LAYERED SPACE-TIME CODING FOR HIGH DATA RATE TRANSMISSIONS*

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ABSTRACT

In the spirit of BLAST systems, this paper proposes a novel layered space-time (ST) coding scheme. By slicing the two-dimensional ST code into a stack of one-dimensional so-called layer codes, ST coding and decoding are performed in a layer by layer fashion. Because different layers are transmitted independently, the proposed scheme is capable of supporting considerably higher transmission rates than (non-layered) ST coding. In addition to retaining most advantages of BLAST systems, the proposed scheme offers additional flexibility and superior performance that is confirmed by simulations.

I. INTRODUCTION

Recent information-theoretic results have revealed that deploying multiple antennas increases channel capacity significantly without sacrificing bandwidth or power. Motivated by these results, a number of multi-antenna transmission schemes have been advocated (see [5,6] and references therein). Among those, particularly popular is the Bell-laboratories' LAyered Space Time (BLAST) systems [1,2].

BLAST systems are designed for high-data-rate transmissions, and are particularly effective when a large number of antennas are deployed at both the transmitter and the receiver. In BLAST systems, the data stream is demultiplexed into independent substreams that are refereed to as layers. These layers are simultaneously transmitted through multiple transmit-antennas, and at the receiver they are successively detected using the so-termed interference cancellation and nulling algorithms [1,2]. Depending on how layers are transmitted in space and time, two layering structures, namely, diagonal layering and vertical layering, have been proposed. Accordingly, BLAST systems can be categorized as Diagonal BLAST (D-BLAST) [1] and Vertical BLAST (V-BLAST) [2]. D-BLAST spreads each layer diagonally in space and time, and relies on layer encoding to achieve transmit diversity gain. However, diagonal layering induces considerable bandwidth efficiency loss especially for short-burst communications. In addition, because all layers are virtually identical in D-BLAST, successive detection may experience severe error-propagation. In V-BLAST, each layer is transmitted through a particular transmit-antenna. Without transmit diversity gain, V-BLAST normally requires several more receive-antennas than transmit-antennas, which is not desirable in many applications.

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Targeting high transmission rate and low complexity, this paper develops novel Layered ST (LST) coding for wireless systems with a large number of transmit- and receive-antennas. We propose a *circulant* layering structure that enables each layer to access fully space and time (similar to D-BLAST) without bandwidth efficiency loss. Based on circulant layering, the design of ST codes is treated as a set of layer coding designs. Accounting for error propagation across layers, we show that not all layers should be necessarily encoded in our scheme. Thus, high bandwidth efficiency can be expected, as compared to D-BLAST. In addition to retaining most advantages of BLAST systems, the proposed scheme offers flexibility to tradeoff among transmission rate, performance and complexity. The rest of paper is organized as follows. In Section II, we describe a general system model for multiple antenna communications. Based on this model, our LST coding and decoding are developed in Sections III and IV, respectively. The layer codes are designed in Section V. Section VI provides one design example and supporting simulations.

II. SYSTEM MODEL

Consider a single-user communication system with M transmit-antennas and N receive-antennas. At the transmitter, the information symbol sequence s_k belonging to the constellation set $\mathcal{A}_s$ is parsed into blocks $\mathbf{s} := [s_1, \ldots, s_K]^T$ of length K. Then, each information symbol block $\mathbf{s}$ is *uniquely* ST encoded into

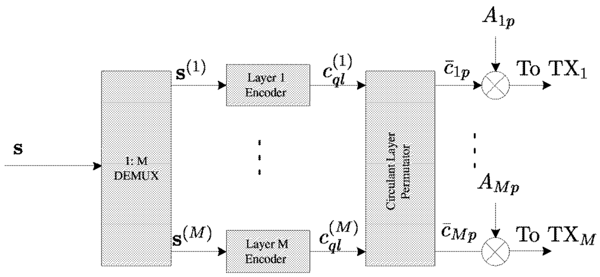

Fig. 1. Layered space-time encoder

an $M \times P$ codematrix $\mathbf{C}$:

$$\mathbf{C} := \begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} & \cdots & \bar{c}_{1P} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{c}_{M1} & \bar{c}_{M2} & \cdots & \bar{c}_{MP} \end{bmatrix} \begin{matrix} \rightarrow \text{ time} \\ \\ \downarrow \text{ space} \end{matrix} \quad (1)$$

where the (m,p)th element $\bar{c}_{mp}$ belongs to the constellation set $\mathcal{A}_c$ whose average energy is normalized to 1. The P columns of $\mathbf{C}$ are generated in P successive time intervals with each one of the M entries in a given columns being forwarded to one of the M transmit-antennas simultaneously.

We assume quasi-static flat fading channels and perfect timing and synchronization at the receiver, the received sample at the nth receive-antenna is given by

$$y_{np} = \sum_{m=1}^{M} h_{mn} A_{mp} \bar{c}_{mp} + w_{np} \ , \quad (2)$$

where, h_{mn} denotes the channel from the mth transmit-antenna to the nth receive-antenna; A_{mp} is the amplitude for $\bar{c}_{mp}$; and w_{np} is a white complex Gaussian variable with variance $N_0/2$ per dimension.

So far, we have described a (quite) general system model for multiple antenna communications. Under this model, we next present a novel LST coding scheme, as an enhancement of the BLAST systems proposed in [1, 2].

III. LAYERED SPACE-TIME CODING

Targeting high-rate transmissions and low complexity implementations, we develop herein a novel LST Coding (LSTC) scheme.

A. Layered ST Encoder

As illustrated in Fig. 1, our LST encoder consists of three units, namely, one demultiplexer, a set of M layer encoders and what we term circulant layer permutator. The three units perform separate roles, and correspond to three successive steps in LSTC, as we describe next.

A.1 Layer Demultiplexing

Similar to BLAST systems [1, 2], each information (super) block $\mathbf{s}$ is first demultiplexed into M independent layer information (sub) blocks (messages) $\mathbf{s}^{(l)} := [s_1^{(l)}, \ldots, s_{K^{(l)}}^{(l)}]^T$, $l = 1, \ldots, M$, of lengths $K^{(l)}$ that satisfy $K = \sum_{l=1}^{M} K^{(l)}$. The layer information symbols $s_k^{(l)}$ are related to the information symbols s_k through $\mathbf{s} = [\mathbf{s}^{(1)T}, \ldots, \mathbf{s}^{(M)T}]^T$. Unlike [1, 2] where all layer information blocks have the same length, notice that the lengths $K^{(l)}$'s in our LSTC are generally different.

A.2 Layer Encoding

For simplicity, we adopt linear block encoders as our layer encoders. After demultiplexing $\mathbf{s}$ into $\mathbf{s}^{(l)}$'s, the lth layer information block $\mathbf{s}^{(l)}$ is encoded by the lth layer encoder to yield the Layer CodeWord (LCW) $\mathbf{c}^{(l)} := [c_1^{(l)}, \ldots, c_P^{(l)}]^T$ of length P. The coded layer symbol $c_p^{(l)}$ belongs to the same constellation $\mathcal{A}_c$ as $\bar{c}_{mp}$ in (1). The relationship between $c_p^{(l)}$ and $\bar{c}_{mp}$ will be clarified in Section III-A.3. Note that the LCW length P is common to all layer encoders whose message lengths $K^{(l)}$'s may be different. Thus, rather than choosing the same layer encoders as in [1], the layer encoders in our scheme are in general distinct, which plays a critical role in reducing possible error propagation at the decoding stage, and offers flexibility to strike the best compromise among performance, complexity and transmission rate as discussed in [4]. We find it convenient to denote the lth layer encoding and the corresponding decoding as

$$\mathbf{c}^{(l)} = \Psi^{(l)}(\mathbf{s}^{(l)}), \quad (3a)$$

$$\mathbf{s}^{(l)} = \bar{\Psi}^{(l)}(\mathbf{c}^{(l)}) \ , \quad (3b)$$

respectively. Different layer encoders operate independently and can thus be designed separately. We remark that designing ST codes in our scheme is equivalent to designing $\{\Psi^{(l)}(\cdot)\}_{l=1}^{M}$. This equivalence enables one to exploit conventional channel coding techniques in the ST context, rather than developing new ones. However, it is still important to account for ST transmissions in designing our layer encoders.

A.3 Circulant Layering

Having obtained coded layer symbols $c_p^{(l)}$'s, we are in a position to decide when and from which transmit-antenna $c_p^{(l)}$'s are transmitted. Before we proceed, let us choose $P = MQ$, where Q is a positive integer. Similar to V-BLAST and D-BLAST, $\{c_p^{(l)}\}_{l=1}^M$ are simultaneously transmitted through M transmit-antennas at the time interval p. Unlike V-BLAST and D-BLAST, the association between the coded layer symbols and the transmit-antennas is *circularly* shifted once every Q time intervals. To be more specific, we divide $\mathbf{c}^{(l)}$ equally into M layer sub-codewords of length Q. For convenience, we denote $\mathbf{c}_\nu^{(l)} := [c_{\nu 1}^{(l)}, \ldots, c_{\nu Q}^{(l)}]^T$ the νth Layer Sub-CodeWord (LSCW), and express $\mathbf{c}^{(l)} := [\mathbf{c}_1^{(l)}, \ldots, \mathbf{c}_M^{(l)}]$. The coded layer symbol $c_{\nu q}^{(l)}$ is transmitted through the $[(l+\nu-2) \bmod M+1]$th transmit antenna at the $[(\nu-1)Q+q]$th time interval. Equivalently, the ST codematrix $\mathbf{C}$ in (1) is constructed as

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_1^{(1)T} & \mathbf{c}_2^{(M)T} & \mathbf{c}_3^{(M-1)T} & \cdots & \mathbf{c}_M^{(2)T} \\ \mathbf{c}_1^{(2)T} & \mathbf{c}_2^{(1)T} & \mathbf{c}_3^{(M)T} & \cdots & \mathbf{c}_M^{(3)T} \\ \mathbf{c}_1^{(3)T} & \mathbf{c}_2^{(2)T} & \mathbf{c}_3^{(1)T} & \cdots & \mathbf{c}_M^{(4)T} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{c}_1^{(M)T} & \mathbf{c}_2^{(M-1)T} & \mathbf{c}_3^{(M-2)T} & \cdots & \mathbf{c}_M^{(1)T} \end{bmatrix}, \quad (4)$$

where, we observe that the M sub-codewords $\mathbf{c}_\nu^{(l)}$ of a given layer codeword are circularly spread in $\mathbf{C}$.

Thus far, three notations, namely, $\bar{c}_{mp}$ in (1), $c_p^{(l)}$ in $\mathbf{c}^{(l)}$ and $c_{\nu q}^{(l)}$ in $\mathbf{c}_\nu^{(l)}$, have been introduced to denote the coded layer symbols. We will continue using all three notations for clarity. Their conversions are given by:

$$\bar{c}_{mp} = c_{\nu q}^{(l)} = c_p^{(l)}, \quad (5)$$

where, the antenna index m, the time index p, the layer index l, the sub-codeword index ν and the index q are related by

$$\begin{aligned} \nu &= \lfloor \frac{p-1}{Q} \rfloor + 1 \\ q &= (p-1) \bmod Q + 1 \\ l &= \left(\lfloor \frac{p-1}{Q} \rfloor + m - 1 \right) \bmod M + 1 \end{aligned}, \quad (6)$$

where $\lfloor \cdot \rfloor$ stands for integer-floor.

With (l, ν) denoting the νth LSCW of the lth LCW, Fig. 2 illustrates the proposed so-termed circulant layering. Similar to D-BLAST, the M LSCWs of any given LCW experience a balanced presence over all transmit-antennas, and none of the individual LCWs suffers entirely from (possible) deep channel fading, which is not the case for V-BLAST. In D-BLAST, a significant portion of ST transmissions is inactive at the beginning and the end of each block, causing considerable efficiency loss. To reduce this loss, one way is to use burst transmissions (with one burst containing several blocks), so that the underlying block transmissions overlap. However, the effect of error propagation may spread over the entire burst in D-BLAST, while it is restricted to one block in our scheme. Consequently, it is expected that the proposed scheme will provide better performance than D-BLAST.

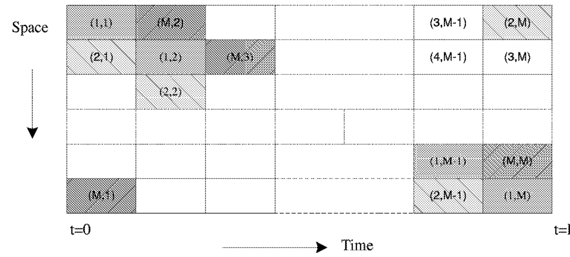

Fig. 2. Transmission flow in space and time

B. Layer Power Loading

In our layer power loading, A_{mp} in (2) is layer dependent, i.e., $A_{mp} = A^{(l)}$, where l is a function of m and p as defined in (6). Combining (2) with (5), we express the received data samples y_{np} in terms of $c_{\nu q}^{(l)}$ as:

$$y_{np(\nu,q)} = \sum_{l=1}^{M} h_{m(l,\nu)n} A^{(l)} c_{\nu q}^{(l)} + w_{np(\nu,q)}, \quad (7)$$

where the indices

$$p(\nu, q) = (\nu - 1)Q + q, \quad (8a)$$
$$m(l, \nu) = (l + \nu - 2) \bmod M + 1, \quad (8b)$$

are obtained from (6). We design next the LST decoding.

IV. LAYERED SPACE-TIME DECODING

LST decoding was first proposed in [1]. The key idea is to treat each layer in LSTC as one user in a multiuser system, and thereafter, use multiuser detection to decode layers successively. To derive LST decoding, we assume:

(as1) The channels h_{mn}'s are known at the receiver.
(as2) The channels h_{mn}'s are independent of one another, and each of them can be modeled as a complex Gaussian variable with zero-mean and unit variance.
(as3) The number of receive-antennas is not less than the number of transmit-antennas, i.e., $N \geq M$.

Note that assumptions (as1)-(as3) were also made in D-BLAST and V-BLAST.

Because all LCWs in our scheme are balanced in the sense that they are sent from all transmit antennas. Layer ordering in V-BLAST is not needed in our scheme. Without loss of generality, we perform LST decoding in an ascending order, i.e., we first decode $\mathbf{c}^{(1)}$, then $\mathbf{c}^{(2)}$, and so on. Observe from Fig. 2 that at any given time interval, the coded layer symbols are transmitted in the same way as in V-BLAST. With minor modifications, we estimate $c_{\nu q}^{(l)}$ by applying the vertical detection process reported in [2]. Due to the limit of space, we omit the detailed development, and write the resulting decision statistics for $c_{\nu q}^{(l)}$, denoted by the $(N-M+l)\times 1$ vector $\bar{\mathbf{y}}_{p(\nu,q)}^{(l)}$, as

$$\bar{\mathbf{y}}_{p(\nu,q)}^{(l)} = \bar{\mathbf{h}}_{\nu}^{(l)} c_{\nu q}^{(l)} + \bar{\mathbf{w}}_{p(\nu,q)}^{(l)}, \tag{9}$$

where, $\bar{\mathbf{h}}_{\nu}^{(l)}$ is a $(N-M+l)\times 1$ vector whose elements are approximately i.i.d., complex Gaussian variables with zero mean and unit variance; and $\bar{\mathbf{w}}_{\nu}^{(l)}$ is a $(N-M+l)\times 1$ noise vector whose elements are i.i.d., complex Gaussian variables with zero mean and variance N_0, if no error propagation has occurred. Two remarks are due regarding (9):

(p1) The receive-diversity gain of order $N-M+l$ is expected when decoding the lth layer.

(p2) Because $\bar{\mathbf{h}}_{\nu}^{(l)}$'s are different for different ν's, every LSCW in one LCW experiences different SIMO channels. Thus, time-diversity resides within LCW, which will be exploited by layer encoding.

So far, we have assumed that the layer encoders $\Psi^{(l)}(\cdot)$ and decoders $\bar{\Psi}^{(l)}(\cdot)$ are known. Next, we proceed to design $\Psi^{(l)}(\cdot)$ and $\bar{\Psi}^{(l)}(\cdot)$.

V. LAYER CODES DESIGN

We start our layer encoder design by first analyzing the layer performance.

A. Pairwise Layer Error Performance

The exact layer performance analysis in terms of the bit-error-rate (BER) is difficult, if not impossible. As in [3, 7], we render analysis tractable by choosing the average pairwise layer error probability as the figure of merit. Here, the average is taken over random channel realizations.

Let us define the pairwise layer error event $\mathbf{c}^{(l)} \rightarrow \mathbf{e}^{(l)}$ as the event that the receiver decodes layer codeword $\mathbf{e}^{(l)}$ erroneously when $\mathbf{c}^{(l)}$ is actually sent, and denote its average probability by $\bar{P}(\mathbf{c}^{(l)} \rightarrow \mathbf{e}^{(l)})$. Following the approach in [3, 7], the performance of the lth layer can be approximated by [4]

$$\bar{P}^{(l)} = \max_{\forall \mathbf{c}^{(l)} \neq \mathbf{e}^{(l)}} \prod_{\nu=1}^{M} \left(1 + \frac{\gamma_c^{(l)}}{4} \|\mathbf{c}_{\nu}^{(l)} - \mathbf{e}_{\nu}^{(l)}\|^2\right)^{-(N-M+l)} \tag{10}$$

where the maximization is taken over all possible pairs of distinct LCWs. Clearly, the layer performance depends on the design of layer codes, which is the focus of the next subsection.

B. Concatenated Layer Coding and Decoding

Observe from (10) that $\bar{P}^{(l)}$ is a function of the Euclidean distances between LSCWs instead of LCWs. Rather than designing the layer codes as a whole, we are motivated to consider each layer code as the concatenation of an inner code and an outer code. The design of the inner code and the outer code is presented next.

Let us assume that there exists Galois field $\mathrm{GF}(|\mathcal{A}_c|)$ of size $|\mathcal{A}_c|$. Without loss of generality, we also assume $|\mathcal{A}_s| = |\mathcal{A}_c|$. Denoting by $K_{\text{in}}^{(l)}$ ($K_{\text{out}}^{(l)}$) the message length for the inner (outer) code, our design starts by taking the inner code to be a $(Q, K_{\text{in}}^{(l)})$ block code over $\mathrm{GF}(|\mathcal{A}_c|)$, and the outer code to be an $(M, K_{\text{out}}^{(l)})$ block code over $\mathrm{GF}(|\mathcal{A}_c|^{K_{\text{in}}^{(l)}})$. The selection of $K_{\text{in}}^{(l)}$ and $K_{\text{out}}^{(l)}$ depends on the tradeoff among transmission rate, system performance and complexity [4]. After specifying the parameters $K_{\text{in}}^{(l)}$, $K_{\text{out}}^{(l)}$, we choose the layer information block length $K^{(l)} = K_{\text{in}}^{(l)} K_{\text{out}}^{(l)}$. The concatenated layer encoding is described as follows.

The layer information block $\mathbf{s}^{(l)}$ is first equally partitioned into $K_{\text{out}}^{(l)}$ layer information sub-blocks of length $K_{\text{in}}^{(l)}$. Recalling that $|\mathcal{A}_s| = |\mathcal{A}_c|$, each of the $K_{\text{out}}^{(l)}$ layer information sub-blocks can be uniquely mapped to one element in $\mathrm{GF}(|\mathcal{A}_c|^{K_{\text{in}}^{(l)}})$. Therefore, $\mathbf{s}^{(l)}$ corresponds to a $K_{\text{out}}^{(l)}$-long block in $\mathrm{GF}(|\mathcal{A}_c|^{K_{\text{in}}^{(l)}})$ which is then encoded by the outer coder. Each of the M output symbols from the outer coder belongs to $\mathrm{GF}(|\mathcal{A}_c|^{K_{\text{in}}^{(l)}})$ and thus, it can be represented in $\mathrm{GF}(|\mathcal{A}_c|)$ as a $K_{\text{in}}^{(l)}$-long block, denoted by $\check{\mathbf{c}}_{\nu}^{(l)}$, $\nu = 1, \ldots, M$. Finally, the inner coder takes $\check{\mathbf{c}}_{\nu}^{(l)}$ as input to yield the LSCW $\mathbf{c}_{\nu}^{(l)}$.

To reduce decoding complexity, one way is to perform ML decoding for inner codes and outer codes successively. To further reduce complexity, we will use a *suboptimal* concatenated algebraic decoding algorithm, which is summarized in the following steps:

M	Q	$K_{\text{out}}^{(2)}$	$K_{\text{out}}^{(3)}$	$K_{\text{out}}^{(4)}$	$K_{\text{out}}^{(5)}$
15	2	5	13	13	13

TABLE I
DESIGN PARAMETERS

(s1): Make hard decision $\hat{c}_{\nu q}^{(1)} \in \mathcal{A}_c$ on $c_{\nu q}^{(l)}$ from $\bar{\mathbf{y}}_{p(\nu,q)}^{(l)}$;
(s2): Use the inner decoder to correct errors in $[\hat{c}_{\nu 1}^{(1)}, \ldots, \hat{c}_{\nu Q}^{(1)}]$ to obtain $\hat{\mathbf{c}}_{\nu}^{(l)}$;
(s3): Use the outer decoder to correct errors in $[\hat{\mathbf{c}}_1^{(l)}, \ldots, \hat{\mathbf{c}}_M^{(l)}]$ and obtain $\hat{\mathbf{c}}^{(l)}$;
(s4): Recover $\mathbf{s}^{(l)}$ from $\hat{\mathbf{c}}^{(l)}$.
The decoding complexity depends on the decoding algorithms used in inner and outer decoders. If we choose well-developed channel codes as the inner and outer codes, it is not difficult to find efficient decoding algorithms [8] to reduce overall decoding complexity.

So far, we have ignored error propagation. We next investigate its effects by simulations.

VI. SIMULATION

In this section, we present one design example and further test its performance by simulations. We choose BER as our figure of merit which we average over 100 channel realizations and 1,000 noise realizations for each E_b/N_0 point.

Design Example: We choose $M = N = 15$ and use QPSK modulation, i.e.,$|A_c| = |A_s| = 2$. We use outer encoding only. We choose Reed-Solomon (RS) as our outer codes. In the ST context, diversity gain is an extremely important performance metric. Although outer coding is helpful to improve diversity gain [4], it has limited error-correcting capability. Because $M = N$, the receive diversity gain of the first layer is 1. Instead of using strong outer encoding for the first layer, we find it more efficient if we just leave the first layer unused, and allocate limited power to the rest layers, which is specified by

$$\frac{A^{(2)}}{\sqrt{7}} = \frac{A^{(3)}}{\sqrt{5}} = \frac{A^{(4)}}{\sqrt{3}} = \frac{A^{(5)}}{\sqrt{2}} = \frac{A^{(6)}}{\sqrt{1.5}} = A^{(7)} = \cdots = A^{(15)}.$$

In addition, we layer-encode layers $l = 2, 3, 4, 5$ and leave the remaining layers uncoded. The parameters of the outer codes are tabulated in Table I. It is easy to obtain that the transmission rate in our design is $R \approx 26$ bits/sec/Hz.

Test Case (Performance comparison with D-BLAST) We compare our design to D-BLAST whose layers use the same layer codes and are loaded with the same power.

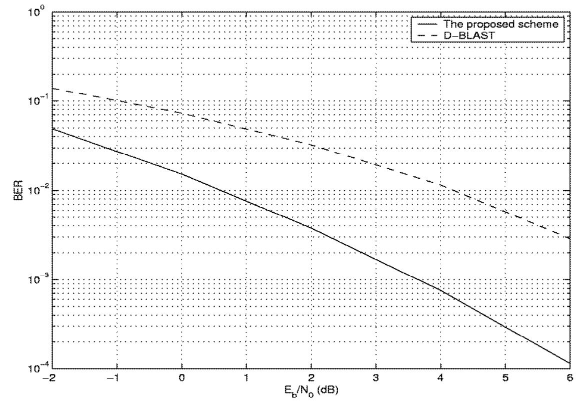

Fig. 3. Performance comparison with D-BLAST

Similar to our design, we only use outer encoding in D-BLAST. The outer codes in D-BLAST are (15, 13) RS codes, so that the overall code rates in both D-BLAST and our design are (almost) same. However, as we mentioned in Section III-A.3, D-BLAST wastes transmissions at the beginning and end of one burst. Thus, our scheme has higher transmission rate than D-BLAST. Fig. 3 shows that our design outperforms D-BLAST considerably.

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