Subspace Methods for Blind Estimation of Time-Varying FIR Channels

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***Abstract*—Novel linear algorithms are proposed in this paper for estimating time-varying FIR systems, without resorting to higher order statistics. The proposed methods are applicable to systems where each time-varying tap coefficient can be described (with respect to time) as a linear combination of a finite number of basis functions. Examples of such channels include almost periodically varying ones (Fourier Series description) or channels locally modeled by a truncated Taylor series or by a wavelet expansion. It is shown that the estimation of the expansion parameters is equivalent to estimating the second-order parameters of an unobservable FIR single-input-many-output (SIMO) process, which are directly computed (under some assumptions) from the observation data. By exploiting this equivalence, a number of different blind subspace methods are applicable, which have been originally developed in the context of time-invariant SIMO systems. Identifiability issues are investigated, and some illustrative simulations are presented.**

I. Introduction

LINEAR parametric models have found widespread use as basic tools in the analysis of physical phenomena and engineering systems. ARMA models have been employed in the mathematical analysis of such diverse signals and systems as seismic responses [14], speech recordings [9], and communication links (e.g., [10] and [17]). When only output information is available, however, the identification of MA models (or MA parts of ARMA models) imposes challenging theoretical questions and/or considerable computational burden.

A great deal of research effort has focused in the past on the problem of estimating the parameters $h(k)$ of an MA model

$$x(n) = \sum_{k=0}^{q} h(k)w(n-k) \tag{1}$$

of order q driven by an i.i.d. sequence $w(k)$ when only measurements of $x(n)$ are available [3], [5], [15], [28]. In fact, the renewed interest in higher order statistics (HOS) is to a great extend motivated by their ability to provide output-only (linear and nonlinear) solutions to the MA identification problem [15].

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While in many cases the linear time-invariant (TI) model of (1) may provide satisfactory description of the underlying system, in certain applications, the time-invariance assumption is not warranted. For example, in wireless communications, the multipath environment changes with time, as the mobiles move. As a consequence, the system parameters $h(k)$ change with time [26]. Other applications where nonstationary signals appear include econometrics [18] and speech processing [9], to name but a few examples.

The situation is even more pronounced when HOS-based methods are employed. Due to their notoriously slow convergence, these methods require very long data records and, hence, are prone to a violation of the TI assumption even for very slowly changing systems. If the TI description of (1) does not provide a satisfactory approximation to the system's behavior, a time-varying (TV) model has to be considered

$$x(n) = \sum_{k=0}^{q} h(n;k)w(n-k) \tag{2}$$

where the impulse response $h(n;k)$ now depends explicitly on time n.

One way of handling such system variations in the identification procedure is by employing adaptive algorithms. If the system variations are slow (when compared with the algorithm's convergence time), then the adaptive algorithm will track the TV system parameters. The popularity of adaptive signal processing is an indication of the importance of TV systems in engineering applications. When output-only identification is considered, however, the very long convergence time of adaptive algorithms (e.g., [7]) renders them impractical for all but the most slowly changing systems. For this reason, the Godard algorithm [7] (a popular blind-channel equalization technique) is, in most cases, studied only for TI channels.

It appears that a basis expansion approach would be the natural way of modeling the TV parameters $h(n;k)$. Prompted by the general applicability of basis expansion approximations (e.g., Fourier and Taylor series expansion, polynomial approximation, etc.), we consider in this paper the parameters $h(n;k)$ to be given by a linear combination of some known sequences. Hence, the system identification problem is equivalent to estimating the coefficients of this expansion.

Basis expansion ideas have been used in the past to estimate TV-AR systems in the context of speech processing [9], [12], [25]. Similarly, Taylor expansion of the TV parameters were used in [11] and [18] in the context of economic time series analysis. However, only I/O cases or AR linear prediction scenarios have been considered, to the best of our knowledge. In contrast, the literature on output only (or blind) TV identification problems is not equally abundant. An exception to that rule is a cumulant matching approach, proposed in [27], which employs the output TV cumulants. Although identifiability and consistency were established in [27], the computational burden is considerable since a nonlinear minimization problem has to be solved. Moreover, that approach is also based on HOS, thereby suffering from inaccuracies in estimating the higher order cumulants and requiring long data records.

In the TI case and in the context of blind channel equalization, it has recently been established that a single input, many output (SIMO) setup offers more flexibility than the classical single input, single output (SISO) one. In fact, a large class of even nonminimum-phase channels can be blindly identified using only second-order statistics [6], [16], [22], [24], [29] in the SIMO framework. Since these techniques do not resort to HOS, they have better accuracy and shorter convergence times. It would be interesting, therefore, to investigate whether similar, second-order-based techniques are applicable to the TV case.

In this paper, we provide a link between the TV-SISO and the TI-SIMO case by establishing an equivalence between each basis sequence and a different information channel. In this way, we are able to apply second-order-based subspace techniques (which were originally developed for TI-SIMO channels [16]) to estimate the basis expansion coefficients of a TV system (see Fig. 1).

The proposed identification procedure is divided into two steps. In the first step, a novel linear least-squares approach is proposed to recover the correlation information of the expansion coefficients. In the second step, a subspace method is applied to the coefficients' correlation matrix to recover those expansion coefficients.

Identifiability issues are studied in this paper, and consistency of the proposed technique is shown under some assumptions. It is shown that a large class of even nonminimum phase TV systems, or systems that change from minimum phase to nonminimum phase, can be identified using only second-order statistics.

The proposed technique can be applied to general TV-MA modeling problems. In the simulations section, however, we will focus on communication applications and channel equalization problems.

The rest of the paper is organized as follows. In the next section, the problem statement and the relevant assumptions are detailed. In Section III, the link to multichannel TI systems is explained, whereas in Section IV, the first step of the proposed algorithm is developed. In Section V, the subspace identification method is applied, whereas identifiability and consistency issues are discussed in Section VI. Some extensions are discussed in Section VII, and some simulation examples are presented in Section VIII.

II. Problem Statement

In this paper, we consider the problem of identifying the TV system parameters $h(n;k)$ in (2) when only measurements of $x(n),\ n=0,1,\dots,N-1$ are available. It is clear from (2) that the problem is ill posed, unless some more restrictions are applied to the variations of $h(n;k)$. Indeed, if $h(n;k)$ is allowed to vary arbitrarily, then (2) may be satisfied for an infinite number of different parameter trajectories $h(n;k)$, even if both $x(n)$ and $w(n)$ are given. For this reason, in the sequel, we constrain $h(n;k)$ to be given by a truncated basis expansion

$$h(n;k)=\sum_{l=0}^{L-1}\theta_{kl}f_l(n). \tag{3}$$

The sequences $f_l(n)$ are not required to be orthogonal (in l_2) but should satisfy certain assumptions, as explained later. The choice of the basis sequences $f_l(n),\ l=0,1,\dots L-1$ as well as the truncation order L is clearly application dependent. For example, for certain signals like radar returns and fading communications links, there is evidence that the system parameters $h(n;k)$ are given by a linear combination of complex exponentials [19], [21], [26], [27]. In other cases, where a linear or quadratic trend is suspected on the system parameters, a polynomial approximation should be preferred.

In view of (3), the problem of estimating the TV coefficients $h(n;k),\ k=0,1,\dots,q$ in (2) is equivalent to estimating the expansion parameters θ_{kl}. In the sequel, we will show how θ_{kl} can be estimated from the output data $x(n),\ n=0,\dots,N-1$. Other parameters associated with $f_l(n)$ (e.g., Doppler shifts) may need to be estimated but will not be considered in this paper (see [1] for more details). The following conditions are assumed to hold throughout the paper.

- **AS1)** $w(k)$ is an i.i.d., zero mean sequence with finite cumulants of all orders and variance $\sigma_w^2=E\{|w(k)|^2\}$.
- **AS2)** The sequences $f_l(n),\ l=0,\dots,L-1$ are linearly independent.

Additional assumptions on the sequences $f_l(n)$ will be necessary to establish consistency of the proposed method. We defer this discussion, however, to Section VI. Finally, in Appendix A, we show how **AS1)** may be relaxed.

Let us now develop a relationship between the TV correlation of the nonstationary signal $x(n)$ and an equivalent TI-SIMO system. This relationship will be instrumental in the derivation of the proposed algorithm.

III. From TV-SISO to TI-SIMO

Let us assume for the time being that the TV signal correlation is known *a priori*

$$r_x(n;\tau)=E\{x(n)x^*(n+\tau)\},\quad \tau=0,\dots,q. \tag{4}$$

Issues related to the estimation of the signal statistics are discussed in Section IV. Notice in (4) that $r_x(n;\tau)$ depends explicitly on time n since the stationarity assumption is violated. By substituting $x(n)$ from (2) in (4), we obtain

$$r_x(n;\tau)=\sigma_w^2\sum_{k=0}^{q}h(n;k)h^*(n+\tau;k+\tau) \tag{5}$$

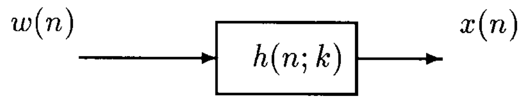

Fig. 1. Time-varying system.

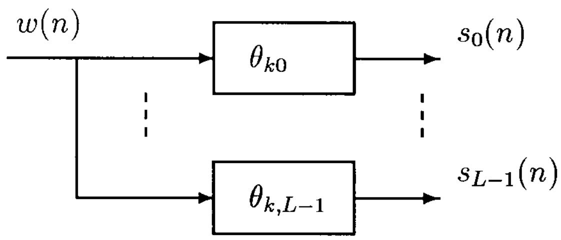

Fig. 2. Corresponding TI-SIMO channel.

where the independence of $w(n)$ in AS1) was used. By further substituting $h(n;k)$ in (5) from (3), we obtain

$$\begin{aligned} r_x(n;\tau) &= \sigma_w^2 \sum_{k=0}^{q} \sum_{l_1,l_2=0}^{L-1} \left[\theta_{kl_1} f_{l_1}(n)\right]\left[\theta^*_{k+\tau,l_2} f^*_{l_2}(n+\tau)\right] \\ &= \sum_{l_1,l_2=0}^{L-1} \left[\sigma_w^2 \sum_{k=0}^{q} \theta_{kl_1}\theta^*_{k+\tau,l_2}\right] f_{l_1}(n) f^*_{l_2}(n+\tau). \quad (6) \end{aligned}$$

If we define the terms in brackets as

$$r_\theta(\tau;l_1,l_2) = \sigma_w^2 \sum_{k=0}^{q} \theta_{kl_1}\theta^*_{k+\tau,l_2}, \quad \tau = 0,\dots,q \qquad l_1,l_2 = 0,\dots,L-1 \quad (7)$$

then (6) provides a linear relationship between $r_\theta(\tau;l_1,l_2)$ and $r_x(n;\tau)$

$$r_x(n;\tau) = \sum_{l_1,l_2=0}^{L-1} r_\theta(\tau;l_1,l_2) f_{l_1}(n) f^*_{l_2}(n+\tau). \quad (8)$$

Notice that the parameters $r_\theta(\tau;l_1,l_2)$ are defined in such a way [c.f., (7)] that they can be regarded as the output cross-correlations of a hypothetical SIMO system with L channels and impulse response of channel l, $l = 0,1,\dots,L-1$ equal to θ_{kl}, $k = 0,\dots,q$.

This correspondence can be better understood using Fig. 2. Consider a bank of L filters $h_l(k)$, $l = 0,\dots,L-1$, where each filter's impulse response equals $h_l(k) = \theta_{kl}$, $k = 0,\dots,q$. Then, if $w(n)$ is applied as an input to this hypothetical system, the output cross-correlation between channels l_1 and l_2 is given by the $r_\theta(\tau;l_1,l_2)$ of (7). Hence, (8) provides a relationship between the statistics of a TV-SISO system and a TI-SIMO one.

We should emphasize at this point that the TI-SIMO system of Fig. 2 is purely hypothetical; no such filtering operations have been performed on $w(n)$, and no data $s_1(n),\dots,s_{L-1}(n)$ are available. Nevertheless, this correspondence is an interesting observation that will be used in the sequel.

The proposed algorithm for estimating θ_{kl} will be developed in two steps. In the first step, the correlations $r_\theta(\tau;l_1,l_2)$ will be estimated based on $r_x(n;\tau)$ and the relationship provided by (8). In the second step, the parameters θ_{kl} will be obtained by solving the equivalent TI-SIMO identification problem using the statistics $r_\theta(\tau;l_1,l_2)$. The first step of the algorithm will be analyzed next, whereas the second step will be discussed in Section V.

IV. Recovering the TI Correlations

Let us fix τ in the range $\tau = 0,\dots,q$ and write (8) in a vector form

$$r_x(n;\tau) = \boldsymbol{\phi}^{*T}(n;\tau)\mathbf{r}_\theta(\tau), \quad n = 0,\dots,N-1 \quad (9)$$

where ^{T} and * denote transpose and conjugate, respectively, and

$$\begin{aligned} \mathbf{r}_\theta(\tau) &= [r_\theta(\tau;0,0),\dots,r_\theta(\tau;0,L-1),r_\theta(\tau;1,0),\dots \\ &\quad r_\theta(\tau;L-1,L-1)]^T, \quad (10) \\ \boldsymbol{\phi}^{*T}(n;\tau) &= [f_0(n)f_0^*(n+\tau),\dots,f_0(n)f^*_{L-1}(n+\tau), \\ &\quad f_1(n)f_0^*(n+\tau),\dots,f_{L-1}(n)f^*_{L-1}(n+\tau)]. \quad (11) \end{aligned}$$

Furthermore, if

$$\boldsymbol{\Phi}(\tau) = [\boldsymbol{\phi}(0;\tau),\dots,\boldsymbol{\phi}(N-1;\tau)]^{*T} \quad (12)$$

then (9) can be written in a compact matrix form as

$$\mathbf{r}_x(\tau) = \boldsymbol{\Phi}(\tau)\mathbf{r}_\theta(\tau) \quad (13)$$

where

$$\mathbf{r}_x(\tau) = [r_x(0;\tau),\dots,r_x(N-1;\tau)]^{*T}. \quad (14)$$

Matrix $\boldsymbol{\Phi}(\tau)$ in (13) consists of elements constructed from the basis functions $f_l(n)$, which are known *a priori*. Hence, $\boldsymbol{\Phi}(\tau)$ is given; if it also has full column rank, (13) can be solved to yield the desired $\mathbf{r}_\theta(\tau)$.

The identifiability issues related to the rank of $\boldsymbol{\Phi}(\tau)$ will be discussed in Section VI. As far as trivial violations of the full-rank property are concerned, the construction of $\boldsymbol{\Phi}(\tau)$ described above should be modified accordingly to include the nonredundant pairs (l_1,l_2) in (8) only. For example, for real signals and $\tau = 0$, only the $0 \le l_1 \le l_2 \le L-1$ terms in (8) are nonredundant, necessitating a modified definition of (13).

Next, let us concentrate on an equally important issue related to the availability of the statistics $\hat{\mathbf{r}}_x(\tau)$. It is common practice when developing an algorithm for the TI stationary case to substitute the ensemble correlations with their sample estimates obtained through averaging over the data record. In the current situation, however, the ensemble quantities do not remain the same throughout the data record, and hence, sample averaging techniques are not applicable.

In order to obviate this obstacle, we propose to use instantaneous approximations for $r_x(n;\tau)$ at every time point n, $n = 0,\dots,N-1$. Consider the estimate $\hat{r}_x(n;\tau) = x(n)x^*(n+\tau)$, and let us write it as

$$\begin{aligned} \hat{r}_x(n;\tau) &= E\{x(n)x^*(n+\tau)\} + e(n;\tau) \\ &= r_x(n;\tau) + e(n;\tau) \quad (15) \end{aligned}$$

where $e(n;\tau)$ is the estimation error. It is clear from (15) that $e(n;\tau)$ has zero mean, and hence, $\hat{r}_x(n;\tau)$ is unbiased. However, since $\hat{r}_x(n;\tau)$ is only an instantaneous approximation of the ensemble statistics, one might question the accuracy of this approach or, equivalently, be concerned with the variance of the estimation error $e(n;\tau)$. Fortunately, a large collection of such estimates $\hat{r}_x(n;\tau)$ will be used in our problem (one for every data point $n = 0,\dots,N-1$). Therefore, the resulting solution turns out to be significantly more accurate, despite the inaccuracies of each individual estimated $\hat{r}_x(n;\tau)$. In fact, in Section VI, we will quantify this claim by showing consistency of the proposed algorithm.

Consider the following linear regression problem.

$$\hat{r}_x(n;\tau) = x(n)x^*(n+\tau) = \boldsymbol{\phi}^{*T}(n;\tau)\hat{\mathbf{r}}_\theta(\tau) + e_{\mathrm{LS}}(n;\tau) \qquad n = 0,\dots N-1 \quad (16)$$

over the regressors' vector $\boldsymbol{\phi}^{*T}(n;\tau)$ and with an unknown coefficient vector $\hat{\mathbf{r}}_\theta(\tau)$. In matrix form, (16) may be written equivalently as

$$\hat{\mathbf{r}}_x(\tau) = \boldsymbol{\Phi}(\tau)\hat{\mathbf{r}}_\theta(\tau) + \mathbf{e}_{\mathrm{LS}}(\tau) \quad (17)$$

with obvious notation following that of (13). If we denote by $\hat{\mathbf{r}}_{\theta,N}(\tau)$ the least-squares (LS) solution of (17) based on N data points, then it can be shown (under some conditions) that as N grows large, $\hat{\mathbf{r}}_{\theta,N}(\tau)$ converges strongly to the desired $\mathbf{r}_\theta(\tau)$ (see Section VI).

The intuitive reason for this behavior is that as N grows large, more equations are involved in the LS problem of (16), providing a more accurate solution.

In conclusion, the proposed LS approach provides a linear method for transforming a TV-SISO problem into a TI-SIMO one. The estimation of θ_{kl} can now be completed using the estimated statistics $\hat{r}_\theta(\tau;l_1,l_2)$ in a SIMO framework.

V. Subspace Estimation Methods

Once the statistics $\hat{r}_\theta(\tau;l_1,l_2)$ of the SIMO system are obtained, a number of different techniques can be employed to recover θ_{kl} [6], [16], [22], [24]. Of all these methods, we will focus on the subspace approach of [16] in the sequel, which appears to have improved performance when compared with other linear methods [23]. For reasons of completeness, let us briefly revisit the method of [16] and describe how it can be adapted in the current framework.

With reference to Fig. 2, let us define the vectors $\mathbf{s}_l(n) = [s_l(n)\dots s_l(n-M+1)]^T$, $l = 0,\dots,L-1$ for some order M and the vector $\boldsymbol{s}_M(n) = [\mathbf{s}_0^T(n)\cdots\mathbf{s}_{L-1}^T(n)]^T$. Then, from the filtering operations of Fig. 2, we may write

$$\boldsymbol{s}_M(n) = \mathcal{S}_M(\boldsymbol{\theta})\mathbf{w}_M(n) \quad (18)$$

where $\boldsymbol{\theta} = [\boldsymbol{\theta}_{00}\cdots\boldsymbol{\theta}_{q0}|\cdots|\boldsymbol{\theta}_{0,L-1}\cdots\boldsymbol{\theta}_{q,L-1}]^T$, $\mathbf{w}_M(n) = [w(n)\cdots w(n-M+1)]$, and

$$\mathcal{S}_M(\boldsymbol{\theta}) = \left[\mathcal{T}_M^T(\boldsymbol{\theta}_0),\dots,\mathcal{T}_M^T(\boldsymbol{\theta}_{L-1})\right]^T \quad (19)$$

is a Sylvester matrix, whereas

$$\mathcal{T}_M(\boldsymbol{\theta}_l) = \begin{bmatrix} \theta_{0l} & \cdots & \theta_{ql} & & 0 \\ & \ddots & & \ddots & \\ 0 & & \theta_{0l} & \cdots & \theta_{ql} \end{bmatrix} \quad (20)$$

is a Toeplitz matrix with M rows. From (18), the data correlation matrix $\mathbf{R_{ss}} = E\{\boldsymbol{y}_M(n)\boldsymbol{y}_M^{*T}(n)\}$ can be expressed as

$$\mathbf{R_{ss}} = \mathcal{S}_M(\boldsymbol{\theta})\mathbf{R_{ww}}\mathcal{S}_M^{*T}(\boldsymbol{\theta}) \quad (21)$$

in terms of the input correlation matrix $\mathbf{R_{ww}} \triangleq E\{\mathbf{w}_M(n)\mathbf{w}_M^{*T}(n)\}$. It was shown in [16] that under certain identifiablity conditions and for M large enough, $\mathcal{S}_M(\boldsymbol{\theta})$ has full column rank. Then, range$\{\mathcal{S}_M(\boldsymbol{\theta})\}$ = range$\{\mathcal{S}_M(\boldsymbol{\theta})\mathbf{R_{ww}}\mathcal{S}_M^{*T}(\boldsymbol{\theta})\}$, provided that $\mathbf{R_{ww}}$ has full rank. If we define the noise subspace of $\mathbf{R_{ss}}$ to be the space generated by the eigenvectors corresponding to the smallest eigenvalue and let $\mathbf{U}_n = [\mathbf{u}_1\cdots\mathbf{u}_{ML-M-q}]$ be the matrix containing those eigenvectors, then $\mathbf{U}_n$ spans the null space of $\mathcal{S}_M(\boldsymbol{\theta})\mathbf{R_{ww}}\mathcal{S}_M^{*T}(\boldsymbol{\theta})$ and is orthogonal to the range space

$$\mathbf{U}_n^{*T}\mathcal{S}_M(\boldsymbol{\theta}) = \mathbf{0}. \quad (22)$$

Equation (22) is linear in the parameters $h(k)$ and can be equivalently written as

$$\mathcal{S}_{q+1}^{*T}(\mathbf{U}_n)\boldsymbol{\theta} = \mathbf{0}, \quad j = 1,\dots,J \quad (23)$$

where

$$\mathcal{S}_{q+1}(\mathbf{U}_n) = [\mathcal{S}_{q+1}(\mathbf{u}_1),\dots,\mathcal{S}_{q+1}(\mathbf{u}_{MP-M-q})]. \quad (24)$$

It was shown in [16] that the solution of (22) uniquely identifies θ_{kl} (within a scaling ambiguity) if $M > q$.

In the current framework, the subspace estimation algorithm should follow these steps.

Step 1) Construct an estimate of correlation matrix $\mathbf{R_{ss}}$ using the estimated $\hat{r}_\theta(\tau;l_1,l_2)$. It is clear by comparing (18) and (7) that

$$\mathbf{R_{ss}} = E\{\mathbf{s}(n)\mathbf{s}^{*T}(n)\} = \begin{bmatrix} \mathbf{R}_{00} & \mathbf{R}_{01} & \cdots & \mathbf{R}_{0,L-1} \\ \mathbf{R}_{10} & \mathbf{R}_{11} & \cdots & \mathbf{R}_{1,L-1} \\ \vdots & & \ddots & \vdots \\ \mathbf{R}_{L-1,0} & & \cdots & \mathbf{R}_{L-1,L-1} \end{bmatrix} \quad (25)$$

is an $LM \times LM$ block matrix, where each block is given by (26), shown at the bottom of the next page. Therefore, an estimate $\hat{\mathbf{R}}_\mathbf{s}$ can be obtained by substituting the estimated $\hat{r}_\theta(\tau;l_1,l_2)$ in (26). Notice that the identity $\hat{r}_\theta(-\tau;l_1,l_2) = \hat{r}_\theta^*(\tau;l_2,l_1)$ can be used to obtain the required correlations with negative lags τ.

Step 2) Perform SVD on $\hat{\mathbf{R}}_\mathbf{s}$, and obtain the matrix $\hat{\mathbf{U}}_n$.

Step 3) Solve the system of (23) in the least squares sense to obtain $\hat{\boldsymbol{\theta}}_N$.

VI. Consistency/Identifiability

In this section, we study the behavior of the proposed algorithm and derive sufficient conditions under which reasonable accuracy of the proposed method should be expected. In particular, we are concerned with the following two issues. First, we examine the question of whether the parameter vector $\boldsymbol{\theta}$ is uniquely identifiable once the correlations $\hat{r}_x(n;\tau)$ are given. Then, based on the identifiability results, we derive the conditions so that $\hat{\boldsymbol{\theta}}_N$, which is obtained through the proposed algorithm and based on N data points, strongly converges to true parameter vector as $N \to \infty$.

A. Identifiability

In the sequel, it will be useful to denote the TV correlations by $r_x(n;\tau \mid \boldsymbol{\theta})$ and $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$ in order to emphasize the fact that they depend on the true parameter vector $\boldsymbol{\theta}$.

The proposed algorithm proceeds in two steps. In the first step, the correlations $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$ are estimated based on the estimates of $r_x(n;\tau \mid \boldsymbol{\theta})$. Then, θ_{kl} is estimated from the recovered correlations $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$. Following the same logic, in this section, we show identifiability by establishing uniqueness for each one of the two steps. The following Lemma discusses the uniqueness conditions for the first step.

Lemma 1: If $x(n)$ is generated by the system (2), (3) with parameter vector $\boldsymbol{\theta}$ and $r_x(n;\tau \mid \boldsymbol{\theta})$ is given, then under the assumptions AS1), AS2) and

AS3) for every fixed $\tau, \tau = 0, \ldots, q$, the product sequences $f_{l_1}(n) f^*_{l_2}(n+\tau)$, $l_1, l_2 = 0, \ldots, L-1$ are linearly independent

then, there is a unique $\mathbf{r}_\theta(\tau \mid \boldsymbol{\theta})$ satisfying (9) $\forall n$. □

It is clear from (9) and (11) that AS3) is crucial in order to guarantee identifiability. Under that assumption, the columns of $\boldsymbol{\Phi}(\tau)$ are linearly independent, and therefore, the lemma is true since (9) admits a unique solution. AS3) presents the most important limitation of the proposed method in the sense that it cannot be applied to problems with arbitrary expansion sequences $f_l(n)$ that may not satisfy AS3). More discussion on the advantages and limitations of the proposed method will be given in Section VII.

The next lemma establishes uniqueness of the second part of the algorithm and requires no extra identifiability conditions than those needed in the analysis of SIMO time-invariant systems [16], [24].

Lemma 2: If $x(n)$ is generated by the system (2), (3) with parameter vector $\boldsymbol{\theta}$ and $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$ is given, then under AS1) and the following assumption for θ_{kl}

AS4) i) the polynomials $\theta_l(z) = \sum_{k=0}^{q} \theta_{kl} z^{-k}$ have no common roots;
ii) $\theta_{0l} \neq 0$ for at least one l, $0 \le l \le L-1$;
iii) $\theta_{ql} \neq 0$ for at least one l, $0 \le l \le L-1$

there exists (within a scaling ambiguity) a unique parameter vector $\boldsymbol{\theta}$ generating $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$. □

Proof: See [16].

AS4) represents one more restriction on the applicability of the proposed method. By combining Lemmas 1 and 2, we arrive at the following identifiability result.

Proposition 1: If $x(n)$ is generated by the system (2), (3) with parameter vector $\boldsymbol{\theta}$, then under AS1), AS3), and AS4), $\boldsymbol{\theta}$ is uniquely identifiable from the TV correlations $r_x(n;\tau \mid \boldsymbol{\theta})$, $\tau = 0, \ldots q$, $n = 0, 1, \ldots$ □

Proposition 1 provides the sufficient conditions such that identifiability is guaranteed. It would be interesting to investigate if those conditions [especially AS3) and AS4)] are also necessary in order to recover θ_{kl} from second-order information or if they are only inherent to the particular method used here.

It can be easily seen that AS4) is indeed necessary for identifiability. If AS4) is violated, then there exist two different parameter vectors $\boldsymbol{\theta}^{(1)}$ and $\boldsymbol{\theta}^{(2)}$ that generate the same correlations $r_\theta(\tau; l_1, l_2)$ [16]; then, from (8), those two systems will have identical TV correlations $r_x(n;\tau)$ and are clearly nonidentifiable. Violation of AS4) is also the reason that the proposed method breaks down if applied to time-invariant systems.

On the other hand, if AS3) is violated, then only partial information about the correlations $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$ can be recovered from $r_x(n;\tau)$. Hence, the identifiability question depends on whether θ_{kl} can be uniquely identified from only partial knowledge of $r_\theta(\tau; l_1, l_2 \mid \boldsymbol{\theta})$; the answer appears to be application dependent.

The identifiability condition AS3) may be further relaxed if one is willing to employ a nonlinear correlation matching algorithm [27]; $\boldsymbol{\theta}$ may be estimated by minimizing the cost function

$$J = \sum_{\tau=-q}^{q} \|\hat{\mathbf{r}}_\theta(\tau) - \mathbf{r}_\theta(\tau \mid \boldsymbol{\theta})\|^2 = \sum_{\tau=-q}^{q} \sum_{n=0}^{N-1} [x(n)x^*(n+\tau) - r_x(n;\tau \mid \boldsymbol{\theta})]^2 \quad (27)$$

where $r_x(n;\tau \mid \boldsymbol{\theta})$ is computed from (6). This approach has higher computational complexity, but its consistency does not depend on AS3).

Other ways of relaxing AS3) are currently under investigation. Preliminary results show that if more than one output sensor is available, the conditions become milder [2], [13]. In the communications context, it is also possible to replace or complement spatial (antenna) diversity with oversampling [2], [4].

B. Consistency

Based on the identifiability results of the previous section, we will show next that the linear estimate $\hat{\boldsymbol{\theta}}_N$ converges to $\boldsymbol{\theta}$ w.p. 1 as $N \to \infty$. Two more assumptions will be needed to obtain consistency of the LS step of the algorithm:

AS5) The product sequences $f_{l_1}(n) f^*_{l_2}(n+\tau)$ are bounded, i.e., $\exists B$ such that $|f_{l_1}(n) f^*_{l_2}(n+\tau)| < B$, $\forall n$, $l_1, l_2 = 0, \ldots, L-1$.

$$\mathbf{R}_{l_1,l_2} = E\{\mathbf{s}_{l_1}(n)\mathbf{s}^{*T}_{l_2}(n)\} = \begin{bmatrix} r_\theta(0;l_1,l_2) & r_\theta(1;l_1,l_2) & \cdots & r_\theta(M-1;l_1,l_2) \\ r_\theta(-1;l_1,l_2) & r_\theta(0;l_1,l_2) & \cdots & r_\theta(M-2;l_1,l_2) \\ \vdots & & \ddots & \vdots \\ r_\theta(-M+1;l_1,l_2) & & \cdots & r_\theta(0;l_1,l_2) \end{bmatrix}. \quad (26)$$

AS6) If

$$\mathbf{R}_{\phi,N} = \frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{\phi}(n;\tau)\boldsymbol{\phi}^{*T}(n;\tau) \tag{28}$$

then the limit

$$\mathbf{R}_{\phi} = \lim_{N\to\infty}\mathbf{R}_{\phi,N}(n;\tau) \tag{29}$$

exists, and $\mathbf{R}_{\phi}$ is nonsingular.

AS6) is somewhat stronger than AS3) as it not only requires the columns of $\mathbf{\Phi}(\tau)$ to be linearly independent but also to be sequences of finite power; in other words, they are not allowed to become zero or tend to zero as $N \to \infty$.

AS5) and AS6) are used here because of their simplicity. They can be relaxed, however, to cover cases of basis functions that grow in an unbounded fashion (for details, see [8]). Based on these assumptions, the following proposition shows consistency of the proposed method.

Proposition 2: Let $x(n)$, $n = 0,\ldots,N-1$ be generated by the system (2), (3) with parameter vector $\boldsymbol{\theta}$; in addition, let $\hat{\mathbf{r}}_{\theta,N}(\tau)$ denote the LS solution of (17) and $\boldsymbol{\theta}_N$ the estimate from the subspace algorithm (22). Then, under assumptions AS1) and (AS4)–AS6), the following hold:

i) $\hat{\mathbf{r}}_{\theta,N}(\tau) \to \mathbf{r}_{\theta}(\tau)$ w.p. 1 as $N \to \infty$.
ii) $\boldsymbol{\theta}_N \to \boldsymbol{\theta}$ w.p. 1 as $N \to \infty$.

Proof: Let us first establish i). The LS solution of (17) can be written as

$$\hat{\mathbf{r}}_{\theta,N}(\tau) = \left[\frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{\phi}(n;\tau)\boldsymbol{\phi}^{*T}(n;\tau)\right]^{-1} \times \left[\frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{\phi}(n;\tau)\hat{r}_x(n;\tau)\right]. \tag{30}$$

By combining (30) with (9), we obtain [c.f. (15)]

$$\hat{\mathbf{r}}_{\theta,N}(\tau) - \mathbf{r}_{\theta}(\tau) = \left[\frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{\phi}(n;\tau)\boldsymbol{\phi}^{*T}(n;\tau)\right]^{-1} \times \left[\frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{\phi}(n;\tau)e(n;\tau)\right]. \tag{31}$$

For a fixed τ, $e(n;\tau)$ is a sequence with zero mean, finite memory and finite cumulants of every order. Based on this and AS5), it follows that the last factor in brackets in the right-hand side of (31) converges to 0 w.p. 1 as $N \to \infty$.

Furthermore, under AS6), the first factor in the right-hand side of (31) converges to $\mathbf{R}_{\phi}^{-1}$ and, hence, is bounded. Therefore, the left-hand side of (31) converges strongly to 0 as $N \to \infty$, completing the proof of i).

Based on i) and AS4), part ii) can be established (see [16]).

VII. Discussion

In the previous section, we developed the conditions under which $\boldsymbol{\theta}$ is identifiable and the proposed solution is expected to converge to the desired result. AS3) seems to be the most crucial as well as restrictive assumption in this approach. Let us consider a specific example to understand in which cases this assumption may fail. Let the basis sequences be complex exponentials $f_l(n) = \exp(j\omega_l n)$ with frequencies ω_l. Then, the product sequences are $f_{l_1}(n)f_{l_2}^*(n+\tau) = \exp(-j\tau\omega_{l_2})\exp(jnd_{l_1,l_2})$, $l_1, l_2 = 0,\ldots,L-1$, where $d_{l_1,l_2} = \omega_{l_1} - \omega_{l_2}$. These sequences are independent if all d_{l_1,l_2}'s are distinct for $\forall l_1, l_2$. However, even if the frequencies ω_l are distinct and not multiples of each other, for $l_1 = l_2 = l$, $\delta_{l,l} = 1$ will be identical $\forall l$. Hence, only the parameters $r_\theta(\tau; l_1, l_2)$ corresponding to the linearly independent regressors can be recovered; in contrast, only partial information can be obtained for the parameters $r_\theta(\tau; l, l)$. It is an interesting question whether θ_{kl} can still be recovered from the incomplete information on $r_\theta(\tau; l_1, l_2)$.

We close this discussion on the behavior of the algorithm with a note regarding additive noise. In certain applications, only a noisy version of the $x(n)$ is available, i.e.,

$$y(n) = x(n) + v(n) \tag{32}$$

where $v(n)$ is zero mean white noise (e.g., communications applications). In this case, (8) becomes

$$r_y(n;\tau) = \sum_{l_1,l_2=0}^{L-1} r_\theta(\tau;l_1,l_2)f_{l_1}(n)f_{l_2}^*(n+\tau) + \sigma_v^2\delta(\tau) \tag{33}$$

where $\sigma_v^2 = E\{|v(n)|^2\}$. In matrix form

$$\mathbf{r}_y(\tau) = \mathbf{\Phi}(\tau)\mathbf{r}_\theta(\tau) + \sigma_v^2\delta(\tau)\mathbf{1} \tag{34}$$

where $\mathbf{1} = [1, 1, \ldots 1]^T$ is a length N vector containing ones. We can see from (34) that the noise affects only the $\tau = 0$ lags. If σ_v^2 is given, these lags can be clearly estimated using $\mathbf{r}_x(\tau) = \mathbf{r}_y(\tau) - \sigma_v^2\delta(\tau)\mathbf{1}$. If σ_v^2 is unknown, then (34) can only be solved if $\mathbf{1}$ is incorporated in the regression matrix $\mathbf{\Phi}(\tau)$, and σ_v^2 is considered to be one more unknown parameter. This approach will only be successful, however, if the vector $\mathbf{1}$ is linearly independent with the columns of the matrix $\mathbf{\Phi}(\tau)$.

VIII. Simulations

In this section, we demonstrate the performance of the proposed algorithm by applying it to a simple linear almost periodically TV system of order $q = 1$.

The data were generated using (2), where the channel's TV parameters $h(n;k)$ were given by linear combination of the basis sequences, c.f. (3)

$$f_0(n) = 1, \quad f_1(n) = \cos\omega_0 n \tag{35}$$

where $\omega_0 = 2\pi/50$, and hence, the system is periodic with period equal to 50 samples; two basis sequences were used; hence, $L = 1$ here. The expansion parameters were $\theta_{00} = 0.5998$, $\theta_{01} = 0.2999$ and $\theta_{10} = -0.7197$, $\theta_{11} = -0.1799$, chosen so that the parameter vector θ has unit norm. The input

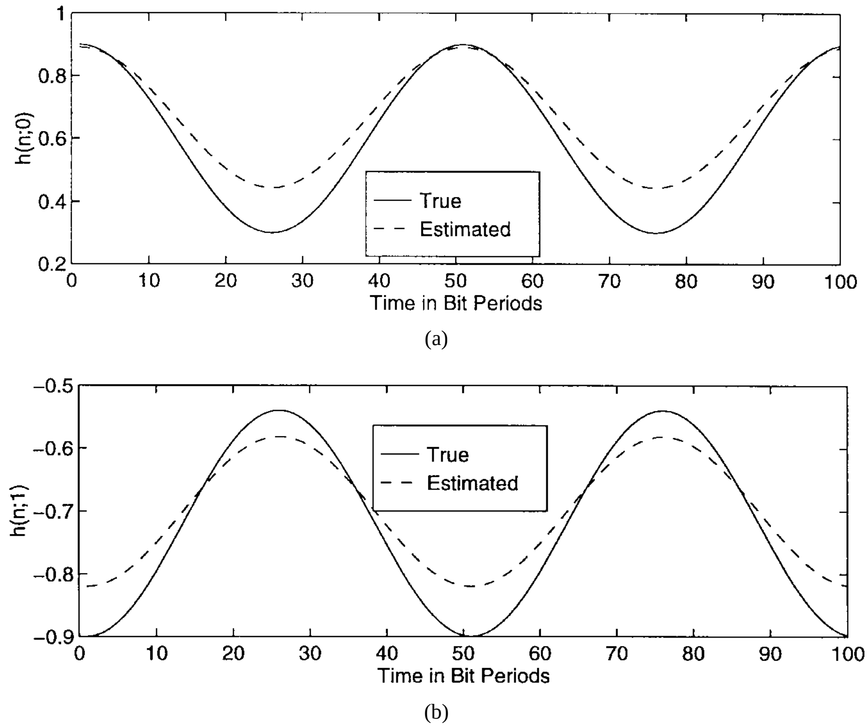

Fig. 3. Time-varying channel coefficients.

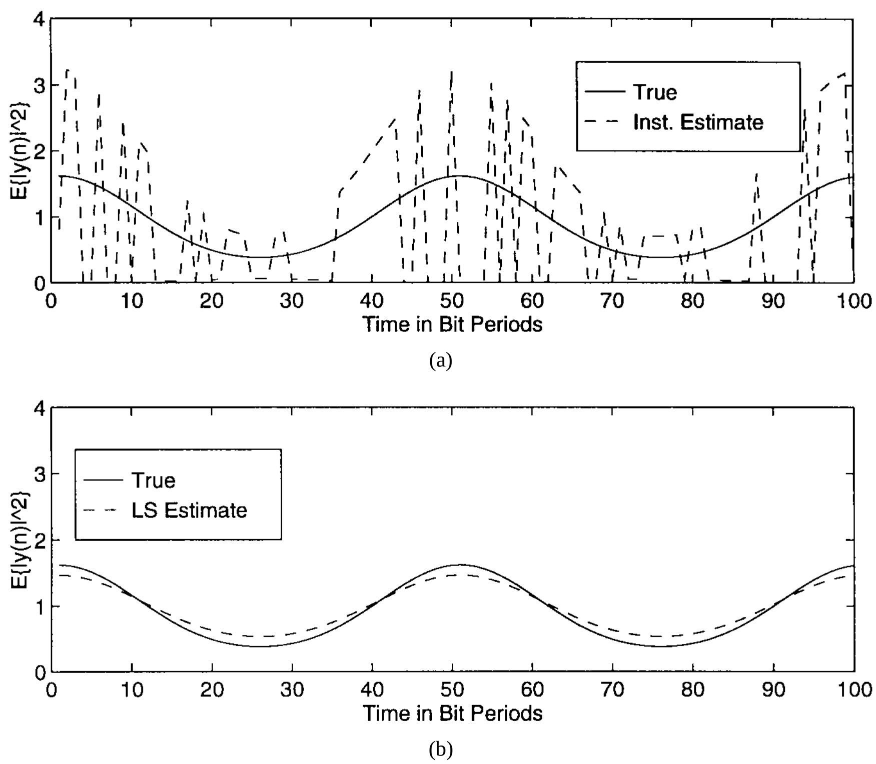

Fig. 4. Time-varying signal power.

was an i.i.d. binary (BPSK) ± 1 sequence, and the average SNR was 30 dB.

This system could represent a periodically TV communications channel. This kind of periodicity could arise if the transmission propagated through a periodically varying medium or under idealized Doppler shift and multipath conditions [27]. However, this example is only intended to illustrate the use of the proposed algorithm and should not be regarded

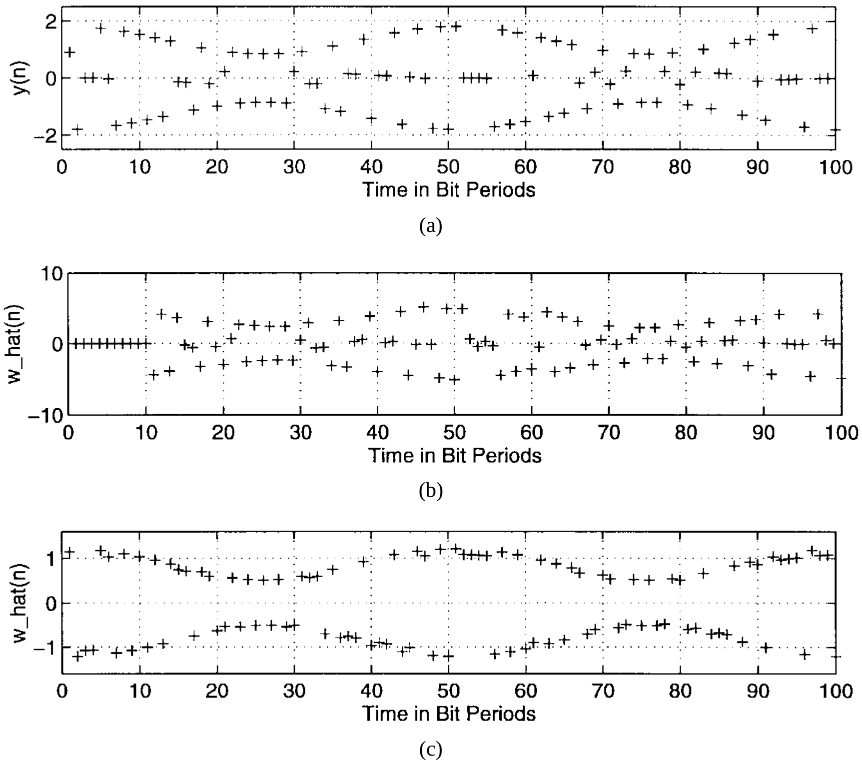

Fig. 5. Received and equalized symbols.

as a detailed simulations study of a particular communication system or standard. Fig. 3(a) and (b) shows the time history of the channel's TV parameters $h(n;0)$ and $h(n;1)$, respectively. The true ones are depicted with continuous line, whereas the reconstructed ones using the estimates $\hat{\theta}_{kl}$ obtained through the proposed algorithm are depicted with the dashed line. The data size was $N = 100$. Fig. 4 illustrates the fact that the LS estimates for $r_\theta(\tau; l_1, l_2)$ may be very reliable despite the inaccuracies of the instantaneous approximations $\hat{r}_x(n;\tau)$. The true TV signal power $r_x(n;0) = E\{|x(n)|^2\}$ as well as the instantaneous estimate $\hat{r}_x(n;0) = x^2(n)$ are shown in Fig. 4(a) (solid and dashed lines, respectively). In Fig. 4(b), the true and LS reconstructed signal power $\hat{r}_{x,\mathrm{LS}}(n;0) = \mathbf{\Phi}(0)\hat{r}_{\theta,N}(0)$ is shown.

The proposed method was not directly compared with any other technique since no other second-order based method is available that is capable of blindly identifying this TV channel.

However, we tried the Godard algorithm, which is an adaptive technique, and could perhaps follow the channel's variations. Unfortunately, as Fig. 5 shows, this particular channel is too fast, and the Godard algorithm cannot track its variations. In Fig. 5(a), the received data symbols $x(n)$ are plotted, whereas in Fig. 5(b), the output of the Godard equalizer is plotted. We observe that severe intersymbol interference (ISI) is present in the received data, which is not corrected by the Godard equalizer. In contrast, a decision feedback (DF) equalizer based on the estimated $\hat{h}(n;k)$ from the proposed method manages to remove most of the ISI [Fig. 5(c)]. A Godard equalizer of 10 taps was used for part b), whereas a simple DF equalizer of the form

$$\hat{w}(n) = [x(n) - \hat{h}(n;1)\tilde{w}(n-1)]/\hat{h}(n;0) \qquad (36)$$

was used for part c), where $\tilde{w}(n-1)$ is the previously decoded symbol. The periodic variations in the amplitude of $\hat{w}(n)$ are due to inaccuracies in the estimation of the expansion parameters.

Finally, in order to gain some intuition on the average performance of the proposed technique, Table I presents some Monte Carlo results. The true parameter vector $\boldsymbol{\theta} = \{\{\theta_{kl}\}_{l=0}^{L-1}\}_{k=0}^{q}$ is depicted, as well as the mean $\pm$ standard derivation of 100 Monte Carlo iterations. The data size was $N = 100$.

IX. Conclusion

A novel approach to blind estimation of TV-FIR systems is presented, which does not resort to higher order statistics. It exploits time-diversity information, which is not necessarily of the fractional sampling type, and allows blind TV-SISO channel identification. Further research is needed to broaden the applicability of the proposed method and relax the identifiability assumptions. In particular, the development of a direct method (which obviates the two-stage procedure) is perhaps possible, especially in the antenna array case.

Appendix

The proposed alogrithm has been derived under the i.i.d. assumption for the input AS1). For cases where this assump-

TABLE I
MONTE CARLO RESULTS

SNR	θ_{00}			θ_{10}		
	true	mean	variance	true	mean	variance
30dB	0.5998	0.6142	0.0307	-0.7197	-0.5757	0.0922
15dB	0.5998	0.6211	0.0261	-0.7197	-0.5509	0.1065
SNR	θ_{01}			θ_{11}		
	true	mean	variance	true	mean	variance
30dB	0.2999	0.2254	0.0353	-0.1799	-0.2000	0.0434
15dB	0.2999	0.2420	0.0339	-0.1799	-0.1860	0.0521

tion may not be realistic, the derivation can be modified to incorporate colored inputs.[1]

Let $r_w(\tau) = E\{w(n)w^*(n+\tau)\}$ be the correlation of the input. Then, (5) becomes

$$r_x(n;\tau) = \sum_{k_1,k_2=0}^{q} h(n;k_1)h^*(n+\tau;k_2)r_w(\tau+k_1-k_2). \tag{37}$$

Substituting from (3), we obtain

$$r_x(n;\tau) = \sum_{l_1,l_2=0}^{L-1} f_{l_1}(n)f_{l_2}^*(n+\tau)r_\theta(\tau;l_1,l_2) \tag{38}$$

where

$$\begin{aligned} r_\theta(\tau;l_1,l_2) &= \sum_{k_1,k_2=0}^{q} \theta_{k_1,l_1}\theta_{k_2,l_2}^* r_w(\tau+k_1-k_2) \\ &= \sum_{k=0}^{q}\sum_{m=k+\tau-q}^{k+\tau} \theta_{k,l_1}\theta_{k+\tau-m,l_2}^* r_w(m). \end{aligned} \tag{39}$$

Hence, (21) is still valid with $\mathbf{R_{ww}}$ being a Toeplitz matrix with lags $\{r_w(\tau)\}_{\tau=0}^{M-1}$. As shown in [16], the subspace estimation method is still applicable as long as $\mathbf{R_{ww}}$ has full rank. Hence, the proposed method is not sensitive to the color of the input.

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