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Distributed Scheduling and Resource Allocation for Cognitive OFDMA Radios

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Abstract Scheduling spectrum access and allocating power and rate resources are tasks affecting critically the performance of wireless cognitive radio (CR) networks. The present contribution develops a primal-dual optimization framework to schedule any-to-any CR communications based on orthogonal frequency division multiple access and allocate power so as to maximize the weighted average sum-rate of all users. Fairness is ensured among CR communicators and possible hierarchies are respected by guaranteeing minimum rate requirements for primary users while allowing secondary users to access the spectrum opportunistically. The framework leads to an iterative channel-adaptive distributed algorithm whereby nodes rely only on local information exchanges with their neighbors to attain global optimality. Simulations confirm that the distributed online algorithm does not require knowledge of the underlying fading channel distribution and converges to the optimum almost surely from any initialization.

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1 Introduction

The broadcast nature of wireless radios comes with interference among communicators contending for the shared air-interface. Fixed access and resource allocation approaches to cope with interference have led to scarcity of the available bandwidth, expensive licenses and under-utilization of the spectrum in space, frequency and time [1]. Spectrum under-utilization has in turn motivated recent research on dynamic spectrum management and wireless cognitive radios (CRs) which are capable of sensing and accessing the spectrum dynamically. Co-existence settings involving licensed and un-licensed CRs have also been proposed [2]. These settings entail primary, i.e., licensed, users as well as secondary users who access the spectrum opportunistically, provided that they yield to primary users.

A number of challenges arise with such dynamic and hierarchical means of accessing the spectrum [3]. As the secondary CRs communicate opportunistically, they should be capable of sensing the spectrum over a wide range of frequencies—a task requiring sampling and processing at very high rates. Furthermore, dynamic scheduling and allocation of resources must be performed online to improve the bandwidth utilization. Accordingly, CR users must be capable of adapting their transmission and reception parameters to the intended dynamically changing channel while respecting possible hierarchies and adhering to power constraints and diverse quality of service (QoS) requirements. Challenges in CR networks are magnified if instead of centralized operations performed at access point(s), scheduling and resource allocation are to carried in a distributed fashion. Such distributed protocols are particularly well motivated for infrastructureless, i.e., ad hoc CR networks which entail any-to-any communication links.

In this context, the present paper provides a framework for optimal scheduling and resource allocation in ad hoc CR networks that rely on orthogonal frequency division multiple access (OFDMA) at the physical layer. OFDMA is well suited for wireless access because it is agile in allocating sub-carriers dynamically and facilitates decoding at the receiving end of each link. The optimization objective is to maximize the weighted sum-rate (ergodic capacity) subject to maximum average power constraints on individual radios and nonzero minimum average rate constraints on primary users that are possibly present. The novel framework provides fairness and results in an iterative off-line algorithm that is also amenable to distributed implementation but requires knowledge of the underlying fading distribution. More important from a practical perspective is an online alternative algorithm which relies on stochastic approximation iterations and is provably convergent to the off-line optimal solution regardless of the underlying fading distribution and the chosen initialization.

The rest of the paper is organized as follows. The ensuing Section 2 describes the model and operating conditions. Section 3 states analytically the optimization problem at hand and derives the off-line algorithm for scheduling and power allocation. The distributed online solution is the subject of Section 4. Simulated test cases are included in Section 5 and conclusions are drawn in Section 6.

2 Modeling preliminaries

Consider a network of CRs comprising a set of J users $U := \{u_j,\ \ j = 1 : J\}$ who wish to communicate any-to-any over the available system bandwidth B. Assuming OFDMA, this bandwidth is divided in K sub-bands, each having width $\Delta B = B/K$, sufficiently small for the corresponding sub-channel to be accurately approximated by a flat fading coefficient. To schedule access, the K system sub-carriers over the corresponding sub-bands must be assigned to the active pairs of user links and power must be allocated judiciously so that the weighted average sum-rate of the network is maximized and constraints of CRs are satisfied.

Let $\boldsymbol{\gamma} := (\gamma_{ijk},\ \ i, j = 1 : J,\ \ i \neq j,\ \ k = 1 : K)$ denote the vector collecting the sub-channel coefficient gains (magnitude squares), where γ_{ijk} stands for the gain of the wireless link from source u_i to destination u_j over the subcarrier k. Since it can be assumed without loss of generality (w.l.o.g.) that the additive white Gaussian noise at each destination node has unit variance, these gains describe the instantaneous receive-SNR per receiver. They further adhere to a block fading channel model whereby $\boldsymbol{\gamma}$ is a random vector that remains invariant over blocks (coherence time slots) of size T_n and uncorrelated across successive blocks. Formally stated, the resultant assumption is:

(as1) *The receive-SNR vector $\boldsymbol{\gamma}$ is stationary, ergodic, and entry γ_{ijk} is known (via training) at source node u_i $\forall i$.*

The distributed algorithm to be developed requires users to exchange minimal amount of control information. To this end, each node u_i is allowed to exchange control information only with users in its neighborhood $\mathcal{N}_i$. Associated with this control network, let $\mathcal{G} = (\{u_i\}, \{\epsilon_{ij}\})$ denote a graph, where the edge $\epsilon_{ij} \in \mathcal{G}$ if and only if $u_j \in \mathcal{N}_i$. For control information to percolate the entire network based only on local exchanges, it is further assumed that:

(as2) *The control graph $\mathcal{G}$ is connected.*

During each coherence interval T_n different users are allowed at the outset to time share a subcarrier provided that the time-sharing sub-intervals are non-overlapping across users. This ensures orthogonality of user access in frequency and/or time and prevents interference from link to link. If α_{ijk} denotes the fraction of time user u_i transmits to user u_j over the sub-carrier k, the time sharing vector to optimize over is $\boldsymbol{\alpha} := (\alpha_{ijk},\ \ i, j = 1 : J,\ \ i \neq j,\ \ k = 1 : K)$. Clearly, its entries must satisfy

$$\sum_{i,j=1,\ i\neq j}^{J} \alpha_{ijk} \leq 1. \tag{1}$$

If user u_i is scheduled to transmit to user u_j over the subcarrier k, then $\alpha_{ijk} > 0$ and during this fraction of time u_i will load the subcarrier k with power p_{ijk}. For future use, let $\boldsymbol{p} := (p_{ijk},\ \ i, j = 1 : J,\ \ i \neq j,\ \ k = 1 : K)$ denote the vector of transmit-powers across the network. The goal is to maximize the network welfare of weighted sum-rate, where attainable rates are expressed in terms of the ergodic capacity of user links. Specifically, for the wireless channel from u_i to u_j over the sub-carrier k the ergodic capacity is given by

$$c_{ijk} = \mathbb{E}_{\boldsymbol{\gamma}}[\alpha_{ijk} \log_2(1 + \gamma_{ijk} p_{ijk})] \tag{2}$$

where $\mathbb{E}_{\boldsymbol{\gamma}}[\cdot]$ denotes expectation over $\boldsymbol{\gamma}$.

Having introduced the notation and stated the main assumptions and ultimate goal, we are ready to specify the optimization objective based on which the distributed scheduling and resource allocation algorithms will be derived in Sections 3 and 4. These algorithms must schedule access by optimizing the user-specific time fractions $\boldsymbol{\alpha}$ and the corresponding transmit-power levels $\boldsymbol{p}$.

3 Batch off-line algorithm

In this section, the optimization problem at hand is formulated analytically, recast as a convex problem and solved using a primal-dual iterative approach. This solution lends itself naturally to a distributed algorithm which is the subject of Section 4.

3.1 Weighted sum-rate optimization

The objective is to maximize the weighted sum-rate of all active links, i.e., $\sum_{i,j=1,\ i\neq j}^{J} w_{ij}\sum_{k=1}^{K} c_{ijk}$, where c_{ijk} is the ergodic capacity in Eq. 2 and $w_{ij},\ i\neq j=1:J$ denote rate-reward weights allowing the scheduler to weigh differently the rate attainable over different links.

When the CR network is hierarchical with primary and secondary users, the scheduler must guarantee a prescribed non-zero minimum average rate for primary links while for secondary ones this minimum rate can be zero. In this setup, a primary user can specify a minimum rate, whether acting as transmitter or receiver. Such rate constraints are expressed as $\sum_{k=1}^{K} c_{ijk}\geq \check{R}_{ij}$ for a set of channels selected by primary users.

Since power per node is finite, besides minimum average rate, maximum average power constraints are also in effect. Suppose that u_i is scheduled to transmit to u_j over the subcarrier k during the time fraction α_{ijk} of the coherence interval T_n. Also suppose that the scheduler assigns u_i to transmit over this subcarrier with power level p_{ijk} during this time fraction. Then the power spent by user u_i during the interval T_n over the subcarrier k is $\alpha_{ijk}p_{ijk}$. Noticing that u_i must divide its power to transmit over all the sub-carriers it is scheduled for, the power constraint takes the form $\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}\alpha_{ijk}p_{ijk}\right]\leq \check{P}_i$, where w.l.o.g. we set $T_n=1$. Note that the scheduler will adapt $\boldsymbol{\alpha}$ and $\boldsymbol{p}$ to the channel state information, which explains the expectation operator over the random channel vector $\boldsymbol{\gamma}$.

In addition to the rate and power constraints, the trivial constraints on non-negative power levels and time fractions are needed along with Eq. 1. For notational brevity, we collect the latter in the set

$$\mathcal{A}\triangleq\{\boldsymbol{\alpha},\ \boldsymbol{p}:\alpha_{ijk}\geq 0,\ \ p_{ijk}\geq 0\ \forall ijk;\ \sum_{i,j=1,\ i\neq j}^{J}\alpha_{ijk}\leq 1,\ \forall k\}$$

Recalling all the constraints introduced as well as the weighted average sum-rate objective, we end up with the following well defined constrained optimization problem

$$\begin{aligned}
\max_{(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}}\ & \sum_{i,j=1,\ i\neq j}^{J} w_{ij}\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2(1+\gamma_{ijk}p_{ijk})\right]\\
\text{s.t.}\ \ & \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1}^{J}\sum_{k=1}^{K}\alpha_{ijk}p_{ijk}\right]\leq\check{P}_i,\\
& \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2(1+\gamma_{ijk}p_{ijk})\right]\geq\check{R}_{ij}
\end{aligned}\tag{3}$$

where rate-reward weights w_{ij}, minimum average rates $\check{R}_{ij}$ per link and maximum average power $\check{P}_i$ per user are all given.

In order to transform Eq. 3 into a convex optimization problem we rewrite Eq. 3 using auxiliary powers $q_{ijk}:=p_{ijk}\alpha_{ijk}$ as[1]

$$\begin{aligned}
\max_{(\boldsymbol{\alpha},\boldsymbol{q})\in\mathcal{A}}\ & \sum_{i,j=1,\ i\neq j}^{J} w_{ij}\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2\left(1+\gamma_{ijk}\frac{q_{ijk}}{\alpha_{ijk}}\right)\right]\\
\text{s.t.}\ \ & \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1}^{J}\sum_{k=1}^{K}q_{ijk}\right]\leq\check{P}_i,\\
& \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2\left(1+\gamma_{ijk}\frac{q_{ijk}}{\alpha_{ijk}}\right)\right]\geq\check{R}_{ij}.
\end{aligned}\tag{4}$$

Problem Eq. 4 is equivalent to Eq. 3 as the change of variables is invertible except for $\alpha_{ijk}=0$ in which case every $p_{ijk}\geq 0$ solves Eq. 3. Furthermore, for any practical fading distribution of $\boldsymbol{\gamma}$, the problem in Eq. 4 is convex and can thus be efficiently tackled using modern convex programming schemes. However, for resource allocation to be practical, the resultant optimization algorithm must be amenable to distributed implementation. To this end, we follow the approach in [4] and [5] which is based on the dual decomposition of the problem in Eq. 4. The optimal solution will lead us to the distributed dynamic allocation algorithm of Section 4.

[1]The function $\alpha\log_2(1+\gamma\frac{q}{\alpha})$ is defined at $\alpha=0$ by continuity as $0\log_2(1+\gamma\frac{q}{0})=\lim_{\alpha\to 0}\alpha\log_2(1+\gamma\frac{q}{\alpha})=0$

3.2 Water-filling power and winner-takes-all scheduling

In order to develop the primal-dual iteration which solves Eq. 4, we need the Lagrangian function. The latter entails a vector of Lagrange multipliers $\boldsymbol{\lambda}$ corresponding to the power constraints and a matrix $\boldsymbol{\mu}$ of Lagrange multipliers associated with the rate constraints (those defined by the set $\mathcal{A}$ are purposely not included at this point). Thus, we have

$$\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}, \boldsymbol{q}) = \sum_{i,j=1,\ i\neq j}^{J} w_{ij}\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2\left(1+\gamma_{ijk}\frac{q_{ijk}}{\alpha_{ijk}}\right)\right] + \sum_{i=1}^{J}\lambda_i\left[\check{P}_i - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1,\ j\neq i}^{J}\sum_{k=1}^{K} q_{ijk}\right]\right] - \sum_{i,j=1,\ i\neq j}^{J}\mu_{ij}\left[\check{R}_{ij} - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2\left(1+\gamma_{ijk}\frac{q_{ijk}}{\alpha_{ijk}}\right)\right]\right]. \tag{5}$$

The Lagrange dual function is then given by

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{(\boldsymbol{\alpha},\boldsymbol{q})\in\mathcal{A}} \mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}, \boldsymbol{q}) \tag{6}$$

and the dual problem of Eq. 4 is therefore

$$\min_{\boldsymbol{\lambda}\geq 0,\ \boldsymbol{\mu}\geq 0} D(\boldsymbol{\lambda},\ \boldsymbol{\mu})\ . \tag{7}$$

For any given values of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, we need first to obtain the dual function by solving the optimization problem Eq. 6. Towards this goal we first decouple α_{ijk} and q_{ijk} in Eq. 5 by substituting $\tilde{p}_{ijk} := q_{ijk}/\alpha_{ijk}$, and rewrite Eq. 6 as

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{(\boldsymbol{\alpha},\tilde{\boldsymbol{p}})\in\mathcal{A}} \mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}, \tilde{\boldsymbol{p}}) \tag{8}$$

with

$$\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}, \tilde{\boldsymbol{p}}) = \sum_{i,j=1,\ i\neq j}^{J} w_{ij}\mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2(1+\gamma_{ijk}\tilde{p}_{ijk})\right] + \sum_{i=1}^{J}\lambda_i\left[\check{P}_i - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1,\ j\neq i}^{J}\sum_{k=1}^{K}\alpha_{ijk}\tilde{p}_{ijk}\right]\right] - \sum_{i,j=1,\ i\neq j}^{J}\mu_{ij}\left[\check{R}_{ij} - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K}\alpha_{ijk}\log_2\left(1+\gamma_{ijk}\tilde{p}_{ijk}\right)\right]\right]. \tag{9}$$

Problem Eq. 8 is equivalent to Eq. 6 as the solution of Eq. 6 is $(\alpha^*_{ijk}, q^*_{ijk})$=$(\alpha^*_{ijk}, \alpha^*_{ijk}\tilde{p}^*_{ijk})$ where $(\alpha^*_{ijk}, \tilde{p}^*_{ijk})$ is a solution for Eq. 8. Note that $\tilde{p}_{ijk} = p_{ijk}$ for the case of interest, that is $\alpha_{ijk} \neq 0$, and for this reason we will henceforth use $\tilde{p}_{ijk} = p_{ijk}$ with a slight abuse of notation when $\alpha_{ijk} = 0$.

Upon rearranging terms and interchanging expectation and summation operators, the Lagrangian can be re-written as

$$\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}, \boldsymbol{p}) = \sum_{i,j=1,\ i\neq j}^{J}\sum_{k=1}^{K}\mathbb{E}_{\boldsymbol{\gamma}}\left[\alpha_{ijk}\varphi_{ijk}(p_{ijk})\right] + \sum_{i=1}^{J}\lambda_i\check{P}_i - \sum_{i,j=1,\ i\neq j}^{J}\mu_{ij}\check{R}_{ij} \tag{10}$$

where φ_{ijk} are aggregate link quality indicators defined as

$$\varphi_{ijk}(p_{ijk}) := (w_{ij}+\mu_{ij})\log_2(1+p_{ijk}\gamma_{ijk}) - \lambda_i p_{ijk}. \tag{11}$$

Based on Eq. 11, the dual in Eq. 8 takes the form

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{(\boldsymbol{\alpha},\boldsymbol{p})\in\mathcal{A}} \mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}, \boldsymbol{p}) = \max_{\boldsymbol{\alpha}\in\mathcal{A}}\sum_{i,j,k}\left\{\max_{\boldsymbol{p}\geq 0}\left\{\mathbb{E}_{\boldsymbol{\gamma}}\left[\alpha_{ijk}\varphi_{ijk}(p_{ijk})\right]\right\}\right\} + \sum_{i=1}^{J}\lambda_i\check{P}_i - \sum_{i,j=1,\ i\neq j}^{J}\mu_{ij}\check{R}_{ij} \tag{12}$$

Toward the solution of Eq. 12, our initial step is to tackle the inner maximization, that is to find

$$p^*_{ijk} = \arg\max_{p_{ijk}\geq 0}\ \mathbb{E}_{\boldsymbol{\gamma}}\left[\alpha_{ijk}\varphi_{ijk}(p_{ijk})\right]. \tag{13}$$

Differentiating Eq. 11 with respect to p_{ijk} and equating the derivative to zero leads to the standard water-filling solution per fading state realization (and thus for the average over $\boldsymbol{\gamma}$ since power is non-negative). With $[\cdot]^+$ denoting projection onto non-negative reals, this optimal power allocation is

$$p^*_{ijk} = \left[\frac{w_{ij}+\mu_{ij}}{\lambda_i\ln 2} - \frac{1}{\gamma_{ijk}}\right]^+ \quad \forall i,\ j,\ k\ . \tag{14}$$

Substituting Eq. 14 into Eq. 6, the dual function is expressed as

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{\boldsymbol{\alpha}\in\mathcal{A}}\sum_{i,j=1,\ i\neq j}^{J}\sum_{k=1}^{K}\mathbb{E}_{\boldsymbol{\gamma}}\left[\alpha_{ijk}\varphi_{ijk}(p^*_{ijk})\right] + \sum_{i=1}^{J}\lambda_i\check{P}_i - \sum_{i,j=1}^{J}\mu_{ij}\check{R}_{ij}\ . \tag{15}$$

Our next step is to carry out the maximization over $\boldsymbol{\alpha}$ in Eq. 15. This will specify the dual function and lead us to what is termed "winner-takes-all" scheduling. Because $\sum_{i,j=1,\ i\neq j}^{J} \alpha_{ijk} \leq 1 \quad \forall k$ and $\alpha_{ijk} \geq 0$, it follows that

$$\sum_{i,j=1,\ i\neq j}^{J} \sum_{k=1}^{K} \alpha_{ijk} \varphi_{ijk}(p^*_{ijk}) \leq \sum_{k=1}^{K} \max_{i\neq j} \varphi_{ijk}\left(p^*_{ijk}\right) . \qquad (16)$$

Equality in the latter yields the optimum $\boldsymbol{\alpha}$ as

$$\alpha^*_{ijk} = \begin{cases} 1 & (i,j) = (i^*_k, j^*_k), \\ 0 & \text{otherwise} \end{cases} \qquad (17)$$

where $(i^*_k, j^*_k) := \arg\max_{(i,j)} \varphi_{ijk}(p^*_{ijk})$. In words, for each subcarrier k the link with the maximum $\varphi_{ijk}(p^*_{ijk})$ for a given γ_{ijk}, i.e., the winner link per fading realization, will be picked and all resources will be assigned to it.

So far we established that Eqs. 14 and 17 provide the maximizing pair $(\boldsymbol{\alpha}, \boldsymbol{p})$ which specifies the dual function in Eqs. 6 and 8 in terms of the Lagrange multipliers $(\boldsymbol{\lambda}, \boldsymbol{\mu})$. By convexity, replacing $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ in Eqs. 14 and 17 with the optimal solution $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ of the dual problem Eq. 7 yields the optimal power allocation and scheduling solutions of the original problem in Eq. 3. Hence, to complete the solution it suffices to obtain the optimum Lagrange multipliers $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$. This is accomplished by solving Eq. 7 using a sub-gradient based iterative approach. For any sub-gradient $(\boldsymbol{g}_\lambda, \boldsymbol{g}_\mu)$, the pertinent iterations are

$$\begin{aligned} \boldsymbol{\lambda}^{(n+1)} &= \left[\boldsymbol{\lambda}^{(n)} - \beta_n \boldsymbol{g}^{(n)}_\lambda\right]^+ \\ \boldsymbol{\mu}^{(n+1)} &= \left[\boldsymbol{\mu}^{(n)} - \beta_n \boldsymbol{g}^{(n)}_\mu\right]^+ \end{aligned} \qquad (18)$$

where n denotes iteration index and β_n a step size that must be properly selected to guarantee convergence regardless of the initialization. The chosen sub-gradients for the problem at hand are (subscripts i and ij index entries of the corresponding sub-gradient vector and matrix)

$$\begin{aligned} g^{(n)}_{\lambda i} &= \check{P}_i - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1,\ j\neq i}^{J} \sum_{k=1}^{K} \alpha^{*(n)}_{ijk} p^{*(n)}_{ijk}\right] \\ g^{(n)}_{\mu ij} &= \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} \alpha^{*(n)}_{ijk} \log_2\left(1 + p^{*(n)}_{ijk} \gamma^{(n)}_{ijk}\right)\right] - \check{R}_{ij} \end{aligned} \qquad (19)$$

where $\alpha^{*(n)}_{ijk}$ and $p^{*(n)}_{ijk}$ are the optimal solutions in Eqs. 17 and 14, with $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ replaced by $(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu}^{(n)})$.

Notice that Eqs. 17 and 14 are also functions of $\boldsymbol{\gamma}$. This means that evaluating the expectations in Eq. 19 requires knowledge of the fading distribution functions. If these are available, then by generating a large enough number of $\boldsymbol{\gamma}$ deviates and using each of them in Eqs. 17 and 14 it is possible to obtain the expectations (ensemble averages) via sample averages. In the ensuing section we will see that it is possible to learn the required expectations online, thereby relaxing the need to know the distribution functions of the fading links.

With a means of evaluating expectations available, we proceed to insert the sub-gradient expressions into Eq. 18 to arrive at

$$\begin{aligned} \lambda^{(n+1)}_i &= \left[\lambda^{(n)}_i - \beta_n\left(\check{P}_i - \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{j=1,\ j\neq i}^{J} \sum_{k=1}^{K} \alpha^*_{ijk} p^*_{ijk}\right]\right)\right]^+ \\ \mu^{(n+1)}_{ij} &= \left[\mu^{(n)}_{ij} - \beta_n\left(r(\alpha^*_{ijk}, p^*_{ijk}) - \check{R}_{ij}\right)\right]^+ \end{aligned} \qquad (20)$$

where $r(\alpha^*_{ijk}, p^*_{ijk}) := \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=1}^{K} \alpha^*_{ijk} \log_2(1 + p^*_{ijk}\gamma_{ijk})\right]$. The iterations in Eq. 20 are provably convergent to the optimal λ^*_i and μ^*_{ij} provided that the step sizes are selected to satisfy, see e.g., [6],

$$\sum_{n=1}^{\infty} \beta_n = \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \beta^2_n = c < \infty . \qquad (21)$$

A couple of remarks are now in order.

Remark 1 It is interesting to note that the optimal solution of Eq. 3 achieves the QoS constraints by adding the non-negative Lagrange multiplier μ_{ij} to the rate-reward weight w_{ij} for the link $i \to j$ in the objective function Eq. 3. This extra weight μ_{ij} increments the quality indicator and the power to be transmitted over this link (cf. Eqs. 11 and 14). The link quality indicator is further influenced by the multiplier λ_i, In fact, from the definition of the indicator Eq. 11, λ_i can be interpreted as the power cost and μ_{ij} as rate reward. Accordingly, if two links have identical channel gains over a certain time-slot, the one with greater reward and lower cost will have the highest quality indicator and win access to the spectrum.

Remark 2 Paralleling the ideas in the previous remark and contrary to what the term "winner-takes-all" may suggest, the optimal scheduler is fair. This is because the "winner link" does not necessarily coincide with the link having maximum channel gain. Indeed, fairness is ensured by scheduling the maximum link quality indicator which takes into account rate reward weights w_{ij} along with channel gains γ_{ijk} and average rate as well as power requirements through the prices and rewards λ_i and μ_{ij}. Links with high channel gains are in principle those taking most sub-carriers. But if they take all sub-carriers all the time, their power will be consumed allowing other users with weaker channel gains but more available power to contributed their rates and further increase the network welfare. The value of $\boldsymbol{\lambda}$ progresses to a point where links with lower channel gains are allowed to access a few sub-carriers occasionally (typically in bursts as simulations indicate).

This intuitive interpretation of the optimal scheduling and power allocation algorithm will be corroborated by the simulated tests in Section 5.

4 Distributed on-line algorithm

In this section, distributed scheduling and power allocation schemes are derived to solve the original problem Eq. 3 online. The optimal solution implements the winner-takes-all strategy Eq. 17 and the water-filling strategy Eq. 14 using a stochastic version of the dual iteration in Eq. 20. This stochastic iteration learns the expected values in Eq. 20 "on the fly" without requiring knowledge of the fading channels distribution and lends itself naturally to an online algorithm. Thanks to the primal-dual approach, the online algorithm can be implemented in a distributed fashion where nodes need to exchange only minimal control information with their immediate (one-hop) neighbors.

Relative to the dual iteration of the previous section, each iteration here will include two phases. In the first phase, the scheduling policy in Eq. 17 and the power allocation in Eq. 14 will be carried using $(\boldsymbol{\lambda}^{(t)}, \boldsymbol{\mu}^{(t)})$ and the current state of the channel gains to obtain $\alpha_{ijk}^{*(t)}$ and $p_{ijk}^{*(t)}$. (Here the discrete-time index t replaces the off-line iteration index n in the previous section and indicates the current time-slot.) In the second phase, $\alpha_{ijk}^{*(t)}$ and $p_{ijk}^{*(t)}$ will be used to update the multipliers and find $(\boldsymbol{\lambda}^{(t+1)}, \boldsymbol{\mu}^{(t+1)})$ without having to evaluate the expected values involved. Starting with the first phase, let us substitute Eq. 14 into Eq. 11 to obtain

$$\varphi_{ijk}(p_{ijk}^*) = \begin{cases} \frac{w_{ij}+\mu_{ij}}{\ln(2)} \ln\left(\frac{(w_{ij}+\mu_{ij})\gamma_{ijk}}{\lambda_i \ln(2)}\right) \\ \qquad -\frac{w_{ij}+\mu_{ij}}{\ln(2)} + \frac{\lambda_i}{\gamma_{ijk}}, \quad \text{if} \\ \qquad (w_{ij}+\mu_{ij})\gamma_{ijk} > \lambda_i \ln(2) \\ 0, \qquad \text{otherwise}\,. \end{cases} \tag{22}$$

As in Eq. 17, the pair of users scheduled to use sub-carrier k correspond to the winner link

$$(i_k^*, j_k^*) := \arg\max_{(i,j)} \varphi_{ijk}(p_{ijk}^*)\,. \tag{23}$$

Furthermore, the transmitter of this winner link will allocate power on the given subcarrier according to Eq. 14 as

$$p_{ijk}^* = \left[\frac{w_{ij}+\mu_{ij}}{\lambda_i \ln 2} - \frac{1}{\gamma_{ijk}}\right]^+\,.$$

Aiming next at a distributed implementation, it is instructive to consider which variables and parameters in Eqs. 14 and 22 are available locally to each user. The second phase of the algorithm (to be detailed soon), shows that the multipliers λ_i and μ_{ij}, $j=1:J$, $j\neq i$ can be updated locally at u_i. Furthermore, as per (as1), the channel gain $\gamma_{ijk}\forall k$, $j\neq i$ is known at the location of user u_i. Finally, the weights w_{ij} are prescribed.

Summarizing, user node u_i knows w_{ij}, γ_{ijk}, λ_i and μ_{ij} for all $j=1:J$, $j\neq i$, $k=1:K$. Hence, it can find p_{ijk}^* and locally allocate the optimum power, if scheduled. For node u_i to know whether it is scheduled, it should be able to solve the maximization problem Eq. 23 in a distributed manner. Toward this objective, the only control protocol needed is for users to consent on the winner link Eq. 23 for each subcarrier. To effect such a consensus with single-hop node communications, let be

$$\varphi_{ik}^* := \max_j \varphi_{ijk}(p_{ijk}^*)$$
$$\varphi_k^* := \max_i \varphi_{ik}^*\,. \tag{24}$$

It can be seen from Eq. 22 that $\varphi_{ijk}(p_{ijk}^*)$ depends only on w_{ij}, μ_{ij}, λ_i and γ_{ijk}, all of which are available to u_i. Thus, φ_{ik}^* can be obtained locally at u_i per sub-carrier k by simply comparing the quality indicators of the links starting at i and ending at all possible j's. Picking the largest of these indicators, i.e., φ_{ik}^*, the second step for u_i is to compare this φ_{ik}^* with φ_k^* so as to judge whether or not $i=i_k^*$. Connectivity of the control graph assumed under (as2) comes handy here to ensure that J nodes can reach consensus on φ_k^* per subcarrier k. The communication protocol used to percolate this φ_k^* across all user nodes proceeds as outlined next.

Protocol 1 (Consensus on the maximum quality indicator)

* *Initialize with* $x_{ik}^{(0)} = \varphi_{ik}^*$*;*
* *Communicate with neighboring nodes; and*
* *Run the iteration* $x_{ik}^{(m+1)} = \max_{j\in\mathcal{N}_i} x_{ik}^{(m)}$ *per node* i *and sub-carrier* k*.*

Performance of Protocol 1 can be summarized as follows.

Proposition 1 *With local communications over the control network obeying* (as2)*, Protocol 1 drives* x_{ik}*'s to* φ_k^*

after N iterations, where $N \leq J$ is the diameter of the graph $\mathcal{G}$.

Using Protocol 1, φ_k^* becomes available to each u_i. Those user nodes u_i with $\varphi_{ik}^* < \varphi_k^*$, will set their power p_{ijk}^* to zero; whereas the node with $\varphi_{ik}^* = \varphi_k^*$ will know it is the winner. For this winner node, the next task is to determine locally j_k^* using (cf. Eq. 23)

$$j_k^* = \arg\max_j \varphi_{i_k^* jk}(p_{ijk}^*) \tag{25}$$

and then assign power $p_{i_k^* j_k^* k}$ in accordance with Eq. 14. This procedure assumes a unique winner node which holds true almost surely since practical fading distributions are smooth and continuous.

Having completed the first phase of the distributed algorithm, we proceed to the second phase which involves updating online the Lagrange multipliers. To avoid the expectation operators in Eq. 19, the idea is to rely on stochastic approximation where expectations are estimated on the fly using the iterations

$$\lambda_i^{(t+1)} = \left[\lambda_i^{(t)} - \beta_t\left(\check{P}_i - \sum_{j=1}^{J}\sum_{k=1}^{K}\alpha_{ijk}^{*(t)}\, p_{ijk}^{*(t)}\right)\right]^+$$

$$\mu_{ij}^{(t+1)} = \left[\mu_{ij}^{(t)} - \beta_t\left(\sum_{k=1}^{K}\alpha_{ijk}^{*(t)}\log_2\left(1+p_{ijk}^{*(t)}\gamma_{ijk}^{(t)}\right) - \check{R}_{ij}\right)\right]^+ \tag{26}$$

Relative to Eq. 18 which can only be computed off-line, the last iterations do not include expectation and can be run online. Their convergence is asserted as follows.

Proposition 2 *Under (as1) and with the step-size β_n selected to satisfy Eq. 21, the iterations in Eq. 26 converge in the mean-square sense (as $t \to \infty$) to their optimum values λ_i^* and μ_{ij}^*.*

The detailed proof of Proposition 2 is omitted due to lack of space but relies on verifying that the iterations in Eq. 26 satisfy the conditions of the Robbins–Monro algorithm in [7].

The two phases of the distributed online scheme are summarized in the following algorithm that is run locally at each node u_i.

Convergence of the overall algorithm is guaranteed because two facts hold true: (i) the optimization problem in Eq. 4 is strictly convex, which implies that the sub-gradient iterations in Eq. 18 converge; and (ii) Proposition 2 ensures convergence of the stochastic approximation iterates in Eq. 26 to the optimal pair $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$. Upon convergence to the optimal multiplier values, we can continue the iteration after disconnecting Step 7.

Algorithm 1 (Distributed online scheduling and allocation)

Initialize with $\lambda_i^{(0)} = \mu_{ij}^{(0)} = 0\ \forall j = 1:J$;

LOOP
Step 1: Acquire instantaneous γ_{ijk} via training;
Step 2: Obtain φ_{ijk}^* for $j=1:J$, $k=1:K$ via Eq. 22;
Step 3: Find φ_{ik}^* for $k=1:K$ using Eq. 24;
Step 4: Run Protocol 1 to obtain φ_k^* as in Eq. 24;
Step 5: Compare φ_k^* with φ_{ik}^* and decide if $i = i_k^*$;
*Step 6: **IF** $(i = i_k^*)$*

Step 6.1 : Set $j_k^* = \arg\max_j \varphi_{ijk}^*$; and
Step 6.2: Determine p_{ijk}^* using Eq. 14.

FI
Step 7: Update λ_i and μ_{ij} for $j=1:J$ via Eq. 26; and
Step 8: Return to Step 1 after coherence time elapses.

5 Simulations

To corroborate the analytical results derived, three simulated test cases are presented in this section. The first two pertain to a simplified scenario where channel links can take only two possible states (good or bad). This scenario allows one to evaluate the expected values in Eq. 19 analytically. In the second case, the distributed online Algorithm 1 is tested and the results are compared with those obtained in the first case. Algorithm 1 in the third test case is implemented for the pragmatic scenario involving Rayleigh fading channel links.

All three cases are implemented for the network depicted in Fig. 1, where the square shown with dashed lines has area 1km^2 and contains a group of five nodes $(J = 5)$ located at positions

$$\left\{\left(\frac{3}{4},1\right),\left(0,\frac{1}{2}\right),\left(\frac{1}{4},\frac{3}{4}\right),\left(\frac{3}{4},0\right),\left(\frac{1}{2},0\right)\right\}.$$

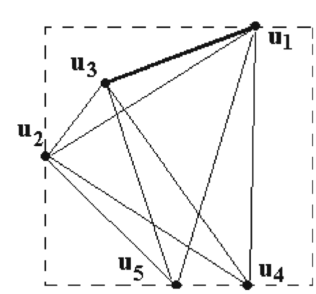

Figure 1 Network topology used in simulations

The weights are collected in the following matrix $\mathbf{W}$ with 0, 1 entries chosen so that the links contribute equally to the utility function if they are connected in a specific direction

$$\mathbf{W} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Note that the gains of non-existing links (from u_i to u_i) are also set to 0.

The available system bandwidth of 10 MHz is split in two 5 MHz sub-bands over which links content to transmit over two sub-carriers ($K = 2$). User u_1 is a primary user with required minimum average rate 10 Mbps linked with user u_3. The corresponding minimum average rate to guarantee in this link is therefore $\check{R}_{13} = 2$ bits/Hz. For simplicity, channels are assumed independent. Each user cannot exceed an average power of 1 W.

5.1 Off-line algorithm for binary channels

In this first example we consider a binary fading channel model with states $\gamma_{ijk}^{(1)} = \overline{\gamma}_{ij} - 3\mathrm{d}B$ and $\gamma_{ijk}^{(2)} = \overline{\gamma_{ij}} + 3\mathrm{d}B$, where $\overline{\gamma}_{ij}$ is a constant inversely proportional to the cubic distance from node i to node j. Note that we can find the expectations in Eq. 19 analytically by evaluating $\alpha_{i_k^* j_k^* k}$ and $p_{i_k^* j_k^* k}$ for each of the $N_s = (2^{C_2^5})$ states of the network, averaging and multiplying by K (C_2^5 stands for the number of 5-choose-2 combinations). Hence, iterations Eq. 18 can be run off-line to obtain the optimal multipliers.

Figure 2 refers to the primal iteration and shows how the weighted average sum-rate increases with iterations. The optimal $\boldsymbol{\lambda}^*$ in this case was found to be

$$\boldsymbol{\lambda}^* = [0.3592,\ 0.7190,\ 0.1813,\ 1.5772,\ 0.0841].$$

The optimal μ_{13}^* corresponding to the channel with the minimum rate requirement approaches zero. The rate constraint is not active here and the average capacity over this channel is

$$c_{13} = 2.2104 \text{ bits/Hz}.$$

The overall optimal weighted average sum-rate achieved by the off-line algorithm is

$$C = 19.3815 \text{ bits/Hz}.$$

Upon convergence to the optimal point, all users are scheduled to spend their maximum available power.

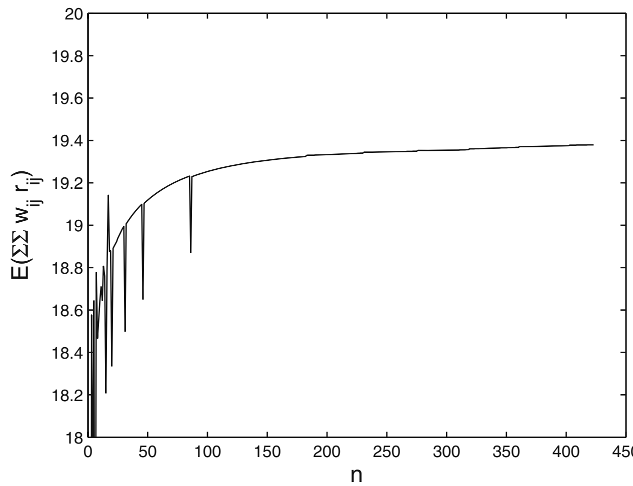

Figure 2 Evolution of the weighted average sum-rate in the off-line iterative algorithm applied to binary channels

Indeed, by simply inspecting the iterations we verify that the algorithm ensures fairness, as asserted in Section 3. To gain additional insight in this regard, we list the matrices

$$\boldsymbol{\gamma}^{(1)} = \begin{pmatrix} 0 & 27.2 & 114.2 & 20.0 & 18.2 \\ 0 & 0 & 451.5 & 27.2 & 56.4 \\ 0 & 0 & 0 & 27.2 & 40.4 \\ 0 & 0 & 0 & 0 & 1,277.0 \\ 18.2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\gamma}^{(2)} = \begin{pmatrix} 0 & 6.8 & 28.6 & 5.0 & 4.5 \\ 0 & 0 & 113.4 & 6.8 & 14.1 \\ 0 & 0 & 0 & 6.8 & 10.1 \\ 0 & 0 & 0 & 0 & 320.7 \\ 4.5 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where the entry $\boldsymbol{\gamma}_{ij}^{(1)}$ denotes the gain of the $i \rightarrow j$ link when the corresponding channel is up by 3dB and respectively down by 3dB for $\boldsymbol{\gamma}_{ij}^{(2)}$.

Clearly, entries corresponding to null weights ($w_{ij} = 0$) are not considered as they are not scheduled by the algorithm. This is because the corresponding links neither add to the network welfare nor they present any minimum rate guarantee. Possible channel realizations are combinations of the values of these two matrices. We see that the maximum gains for different realizations of these channels will always be one of the links $4 \rightarrow 5$ or $2 \rightarrow 3$.

At first sight one might expect the winner user to be always the one with the most reliable channel gain, in which case users u_1, u_3 and u_5 are never scheduled. However, the multiplier vector $\boldsymbol{\lambda}$ converges to $\boldsymbol{\lambda}^*$ that implies a different behavior of the optimal scheduler. In

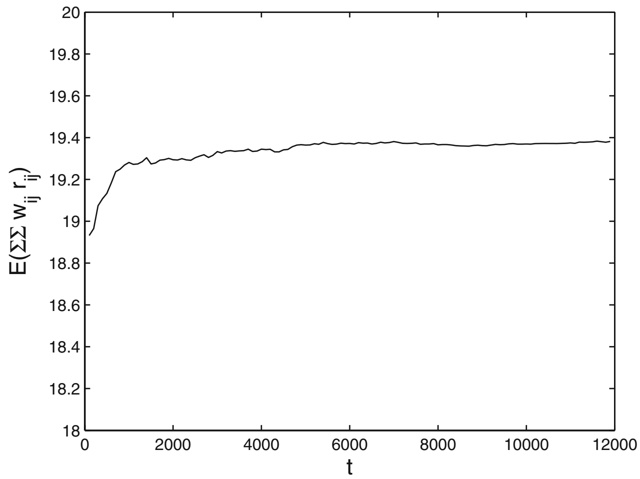

Figure 3 Evolution of the weighted average sum-rate in the online iterative algorithm applied to binary channels

fact, the optimal $\boldsymbol{\lambda}^*$ allows users u_1, u_3 and u_5 to transmit in a few network states. When they are scheduled, they burst power until their resources are exhausted. This in turn implies that the power constraints for all users are active at the optimum. The intuition behind this behavior is as follows. Power resources of users u_2 and u_4 are low after they transmit over several coherence intervals. Even if their channel gains remain better than those of users u_1, u_3 and u_5 in subsequent intervals, the power reserves of u_1, u_3 and u_5 are plentiful, rendering it more beneficial for the network welfare to have them transmit (in bursts) even if their channel gains are worse than those of u_2 and u_4. Eventually all users are scheduled to transmit until the whole power of the network is used. Links with reliable channel gains will be scheduled more frequently while those with weaker channel gains will be active in bursts. In this sense, this test case confirms that the novel scheduling and resource allocation algorithm is fair as asserted in Section 3.

5.2 On-line algorithm for binary channels

This second test case pertains to the distributed online scheme which relies on the stochastic approximation iterations summarized in Algorithm 1. The setup here is identical to that of the previous test case. As predicted by the theory, the online algorithm converges to the optimum attained by its off-line counterpart. However, convergence is slower here since ensemble averages are approximated online by sample averages. The weighted average sum-rate is depicted in Fig. 3. As in the previous test case, the rate constraint at the optimum is not active here too. Even in this case the evolution of μ_{13} is important as it reduces up the time elapsed until the rate constraint is met. Indeed, while the constraint is violated μ_{13} increases according to Eq. 26 and the link quality indicator is higher for this channel.

Iterations for binary channels may exhibit oscillatory behavior when an active rate constraint is added. This happens because the network has finite number of states. As a result, the parameters $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ are adjusted to schedule the primary user to transmit over a channel shared with a secondary user, which renders the maximum in Eq. 24 non-unique with nonzero probability. Fortunately, practical fading channels have infinite number of states and this event occurs with probability of measure zero.

5.3 On-line algorithm for Rayleigh channels

In this test case, user locations and link weights are as in the previous test cases. User u_1 is still the primary user but with higher rate requirement $\check{R}_{13} = 3$ bits/Hz. This is done in order to activate the rate constraint. Channel gains are now exponentially distributed (Rayleigh fading model); i.e.,

$$\gamma_{ij} = \exp(\overline{\gamma}_{ij})$$

where the mean of each gain, namely $\overline{\gamma}_{ij}$, is identical to that in the previous two examples. The Lagrange multiplier iterations converge to the optimum point

$$\boldsymbol{\lambda}^* = [0.8480,\ 0.8351,\ 0.1964,\ 1.5328,\ 0.1003]$$

$$\mu_{13}^* = 0.2289$$

which shows that in terms of power the best user is again u_4 and the rate constraint is now active.

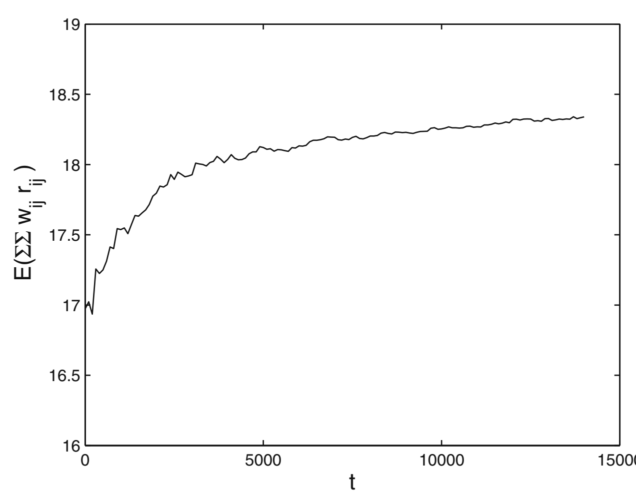

Figure 4 Evolution of the weighted average sum-rate in the online iterative algorithm applied to Rayleigh fading channel links

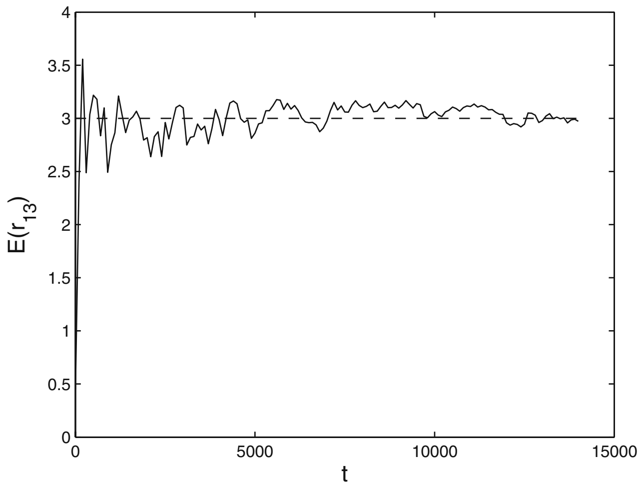

Figure 5 Evolution of the average rate attained by the primary user u_1 transmitting to u_3

Along with these online iterations, transient values of the multipliers were recorded to analyze the evolution of the algorithm off-line. These transient values were picked every 100 iterations and for each point the expected power and rates were estimated by averaging over Monte Carlo realizations. Specifically, 500,000 Monte Carlo realizations of all channel gains were drawn and for each realization water-filling power allocation was performed as in Eq. 14 and winner-takes-all scheduling was followed in accordance with Eq. 17. The resultant power levels and rates were subsequently averaged. Using this Monte Carlo approach for evaluating averages, we generated Fig. 4 which depicts the weighted average sum-rate as it evolves with primal iterations. Figure 5 shows the evolution of the average rate for the link $1 \to 3$ along with the pre-specified lower bound $\check{R}_{13} = 3$ bits/Hz.

Unlike channels assuming a finite number of states, all users here transmit over all links with nonzero weights. In the previous examples, all users are scheduled but they only transmit over their best channels. User u_1 for instance, is never scheduled to transmit to user u_5 because the gain of the link $1 \to 4$ is always higher than all other gains. This is not the case with Rayleigh fading channels which can take infinite number of states and illustrates that the novel scheduler becomes increasingly fair with pragmatic fading channel models.

6 Concluding summary

We developed a distributed online scheduling and resource allocation algorithm for OFDMA based networks of CRs entailing any-to-any communication links. The algorithm is iterative and runs locally at each CR node to allocate power and frequency resources dynamically. Iterations were shown convergent to an optimal point that maximizes under average power and minimum average rate constraints the common welfare taken to be the weighted average sum-rate of the network. The overall approach offers fairness among users of the same hierarchy in the sense of ensuring not only that users with reliable channel gains are scheduled more often but also that all users are scheduled, so long as their power budget can afford access.

When both secondary and primary users are present, primary users can specify non-zero minimum average rates and select larger weights than secondary users who can afford zero minimum rates. This way, the present framework is flexible to accommodate hierarchies in QoS requirements encountered with CR networks. Even when the minimum rate constraints are inactive at the optimal point, the weights reduce the time until QoS requirements are assured. In every step of the iterative algorithm, nodes reach consensus on the scheduled user to transmit over each OFDM sub-carrier. Although different users were allowed a priori to use the same sub-carrier over non-overlapping time intervals, the optimal solution assigns almost surely each sub-carrier only to a single winner user per time slot. This facilitates implementation since time sharing a sub-carrier is not easy to implement in practice.

The algorithm was successfully tested in a small network of users communicating over Rayleigh fading channels.

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