H}_{22}(z) \end{bmatrix} \begin{bmatrix} \boldsymbol{w}_M(z) \\ \boldsymbol{w}_M(z) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{H}_{11}(z) + \boldsymbol{H}_{12}(z) \\ \boldsymbol{H}_{21}(z) + \boldsymbol{H}_{22}(z) \end{bmatrix} \boldsymbol{w}_M(z) + \boldsymbol{v}_{2M}(z) \end{aligned} \tag{9}$$

where from (8)

$$\boldsymbol{H}_{11}(z) = \begin{bmatrix} h_0(z) & z^{-1}h_{2M-1}(z) & \cdots & z^{-1}h_{M+1}(z) \\ h_1(z) & h_0(z) & \ddots & \vdots \\ \vdots & \vdots & \ddots & z^{-1}h_{2M-1}(z) \\ h_{M-1}(z) & h_{2M-2}(z) & \cdots & h_0(z) \end{bmatrix}$$

$$\boldsymbol{H}_{21}(z) = \begin{bmatrix} h_M(z) & \cdots & h_1(z) \\ \vdots & \ddots & \vdots \\ h_{2M-1}(z) & \cdots & h_M(z) \end{bmatrix}. \tag{10}$$

Due to the pseudocirculant nature of $\boldsymbol{H}_{2M}(z)$, we have

$$\boldsymbol{H}_{22}(z)=\boldsymbol{H}_{11}(z) \quad \text{and} \quad \boldsymbol{H}_{12}(z)=z^{-1}\boldsymbol{H}_{21}(z).$$

Hence, (9) can be written as

$$\boldsymbol{y}_{2M}(z)=\begin{bmatrix}\boldsymbol{H}_{11}(z)+z^{-1}\boldsymbol{H}_{21}(z)\\ \boldsymbol{H}_{21}(z)+\boldsymbol{H}_{11}(z)\end{bmatrix}\boldsymbol{w}_M(z)+\boldsymbol{v}_{2M}(z). \quad (11)$$

The special structure of this input/output relationship can be exploited to blindly estimate the channel $h(z)$. Equation (11) can be considerably simplified if the channel order is less than the block length, $q<M$. This may not pose any severe restriction in practice since M is a design parameter and can be chosen arbitrarily. Moreover, some communication channels have a dispersion of only a few symbols, and hence, in certain applications, q may be a small number. For example, in the North American cellular standard IS-54, the delay spread varies from 0.5 to one symbol period, and hence, $q=1$.

When $M>q$, the channel polyphase components become scalar constants

$$h_k(z)=h(k) \quad \text{and} \quad \boldsymbol{h}_{2M}(z)=[h(0),\cdots,h(q),0,\cdots,0]^T.$$

Notice that this behavior cannot be observed in the FSE framework, even if we let the oversampling rate grow indefinitely (P very large) since every increase in P results in an equivalent increase of the underlying channel order q.

With $q<M$, all matrices involved in (11) are constant matrices. Hence, (11) can be easily written in the time domain as

$$\boldsymbol{y}_{2M}(n)=\begin{bmatrix}\boldsymbol{H}_{11}\\ \boldsymbol{H}_{11}+\boldsymbol{H}_{21}\end{bmatrix}\boldsymbol{w}_M(n)+\begin{bmatrix}\boldsymbol{H}_{21}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{w}_M(n-1)+\boldsymbol{v}_{2M}(n) \quad (12)$$

where [c.f. (10)][1] $\boldsymbol{H}_{11}$ is lower triangular, and $\boldsymbol{H}_{21}$ is upper triangular with first all-zero column

$$\boldsymbol{H}_{11}=\begin{bmatrix}h(0) & \cdots & \boldsymbol{0}\\ \vdots & \ddots & \vdots\\ h(M-1) & \cdots & h(0)\end{bmatrix}$$
$$\boldsymbol{H}_{21}=\begin{bmatrix}0 & h(M-1) & \cdots & h(1)\\ \vdots & \ddots & \ddots & \vdots\\ 0 & & 0 & h(M-1)\\ 0 & \cdots & & 0\end{bmatrix}. \quad (13)$$

Let us consider the output autocorrelation matrix in (14), shown at the bottom of the page, where $\sigma_w^2=E\{|w(n)|^2\}$, and * denotes congugation. Notice that the first column of $\sigma_w^2(\boldsymbol{H}_{11}+\boldsymbol{H}_{21})\boldsymbol{H}_{11}^{*T}$ in (14) equals [c.f. (13)]

$$[\gamma h(0),\gamma h(1),\cdots,\gamma h(M-1)]^T=\gamma\boldsymbol{h}_M$$

[1] If $q<M-1$, the samples $h(q+1),\cdots,h(M-1)$ equal zero and simply denote the zero padded extention of $h(n)$ in what follows.

where $\gamma:=\sigma_w^2 h^*(0)$, i.e., it is a multiple of the impulse response.

This observation provides the simplest perhaps way of estimating the channel in the current framework. If $\boldsymbol{R}_v(0)$ is a banded matrix, i.e., the correlation of the noise is zero for lags greater than $M-1$ (a weaker assumption than the white noise one), then this first column of $\sigma_w^2(\boldsymbol{H}_{11}+\boldsymbol{H}_{21})\boldsymbol{H}_{11}^{*T}$ coincides with the corresponding column of $\boldsymbol{R}_y(0)$, that is $E\{y_0^*(n)[y_M(n),\cdots,y_{2M-1}(n)]^T\}$, or equivalently

$$\boldsymbol{h}_M=\gamma^{-1}E\{y^*(2Mn)[y(2Mn+M),\cdots y(2Mn+2M-1)]^T\}. \quad (15)$$

Hence, the channel impulse response can be estimated (within a scaling ambiguity inherent to all blind methods) by the sample average

$$\hat{\boldsymbol{h}}=\frac{1}{N_b}\sum_{n=0}^{N_b-1}y^*(2Mn)[y(2Mn+M),\cdots y(2Mn+2M-1)]^T \quad (16)$$

where $N_b=\lfloor N/2M\rfloor$ is the number of available data blocks if N denotes the total data length. Notice that (16) offers a closed-form solution with no restriction imposed on the location of the channel's zeros. Indeed, as verified by computer simulations in Section VI, this approach is successful even for channels not identified by FSE methods. Moreover, since only correlation lags $\tau\geq M$ are used in (16), certain types of colored additive noise can be tolerated (as long as the noise correlation becomes zero for $\tau\geq M$).

Due to its simplicity, (16) is well suited for adaptive implementation. From (16), a recursive computation of $\hat{\boldsymbol{h}}$ is given by

$$\hat{\boldsymbol{h}}(n)=\frac{n-1}{n}\hat{\boldsymbol{h}}(n-1)+y^*(2Mn)[y(2Mn+M),\cdots y(2Mn+2M-1)]^T \quad (17)$$

which suggests an LMS-type recursive algorithm with variable step size. Following common practice in the design of adaptive algorithms, a fixed step-size version of (17) is

$$\hat{\boldsymbol{h}}(n)=\lambda\hat{\boldsymbol{h}}(n-1)+(1-\lambda)y^*(2Mn)[y(2Mn+M),\cdots y(2Mn+2M-1)]^T \quad (18)$$

where $0<\lambda<1$ (see also Table I). The parameter λ is typically chosen close to 1 so that $(1-\lambda)$ is a small step size. The algorithm of (18) can adapt to slow changes of the underlying channel at the expense of a steady-state misadjustment error.

IV. An Exact Subspace Method

In the previous section, a channel estimation algorithm was developed by exploiting the statistical properties of the transmitted and the received signals. Other methods are also

$$\boldsymbol{R}_y(0)=E\{\boldsymbol{y}_{2M}(n)\boldsymbol{y}_{2M}^{*T}(n)\}=\sigma_w^2\begin{bmatrix}\boldsymbol{H}_{11}\boldsymbol{H}_{11}^{*T}+\boldsymbol{H}_{21}\boldsymbol{H}_{21}^{*T} & \boldsymbol{H}_{11}(\boldsymbol{H}_{11}+\boldsymbol{H}_{21})^{*T}\\ (\boldsymbol{H}_{11}+\boldsymbol{H}_{21})\boldsymbol{H}_{11}^{*T} & (\boldsymbol{H}_{11}+\boldsymbol{H}_{21})(\boldsymbol{H}_{11}+\boldsymbol{H}_{21})^{*T}\end{bmatrix}+\boldsymbol{R}_v(0) \quad (14)$$

TABLE I
SIMPLE ADAPTIVE ALGORITHM

Initialization:	$\mathbf{h}(0) = 0,\ 0 < \lambda < 1$
Recursion:	$\hat{\mathbf{h}}(n) = \lambda \hat{\mathbf{h}}(n-1) + (1-\lambda) y^*(2Mn)[y(2Mn+M), \ldots, y(2Mn+2M-1)]^T$

possible [31], using the cyclic signal correlations. Recent work in fractionally spaced equalizers, however, has shown that channel estimation is possible, without any assumption on the input correlation (apart from persistence of excitation) [34], [26]. These methods are called "deterministic" because in the absence of noise, they provide exact solutions from finite sample sizes. Despite the fact that they possess no optimality in the presence of noise, these methods can be very useful in rapidly fading channels at relatively high SNR, where only a small number of samples is available, e.g., in TDMA mobile radio links. It is worthwhile, therefore, to investigate if exact or "deterministic" solutions can be derived within the proposed repetition coding framework. In the sequel, we develop an SVD-based approach with this property for the case where $q < M$.

Let the received data vector be

$$\boldsymbol{y}_{2M}(n) := [\boldsymbol{y}_{M,1}^T(n), \boldsymbol{y}_{M,2}^T(n)]^T$$

in (12), where $\boldsymbol{y}_{M,1}(n)$ $[\boldsymbol{y}_{M,2}(n)]$ denotes the first (last) M components of $\boldsymbol{y}_{2M}(n)$, and similarly

$$\boldsymbol{v}_{2M}(n) := [\boldsymbol{v}_{M,1}^T(n), \boldsymbol{v}_{M,2}^T(n)]^T.$$

It is easy to verify from (12) and (13) that

$$\begin{aligned}\boldsymbol{y}_{M,2}(n) - \boldsymbol{y}_{M,1}(n) \\ = \boldsymbol{h}_{21}[\boldsymbol{w}_M(n) - \boldsymbol{w}_M(n-1)] + \boldsymbol{v}_{M,2}(n) - \boldsymbol{v}_{M,1}(n)\end{aligned} \tag{19}$$

and

$$\begin{aligned}\boldsymbol{y}_{M,1}(n) - \boldsymbol{y}_{M,2}(n-1) \\ = \boldsymbol{H}_{11}[\boldsymbol{w}_M(n) - \boldsymbol{w}_M(n-1)] + \boldsymbol{v}_{M,1}(n) - \boldsymbol{v}_{M,2}(n).\end{aligned} \tag{20}$$

Based on (19) and (20), we will estimate the channel's impulse response. If we let

$$\begin{aligned}\tilde{\boldsymbol{w}}_M(n) &= \boldsymbol{w}_M(n) - \boldsymbol{w}_M(n-1)\\ \tilde{\boldsymbol{y}}_{2M}(n) &= [(\boldsymbol{y}_{M,2}(n) - \boldsymbol{y}_{M,1}(n))^T\\ &\qquad (\boldsymbol{y}_{M,1}(n) - \boldsymbol{y}_{M,2}(n-1))^T]^T \quad \text{and}\\ \tilde{\boldsymbol{v}}_{2M}(n) &= [(\boldsymbol{v}_{M,2}(n) - \boldsymbol{v}_{M,1}(n))^T\\ &\qquad (\boldsymbol{v}_{M,1}(n) - \boldsymbol{v}_{M,2}(n-1))^T]^T\end{aligned}$$

then (19) becomes

$$\tilde{\boldsymbol{y}}_{2M}(n) = \mathcal{T}(\boldsymbol{h})\tilde{\boldsymbol{w}}_M(n) + \tilde{\boldsymbol{v}}_{2M}(n) \tag{21}$$

with the $2M \times M$ matrix $\mathcal{T}(\boldsymbol{h})$ given by

$$\mathcal{T}(\boldsymbol{h}) = \begin{bmatrix}\boldsymbol{H}_{11}\\ \boldsymbol{H}_{21}\end{bmatrix} = \begin{bmatrix} h(0) & & 0\\ \vdots & \ddots & \\ h(M-1) & & h(0)\\ 0 & \ddots & \vdots\\ \vdots & \ddots & h(M-1)\\ 0 & \cdots & 0\end{bmatrix} \tag{22}$$

where the last equality is due to the structure of $\boldsymbol{H}_{11}, \boldsymbol{H}_{21}$ [c.f., (13)]. Notice that $\mathcal{T}(\boldsymbol{H})$ is a Toeplitz filtering matrix. The correlation matrix of $\tilde{\boldsymbol{y}}_{2M}(n)$ is

$$\boldsymbol{R}_{\tilde{\boldsymbol{y}}}(0) = \mathcal{T}(\boldsymbol{h})\boldsymbol{R}_{\tilde{\boldsymbol{w}}}(0)\mathcal{T}^{*T}(\boldsymbol{h}) + \boldsymbol{R}_{\tilde{\boldsymbol{v}}}(0). \tag{23}$$

If $\boldsymbol{R}_{\tilde{\boldsymbol{w}}}(0)$ has full rank M (under a persistence of excitation assumption for $\boldsymbol{w}$ and, hence, for $\tilde{\boldsymbol{w}}$) and since $\mathcal{T}(\boldsymbol{h})$ is full rank (unless $h(k) = 0, \forall k$), the "signal subspace" of (23) has rank M. Therefore, the last M eigenvalues of the first term in the RHS of (23) are equal to zero, and the corresponding eigenvectors span the null subspace. Moreover, the columns of $\mathcal{T}(\boldsymbol{h})$ span the signal subspace and, hence, are orthogonal to the null subspace eigenvectors. It was shown in [22] in a different context that $h(k)$ can thus be uniquely identified (within a scaling ambiguity) from the equations

$$\boldsymbol{G}^{*T}\mathcal{T}(\boldsymbol{h}) = \mathbf{0} \tag{24}$$

where $\boldsymbol{G} := [\boldsymbol{g}_1, \cdots, \boldsymbol{g}_M]$ is a collection of the null subspace eigenvectors. Equation (24) was used in [22] to estimate a FS channel. Similarly, (24) can be used in the current framework after an eigenvalue decomposition of $\boldsymbol{R}_{\tilde{\boldsymbol{y}}}(0)$ in (23). The Toeplitz matrix $\mathcal{T}(\boldsymbol{h})$ is not parametrized in exactly the same way as in [22]. However, the identifiability result of [22] holds here as well. A short proof is given in Appendix B.

Notice that in the absence of noise, (23) holds true even when $\boldsymbol{R}_{\tilde{\boldsymbol{y}}}(0), \boldsymbol{R}_{\tilde{\boldsymbol{w}}}(0)$ are replaced by $\hat{\boldsymbol{R}}_{\tilde{\boldsymbol{y}}}(0), \hat{\boldsymbol{R}}_{\tilde{\boldsymbol{w}}}(0)$, and (24) is still an exact solution as long as $\hat{\boldsymbol{R}}_{\tilde{\boldsymbol{w}}}(0)$ has full rank. Hence, no independence assumption on the input is required, and exact solutions can be found from finite data lengths as long as the input is persistently exciting (see also [34]).

If noise is present, however, the structure of $\boldsymbol{R}_{\tilde{\boldsymbol{v}}}(0)$ needs to be considered. If we assume $v(n)$ to be additive white noise of variance σ_v^2, then from the definition of $\tilde{\boldsymbol{v}}_{2M}(n)$, we can verify that

$$\boldsymbol{R}_{\tilde{\boldsymbol{v}}}(0) = \sigma_v^2 \mathcal{I}_{2M} = \sigma_v^2 \begin{bmatrix} 2\boldsymbol{I} & -\boldsymbol{I}\\ -\boldsymbol{I} & 2\boldsymbol{I}\end{bmatrix}. \tag{25}$$

In this case, the matrix pencil of the matrices $(\boldsymbol{R}_{\tilde{\boldsymbol{y}}}(0), \mathcal{I}_{2M})$ needs to be used. It can be shown that the last M generalized eigenvalues of the two matrices equal σ_v^2, whereas the corresponding generalized eigenvectors span the null subspace of $\mathcal{T}(h)\boldsymbol{R}_{\tilde{\boldsymbol{w}}}(0)\mathcal{T}^{*T}(h)$. Therefore, a QZ decomposition algorithm should be used [15] to obtain the eigenvector matrix $\boldsymbol{G}$ needed in (24).

The resulting subspace algorithm can provide an exact solution from a finite number of data points in the absence of noise. Contrary to FSE subspace solutions [22], the proposed method is not sensitive to channel order overestimation or to the zeros' location (provided that $q < M$). It therefore obviates the need for statistical tests on the eigenvalues to estimate the correct channel order.

V. SYNCHRONIZATION

The two channel estimation methods developed in the previous sections illustrate the versatility of the proposed transmission framework. Both methods, however, rely on the tacit assumption that the receiver has knowledge of the timing instant at which each block of $2M$ symbols begins. While carrier and symbol timing information is crucial in many equalization methods, the current approach requires *block* timing on top of symbol timing information. Since the availability of such timing information in a blind scenario is not obvious, this matter deserves further discussion.[2]

In the sequel, we will show that block timing information can be retrieved from the statistics of the received signal. Following the general style of this paper, we will focus on suboptimal but simpler solutions, as opposed to more involved maximum likelihood formulations.

Let us assume that the observed signal is

$$\tilde{y}(n) = y(n-d), \qquad 0 \le d \le 2M-1 \bmod 2M \tag{26}$$

received with a delay of d symbol periods (symbol synchronization is assumed). Then, the correlation of two data points that are M samples apart is

$$\begin{aligned} r(\tau) &= E\{\tilde{y}^*(2Mn+\tau)\tilde{y}(2Mn+M+\tau)\} \\ &= E\{y^*(2Mn+\tau-d)y(2Mn+M+\tau-d)\}. \end{aligned} \tag{27}$$

We showed in Section III that for $\tau - d = 0$, $r(0) = \sigma_w^2|h(0)|^2$. This is so because the data points $y^*(2Mn), y(2Mn+M)$ share only one common input point $w(Mn)$ multiplying the factors $h^*(0), h(0)$, respectively. If $0<\tau-d<M$, it follows from (5) and (6) that the two data points $y^*(2Mn+\tau-d), y(2Mn+M+\tau-d)$ depend on the common inputs $w(Mn), \cdots, w(Mn+\tau-d)$, whereas if $M \le \tau-d<2M$, they depend on $w(\tau-d-M+1), \cdots, w(M-1)$. Hence, $r(\tau)$ may expressed in terms of the channel parameters as [c.f. (15)]

$$r(\tau) = \begin{cases} \sigma_w^2 \sum_{k=0}^{\tau-d} |h(k)|^2 \\ \quad 0 \le \tau - d < M \bmod 2M \\ \sigma_w^2 \sum_{k=\tau-d-M+1}^{M-1} |h(k)|^2 \\ \quad M \le \tau - d < 2M \bmod 2M. \end{cases} \tag{28}$$

[2] We wish to thank one of the anonymous reviewers, who brought this issue to our attention.

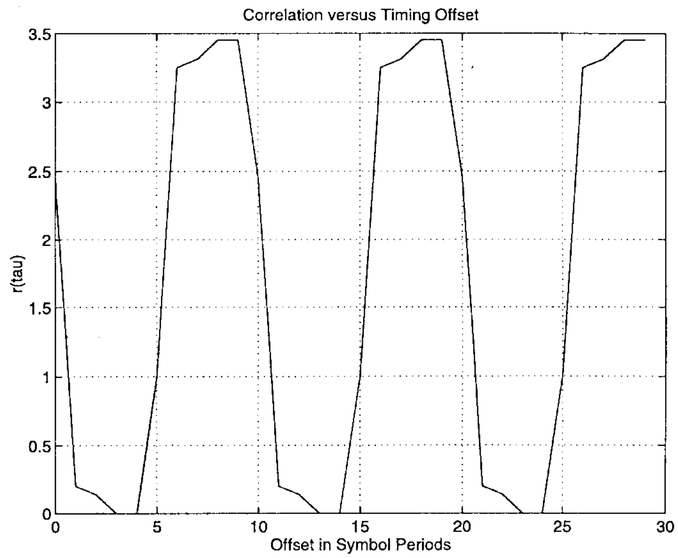

Fig. 6. Correlation-based synchronization.

Notice that $r(\tau)$ is nondecreasing for $\tau \in [0, M-1]$ [more terms are added to (28)], whereas it is nonincreasing for $\tau \in [M, 2M-1]$. Fig. 6 shows the values of $r(\tau)$ for $M=5$ and for a particular channel of order $q = 3$ described in the simulations section. The offset here is $d = 5$, as can be deduced from the monotonicity of the graph in each block of length M.

If an automated procedure for estimating d is desired based on (28), then a statistical test is needed to check the monotonicity of $r(\tau)$. Let $\hat{r}(\tau)$ be the sample estimate

$$\hat{r}(\tau) = \frac{1}{N_b}\sum_{n=0}^{N_b-1} \tilde{y}^*(2Mn+\tau)\tilde{y}(2Mn+M+\tau) \tag{29}$$

and define the differences

$$\hat{e}(\tau) = \hat{r}(\tau) - \hat{r}(\tau-1). \tag{30}$$

Then, d can be estimated by maximizing the cost function

$$\hat{d} = \arg\max_{0\le d' \le 2M-1} \left[\sum_{\tau=d'}^{d'+M-1} \hat{e}(\tau) - \sum_{\tau=d'+M}^{d'+2M-1} \hat{e}(\tau)\right]. \tag{31}$$

The test of (31) exploits the fact that $e(\tau) \ge 0$ for $\tau \in [d, d+M-1]$, and $e(\tau) \le 0$ for $\tau \in [d, d+M-1]$.

Statistical performance analysis of (31) (e.g., evaluation of the probability of incorrect decision) follows standard steps and exploits the asymptotic normality of $\hat{r}(\tau)$ (and, hence, of $\hat{e}(\tau)$). However, we will not pursue it any further here due to lack of space.

VI. EQUALIZATION OPTIONS AND OTHER ISSUES

Up to this point, we have devoted a major part of this paper in exploring different channel estimation methods in the framework of repetition coding. We have demonstrated that the channel estimation procedure is greatly facilitated by the proposed setup and have pointed out several advantages over conventional symbol spaced or fractionally spaced methods.

However, a number of other issues need to be studied including power and bandwidth considerations as well as decoding delay issues.

Due to the repetitive nature of the proposed transmission scheme, it is natural to imagine that the proposed method doubles the data rate and therefore requires twice the bandwidth for a given information symbol rate. This implication, however, is misleading. If the transmitter uses the same spectral shaping pulse $g_c(t)$ and transmits one symbol every $T/2$ s, the bandwidth does not increase since it is only determined by the support of $g_c(\Omega)$, which is the Fourier transform of $g_c(t)$. Indeed, in the uncoded case, where i.i.d. $w(n)$'s are transmitted every T s, it is known that the average correlation of the transmitted continuous time signal $x_c(t)$ is [24, p. 191]

$$\overline{r}_{x_c}(\tau) = E\{x_c^*(t)x_c(t+\tau)\} = \frac{1}{T}\sigma_w^2 r_{g_c}(\tau) \tag{32}$$

where

$$r_{g_c}(\tau) = \int_{-\infty}^{\infty} g_c^*(t)g_c(t+\tau)\,dt$$

is the deterministic correlation of the spectral pulse. Similarly, the spectrum of $x_c(t)$ is

$$S_{x_c}(\Omega) = \frac{1}{T}\sigma_w^2|g_c(\Omega)|^2. \tag{33}$$

In the coded case, where $w_b(n)$ is given by (5) and is transmitted every $T/2$ s, it follows that

$$\overline{r}_{x_c}(\tau) = \frac{2}{T}\sigma_w^2 r_{g_c}(\tau) + \frac{1}{T}\sigma_w^2 r_{g_c}\left(\tau - M\frac{T}{2}\right) + \frac{1}{T}\sigma_w^2 r_{g_c}\left(\tau + M\frac{T}{2}\right) \tag{34}$$

and

$$S_{x_c}(\Omega) = \frac{2}{T}\sigma_w^2|g_c(\Omega)|^2\left[\cos\left(\Omega M\frac{T}{2}\right) + 1\right]. \tag{35}$$

Equation (35) shows that the bandwidth of the coded transmission is the same as the conventional one and equals the support of $g_c(\Omega)$. Equation (34) also implies that if $g_c(\tau)$ is appropriately scaled, the required transmission power is the same. Hence, the resulting SNR is also equal.

The proposed method has the drawback of doubling the discrete time channel's order for a given actual symbol spread since symbols are spaced by $T/2$ instead of T s. However, the same drawback appears when using FSE's and introduces some increase in the equalizer's complexity. The proposed approach deliberately introduces some controlled ISI in the transmitted signal to facilitate the equalization procedure without increasing the bandwidth. In that respect, it is similar to partial response channel modulation techniques (e.g., duobinary modulation) and should be expected to suffer a small performance loss due to the introduced ISI (see, e.g., [24, pp. 616–624]).

We close this discussion on the relative merits of different approaches with a short note on the various equalization options possible once the channel has been estimated. Clearly, a maximum likelihood input estimation procedure based on the Viterbi algorithm is applicable here [c.f., (12)] as in most conventional methods. However, when simpler equalization schemes are desired, FSE's offer the possibility of linear, FIR, zero forcing equalizers, in contrast to symbol spaced schemes where no FIR equalizer can perfectly remove the channel effects [12]. It is interesting, therefore, to investigate whether this desirable property carries over to the proposed framework. The following proposition presents a sufficient condition for zero-forcing equalization, which resembles a similar condition developed in the context of FSE's [26].

Proposition 1: An FIR zero forcing equalizer for (11), i.e., a polyphase matrix $\boldsymbol{F}(z)$ such that

$$\boldsymbol{F}(z)\boldsymbol{H}(z) = \boldsymbol{I} \tag{36}$$

exists, provided that the channel order $q < M$ and $h(z)$ has no zeros at the points $e^{j(2\pi k/M)}, k = 0, \cdots, M-1$ on the unit circle. □

Proof: An FIR $\boldsymbol{F}(z)$ satisfying (36) exists if $\boldsymbol{H}(z)$ is *irreducible* (see [4]) or, equivalently, if we have the following:

i) $\boldsymbol{H}(z)$ has full rank $\forall z \in \boldsymbol{C}, z \neq 0$,
ii) $\boldsymbol{h}(0)$, which is the impulse response matrix at lag 0, has full rank.

Hence, we need to establish i) and ii). From (12), we can see that ii) is satisfied since $\boldsymbol{H}_{11}$ is a lower triangular matrix and, hence, has full rank. In order to establish i), we show that the lower part of $\boldsymbol{H}(z)$, i.e., $\boldsymbol{H}_{11} + \boldsymbol{H}_{21}$, has full rank $\forall z$ [c.f. (12)]. Notice that $\boldsymbol{H}_{11} + \boldsymbol{H}_{21}$ is a circular matrix [see (13)], and hence, its eigenvalues are given by the DFT of the first row (e.g., see [3, p. 73]), i.e., by $h(z_k), z_k = e^{+j(2\pi k/M)}, k = 0, \cdots, M-1$. Under the proposition's conditions, $h(z_k) \neq 0$, and the matrix has full rank. □

Notice that the conditions of Proposition 1 can, in most cases, be satisfied by appropriate choice of the parameter M.

In the presence of noise, a zero forcing equalizer is not optimal, and a MMSE design could be pursued. In this case, the equalizer output in vector form is

$$\hat{\boldsymbol{w}}_M(n) = \sum_{k=-L}^{L} \boldsymbol{F}(k)\boldsymbol{y}_{2M}(n-k) \tag{37}$$

where L is the equalizer's order, and the coefficient matrices $\boldsymbol{F}(k)$ are computed from the orthogonality condition

$$E\left\{\left[w_i(n) - \sum_{k=-L}^{L}\sum_{j=1}^{2M} \boldsymbol{F}^{ij}(k)y_j(n-k)\right]y_m^*(n-l)\right\} = 0 \tag{38}$$

for $m = 1, \cdots, 2M, i = 1, \cdots, M, l = -L, \cdots, L$. Due to space limitations, no more details on solving (38) will be discussed, as they follow standard procedures [24].

VII. Simulations

Some simulation results are presented in this section to illustrate the advantages of the proposed method when compared with alternative fractionally spaced schemes. In all the

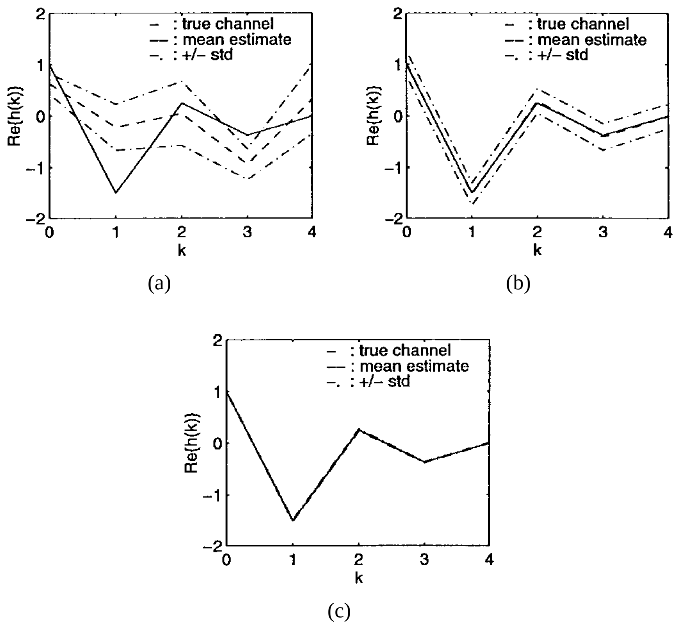

Fig. 7. True and estimated channel tap coefficients.

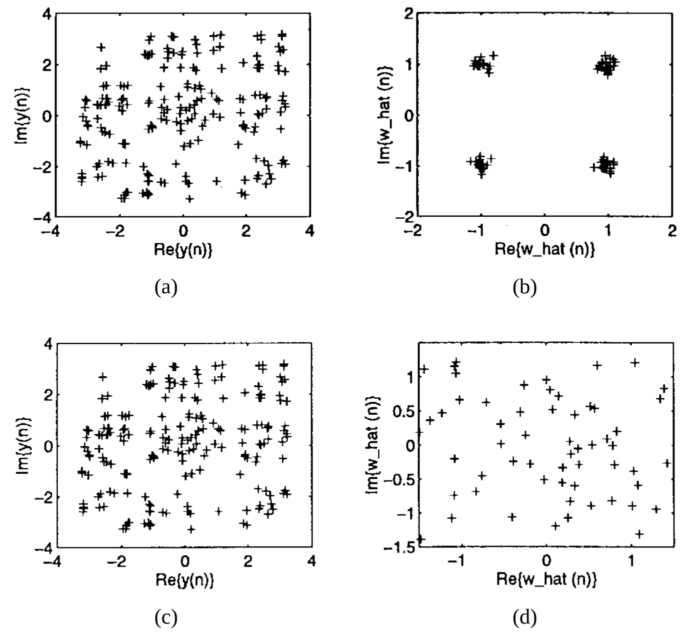

Fig. 8. Received symbols: Before and after equalization.

simulations that follow, 4-QAM i.i.d. symbols were generated, and after being interleaved according to (5) (with $M = 5$), they were transmitted through the channel $h(z) = 1 - 1.5z^{-1} + 0.25z^{-2} - 0.375z^{-3}$ with zeros at $\pm 0.5j$ and 1.5. The channel was specifically chosen so that it is not identifiable using FSE methods (with an oversampling of 2) since the zeros at $\pm 0.5j$ are not resolvable. Moreover, we have overestimated the channel order (assuming $q = 4$, whereas the actual $q = 3$), to show the insensitivity of the proposed methods to model order mismatch.

Fig. 7 shows the true channel coefficients as well as the estimated ones from 100 Monte Carlo realizations (mean $\pm$ standard deviation). In Fig. 7(a), the results of the FSE method of [22] are shown, which are not satisfactory, whereas in Fig. 7(b) and (c), the performance of the proposed methods is depicted. The SNR was 30 dB (relatively high), which explains the superiority of the exact subspace method depicted in Fig. 7(c). The data length was 100 symbols for the exact methods of Fig. 7(a) and (c) and 1000 symbols for the statistical method of Fig. 7(b). Block timing information was obtained from Fig. 6.

The difference in performance between the FSE and the proposed methods can also be seen in Fig. 8. In Fig. 8(a), the received symbols are plotted on a constellation graph, whereas in Fig. 8(b), the equalized symbols are plotted, using the MMSE equalizer based on the channel estimates. Similarly, Fig. 8(c) and (d) shows the same scenario when the equalizer is designed using the channel estimates provided by the FSE algorithm of [22]. Clearly, the equalizer in the latter case does not succeed in removing the ISI.

To obtain a more quantitative description of the above performance comparisons, we tested the FSE and the proposed subspace-based equalizers with respect to the probability of symbol errors for different SNR levels. The resulting curves are shown in Fig. 9, where the FSE method clearly fails to decode the received symbols at any SNR level. In this experiment, 10 000 Monte Carlo realizations were averaged per SNR point.

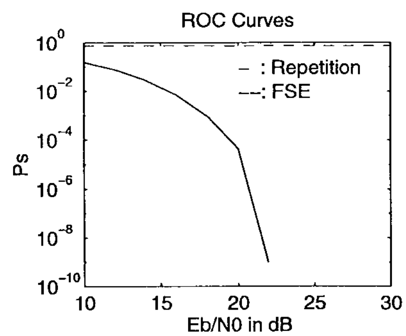

Fig. 9. Probability of symbol error versus SNR.

Finally, in Fig. 10, we show the tracking capabilities of the simple adaptive algorithm of (18) for the same channel. The estimated coefficients (mean $\pm$ standard deviation) are shown versus time (in number of received data blocks of length $2M$). SNR is 30 dB, and $\lambda = 0.995$ here. Faster convergence can be traded for increased misadjustment error by varying the parameter λ.

VIII. Conclusions

Novel blind equalization algorithms are proposed in this paper, which exploit coding information present in the transmitted signal. However, the main contribution of this work lies in emphasizing the importance of transmitter design in facilitating the removal of channel distortions. We have also demonstrated that different coding/interleaving strategies may transform a single channel setup to an equivalent multirate/multichannel formulation, allowing the development of multichannel algorithms with improved performance.

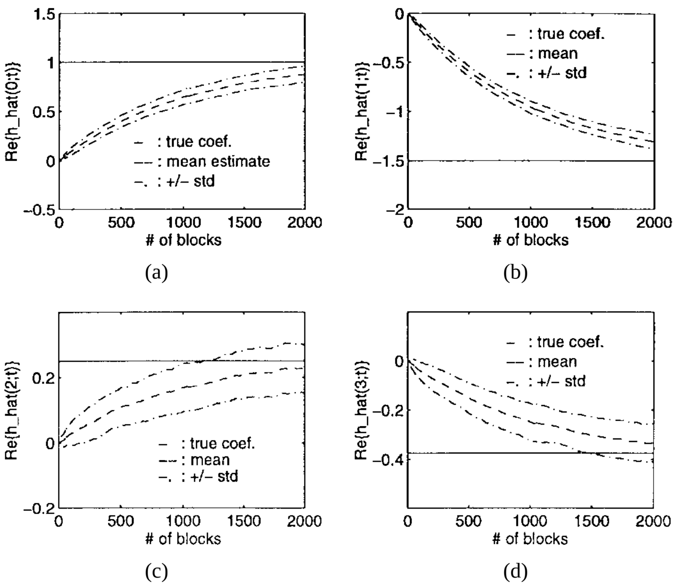

Fig. 10. Coefficient estimates as functions of time.

Appendix A
Cyclostationarity of $w_b(n)$

To establish cyclostationarity of $w_b(n)$ [and, hence, $y(n)$], it suffices to show that its correlation $r_{w_b}(n;\tau)$ is periodic in n with period $2M$, i.e., $r_{w_b}(n+2Mk;\tau) = r_{w_b}(n;\tau)$. To this end, we first observe that for general $c(k)$, the block sequence $w_b(n)$ can be expressed as (c.f. Fig. 5 with $P=2$ and $c(0)=c(1)=1$)

$$w_b(n) = \sum_{m=0}^{M-1}\sum_{l} w(lM+m)c(n-m-2Ml).$$

Hence, the correlation is given by

$$\begin{aligned} r_{w_b}(n;\tau) &= \sigma_w^2 \sum_{m_1,m_2}\sum_{l_1,l_2} c(n-m_1-2Ml_1) \\ &\quad \cdot c(n+\tau-m_2-2Ml_2)\times\delta(l_1-l_2) \\ &\quad \cdot \delta(m_1-m_2) \\ &= \sigma_w^2 \sum_m\sum_l c(n-m-2Ml) \\ &\quad \cdot c(n+\tau-m-2Ml). \end{aligned} \tag{39}$$

For any integer k, the last equation implies

$$\begin{aligned} r_{w_b}(n+2Mk;\tau) &= \sigma_w^2 \sum_m\sum_l c(n-m+2Mk-2Ml) \\ &\quad \cdot c(n+\tau-m+2Mk-2Ml) \\ &= \sigma_w^2 \sum_m\sum_\rho c(n-m-2M\rho) \\ &\quad \cdot c(n+\tau-m-2M\rho) \\ &= r_{w_b}(n;\tau) \end{aligned} \tag{40}$$

where in deriving the second equality, we used the change of variables $\rho = l-k$.

Appendix B
Identifiability of Equaiton (24)

We will show that the parameter vector $\boldsymbol{h}$ is uniquely determined by the range space of $\mathcal{T}(\boldsymbol{h})$ defined in (22) for $M>1$. In particular, following [22], we will show that if for some vector $\boldsymbol{h}' \neq \boldsymbol{0}$, range $[\mathcal{T}(\boldsymbol{h}')] \subset$ range $[\mathcal{T}(\boldsymbol{h})]$, then $\boldsymbol{h}' = \alpha\boldsymbol{h}$, where α is a scalar.

Let $\boldsymbol{h}_1'$ be the first column of $\mathcal{T}(\boldsymbol{h}')$. Since it belongs to the range of $\mathcal{T}(\boldsymbol{h}')$ and, hence, of $\mathcal{T}(\boldsymbol{h})$, we must have $\boldsymbol{h}_1' = \mathcal{T}(\boldsymbol{h})\boldsymbol{\alpha}$ for some parameter vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_M]^T$. Due to the Toeplitz structure of $\mathcal{T}(\boldsymbol{h})$, this system of equations yields $\alpha_M = \alpha_{M-1} = \cdots = \alpha_2 = 0$. Therefore, we must have $\boldsymbol{h}_1' = \alpha_1\boldsymbol{h}_1$. □

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