COMBINING GALOIS WITH COMPLEX FIELD CODING FOR HIGH-RATE SPACE-TIME COMMUNICATIONS

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ABSTRACT

A joint scheme combining error-control coding (ECC) with complex-field coding (CFC) is proposed for space-time communications through flat or frequency-selective fading multiple-input multiple-output (MIMO) channels. It is shown that the diversity gain of this joint coding scheme is the product of the free distance of the ECC, the complex-field block encoder size, and the number of receive antennas, which is computationally difficult to achieve using ECC alone. For two antennas, the scheme can achieve the same performance as the Alamouti scheme with coding. We also present some high rate transmission schemes that can achieve maximum diversity at a rate higher than one symbol per channel use.

1. INTRODUCTION

Future generation communication systems are likely to employ antenna arrays to enhance the data rates and cope with the adverse fading effects of wireless propagation. Linear constellation rotation coding [9] for space-time (ST) communications achieves a rate of one symbol per channel use with diversity gain as high as MN, where M and N are the number of transmit and receive antennas, respectively. Practical decoding, however, relies on near-optimum algorithms such as sphere-decoding to guarantee maximum diversity. Since the complexity of sphere decoding increases dramatically with the number of independent symbols per block, in this case M, the application of constellation rotation codes is restricted to a small number of transmit antennas. Linear constellation rotation coding can be viewed as a counterpart of a Galois-field (GF) block code in complex field (CF), and hence will also be termed Complex field coding (CFC). Instead of having entries from a GF, a CFC generator can have complex entries. It has been shown [9] that CFC allows for large (in many cases, maximum) diversity gain with small or no rate loss. The key point is that in fading channels, the minimum Hamming distance eventually determines the diversity order, or the slope, of the average probability of error curve. Even a square (that is, rate 1) CFC is capable of producing codewords that have minimum Hamming distance as large as the codeword length, which is rarely possible with GF codes.

We propose in this paper a concatenated coding scheme for space-time communications. The idea is to combine conventional error control coding (ECC) and CFC in a serial fashion, with an interleaver in between. Serial concatenation of two ECCs has been used for a long time, where the outer code is usually a Reed-Solomon code, while the inner code can be either a block code or more often a convolutional code. The concatenation of ECC and CFC, however, is motivated by the observation that ECC is usually very good in coping with additive noise channels, while CFC, thanks to its large Hamming distance, is quite effective in coping with fading channels. Transmitting CFC encoded symbols over the channel and detecting them at the receiver convert the fading channel to a "less-fading" one, ideally to an additive white Gaussian noise (AWGN) channel when the CFC encoder size grows to infinity, for which the ECC is known to become most effective.

Notation: we will use bold face lowercase (uppercase) letters to denote column vectors (matrices); $(\cdot)^T$ and $(\cdot)^{\mathcal{H}}$ will denote transpose and hermitian transpose respectively; $\mathrm{diag}(x_1, \ldots, x_n)$ is an $n \times n$ diagonal matrix with the given entries on the diagonal; $|\cdot|$ is the Euclidean norm; and $[\mathbf{X}]_{i,j}$ is the (i, j)th entry of the matrix $\mathbf{X}$.

2. SYSTEM MODELING

Let M denote the number of transmit antennas, and N the number of receive antennas. When the channel is flat fading, and the input-output relationship can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{1}$$

where $\mathbf{x}$, $\mathbf{y}$, $\mathbf{v}$ are $M \times 1$, $N \times 1$, and $N \times 1$ vectors denoting transmitted signal, received signal, and additive noise samples across the multiple antennas during one symbol period, respectively; $\mathbf{H}$ is the channel mixing matrix, whose (n, m)th entry is $h_{n,m}$, the channel gain from the mth transmit to the nth receive antenna. When the channel is frequency selective, we assume that orthogonal frequency-division multiplexing (OFDM) has been used so that (1) may represent the flat-fading channel for one subcarrier. We assume that sufficient interleaving is used to decorrelate the channel.

2.1. Complex Field Coding for Diversity

CFC is one form of *signal space diversity* that was introduced in [2, 3] for flat-fading Rayleigh channels to improve system performance, especially to gain in diversity. The idea is to transmit different linear combinations of uncoded constellation symbols over the flat fading channel, instead of transmitting the constellation symbols directly. These linear combinations are chosen such that the original constellation symbols can be detected even when only one linear combination can be recovered from the channel.

More specifically, suppose $\mathbf{s} \in \mathcal{A}^P$ is a $P \times 1$ vector, where $\mathcal{A}$ denotes the alphabet of the underlying constellation. A CF encoder is represented as a $P \times P$ matrix $\boldsymbol{\Theta}$ with complex entries. The encoding is performed simply through the matrix-vector product: $\mathbf{u} = \boldsymbol{\Theta}\mathbf{s}$. We define the *Hamming distance* between two vectors as the number of non-zero entries in their difference. The encoder $\boldsymbol{\Theta}$ is chosen such that for any two vectors in $\mathcal{A}^P$, their encoded vectors have Hamming distance P. With this property, the entire vector $\mathbf{s}$ can be recovered even from one entry of $\mathbf{u}$. We call a CFC that satisfies this condition *maximum Hamming distance separable (MHDS)*. CFCs that are MHDS achieve maximum diversity [9].

2.2. CFC for ST communications: rate one case

We adopt here the scheme CFC scheme proposed in [9] for our joint space-time ECC-CFC coding setup.

Specifically, we transmit the entries of the CF encoded vector $\mathbf{u}$ serially using the antennas, one entry per symbol period. When there is only one receive antenna, the received signal will be coming from only one transmit antenna at one time (no two antennas are transmitting together). If we define $\mathbf{X}$ to be the space-time signaling matrix, whose (m, t)th entry is transmitted from the mth antenna at time t, then $\mathbf{X}$ is a diagonal matrix whose diagonal entries are outputs of the CFC.

Since the CF coding matrix $\boldsymbol{\Theta}$ is square, it does not introduce rate loss: the rate of transmission is one symbol per channel use. It may seem that the multiple antennas are not used efficiently: one could also transmit using the multiple antennas simultaneously, and thus increase transmission rate. Some high rate options that allow for such parallel transmissions will be presented in Section 2.3. But even for rate one transmissions, antenna switching essentially increases the equivalent channel variation speed by sampling the different channels periodically, and alternately.

There are other ways of converting the multiple-input channel to a single-input channel. Orthogonally designed space-time codes offer one such scheme [7]. However, for complex constellations, a rate one orthogonal design is achievable only for two transmit antennas. For more than two transmit antennas, there is a rate loss factor of 3/4 or 1/2 [7]. Other means of converting the multiple-input channel to a single-input channel include antenna phase-sweeping [4], and delay diversity transmissions [6]. We will compare our CFC antenna switching scheme with orthogonal designs using simulations. We may also combine CFC with orthogonal designs for the two transmit-antenna case. When delay diversity is used, a flat fading ST channel is converted into a frequency-selective channel, and existing results pertaining to OFDM systems are directly applicable.

2.3. CFC for ST communications: high rate cases

It is possible to achieve rate higher than 1 symbol per channel use using CFC.

Compressive CFC. Instead of a square CFC matrix $\boldsymbol{\Theta}$, we can also use a $\boldsymbol{\Theta}$ of size $P \times P'$ that has more columns than rows; i.e., $P < P'$. The CFC encoded block is again serially transmitted through the equivalent SISO channel in Fig. 1. Such a $\boldsymbol{\Theta}$ can be obtained by truncating a larger CFC $\boldsymbol{\Theta}'$ of size $P' \times P'$, and keeping only $P < P'$ rows of $\boldsymbol{\Theta}'$. If $\boldsymbol{\Theta}'$ is designed to achieve full diversity so that any two blocks encoded by $\boldsymbol{\Theta}'$ do not share even a single entry, then $\boldsymbol{\Theta}$ will also inherit full diversity because a block encoded by $\boldsymbol{\Theta}$ is only a sub-block of one encoded by $\boldsymbol{\Theta}'$. The resulting system will then have diversity order PN, the same as in the rate one case.

Triangular ST signaling. Another scheme is to allow for simultaneous (as opposed to alternate) transmissions from the multiple transmit-antennas. In order to guarantee maximum diversity, we propose to transmit using a triangular ST signal matrix. Instead of having a diagonal space-time signaling matrix $\mathbf{X}$, which corresponds to antenna-switching, we let $\mathbf{X}$ be lower (or upper) triangular. The entries of $\mathbf{X}$ are CF coded symbols that are outputs of a size $M(M + 1)/2$ CFC encoder: there are totally $M(M + 1)/2$ non-zero entries in the matrix $\mathbf{X}$. The rate in this case is $(M+1)/2$ symbols per channel use. If the CFC is MHDS, it can be shown that such a signaling scheme can guarantee maximum diversity order of MN. The proof, however, is omitted due to lack of space.

2.4. Joint GF and CF Coding

The complexity of optimum CFC decoding increases exponentially with the size parameter P. Choosing P presents a tradeoff between higher diversity, and lower complexity. The same tradeoff also emerges with GF codes. For example, consider a convolutionally coded transmission through an i.i.d. flat-fading Rayleigh channel. The diversity order is the free distance d_{free} of the GF code. For a fixed rate

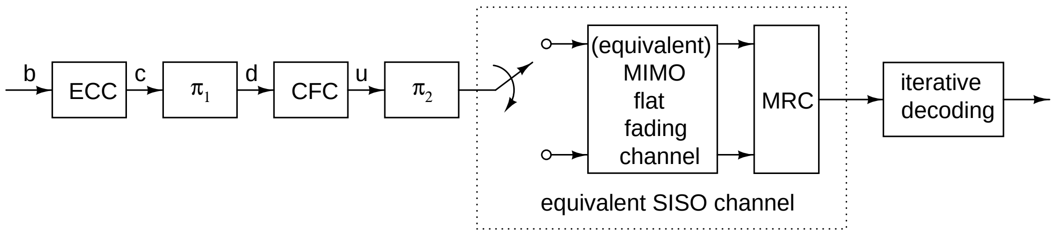

Figure 1: Joint GF and CF coded ST system Model

code, the free distance can be increased by increasing the encoder memory. This memory increase, however, results in an exponential increase in decoding complexity, because the number of states in the trellis of the code is exponential in the encoder memory.

A better tradeoff between performance and complexity is offered when combining GF with CF coding in a serial fashion. We have introduced this combination for single-antenna OFDM transmissions in [8], where the performance of such a joint coding system has been studied using (pairwise error probability) PEP analysis under the assumption of perfect interleaving. In the following, we will tailor the major results of [8], to our space-time setup.

Consider the system diagram in Fig. 1, where a rate one antenna switching scheme is shown. At the transmitter, the information bit stream b_n is first encoded using some GF error control mechanism involving e.g., block, convolutional, or turbo codes. The output stream c_n is then interleaved using Π_1, and mapped to constellation symbols. If Trellis Coded Modulation (TCM) is used as the error-control code, then the constellation mapping is not needed since the mapping is already incorporated in the TCM.

After constellation mapping, the information-bearing symbol stream s_n is encoded by a CFC matrix $\mathbf{\Theta}$. For low-complexity decoding, only small matrices of size 2 or 4 will be used. Specifically, we will use the size 2 and 4 encoders designed using algebraic number theory [8, 9]

$$\mathbf{\Theta}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & \mathrm{e}^{j\frac{\pi}{4}}\\ 1 & \mathrm{e}^{j\frac{5\pi}{4}}\end{bmatrix},\ \mathbf{\Theta}_4 = \frac{1}{2}\begin{bmatrix}1 & \mathrm{e}^{j\frac{\pi}{8}} & \mathrm{e}^{j\frac{2\pi}{8}} & \mathrm{e}^{j\frac{3\pi}{8}}\\ 1 & \mathrm{e}^{j\frac{5\pi}{8}} & \mathrm{e}^{j\frac{10\pi}{8}} & \mathrm{e}^{j\frac{15\pi}{8}}\\ 1 & \mathrm{e}^{j\frac{9\pi}{8}} & \mathrm{e}^{j\frac{18\pi}{8}} & \mathrm{e}^{j\frac{27\pi}{8}}\\ 1 & \mathrm{e}^{j\frac{13\pi}{8}} & \mathrm{e}^{j\frac{26\pi}{8}} & \mathrm{e}^{j\frac{39\pi}{8}}\end{bmatrix}.$$

We may also use Hadamard matrices as $\mathbf{\Theta}$. Hadamard matrices have been advocated in [5], where code-division multiplexing is combined with OFDM to exploit frequency domain diversity.

After the CF block encoding by $\mathbf{\Theta}$, the symbol stream u_n is interleaved by Π_2, and the interleaver output $\bar{u}_n$ is transmitted over the multiple antennas alternately, using one transmit-antenna per time slot. When the channel is frequency-selective, we assume that OFDM modulation/demodulation has been used to convert the channel into parallel (correlated) flat fading channels.

The choice of the size of Π_2 depends on the channel variation. For slowly varying channels, Π_2 needs to have a large enough size so as to sufficiently decorrelate the channel. For performance analysis, we will assume that the channel has been perfectly decorrelated so that the equivalent SISO channel in Fig. 1 is i.i.d., Nakagami-m distributed with parameter m$=N$, due to maximum ratio combining (MRC) at the receiver.

Let R denote the information rate of the ECC, and Q denote the size of the constellation used before CFC. The rate of the overall system is therefore $KR\log_2(Q)$ bits per channel use. When the triangular signaling scheme of Section 2.3 is used, the rate can be $(M+1)/2$ times higher, reaching $0.5KR(M+1)\log_2(Q)$ bits per channel use.

Due to space limit, we will not describe the decoding options for our joint GF-CF coding scheme. The iterative decoders of [8] are applicable.

3. PERFORMANCE ANALYSIS

We assume that the ECC is a convolutional code properly terminated. Other choices including block or turbo codes can be treated similarly. We next look into performance using PEP analysis. For simplicity, we assume binary phase-shift keying (BPSK), although other constellations can be dealt with likewise.

Consider a pair of different information sequences that are encoded using ECC. They may result in two ECC codewords that are different in w ECC symbols, where $w \geq d_{\text{free}}$, and d_{free} is the free distance of the ECC. After interleaving, these w non-zero symbols in the error sequence will be spread far away from each other (or at least so in a statistical sense when a random interleaver Π_1 is used). The CFC is then used to spread each of these w symbols to P independent flat-fading channel attenuations. The remaining symbols in one CFC block are all zeros, under the assumption that no two of the w non-zero symbols in the ECC error event enter the same CFC block. Each of the wP non-zero symbols at the CFC output will have an amplitude $\sqrt{E_s/P}$, where E_s is the energy per ECC symbol, and the factor $\sqrt{P}$ comes from the energy normalization factor of each CFC matrix. Each of the wP symbols sees a different realization of the equivalent SISO channel in Fig. 1. The resulting received error sequence will thus have wP non-zero symbols:

$$\bar{y}_i = \alpha_i u_i, \quad i = i(j),\ j = 1, 2, \ldots, wP, \qquad (2)$$

where u_i is ith non-zero symbol in the CFC output of the error event; α_i is the flat SISO symbol that u_i sees after MRC, which is the square root of the squared sum of N Rayleigh random variables that gives $\mathrm{E}[\alpha_i^2] = N$ (i.e., Nakagami-m distributed with parameter m=N); and y_i is the corresponding received signal from the equivalent SISO channel of Fig. 1. The additive noise term that can confuse the decision between the two sequences has variance N_0. The probability of a pairwise error between the pair of information sequences is then given by the function

$$Q(\sqrt{\sum_{i=1}^{wP} |y_i|^2/(2N_0)}). = Q(\sqrt{\sum_{i=1}^{wP} |\alpha_i u_i|^2/(2N_0)}). \quad (3)$$

We can Write each $|\alpha_i|^2$ as the squared sum of N independent Rayleigh random variables. Since $|u_i|^2 = E_s/P$, each $|y_i|^2$ term in (3) will be equivalent to a squared sum of N Rayleigh random variables each having second moment equal to E_s/P. The variable $\sum_{i=1}^{wP} |\alpha_i u_i|^2/(2N_0)$ will then be chi-square distributed with $2wPN$ degrees of freedom, giving rise to a diversity order of wPN.

The minimum of this diversity order among all error events is $d_{\text{free}}PN$, and will dominate the overall system performance. From this, we conclude that the system diversity order is $d_{\text{free}}PN$. We call this the *multiplicative diversity effect*. That is the system diversity is the product of d_{free}, the diversity achieved by the convolutional code when used alone, times P, the diversity achieved by the CFC when used alone, times the receive diversity N. The number of transmit antennas, M, does not play an apparent role here, because we have assumed that antenna switching is used, and the equivalent SISO channel has been sufficiently decorrelated. However, M will show up if we consider the performance of the high-rate triangular signaling scheme of Section 2.3.

The coding gain and the gain of using CFC have also been quantified in the single-antenna OFDM setup in [8]. We will not re-derive them here, but only mention that using CFC after ECC results in a gain that increases with the CFC encoder size. And with size 4 CFC encoders, most (70-80%) of the ultimate gain achievable by using an infinitely large CFC can already been achieved. Thus, from a performance-complexity tradeoff point of view, using a CFC encoder of size larger than 4 is not recommended.

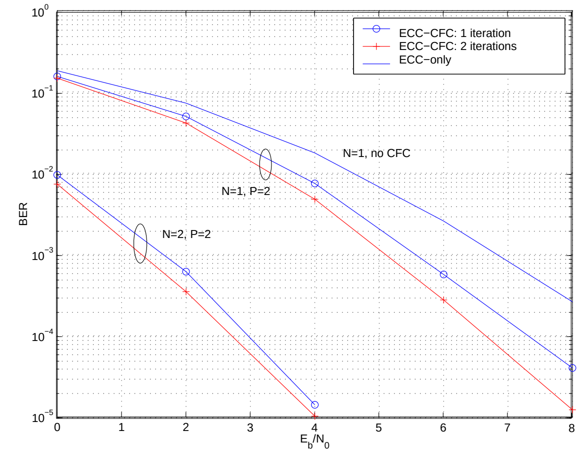

Figure 2: Multiplicative diversity effect

4. SIMULATION RESULTS

In this section, we show numerical simulation results of our joint ECC-CFC scheme for different setups. We use BPSK constellation, and E_b/N_0 to denote the bit signal-to-noise ratio (SNR) per receive antenna. Unless otherwise mentioned, we adopt throughout a random interleaver Π_1 of length corresponding to a delay of 1024 information bits, which we call a frame of symbols. Interleaver Π_2 is assumed to be perfect so that the channel is completely decorrelated in time. We stop the bit-error rate (BER) count when either 50 frame errors are detected, or, 2000 frames are transmitted. Except for Test Case 1, we use one receive antenna: $N = 1$.

Test case 1 (Multiplicative diversity effect): In this simulation, we assume that the channel is decorrelated across time and antennas. We test the antenna-switching scheme of Fig. 1, where the MIMO channel is converted to a flat-fading i.i.d., SISO channel. We use two iterations for decoding the joint ECC-CFC system. More iterations did not provide additional gains. The ECC was a rate 1/2 convolutional code with constraint length 3, generating polynomials in octal form (7, 5), and $d_{\text{free}} = 5$.

We tested the ECC without CFC, and ECC with CFC of size $P = 2$, both for one receive antenna ($N = 1$), as well as ECC with CFC of size $P = 2$ for two received antennas. From the BER performance in Fig. 2, it can be seen that there is a performance improvement by using CFC. Even one iteration provides quite noticeable gain, while the diversity orders approximately comply with the multiplicative diversity prediction: the ECC-only system has diversity equal to $d_{\text{free}} = 5$, the ECC-CFC with $N = 1$ and $P = 2$ has diversity 10, and the ECC-CFC with $N = 2$ and $P = 2$ has diversity order close to 20. The huge gain in the $N = 2$ case is also due to the additional receive antenna, which in-

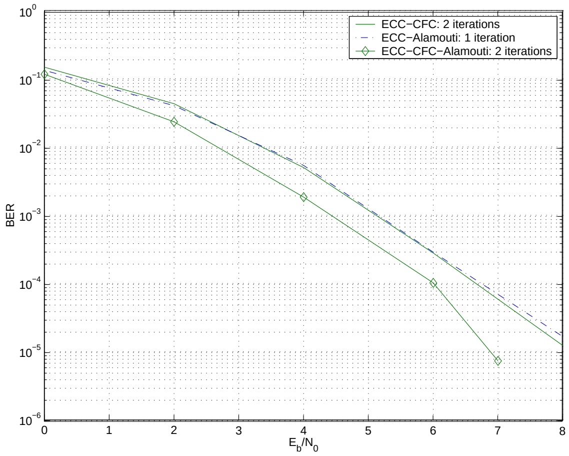

Figure 3: Comparison with the Alamouti code

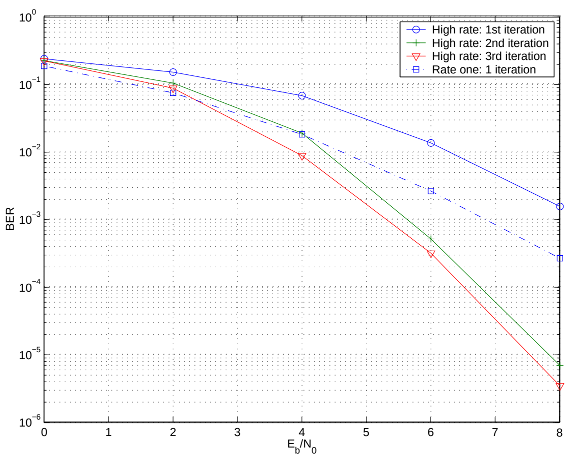

Figure 4: High rate triangular signaling

creases the total received power by 3dB.

Test case 2 (Comparison and combination with Alamouti's code): Relying on two transmit antennas, we compare here our ECC-CFC antenna switching scheme with the Alamouti code [1], which also has rate one symbol per channel use. Both schemes were encoded using a rate $1/2$ convolutional code with generators $(7,5)$. We depict the performance of our ECC-CFC scheme with two iterations along with that of the ECC encoded Alamouti code in Fig. 3. It can be seen that there is virtually no difference in the BER. We can also consider using CFC and Alamouti coding jointly, in which case, we will replace our antenna-switching strategy with the Alamouti code. The result is also depicted in Fig. 3. Although the additional diversity gain shows up at high SNR, combining the two only offers coding gain of about 0.5dB for low SNRs.

Test case 3 (High rate triangular signaling): We simulated in this test the performance of the high rate triangular signaling scheme of Section 2.3. We chose $M = 3$ and $N = 1$. According to Section 2.3, the transmission rate is $(M+1)/2 = 2$ symbols per channel use. Using the rate 1/2 $(7,5)$ convolutional code, the information rate was 1 bit per channel use. At the receiver, we again used siso modules for iterative decoding. We depict in Fig. 4 the BER performance of the ECC-CFC coded transmission with antenna switching and with 3 decoder iterations, together with the performance of an ECC-only system with antenna switching but without CFC high-rate encoding. Although the rate of the ECC-only system is only 0.5 bit per channel use, half of that of the ECC-CFC system, the ECC-CFC has better performance for $E_b/N_0 > 2$dB. The price paid is slightly increased complexity.

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