SPACE-TIME CODING AND KALMAN FILTERING FOR DIVERSITY TRANSMISSIONS THROUGH TIME-SELECTIVE FADING CHANNELS*

Zhiqiang Liu, *Xiaoli Ma* and *Georgios B. Giannakis*

Dept. of ECE, Univ. of Minnesota; 200 Union Str. SE, Minneapolis, MN 55455;
Tel/Fax: (612) 626-7781/625-4583; Emails: {lzq,xiaoli,georgios}@ece.umn.edu.

ABSTRACT

Most existing space-time (ST) coding schemes have been developed for flat fading channels. To offer antenna diversity gains, they rely on channel state information (CSI) acquired at the receiver through training, or, via blind channel estimation. Differential ST modulation obviates the need for CSI but is less effective in rapidly fading environments. This paper develops novel ST coders and decoders that enjoy transmit-diversity gains in the presence of time-selective fading channels. Modeling the time-selective channels as random processes, we employ Kalman filtering to estimate them and mitigate their effects so that symbol detection benefits from transmit-diversity. Computer simulations confirm that the proposed scheme achieves robust performance in time-selective channels with a few training symbols.

I. INTRODUCTION

In recent years, space-time (ST) coding has been shown to be very effective in combating fading and increasing channel capacity significantly without necessarily sacrificing bandwidth efficiency (see [6] and references therein). Remarkably, ST block coding [1] with two transmit-antennas achieves full diversity gains using a linear maximum-likelihood (ML) decoder. So far, the effectiveness of most ST coding schemes relies on accurate multi-channel estimates at the receiver, which are acquired either through training or via blind channel estimation. Multi-channel estimation requires that the underlying channels remain invariant for sufficiently long time which is difficult or impossible to satisfy in many applications. Differential ST modulation [5] forgoes channel estimation and allows for slowly changing channels that have to remain invariant within two consecutive blocks. However, its performance degrades dramatically in time-selective channels. Based on the expectation-maximization (EM) algorithm, an iterative ST receiver for time-selective channels was proposed in [2].

*This work was supported by ARO grant no. DAAG55-98-1-0336.

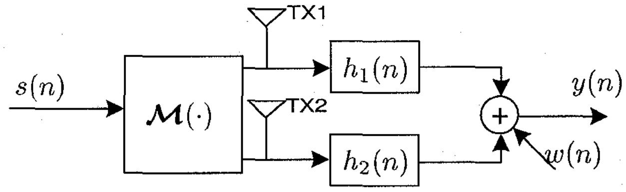

Fig. 1. Discrete time system model

In wireless mobile communications, time-selectivity is mainly caused by Doppler shifts and carrier frequency offsets. Time-selective channels can be modeled either deterministically through basis function expansions [4] , or randomly as autoregressive (AR) processes [3, 8, 9]. Typically, deterministic channel models require estimates of more parameters than random models, which translates to sensitivity due to over-parameterization. For the remainder of the paper, random channel models will be adopted.

In this paper we develop novel ST coders and decoders that achieve transmit-diversity gains through time-selective channels. Consider a system with two transmit antennas and one receive antenna. Modeling the time-selective channels as AR processes of order one (AR(1)), we use Kalman filtering to track the channel variations and decode information symbols with diversity gains. Thorough simulations illustrate the robust performance of our scheme in the presence of time-selective channels. The rest of paper is organized as follows. In Section II we describe the system model while in Section III we present the ST decoder design. The tracking of time-selective channels is developed in Section IV. The performance analysis and simulations are presented in Section V. Section VI concludes this paper.

II. SYSTEM MODEL

Consider a wireless system equipped with two transmit-antennas and one receive-antenna. The baseband discrete-time equivalent transmitter and receiver model is depicted in Figure 1. At the transmitter, the information sequence $s(n)$ at symbol rate $1/T_s$

is first parsed into blocks $\mathbf{s}(n) := [s(2n)\ s(2n+1)]^T$. The ST encoder $\boldsymbol{\mathcal{M}}(\cdot)$ encodes $\mathbf{s}(n)$ and yields at its output the 2×2 code matrix:

$$\mathbf{C}(n) = \begin{pmatrix} s(2n) & -s^*(2n+1) \\ s(2n+1) & s^*(2n) \end{pmatrix} \begin{matrix} \rightarrow & \text{time} \\ \downarrow & \text{space} \end{matrix} \quad (1)$$

whose two columns are transmitted in successive time intervals with two elements sent through two transmit-antennas respectively. We assume that the channel delay spread is smaller than T_s but the channel coherence time is comparable to T_s. Based on these assumptions, channels are frequency-flat but time-selective. Note that the ST code in (1) is Alamouti's block code [1] that achieves full diversity gains in flat fading channels. We will see here how (1) can be applied in time-selective channels.

Denote by $h_i(n)$, $i = 1, 2$, the time-selective channel from the ith transmit-antenna to the receive-antenna. At the receive-antenna, the data $y(n)$ two successive samples are:

$$y(2n) = h_1(2n)s(2n) + h_2(2n)s(2n+1) + w(2n) \quad (2a)$$

$$\begin{aligned} y(2n+1) = &- h_1(2n+1)s^*(2n+1) \\ &+ h_2(2n+1)s^*(2n) + w(2n+1), \end{aligned} \quad (2b)$$

where $w(n)$ is the additive white Gaussian noise (AWGN) with covariance σ_w^2. Given two consecutive received samples $y(2n)$, $y(2n+1)$, our goal is to recover $\mathbf{s}(n)$ with diversity gains. Without imposing structures on $h_i(n)$, this goal is ill-posed because for every two incoming received data two more unknowns, $h_1(n)$ and $h_2(n)$, appear in addition to the unknown $\mathbf{s}(n)$. In wireless mobile communications, channel time-variations arise mainly due to Doppler shifts and carrier frequency offsets. Consequently, channel variations evolve in a progressive fashion and hence fit in some time-evolution models. Among different models, the AR(1) model was shown to be sufficient for capturing slow channel variations [3, 9] and will be adopted herein. Specifically, h_i is modeled as:

$$h_i(n) = \alpha\ h_i(n-1) + v_i(n), \quad i = 1, 2, \quad (3)$$

where: $v_i(n)$ is an i.i.d circular complex Gaussian variable with covariance σ_v^2; the AR(1) coefficient α is assumed to have been estimated as explained in [8]. Because two transmit-antennas share the transmit-oscillator, the two channels will induce the same carrier frequency offsets. The Doppler shifts for the two channels may be different if the multipath components arrive at the receiver at different angles of arrival. However, this difference is negligible if the multipath components originate far away from the receiver. In (3), α captures the common part of time-variations in both channels and $v_i(n)$ stands for unmodeled differences. Given the received sample $y(n)$ and based on channel model (3), we derive next our ST decoders.

III. SPACE-TIME DECODING

We design our ST decoders based on the following assumptions:

(a1) The channels $h_i(n)$, $i = 1, 2$, are stationary, complex Gaussian processes with zero mean and covariance $\sigma_h^2 = 1$;

(a2) The two channels $h_i(n)$ have very similar time-variations, i.e., $|\alpha|^2 \gg \sigma_v^2$. This assumption is valid if the origins of multipath components are far away from the receiver so that the Doppler shifts for both channels are close to each other.

As mentioned before, ST codes in (1) were originally designed for flat fading channels [1] where a simple linear maximum-likelihood (ML) decoder was derived to achieve full diversity gains. Based on (a1) and (a2), we show next that ST codes in (1) also enable a near-ML decoder in time-selective channels.

Let us define $\mathbf{y}(n) := [y(2n)\ y^*(2n+1)]^T$ and rewrite (2a), (2b) into a matrix/vector form as:

$$\mathbf{y}(n) = \mathbf{H}(n)\ \mathbf{s}(n) + \mathbf{w}(n), \quad (4)$$

where the channel matrix $\mathbf{H}(n)$ is given by:

$$\mathbf{H}(n) := \begin{pmatrix} h_1(2n) & h_2(2n) \\ h_2^*(2n+1) & -h_1^*(2n+1) \end{pmatrix}, \quad (5)$$

and $\mathbf{w}(n) := [w(2n)\ w^*(2n+1)]^T$. With $^{\mathcal{H}}$ denoting Hermitian transpose, we use (3) and (5) to form:

$$\mathbf{H}^{\mathcal{H}}(n)\mathbf{H}(n) = \begin{pmatrix} \rho_1(n) & \epsilon(n) \\ \epsilon^*(n) & \rho_2(n) \end{pmatrix} \quad (6)$$

where $\rho_1(n) := |h_1(2n)|^2 + |h_2(2n+1)|^2$, $\rho_2(n) := |h_1(2n+1)|^2 + |h_2(2n)|^2$ and $\epsilon(n) := h_1^*(2n)h_2(2n) - h_1^*(2n+1)h_2(2n+1)$. Using (a1), it follows from (3) that $|\alpha|^2 + \sigma_v^2 = 1$ which implies from (a2) that:

$$|\alpha|^2 \approx 1 \quad \text{and} \quad \sigma_v^2 \approx 0\ . \quad (7)$$

Using (7), it can be readily proved that: $\epsilon(n) \approx 0$. We infer from (6) that $\mathbf{H}(n)$ is near-unitary. Recall that $\mathbf{H}(n)$ is unitary for flat fading channels where $\alpha = 1$ and $v_i(n) = 0$. We underscore that $\mathbf{H}(n)$ is unitary when $|\alpha|^2 = 1$ and $v_i(n) = 0$. Let us denote with f_o and f_d the carrier frequency offset and the Doppler shift, respectively. If f_d is common to both channels, we have $\alpha = \exp[j2\pi(f_o + f_d)]$ and $v_i(n) = 0$. In this case, $\mathbf{H}(n)$ is unitary. This implies that ST codes in (1) are insensitive to carrier offsets and Doppler-induced time-selective effects. Based on these implications, we are motivated to use Alamouti's ST codes in time-selective channels and decode the information symbols using the same scheme as in [1]. Specifically, we construct the decision vector $\mathbf{z}(n) := [z(2n),\ z(2n+1)]^T = \mathbf{H}^{\mathcal{H}}(n)\mathbf{y}(n)$ as:

$$\mathbf{z}(n) = \begin{pmatrix} \rho_1(n) & \epsilon(n) \\ \epsilon^*(n) & \rho_2(n) \end{pmatrix} \mathbf{s}(n) + \mathbf{H}^{\mathcal{H}}(n)\mathbf{w}(n), \quad (8)$$

from which we decode $\mathbf{s}(n)$. If $\epsilon(n)$ is negligible, it is readily proved that the detection (8) is maximum-likelihood. The effects of $\epsilon(n)$ will be investigated by theoretical analysis and simulations in Section V.

So far, the ST decoding in (8) has relied on estimates of $h_i(n)$, $i = 1, 2$, at the receiver. Multichannel estimation is challenging especially in our time-varying MISO system setup. Thanks to the AR model (3) which leads to state-space representation, we proceed to employ Kalman filtering to track the channel variations.

IV. CHANNEL TRACKING

In this section, we resort to Kalman filtering and develop an adaptive algorithm to track the time-selective channels on-line. The implementation of our adaptive algorithm requires a training session to adjust to the real channels, after which it switches to the decision-directed mode. During the training mode, the receiver knows the transmitted symbols while in the decision-directed mode, the decoded symbols take their place. In the following discussions, we will focus on the decision-directed mode and assume that the initial channel estimates are available.

Let us define the state vector $\mathbf{h}(n) := [h_1(n),\ h_2(n)]^T$ and rewrite (3) to arrive at the state equation as:

$$\mathbf{h}(n) = \mathbf{A}\ \mathbf{h}(n-1) + \boldsymbol{v}(n), \quad (9)$$

where $\mathbf{A} := \text{diag}(\alpha,\ \alpha)$ and $\boldsymbol{v}(n) := [v_1(n)\ v_2(n)]^T$. In order to obtain the measurement equation, we define:

$$\bar{\mathbf{s}}(n) := \begin{cases} [s(n),\ s(n+1)]^T & \text{if } n \text{ is even} \\ [-s^*(n+1),\ s^*(n)]^T & \text{if } n \text{ is odd} \end{cases}, \quad (10)$$

and obtain from (4) the measurement equation as:

$$y(n) = \bar{\mathbf{s}}^T(n)\mathbf{h}(n) + w(n). \quad (11)$$

Under (9) and (11), with the knowledge of the decoded $\bar{\mathbf{s}}(n)$ and the observation $y(n)$, the standard Kalman filter based on one-step prediction [7, page 321] can be applied to predict the channel vector $\mathbf{h}(n)$. However, the detection of $\bar{\mathbf{s}}(n)$ relies on the estimates of $\mathbf{h}(n)$ that in turn requires the knowledge of $\bar{\mathbf{s}}(n)$. This implies that an alternative method should be sought to obtain $\bar{\mathbf{s}}(n)$ or $\mathbf{h}(n)$. Under (a2), our time-evolution model in (3) suggests that a coarse prediction of $\mathbf{h}(n)$ can be retrieved using (3) directly. Denote by $\mathbf{h}(n|m)$ the predicted channel at time n based on the state and/or the observation at time m. Our coarse channel prediction can be expressed as:

$$\mathbf{h}(2n|2n-1) = \alpha\ \mathbf{h}(2n-1|2n-1) \quad (12a)$$

$$\mathbf{h}(2n+1|2n-1) = \alpha^2\ \mathbf{h}(2n-1|2n-1), \quad (12b)$$

which are plugged into (4) to retrieve from (8) the coarse estimates of $\bar{\mathbf{s}}(2n)$ and $\bar{\mathbf{s}}(2n+1)$, denoted by $\bar{\mathbf{s}}^{(c)}(2n)$ and $\bar{\mathbf{s}}^{(c)}(2n+1)$ respectively.

Substituting $\bar{\mathbf{s}}(2n)$ with $\bar{\mathbf{s}}^{(c)}(2n)$, we combine (11) with (9) to perform Kalman filtering and obtain the refined channel estimate $\mathbf{h}(2n|2n)$. Then, we use $\mathbf{h}(2n|2n)$ to perform Kalman filtering again to obtain $\mathbf{h}(2n+1|2n+1)$. With refined $\mathbf{h}(2n|2n)$ and $\mathbf{h}(2n+1|2n+1)$, refined estimates $\bar{\mathbf{s}}^{(r)}(2n)$ and $\bar{\mathbf{s}}^{(r)}(2n+1)$ are obtained from (8) with diversity gains. Our channel tracking algorithm is summarized as follows:

Initialization: Obtain $\mathbf{h}(0|0)$ from training;
Step 1: Obtain $\mathbf{h}(2n|2n-1)$ and $\mathbf{h}(2n+1|2n-1)$ using (12a) and (12b);
Step 2: Use (4) and (8) to decode $\bar{\mathbf{s}}^{(c)}(2n)$ and $\bar{\mathbf{s}}^{(c)}(2n+1)$;
Step 3: Perform Kalman filtering to retrieve $\mathbf{h}(2n|2n)$ and $\mathbf{h}(2n+1|2n+1)$ using $\bar{\mathbf{s}}^{(c)}(2n)$ and $\bar{\mathbf{s}}^{(c)}(2n+1)$;
Step 4: Decode $\bar{\mathbf{s}}^{(r)}(2n)$ and $\bar{\mathbf{s}}^{(r)}(2n+1)$ based on $\mathbf{h}(2n|2n)$ and $\mathbf{h}(2n+1|2n+1)$; If necessary, iterate steps 3 and 4 more times to improve the tracking;
Step 5: Repeat from step 2 for $n+1 \leftarrow n$.

Like most adaptive algorithms, our Kalman filtering channel estimators may lose tracking and cause disastrous performance loss. In order to avoid divergence, similar to [9], pilot symbols are periodically inserted into the information sequence. With known pilot symbols, channel estimates are improved and persistent divergence is prevented. Having developed the channel tracking scheme, we proceed to test its performance with computer simulations.

V. PERFORMANCE ANALYSIS AND SIMULATIONS

Assuming that $h_1(n)$, $h_2(n)$, $v(n)$ and $w(n)$ are mutually independent, bit-error-rate (BER) performance analysis of the detector (8) is possible for a given constellation under perfect channel tracking conditions. Without loss generality, we first obtain from (8) the decision statistics for $s(2n)$ as:

$$z(2n) = \rho_1(n)s(2n) + \epsilon(n)\ s(2n+1) + \mathbf{H}^{\mathcal{H}}(n)(1,:)\mathbf{w}(n), \tag{13}$$

where $\mathbf{H}^{\mathcal{H}}(n)(1,:)$ denotes the first row of $\mathbf{H}^{\mathcal{H}}(n)$. Eq. (13) consists of three terms: the first term is useful information from $s(2n)$; the second term is the interference from $s(2n+1)$ caused by time-selective channels; and the third term is the additive noise. Treating the interference as noise and after conditioning on $h_1(2n)$ and $h_2(2n+1)$, we compute the signal-noise-ratio (SNR) $\gamma(n)$ as:

$$\gamma(n) = \frac{[|h_1(2n)|^2 + |h_2(2n+1)|^2]E_s}{2(\sigma_v^2 E_s + \sigma_w^2)}, \tag{14}$$

where E_s denote the symbol energy of $s(n)$. In deriving (14), we neglected the fourth order term σ_v^4 and divided the transmit power by two for each transmit antenna because each symbol is transmitted twice. A similar equation can be obtained for $s(2n+1)$. Supposing that QPSK modulation is used, the BER $P_b(n)$ can be expressed as:

$$P_b(n) = Q\left(\sqrt{\frac{[|h_1(2n)|^2 + |h_2(2n+1)|^2]E_s}{2(\sigma_v^2 E_s + \sigma_w^2)}}\right). \tag{15}$$

Clearly, a diversity gain of order two is achieved if the two channels are independent. When $E_s \gg \sigma_w^2$, we observe from (15) that $P_b(n)$ does not increase with E_s but approaches an error floor given by:

$$P_b(n) = Q\left(\sqrt{\frac{|h_1(2n)|^2 + |h_2(2n+1)|^2}{2\sigma_v^2}}\right). \tag{16}$$

In order to remove or lower this error floor, one approach is not to model the interference in (13) as noise, but treat it as ISI and employ equalization techniques such as ZF or MMSE and their decision feedback variants, to decode $\mathbf{s}(n)$ from $\mathbf{z}(n)$, at the expense of higher receiver complexity. In this paper, we wish to maintain the simplicity of the receiver and will thus focus on the detector (8). So far, our performance analysis assumes perfect channel tracking. In order to test robustness of the overall system performance to this assumption, we resort to simulations.

In all our simulations, QPSK modulation is employed. The time-selective fading channels are generated by initializing $h_i(0)$ as a complex Gaussian variable with variance one and taking $\alpha = 0.998$. We choose BER as our figure of merit which is averaged over 800 channel initializations and 100 Kalman filtering iterations.

Example 1: (performance improvement with channel tracking) In order to appreciate the importance of channel tracking in time-selective channels, we test our proposed scheme in time-selective channels and compare the cases with and without channel tracking. In order to avoid divergence in Kalman filtering, we insert one pilot symbol every 12 symbols and thus introduce 8% bandwidth efficiency loss. To maintain fairness, we set the receiver with perfect current channel estimates every 12 symbols when no channel tracking is employed. The results are shown in Fig. 2 where the system with channel tracking outperforms that without tracking considerably especially at high SNR. Fig. 3 shows the true channels (thick curves) and their corresponding tracked values (thin curves). It is observed that our Kalman filtering yields excellent tracking results. Thanks to transmit-diversity, our proposed scheme demonstrates significant performance gain as compared to the scheme without ST coding in [9]. Note that in addition to transmit-diversity gains, the code structure in (1) helps to eliminate sign ambiguities that occurred in [9] and provides performance robustness against time-selectivity.

Example 2 (performance with perfect channel estimates) We simulate the proposed scheme assuming perfect channel estimates and compare the results with ideal versus actual channel tracking. In addition, we simulate the BER performance in flat fading channels ($\alpha = 1$). Fig. 4 confirms the presence of error floor in time-selective channels. It is also seen that the channel tracking incurs about 5 dB loss at BER $= 10^{-3}$, as compared to perfect channel estimates.

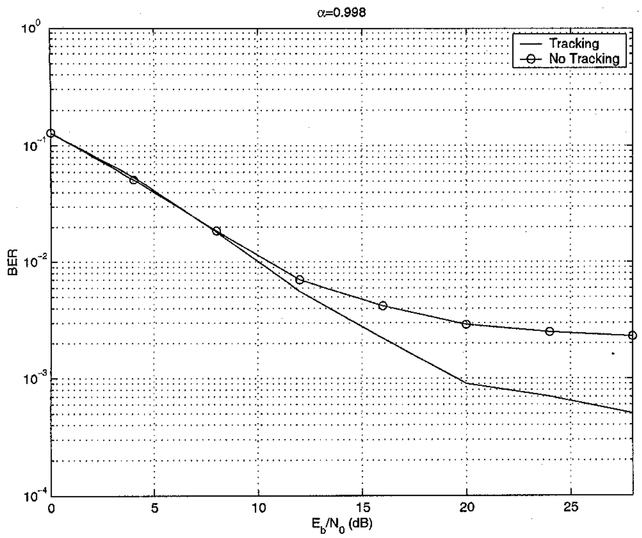

Fig. 2. Improvements with channel tracking

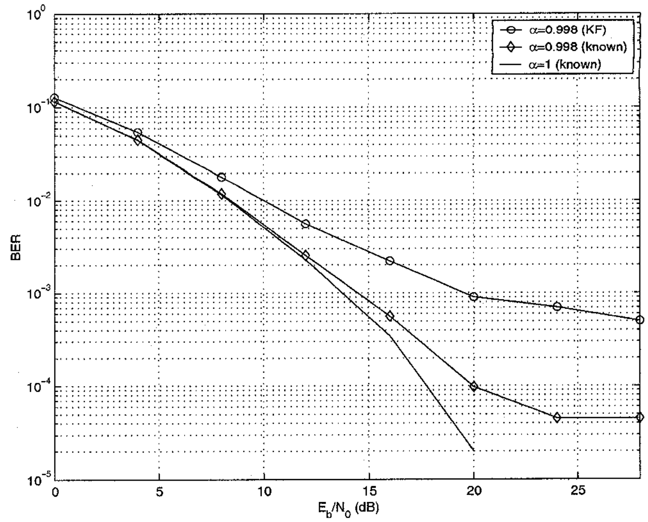

Fig. 4. BER with known and tracked channels

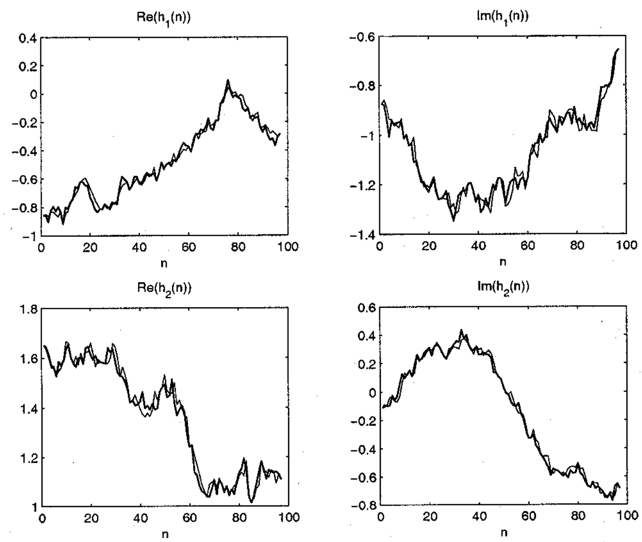

Fig. 3. Performance of channel tracking

VI. CONCLUSIONS

We proposed a novel ST coder/decoder for iterative channel tracking and symbol recovery in time-selective channels. Modeling the time-selective channels as AR(1) processes, Kalman filtering is employed to track the channel variations. Both channel tracking and symbol recovery benefit from ST coding. Future topics include extensions to doubly-selective channels and blind estimation of the AR model parameters.

REFERENCES

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[2] C. Cozzo and B. L. Hughes, "Joint channel estimation and data detection in space-time communications," *IEEE Transactions on Communications*, 1999 (submitted).

[3] A. W. Fuxjaeger and R. A. Iltis, "Adaptive parameter estimation using parallel kalman filtering for spread spectrum code and Doppler tracking," *IEEE Transactions on Communications*, vol. 46, no. 5, pp. 2227–2230, June 1994.

[4] G. B. Giannakis and C. Tepedelenlioglu, "Basis expansion models and diversity techniques for blind equalization of time-varying channels," *Proceedings of the IEEE*, vol. 86, pp. 1969–1986, Oct. 1998.

[5] B. L. Hughes, "Differential space-time modulation," *Proc. of 33rd Annual Asilomar Conference on Signals, Systems, and Computers*, vol. 1, pp. 627-631, Pacific Grove, CA, Oct. 1999.

[6] A. F. Naguib, N. Seshadri and A. R. Calderbank, "Increasing data rate over wireless channels," *IEEE Signal Processing Magazine*, Vol. 17, No. 3, pp. 76-92, May 2000.

[7] S. Haykin, *Adaptive filter theory*, Prentice Hall, 1996.

[8] M. K. Tsatsanis, G. B. Giannakis, and G. Zhou, "Estimation and equalization of fading channels with random coefficients," *Signal Processing*, vol. 53, no. 2/3, pp. 211–228, 1996.

[9] G. T. Zhou, Y. Kim and G. B. Giannakis, "Estimation and Equalization of Time-Selective Channels using Precoding," *Proc. of 33rd Asilomar Conf. on Signals, Systems, and Computers*, vol. 1, pp. 248-252, Pacific Grove, CA, Nov. 1-4, 1999.