# Space–Time Frequency-Shift Keying

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***Abstract*—Frequency-shift keying (FSK) is a popular modulation scheme in power-limited communication links. This paper introduces space–time FSK (ST-FSK), which does not require any channel state information at the transmitter and the receiver, as in conventional noncoherent FSK. ST-FSK can be viewed as a special unitary ST modulation design. However, ST-FSK has a number of advantages over existing unitary ST modulation designs. ST-FSK is easier to design, and enjoys lower decoding complexity. Furthermore, ST-FSK guarantees full diversity. Finally, ST-FSK can be adopted in the digital as well as in the analog domain, and merges very naturally with frequency-hopping multiple access. As expected, all these advantages come at the cost of a decrease in spectral efficiency.**

***Index Terms*—Frequency-shift keying (FSK), orthogonal design, space–time (ST) coding, unitary modulation.**

## I. Introduction

DESPITE the accompanying spectral inefficiency, frequency-shift keying (FSK) and other orthogonal modulation schemes are of interest in power-limited setups, such as those in military and satellite communications. In this letter, we introduce space–time FSK (ST-FSK), which does not require any channel state information (CSI) at the transmitter and the receiver, as in conventional noncoherent FSK. ST-FSK transmits FSK waveforms that are structured according to the full-rate real orthogonal designs of [6]. It is similar in spirit to the unitary ST modulation introduced in [3]. We actually show that ST-FSK can be viewed as a special unitary ST modulation design. However, ST-FSK has a number of advantages over the unitary ST modulation designs of [4].

1) ST-FSK is very simple, whereas the unitary ST modulation designs of [4] require a complex numerical search procedure.
2) The above design complexity advantage might not be such a big issue, because the code design can be carried out offline. However, there is also a complexity advantage in decoding. ST-FSK allows for a simplified maximum-likelihood (ML) detector, whereas the unitary ST modulation designs of [4] require a full-blown ML detector.
3) For two transmit antennas, ST-FSK can be proven to achieve full diversity. For more than two transmit antennas, this is more difficult to prove, but exhaustive search confirms that it still holds. For the unitary ST modulation designs of [4], on the other hand, it depends on the search criterion that is used, whether full diversity is guaranteed or not. However, this is not addressed thoroughly in [4]. We will show in the sequel that the search criterion explored in [4] does not guarantee full diversity.
4) As for conventional FSK, ST-FSK can be adopted in the digital as well as in the analog domain, and merges very naturally with frequency-hopping multiple access (FHMA), which is employed in ad-hoc networks, for instance.

The aforementioned advantages come at a cost. For a fixed bandwidth, the unitary ST modulation designs of [4] may achieve a higher rate, or similarly, for a fixed rate, they may occupy a smaller bandwidth. This is expected, since FSK is known to be power efficient, but not spectrally efficient.

*Notations:* Upper (lower) boldface letters denote matrices (column vectors); $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian, respectively; $\otimes$ is used for the Kronecker product; $\delta_{i,j}$ represents the Kronecker delta; $\|\cdot\|$ represents the Frobenius norm; $[\mathbf{A}]_{m,n}$ denotes the $(m,n)$th entry of the matrix $\mathbf{A}$, and $[\mathbf{a}]_n$ denotes the $n$th entry of the column vector $\mathbf{a}$; $\mathrm{E}\{\cdot\}$ is reserved for statistical average; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}_{M\times N}$ denotes the $M \times N$ all-zero matrix; and finally, $|\mathcal{A}|$ is used to denote the cardinality of the set $\mathcal{A}$.

Paper approved by W. E. Ryan, the Editor for Modulation, Coding, and Equalization of the IEEE Communications Society. Manuscript received October 31, 2002; revised August 11, 2003 and September 15, 2003. This work was supported in part by the National Science Foundation Wireless Initiative under Grant 9979443, in part by the Army Research Lab/Collaborative Technology Alliance (ARL/CTA) under Grant DAAD19-01-2-011, in part by the FWO-Vlaanderen, and in part by the Boğaziçi University Research Fund under Contract 00A202. This paper was presented in part at the 36th Conference on Information Sciences and Systems, Princeton, NJ, March 2002.

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Digital Object Identifier 10.1109/TCOMM.2004.823647

## II. Data Model

We adopt here a similar data model as in [3]. Consider a communication link from $M$ transmit antennas to $N$ receive antennas signaling over a flat-fading channel that is constant over $T$ symbol periods. Denoting the $T \times 1$ vector transmitted from the $m$th transmit antenna as $\mathbf{x}_m$, the $T \times 1$ vector $\mathbf{y}_n$ received at the $n$th receive antenna can be written as $\mathbf{y}_n = \sum_{m=1}^{M} \mathbf{x}_m h_{m,n} + \mathbf{e}_n$, where $h_{m,n}$ is the flat-fading channel coefficient from the $m$th transmit antenna to the $n$th receive antenna, and $\mathbf{e}_n$ is the noise vector received at the $n$th receive antenna. Denoting the $T \times M$ matrix obtained by stacking the

$M$ transmitted vectors as $\mathbf{X} := [\mathbf{x}_1, \ldots, \mathbf{x}_M]$, the $T \times N$ matrix $\mathbf{Y} := [\mathbf{y}_1, \ldots, \mathbf{y}_N]$ obtained by stacking the $N$ received vectors can be expressed as $\mathbf{Y} = \mathbf{XH} + \mathbf{E}$, where $\mathbf{H}$, defined by $[\mathbf{H}]_{m,n} := h_{m,n}$, is the $M \times N$ flat-fading channel matrix obtained by stacking the $MN$ flat-fading channel coefficients, and $\mathbf{E} := [\mathbf{e}_1, \ldots, \mathbf{e}_N]$ is the $T \times N$ noise matrix obtained by stacking the $N$ noise vectors. We assume that the entries of $\mathbf{H}$ and $\mathbf{E}$ are independent and identically distributed (i.i.d.) zero-mean complex Gaussian variables with variance $\sigma_h^2$ and $\sigma_e^2$, respectively.

From [3], we know that it is advantageous from a noncoherent capacity point of view to consider a constellation $\mathcal{A}_\mathbf{X}$ that satisfies $\mathbf{X}^H\mathbf{X} = T\mathbf{I}_M, \forall \mathbf{X} \in \mathcal{A}_\mathbf{X}$. Such a signaling scheme is referred to as *unitary ST modulation*. In [4], a number of different unitary ST modulation designs are presented. In this letter, we consider a much simpler unitary ST modulation design, which we label as *ST-FSK*.

## III. ST-FSK

First, we design a set of $P$ real $P \times M$ matrices $\{\mathbf{A}_p\}_{p=1}^P$ that satisfies $\mathbf{A}_p^T\mathbf{A}_p = \mathbf{I}_M$ and $\mathbf{A}_p^T\mathbf{A}_{p'} = -\mathbf{A}_{p'}^T\mathbf{A}_p$, for $p \neq p'$, also known as a full-rate real orthogonal design [6]. Note that full-rate real orthogonal designs only exist for $M \leq 8$ [6], but these are the cases of practical interest. Furthermore, we take $P$ as small as possible (delay optimal), which results in $P = 2$ for $M = 2, P = 4$ for $M = 3, 4$, and $P = 8$ for $M = 5, 6, 7, 8$ [6]. We then choose $K$, which represents the number of FSK waveforms that we want to include in our design. Defining $\mathcal{A}_k = \{0, \ldots, K-1\}$, the corresponding set of FSK waveforms is $\{\mathbf{f}_k \,|\, k \in \mathcal{A}_k\}$, where $\mathbf{f}_k := [1 e^{j2\pi k/K} \cdots e^{j2\pi k(K-1)/K}]^T$. We finally construct the constellation $\mathcal{A}_\mathbf{X} = \{\mathbf{X}_\mathbf{k} \,|\, \mathbf{k} \in \mathcal{A}_k^{P\times 1}\}$, where

$$\mathbf{X}_\mathbf{k} = \sum_{p=1}^{P} \mathbf{A}_p \otimes \mathbf{f}_{[\mathbf{k}]_p}. \tag{1}$$

Note that this resembles a full-rate real orthogonal ST code written in the form of a linear dispersion code [2], except that the scalar product is now replaced by a Kronecker product, and the real data symbols are replaced by FSK waveforms. The block size of ST-FSK is $T = PK$. The rate of ST-FSK is the same as that of conventional FSK, using the same set of FSK waveforms. Defining $R$ as the rate expressed in bits per symbol, we obtain $R = \log_2(|\mathcal{A}_k^{P\times 1}|)/T = \log_2(K^P)/(KP) = \log_2(K)/K$.

Relying on the fact that $\{\mathbf{A}_p\}_{p=1}^P$ represents a full-rate real orthogonal design and $\{\mathbf{f}_k \,|\, k \in \mathcal{A}_k\}$ represents a set of FSK waveforms, it is clear from (1) that $\mathbf{X}_\mathbf{k}^H\mathbf{X}_\mathbf{k} = T\mathbf{I}_M, \forall \mathbf{k} \in \mathcal{A}_k^{P\times 1}$. Hence, we can view ST-FSK as a special unitary ST modulation design. This allows us to use all the results that were presented in [3].

## IV. ST-FSK Detector

The noncoherent ML detector can be expressed as [3]

$$\hat{\mathbf{k}} = \arg\max_{\mathbf{k}\in\mathcal{A}_k^{P\times 1}} \left\|\mathbf{X}_\mathbf{k}^H\mathbf{Y}\right\|^2.$$

Using (1), we can rewrite $\mathbf{X}_\mathbf{k}^H\mathbf{Y}$ as

$$\begin{aligned}\mathbf{X}_\mathbf{k}^H\mathbf{Y} &= \sum_{p=1}^{P}\left(\mathbf{A}_p^T \otimes \mathbf{f}_{[\mathbf{k}]_p}^H\right)\mathbf{Y}\\ &= \sum_{p=1}^{P}\mathbf{A}_p^T\left(\mathbf{I}_P \otimes \mathbf{f}_{[\mathbf{k}]_p}^H\right)\mathbf{Y}\\ &= \sum_{p=1}^{P}\mathbf{A}_p^T\mathbf{Z}_{[\mathbf{k}]_p}\end{aligned}$$

where $\mathbf{Z}_k := (\mathbf{I}_P \otimes \mathbf{f}_k^H)\mathbf{Y}$ can be viewed as the matched-filter output corresponding to the FSK waveform $\mathbf{f}_k$. Hence, the noncoherent ML detector can be simplified by first computing the $K$ matched-filter outputs $\{\mathbf{Z}_k \,|\, k \in \mathcal{A}_k\}$, which yield a sufficient statistic, and subsequently determining

$$\hat{\mathbf{k}} = \arg\max_{\mathbf{k}\in\mathcal{A}_k^{P\times 1}} \left\|\sum_{p=1}^{P}\mathbf{A}_p^T\mathbf{Z}_{[\mathbf{k}]_p}\right\|^2. \tag{2}$$

In the next section, we will analyze the performance of this simplified noncoherent ML detector.

## V. Performance Analysis

We can upper bound the block-error probability $P_e$ through the union bound as [3]

$$P_e \leq \frac{1}{K^P}\sum_{\mathbf{k},\mathbf{k}'\in\mathcal{A}_k^{P\times 1},\mathbf{k}\neq\mathbf{k}'} P_{\mathbf{k},\mathbf{k}'} \tag{3}$$

where $P_{\mathbf{k},\mathbf{k}'}$ is the pairwise error probability (PEP) of mistaking $\mathbf{k}$ for $\mathbf{k}'$, or vice versa. Furthermore, we can upper bound this PEP through the Chernoff bound as [3]

$$P_{\mathbf{k},\mathbf{k}'} \leq \frac{1}{2}\prod_{m=1}^{M}\left(1 + \frac{(\rho T/M)^2\left(1 - d_{\mathbf{k},\mathbf{k}',m}^2\right)}{4(1+\rho T/M)}\right)^{-N} \tag{4}$$

where $\rho = M\sigma_h^2/\sigma_e^2$ is the signal-to-noise ratio (SNR) at each receive antenna, and $1 \geq d_{\mathbf{k},\mathbf{k}',1} \geq \cdots \geq d_{\mathbf{k},\mathbf{k}',M} \geq 0$ are the singular values of $\mathbf{X}_\mathbf{k}^H\mathbf{X}_{\mathbf{k}'}/T$. Note in this context that we can rewrite $\mathbf{X}_\mathbf{k}^H\mathbf{X}_{\mathbf{k}'}/T$ as

$$\begin{aligned}\mathbf{X}_\mathbf{k}^H\mathbf{X}_{\mathbf{k}'}/T &= \frac{1}{T}\sum_{p=1}^{P}\left(\mathbf{A}_p^T \otimes \mathbf{f}_{[\mathbf{k}]_p}^H\right)\sum_{p'=1}^{P}\left(\mathbf{A}_{p'} \otimes \mathbf{f}_{[\mathbf{k}']_{p'}}\right)\\ &= \frac{1}{P}\sum_{p=1}^{P}\sum_{p'=1}^{P}\mathbf{A}_p^T\mathbf{A}_{p'}\delta_{[\mathbf{k}]_p,[\mathbf{k}']_{p'}}.\end{aligned} \tag{5}$$

For a sufficiently large $\rho$, the Chernoff bound on the PEP $P_{\mathbf{k},\mathbf{k}'}$ depends dominantly on [5]

$$\zeta_{\mathbf{k},\mathbf{k}'} = \left(\prod_{m=1}^{M}(1 - d_{\mathbf{k},\mathbf{k}',m}^2)\right)^{1/2M}$$

which can be interpreted as the geometric mean of the sines of the $M$ principal angles between the subspaces spanned by the columns of $\mathbf{X_k}$ and $\mathbf{X_{k'}}$. Defining the *diversity product* as

$$\zeta = \min_{\mathbf{k},\mathbf{k'} \in \mathcal{A}_k^{P\times 1}, \mathbf{k}\neq\mathbf{k'}} \zeta_{\mathbf{k},\mathbf{k'}}$$

full diversity is achieved if the diversity product $\zeta$ is nonzero [5].

The unitary ST modulation designs of [4] are designed based on some search criterion. A good search criterion could be the maximization of the diversity product (as done in [5] for the special case of $T = M$, because it was used for differential modulation). However, in [4], a different search criterion is explored, which, as we will show later on, does not necessarily lead to a nonzero diversity product, and consequently, does not guarantee full diversity. For $M = 2$, ST-FSK has a nonzero diversity product, and consequently achieves full diversity, as discussed next. For $M > 2$, this is more difficult to prove, but exhaustive search confirms that it still holds.

For $M = 2$, the most general delay-optimal full-rate real orthogonal design is given by [1], [6]

$$\mathbf{A}_1 = \mathbf{I}_2\mathbf{\Phi} := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{\Phi}, \quad \mathbf{A}_2 = \mathbf{J}_2\mathbf{\Phi} := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{\Phi}$$

where $\mathbf{\Phi}$ is an arbitrary real orthogonal $2 \times 2$ matrix. Evaluating (5) for all possible combinations of $\mathbf{k}$ and $\mathbf{k'}$, we then obtain

$$\frac{\mathbf{X_k}^H\mathbf{X_{k'}}}{T} = \begin{cases} \mathbf{I}_2, & k_1 = k_1', k_2 = k_2' \\ (\mathbf{I}_2 - \mathbf{J}_2)/2, & k_1 \neq k_2 = k_1' = k_2' \\ & k_1 = k_2 = k_1' \neq k_2' \\ (\mathbf{I}_2 + \mathbf{J}_2)/2, & k_1' = k_2' = k_1 \neq k_2 \\ & k_1' \neq k_2' = k_1 = k_2 \\ \mathbf{I}_2/2, & k_1 = k_1' \neq k_2', k_2 \neq k_1', k_2 \neq k_2' \\ & k_2 = k_2' \neq k_1', k_1 \neq k_1', k_1 \neq k_2' \\ \mathbf{J}_2/2, & k_1 = k_2' \neq k_1', k_2 \neq k_1', k_2 \neq k_2' \\ -\mathbf{J}_2/2, & k_2 = k_1' \neq k_2', k_1 \neq k_1', k_1 \neq k_2' \\ \mathbf{0}_{2\times 2}, & \text{otherwise} \end{cases} \quad (6)$$

where we have used $k_p := [\mathbf{k}]_p$ and $k_p' := [\mathbf{k'}]_p$. We see that in all cases, the two singular values $d_{\mathbf{k},\mathbf{k'},1}$ and $d_{\mathbf{k},\mathbf{k'},2}$ are equal, i.e., $d_{\mathbf{k},\mathbf{k'},1} = d_{\mathbf{k},\mathbf{k'},2} := d_{\mathbf{k},\mathbf{k'}}$. Hence, $\zeta_{\mathbf{k},\mathbf{k'}}$ can be expressed as $\zeta_{\mathbf{k},\mathbf{k'}} = \sqrt{1 - d_{\mathbf{k},\mathbf{k'}}^2}$. We further observe that the maximum singular value $d_{\mathbf{k},\mathbf{k'}}$ for $\mathbf{k} \neq \mathbf{k'}$ is obtained in cases 2 and 3 of (6), and equals $1/\sqrt{2}$. Hence, the diversity product $\zeta$ can be expressed as $\zeta = 1/\sqrt{2}$. We can thus conclude that for $M = 2$, ST-FSK has a nonzero diversity product, and consequently achieves full diversity.

## VI. Simulation Results

We first compare the performance of ST-FSK for $M = 2$ (and thus, $P = 2$) and $K = 2$ (hence, $T = 4$), which corresponds to rate $R = 1/2$, with the unitary ST modulation design of [4] for $M = 2, T = 4$, and rate $R = 1/2$. We consider only one receive antenna ($N = 1$). First of all, the search criterion used in [4] allows for many solutions, some of which

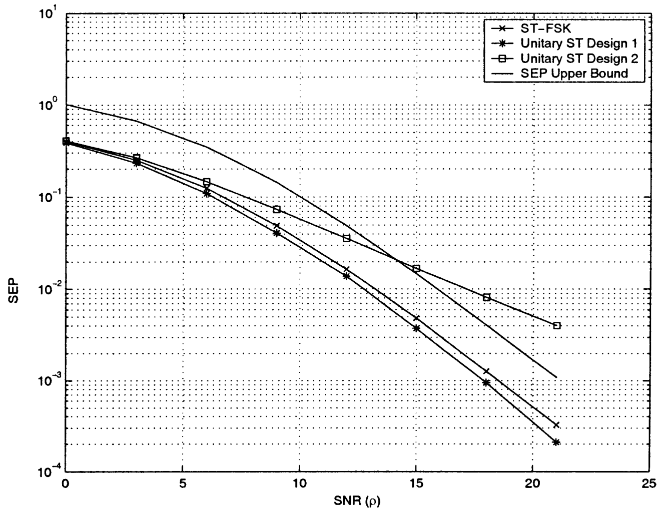


Fig. 1. Performance comparison between ST-FSK and existing unitary ST designs. We consider $M = 2$ transmit antennas, $N = 1$ receive antenna, and rate $R = 1/2$ transmission.

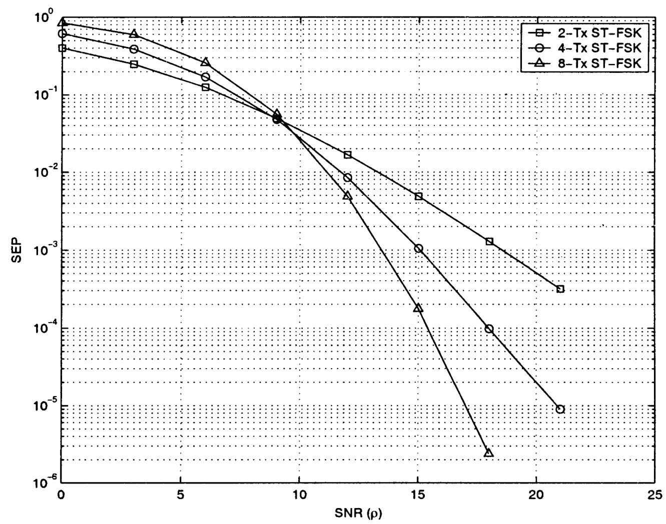


Fig. 2. Performance of ST-FSK with $M = 2, 4, 8$ transmit antennas, $N = 1$ receive antenna, and rate $R = 1/2$ transmission.

have a nonzero diversity product, and thus, achieve full diversity (diversity order 2), and some of which have a zero diversity product, and thus, do not achieve full diversity (diversity order 1). We look at two solutions: one that has the highest diversity product (design 1), and one that has a zero diversity product (design 2). Note that either one of these solutions could be obtained when adopting the search criterion used in [4]. Symbol-error probability (SEP) performance results are shown in Fig. 1. Also shown is the SEP upper bound of the proposed method based on (3) and (4). We observe that ST-FSK and the unitary ST modulation design 1 have a comparable performance. However, ST-FSK is very simple, whereas the unitary ST modulation design 1 requires a complex numerical search procedure. Moreover, ST-FSK allows for a simplified ML detector, whereas the unitary ST modulation design 1 requires a full-blown ML detector. As expected, ST-FSK outperforms the unitary ST modulation design 2.

Let us finally consider ST-FSK for $M = 4, 8$ (and thus, $P = 4, 8$) and $K = 2$ (hence, $T = 8, 16$), which again corresponds to rate $R = 1/2$. By exhaustive search, one can easily show that the diversity product is $\zeta = 1/2$ for $M = 4$ and $K = 2$, and $\zeta = 1/\sqrt{8}$ for $M = 8$ and $K = 2$. Hence, both schemes achieve full diversity, as can also be observed from Fig. 2, where the slope of the SEP at high SNR approaches $-4$ for $M = 4$ and $K = 2$, and $-8$ for $M = 8$ and $K = 2$. As a benchmark, Fig. 2 also repeats the curve for $M = 2$ and $K = 2$ that was shown in Fig. 1. Finally, note that full diversity for $M = 4, 8$ and $K = 2$ also implies full diversity for $M = 3, 5, 6, 7$ and $K = 2$, due to the construction of delay-optimal full-rate real orthogonal designs and the geometrical interpretation of the diversity product.

## VII. Conclusion

In this letter, we have developed a novel unitary ST modulation design coined as ST-FSK, which transmits FSK waveforms that are structured according to the full-rate real orthogonal designs. ST-FSK has a number of advantages over existing unitary ST modulation designs. ST-FSK is easier to design, and enjoys lower decoding complexity. Furthermore, ST-FSK guarantees full diversity. Finally, ST-FSK can be adopted in the digital as well as in the analog domain, and merges very naturally with FHMA. As expected, all these advantages come at the cost of a decrease in spectral efficiency.

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