Space–Time Coding and Kalman Filtering for Time-Selective Fading Channels

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***Abstract*—This letter proposes a novel decoding scheme for Alamouti's space–time (ST) coded transmissions over time-selective fading channels that arise due to Doppler shifts and carrier frequency offsets. Modeling the time-selective channels as random processes, we employ Kalman filtering for channel tracking in order to enable ST decoding with diversity gains. Computer simulations confirm that the proposed scheme exhibits robustness to time-selectivity with a few training symbols.**

***Index Terms*—Diversity methods, Kalman filtering, space–time codes, transmit antennas.**

I. Introduction

Space–time (ST) coding has been shown to be effective in combating channel fading. In particular, Alamouti's ST code with two transmit antennas and one receive antenna [1] has remarkably low decoding complexity and is capable of achieving channel capacity [4].

The effectiveness of [1] and most ST coding schemes relies on accurate multichannel estimation, which may require the insertion of many pilot symbols when the underlying channels are highly time-varying. Differential ST coding (DSTC) forgoes channel estimation and allows for slowly changing channels (see [8] and [9] and references therein). However, its performance may degrade considerably when channels are varying rapidly. Based on the expectation–maximization (EM) algorithm, an ST coded transceiver for time-selective channels was proposed in [3]. Unfortunately, it relies on the invertibility of the channel correlation matrix which is either ill-conditioned in typical time-selective channels (with low-pass Doppler spectra) or becomes rank deficient when the channel is invariant over the block. Double differential ST coding (DDSTC) offers a simple and robust means of handling channel time-selectivity but loses 6 dB in performance [7].

This letter develops a novel decoding scheme for Alamouti's ST code in time-selective fading channels. We identify first the invariance of Alamouti's ST code to multiplicative complex exponential effects such as those arising due to carrier frequency offset (CFO). To account for more general time-selective channels, we model them as autoregressive (AR) processes. We then apply alternately Kalman filtering (KF) and ST decoding to acquire the channel and decode the information symbols with diversity gains. Simulations confirm the robust performance of our scheme.

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II. System Model

Fig. 1 depicts a wireless system with two transmit antennas and one receive antenna, where the information symbols $s(n)$ are transmitted using Alamouti's ST code [1]. Different from [1] where the channels are assumed flat, we consider here more realistic time-selective (but frequency-flat) channels. Let $h_i(n), i = 1, 2$ denote the time-selective channel from the ith transmit antenna to the receive antenna. The two consecutive received samples $y(2n)$ and $y(2n+1)$ are given by (cf. [1, eq (11)])

$$\begin{aligned} y(2n) &= h_1(2n)s(2n) + h_2(2n)s(2n+1) + w(2n), \\ y(2n+1) &= -h_1(2n+1)s^*(2n+1) + h_2(2n+1)s^*(2n) \\ &\quad + w(2n+1) \end{aligned} \tag{1}$$

where the noise $w(n)$ is complex Gaussian distributed with mean zero and variance $\sigma_w^2/2$ per dimension.

Similar to [1], we wish to recover $s(n)$ from $y(n)$ with transmit-diversity gain. Without imposing any structure on $h_i(n)$ however, this goal is ill-posed simply because, for every two incoming received samples, two extra unknowns $h_1(n)$ and $h_2(n)$ appear in addition to the two unknown symbols $s(2n)$ and $s(2n+1)$. Fortunately, many wireless channels exhibit structured variations that can be fit parsimoniously with finitely parameterized yet time-varying models. Among various channel models, the information theoretic results in [11] have shown that the first-order AR model provides a sufficiently accurate model for time-selective fading channels and, therefore, will be adopted henceforth. Specifically, $h_i(n)$ varies according to

$$h_i(n) = \alpha h_i(n-1) + \upsilon_i(n), \quad i = 1, 2 \tag{2}$$

where the noise $\upsilon_i(n)$ is zero-mean complex Gaussian with covariance $\sigma_\upsilon^2/2$ per dimension and is statistically independent of $h_i(n-1)$, and the coefficient α can be estimated as detailed in [10]. Assume that $h_i(n)$ is zero mean, unit-variance complex Gaussian. Using (2), simple manipulations lead to

$$\sigma_\upsilon^2 = 1 - |\alpha|^2 \quad \text{and} \quad \alpha = E[h_i(n)h_i^*(n-1)] \tag{3}$$

where $E(\cdot)$ stands for expectation.

Wireless channel variations are mainly caused by two independent sources: one is due to Doppler effects arising from relative motion between the transmitter and the receiver, and the other is the CFO due to the transmitter–receiver oscillators' mismatch. Denote by f_o the CFO and by T_s the information symbol duration. Consequently, we can factorize $h_i(n)$ into

$$h_i(n) = \bar{h}_i(n)\exp(j2\pi f_o T_s n) \tag{4}$$

where $\bar{h}_i(n)$ accounts for the Doppler effects. According to Jakes' model [5], we have $E[\bar{h}_i(n)\bar{h}_i^*(n-1)] = J_0(2\pi f_d T_s)$,

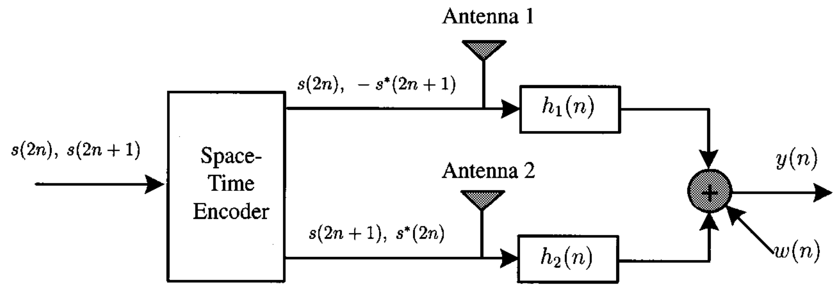

Fig. 1. Discrete time equivalent baseband model.

where $J_0(\cdot)$ is the 0th-order Bessel function and f_d denotes the maximum Doppler shift. Recalling (3), the AR coefficient α is related to f_d and f_o via

$$\alpha = J_0(2\pi f_d T_s) \exp(j2\pi f_o T_s). \tag{5}$$

Having specified the channel model (2), we proceed to design our ST decoding.

III. Alamouti-Based ST Decoding

Let us define $\mathbf{y}(n) := [y(2n), y^*(2n+1)]^T$, and cast (1) in a matrix/vector form:

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{s}(n) + \mathbf{w}(n) \tag{6}$$

where, $\mathbf{w}(n) := [w(2n), w^*(2n+1)]^T$; $\mathbf{s}(n) := [s(2n), s(2n+1)]^T$; and the channel matrix

$$\mathbf{H}(n) := \begin{bmatrix} h_1(2n) & h_2(2n) \\ h_2^*(2n+1) & -h_1^*(2n+1) \end{bmatrix}. \tag{7}$$

To decode $\mathbf{s}(n)$ from $\mathbf{y}(n)$, we form the decision vector $\mathbf{z}(n) := [z(2n), z(2n+1)]^T$ as

$$\mathbf{z}(n) = \mathbf{H}^{\mathcal{H}}(n)\mathbf{y}(n) \tag{8}$$

where $^{\mathcal{H}}$ denotes conjugate transpose. Note that when the $h_i(n)$'s are time-invariant, $\mathbf{H}(n)$ becomes (scaled) unitary and the detection rule (8) is maximum-likelihood (ML), as discussed in [1]. By identifying that $\mathbf{H}(n)$ is near-unitary in the mean sense, we show next that (8) indeed leads to near-ML performance even with time-selective channels. Based on the definition (7), it follows by direct substitution that

$$\mathbf{H}^{\mathcal{H}}(n)\mathbf{H}(n) = \begin{bmatrix} \rho_1(n) & \epsilon(n) \\ \epsilon^*(n) & \rho_2(n) \end{bmatrix} \tag{9}$$

where $\rho_1(n) := |h_1(2n)|^2 + |h_2(2n+1)|^2, \rho_2(n) := |h_1(2n+1)|^2 + |h_2(2n)|^2$, and $\epsilon(n) := h_1^*(2n)h_2(2n) - h_1^*(2n+1)h_2(2n+1)$. For practical wireless applications, the product $f_d T_s$ is typically small (e.g., $f_d T_s < 0.004$ in [11]). Thus, we deduce from (3) and (5) that

$$|\alpha|^2 \approx 1 \quad \text{and} \quad \sigma_v^2 \approx 0. \tag{10}$$

Using (10), it can be readily proved that in the mean sense $E\{\epsilon(n)\} \approx 0$ and $E\{\rho_1(n)\} \approx E\{\rho_2(n)\}$. Thus, we find from (9) that $\mathbf{H}(n)$ is a (scaled) near-unitary matrix in the mean sense.

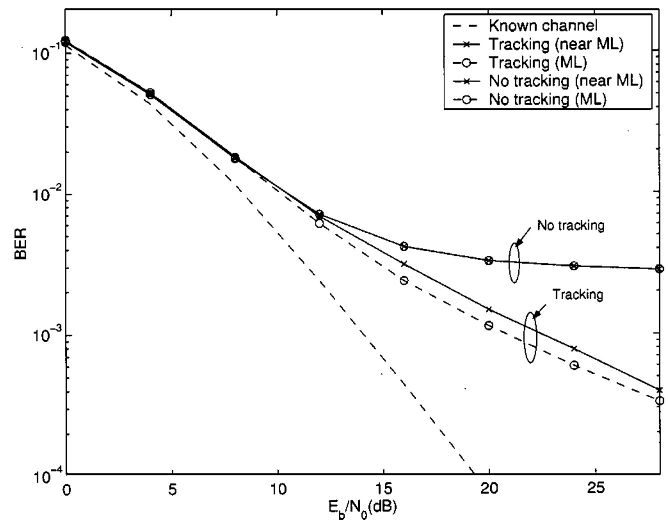

Fig. 2. BER improvement with KF-based channel tracking (the BER curve of no channel tracking with ML decoding overlaps that with near-ML decoding).

Notice that, if $f_d = 0$, then $\mathbf{H}(n)$ is strictly (scaled) unitary regardless of the value of f_o, and *Alamouti's ST block coding is insensitive to CFO*. We are thus motivated to use Alamouti's ST coding for transmissions through more general time-selective channels and, similar to [1], decode the information symbols using (8).

So far, we have tacitly assumed that the ST decoder in (8) has knowledge of the $h_i(n)$'s at the receiver. Multichannel estimation is challenging especially in our time-varying two-input/one-output setup. Fortunately, the AR model in (2) lends itself to a state-space representation that enables application of KF for online tracking of channel variations.

IV. KF-Based Channel Tracking

The implementation of our adaptive algorithm starts with a training mode that is used to acquire initial $h_i(n)$ estimates, after which it reverts to a decision-directed mode. In the training mode, the receiver knows the transmitted symbols, whereas in the decision-directed mode, the decoded symbols replace the information symbols. We will focus on the decision-directed mode and assume that initial channel estimates are available by using any of the approaches developed in [10].

Selecting as state vector $\mathbf{h}(n) := [h_1(n), h_2(n)]^T$, we obtain from (2) the state equation

$$\mathbf{h}(n) = \mathbf{A}\mathbf{h}(n-1) + v(n) \tag{11}$$

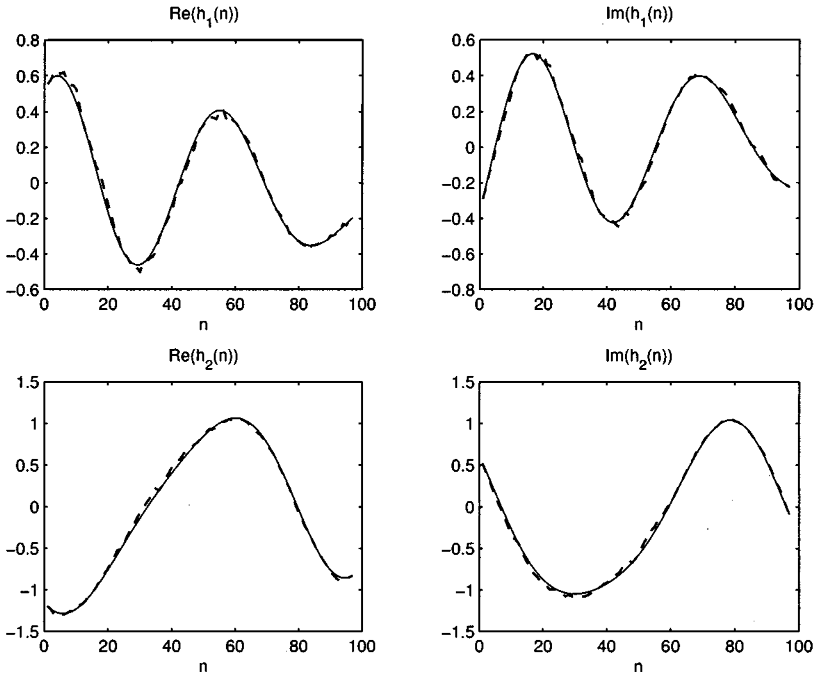

Fig. 3. True (solid curves) and KF-based estimate (dashed curves) of channel variations.

where $\mathbf{A} = \mathrm{diag}(\alpha, \alpha)$ and $\upsilon(n) = [\upsilon_1(n), \upsilon_2(n)]^T$. Using (1), the measurement equation is

$$y(n) = \bar{\mathbf{s}}^T(n)\mathbf{h}(n) + w(n) \qquad (12)$$

where $\bar{\mathbf{s}}(n) = [s(n), s(n+1)]^T$ when n is even and $\bar{\mathbf{s}}(n) = [-s^*(n), s^*(n-1)]^T$ when n is odd. Note that, in the joint detection–estimation problem posed by (8) and (9), both $\mathbf{h}(n)$ and $\bar{\mathbf{s}}(n)$ are unknown. With the knowledge of the decoded symbols $\bar{\mathbf{s}}(n)$ and the observations $y(n)$, $\mathbf{h}(n)$ can be obtained using a standard KF predictor [6, p. 448]. However, the detection of $\bar{\mathbf{s}}(n)$ relies on the estimates of $\mathbf{h}(n)$ that in turn require the knowledge of $\bar{\mathbf{s}}(n)$. This implies that an iterative method should be sought to obtain alternately either $\bar{\mathbf{s}}(n)$ or $\mathbf{h}(n)$. According to (10), a coarse prediction of $\mathbf{h}(n)$ can be obtained directly from (2). Denote by $\mathbf{h}(n \mid m)$ the predicted channel at time n based on the state and/or the observation at time m. The coarse channel prediction obeys the recursions

$$\begin{aligned}\mathbf{h}(2n \mid 2n-1) &= \alpha\mathbf{h}(2n-1 \mid 2n-1)\\ \mathbf{h}(2n+1 \mid 2n-1) &= \alpha^2\mathbf{h}(2n-1 \mid 2n-1)\end{aligned} \qquad (13)$$

that are initialized by $\mathbf{h}(1 \mid 1)$, which is obtained during the training mode. Next, we use the coarse channel estimates and (8) to obtain coarse symbol estimates for $\bar{\mathbf{s}}(2n)$ and $\bar{\mathbf{s}}(2n+1)$ that are denoted by $\bar{\mathbf{s}}^{(c)}(2n)$ and $\bar{\mathbf{s}}^{(c)}(2n+1)$, respectively.

Replacing $\bar{\mathbf{s}}(n)$ by $\bar{\mathbf{s}}^{(c)}(n)$, we rely on the KF to obtain refined channel estimates $\mathbf{h}(2n \mid 2n)$. Based on $\mathbf{h}(2n \mid 2n)$, we perform KF once more to obtain $\mathbf{h}(2n+1 \mid 2n+1)$. With refined $\mathbf{h}(2n \mid 2n)$ and $\mathbf{h}(2n+1 \mid 2n+1)$, refined estimates $\bar{\mathbf{s}}^{(r)}(2n)$ and $\bar{\mathbf{s}}^{(r)}(2n+1)$ are decoded with diversity gain from (8). We summarize our algorithm for channel tracking and symbol decoding, in the following steps.

Initialization) Obtain $\mathbf{h}(1 \mid 1)$ from training;

s1) Obtain $\mathbf{h}(2n \mid 2n-1)$ and $\mathbf{h}(2n+1 \mid 2n-1)$ using (13);

s2) Use (6) and (8) to decode $\bar{\mathbf{s}}^{(c)}(2n)$ and $\bar{\mathbf{s}}^{(c)}(2n+1)$;

s3) Perform KF to retrieve $\mathbf{h}(2n \mid 2n)$ and $\mathbf{h}(2n+1 \mid 2n+1)$ using $\bar{\mathbf{s}}^{(c)}(2n)$ and $\bar{\mathbf{s}}^{(c)}(2n+1)$;

s4) Decode $\bar{\mathbf{s}}^{(r)}(2n)$ and $\bar{\mathbf{s}}^{(r)}(2n+1)$ based on $\mathbf{h}(2n \mid 2n)$ and $\mathbf{h}(2n+1 \mid 2n+1)$;

s5) If necessary, iterate s3) and s4) more times to improve the tracking;

s6) Repeat from s1) for $n+1 \leftarrow n$.

Next, we test the performance of our channel tracker through simulations.

V. Simulations

We use (4) to generate $h(n)$. The generation of $\bar{h}(n)$ follows the Jakes model [5] with the parameters f_d and T_s corresponding to a carrier frequency of 1.9 GHz, a terminal speed of 250 km/h, and a transmission rate of 144 kb/s, as specified for EDGE systems in [2]. The bit error rate (BER) at each SNR (E_b/N_0) point is averaged over 5 000 channel realizations, and 100 KF iterations are performed.

Example 1 (Performance Improvement With Channel Tracking): We compare the BER performance with and without KF-based channel tracking, when $f_o = 1\ 000$ Hz. To avoid divergence in KF, we insert one pilot symbol every 12 symbols which incurs 8% bandwidth efficiency loss. For fairness, when no channel tracking is used, the receiver assumes knowledge of α and updates the channels via $h_i(n+1) = \alpha h_i(n)$, with $h_i(n)$'s reset to their correct values every 12 symbols. Fig. 2 confirms that channel tracking improves the BER performance especially at high SNR, while Fig. 3 shows that the estimated channels (dashed curves) track the true channels (solid curves) well. However, as

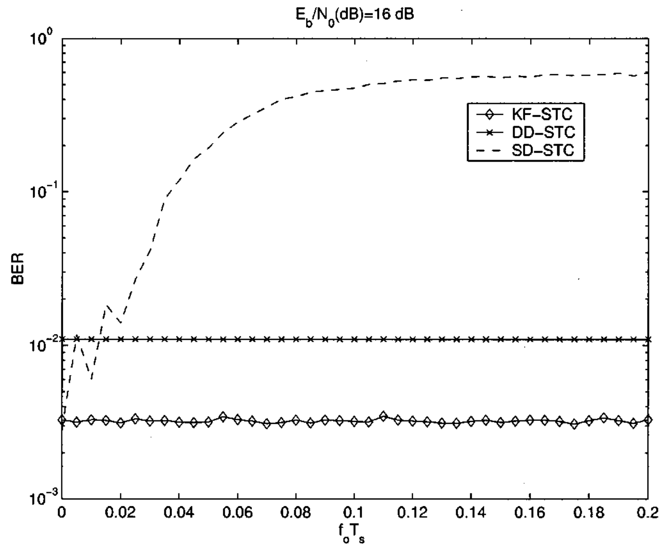

Fig. 4. Comparisons with [9] and [7].

compared to the benchmark performance (when channels are perfectly known), the result in Fig. 2 implies that even small channel estimation errors (as evident in Fig. 3) could induce considerable performance loss, because channel estimation errors cause additional inter-symbol interference between the two transmit antennas. To isolate the effects of our near-ML decoding of (8), we repeat this simulation by replacing the near-ML decoding with the (true) ML decoding (via exhaustive searching). We observe from Fig. 2 that ML decoding yields a small (negligible) performance improvement in the case of channel tracking (no channel tracking).

Example 2 (Comparisons With Competing Schemes): We compare the proposed ST-KF approach with DSTC [9] and with DDSTC [7]. For ST-KF and DSTC, we use QPSK modulation, while for DDSTC we employ the (16;1, 7) codes in [7, Table I] to maintain the same transmission rate. We fix the SNR at 16 dB and vary the normalized CFO f_oT_s from 0 to 0.2. Fig. 4 shows that ST-KF outperforms both DDSTC and DSTC, when $f_oT_s > 0.01$.

VI. Conclusion

We have developed a novel scheme for joint iterative channel tracking and symbol recovery with diversity gains in time-selective channels. Modeling time-selective channels as AR processes, KF was employed to track their time variations. Both channel tracking and symbol recovery benefit from ST coding. The robust performance of our design was illustrated by simulations.

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